

# Dynamical transitions in a pollination–herbivory interaction

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October 30, 2014

- 1 Introduction
  - Ecological interactions
  - Problem
- 2 Mathematical model
  - Interaction mechanics
  - Fast and slow dynamics
- 3 Results
  - Equilibrium and dynamics
  - Herbivory–Mutualism oscillations
  - Alternative food sources
- 4 Summary

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# Interaction compass

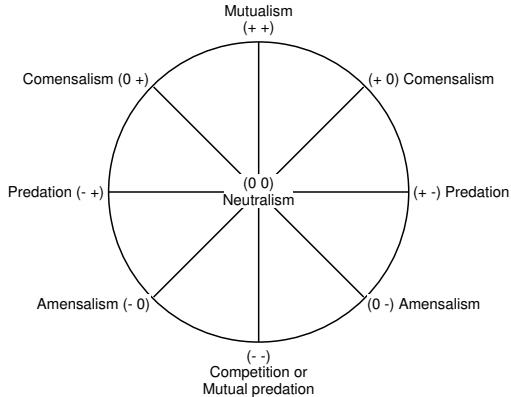


Figure from Holland & DeAngelis (2009)

# Lotka–Volterra equations

$$\frac{dN_1}{dt} = r_1 N_1 + a_{11} N_1^2 + a_{12} N_1 N_2$$
$$\frac{dN_2}{dt} = r_2 N_2 + a_{22} N_2^2 + a_{21} N_1 N_2$$

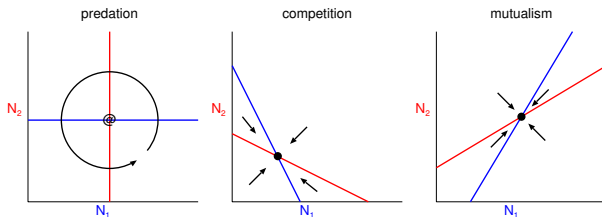
Set the  $r_i$  and the  $a_{ij}$  according to the interaction of interest

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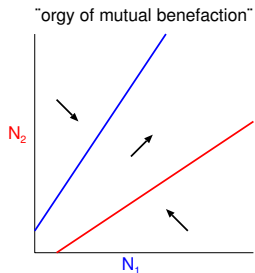
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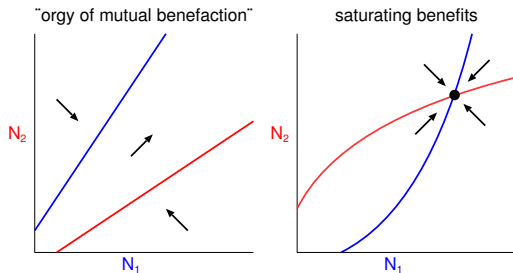
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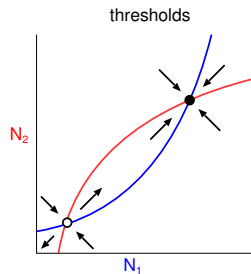
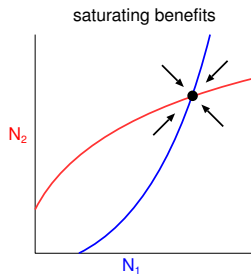
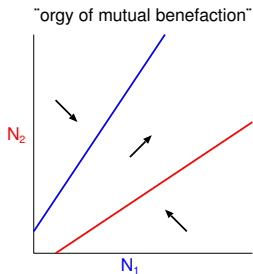
# Lotka–Volterra models and mutualism



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# Variable interactions

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  - Malgas Island: Lobsters are abundant, they predate on snails
  - Marcus Island: Snails are abundant, they predate on lobsters
- Cactus-Moth (several papers of Bronstein & collaborators):
  - Low moth density: pollination benefits overcomes seed parasitism
  - High moth density: seed parasitism overcomes pollination benefits

