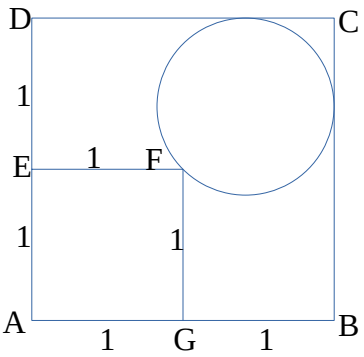
# Solving a Geometry Puzzle

A nice puzzle from Medium with the question if you are smart enough to solve a puzzle. There's a picture of course, and you have to find the radius of a circle giving the length of sides of the squares in the picture. Let us start by assigning letters to vertices:




So, we have:

ABCD is a square whose side length is 2 units.

E,G vertices on sides AD and AB respectively. Such that

$\overline{AG} = \overline{GB} = 1$
and
$\overline{AE} = \overline{ED} = 1$

and

AGFE is a squre.

and

sides BC and CD are tangent to a circle, that passes through vertex F.

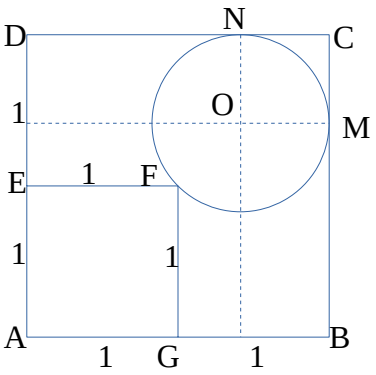
What is the radius of the circle?

## First, let us find the center of the circle

To find the center, let us use some properties of lines tangent to a circle. One of them is that a line that passes through the center of a circle and the point of tangency of one of the lines is perpendicular to that line.

And of course, each line can only have one perpendicular line intersecting with it at the point of tangency. So, the perpendicular lines to the tangents intersect at the center of the circle.

Let us add them perpendicular lines, and assign more letters.




Now, let us call the radius $r$, then:

$\overline{OM} = \overline{ON} = r$

From ABCD being a square:

$\prec OCM = 90^{\circ}$

From CD and BC being tangent to the circle:

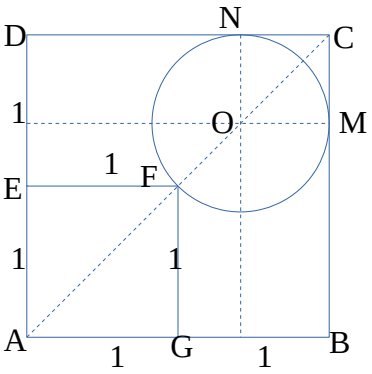
$\prec CMO = \prec CNO = 90^{\circ}$

That leaves $\prec MON$ no choice but to be a right angle, too.

Thus,

OMCN is a square. **Good news**!

## Let us add the diagonal AC




The diagonal AC passes thru O. Thus, it can be divided into 3 section:s

$\overline{AC} = \overline{AF} + \overline{FO} + \overline{OC}$ (1)

Let us calculate the lengths:

$\overline{AC}=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$

and

$overlin\,AF=\sqrt{1^2+1^2}=\sqrt{2}$

and

$\overline{FO}=r$

and

$\overline{OC}\,\sqrt{r^2+r^2}=\sqrt{2r^2}=\sqrt{2}r$

Plug them into (1):

$$\sqrt{2}+r+\sqrt{2}r=2\sqrt{2}\rightarrow$$
$$\rightarrow(1+\sqrt{2})r=\sqrt{2}\rightarrow$$
$$\rightarrow(\sqrt{2}+1)(\sqrt{2}-1)r=\sqrt{2}(\sqrt{2}-1)\rightarrow$$
$$\rightarrow(\sqrt{2})^2-1^2=2-2\sqrt{2}\rightarrow$$
$$\rightarrow(2-1)r=2-\sqrt{2}\rightarrow$$
$$\rightarrow r=2-\sqrt{2}$$