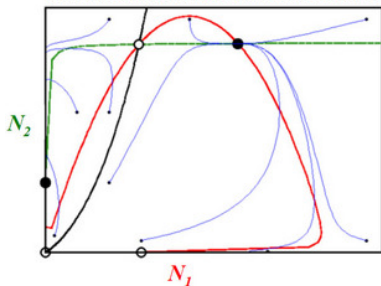
# Holland & DeAngelis (2010) consumer–resource approach

$$\frac{dN_i}{dt} = N_i \left( r_i - d_i N_i + c_i \left[ \frac{a_i N_j}{b_i + N_j} \right] - q_i \left[ \frac{d_i N_j}{h_i + N_i} \right] \right)$$

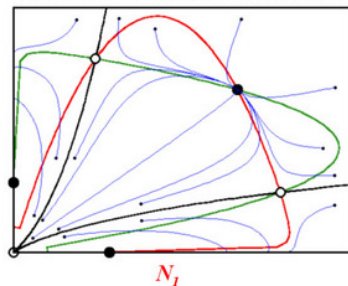
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Uni-directional Consumer-Resource



Bi-directional Consumer-Resource



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# Friend or Foe?



*Il faut bien que je supporte deux ou trois chenilles si je veux connaître les papillons*

*Le Petit Prince – Antoine de Saint-Exupéry*

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- Many lepidopterans have larval phases that feed on the same plant species that they pollinate when adults (Altermatt & Pearse 2011)
- The net outcome of an interaction will depend on population structure, e.g. how many larvae vs. how many adults
- And why a net outcome? why not an alternation:  
mutualism -> antagonism -> mutualism -> antagonism ...

# Goals

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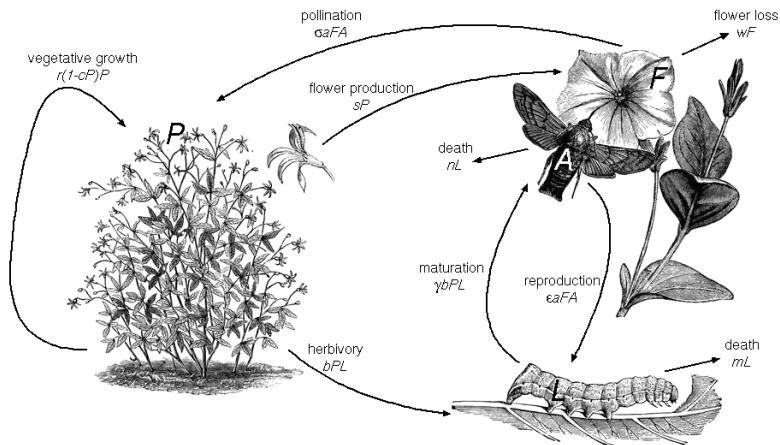
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- Predict potential, indirect effects, of other plants in the costs/benefit balances

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# Interaction mechanism



# Interaction dynamics

Differential equations for plant, flower, larva and adult biomasses

$$\frac{dP}{dt} = rP(1 - cP) + \sigma aFA - bPL$$

$$\frac{dF}{dt} = sP - wF - aFA$$

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- Conversion rates  $\varepsilon$  and  $\gamma$  (maturation rate) are always less than 1
- But plant conversion rates  $\sigma$  can be larger than 1

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# Flower dynamics

*Nous ne notons pas les fleurs, dit le géographe  
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## Assumption

Flower (or nectar) dynamics is much faster than population dynamics. Let us assume that flowers attain a sort of steady-state, compared with plants and insects

$$F \approx \frac{sP}{w + aA}$$

# The PLA model

Now everything is in terms of population biomasses

$$\frac{dP}{dt} = rP(1 - cP) + \sigma \left[ \frac{asA}{w + aA} \right] P - bPL$$

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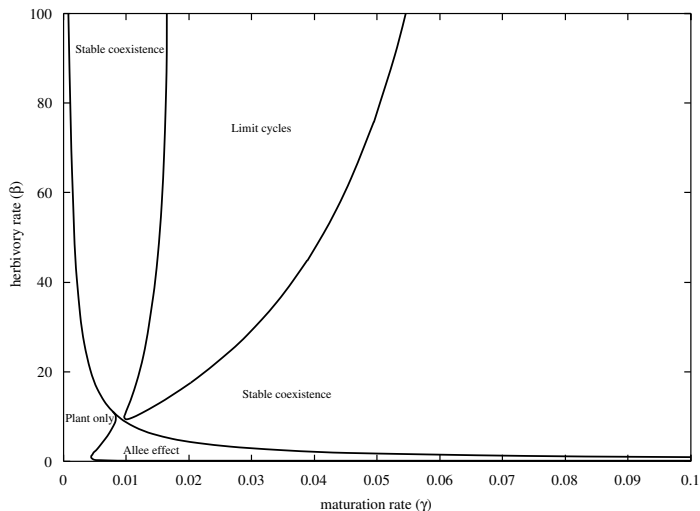
$$\frac{dA}{dt} = \gamma bPL - nA$$

- Pollination benefits saturate with pollinator biomass
- Insects engage in competition for pollination resources
- Let's vary the herbivory rate ( $b$ ) and the maturation rate ( $\gamma$ ), this changes the balance between mutualism and antagonism

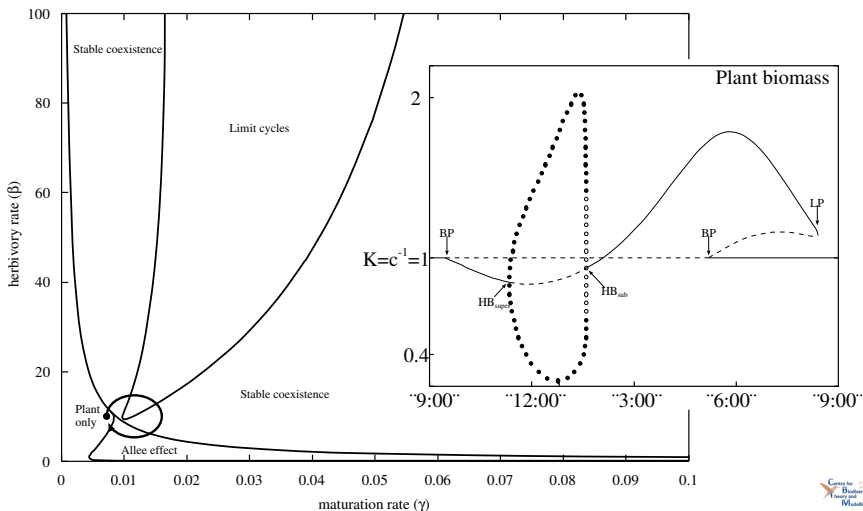
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# Parameter space. Herbivory ( $\beta$ ) vs. Maturation rate ( $\gamma$ )



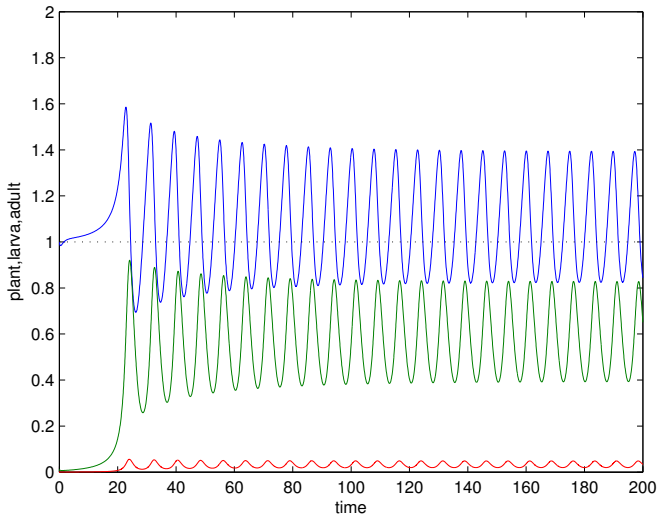
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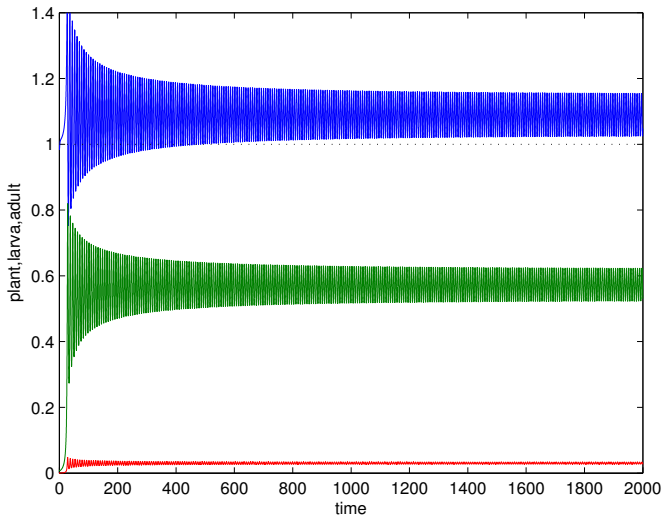
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# Plants can oscillating around the carrying capacity



# Plants can oscillating above the carrying capacity



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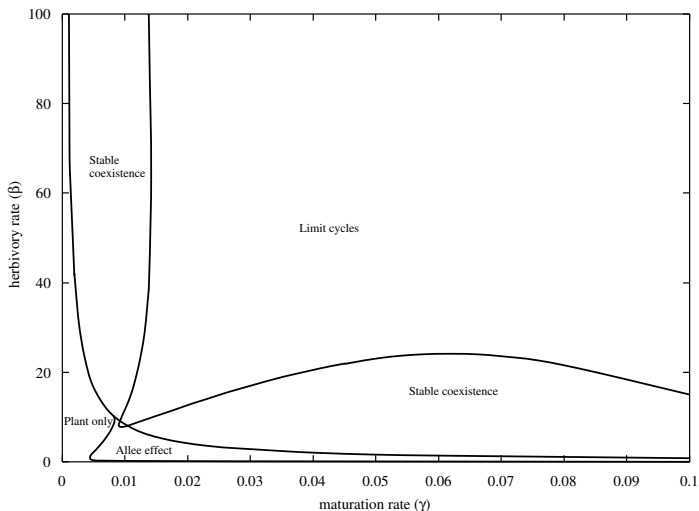
# Alternative food sources

Suppose that the adults can pollinate other plants

$$\begin{aligned}\frac{dP}{dt} &= rP(1 - cP) + \sigma \left[ \frac{asA}{w + aA} \right] P - bPL \\ \frac{dL}{dt} &= gA + \varepsilon \left[ \frac{asP}{w + aA} \right] A - \gamma bPL - mL \\ \frac{dA}{dt} &= \gamma bPL - nA\end{aligned}$$

$g > 0$ : growth rate due to pollination of plants different than species "P"

# Parameter space. Herbivory ( $\beta$ ) vs. Maturation rate ( $\gamma$ )



# Summary

- Population structure determines the balance between the costs of herbivory and the benefits of mutualism
  - The faster the larvae turn into adults, the more stable the association (dynamically)
  - Changes in population structure can be triggered by multiple factors such as temperature, chemicals, food quality

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  - Plants can oscillate **around** their carrying capacities: alternating periods of net mutualism and net exploitation
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- Pollination of other plants increases reproduction, leading to more larvae and loss of stability, i.e. victim-exploiter cycles

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Děkuji!