Stockholm 2004

The Fourth European Congress of Mathematics

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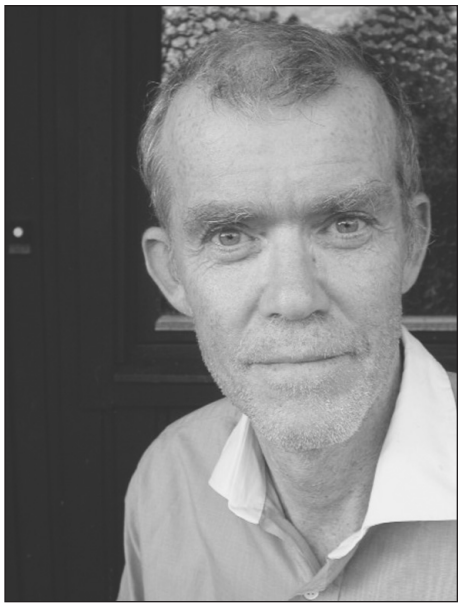

How it came about

In spite of a difficult start, the fourth European Congress in Mathematics (ECM) took place after all. As its President Ari Laptev revealed in an earlier issue of this newsletter, its eventual location in Stockholm was not initially planned but was the result of a crisis perceived in the summer of 2000. Despite what many outsiders may have assumed, finding sufficient funding from Swedish sources was not a straightforward matter (as the first bid made painfully clear). It was only due to the daring and 'savoire faire' of Ari Laptev and his colleague at Kungliga Tekniska Högskolan (KTH), Anders Lindquist, that sufficiently promising leads were extracted from some of the major Swedish funding institutions, thus enabling the Swedish Mathematical Society to send a small delegation to the executive meeting in London later that fall for negotiations. However, a final commitment to hold the congress was not made until just a week before Christmas - after a rather tense encounter with the then President of the European Mathematical Society (EMS), Rolf Jeltsch. The meeting only came to a desired conclusion after a final short interview with the President of the KTH, Anders Flodström, in which the latter agreed to the necessary financial underwriting. The upshot was that the planning and organization of the congress was shortened by at least a year from what has been customary for this kind of event.

Why big congresses? Was it worth it?

This is obviously not the place to delve deeper into that philosophical question, yet a few reflections may not be amiss. In the good old days, which meant up into the fifties, conferences were few and small, and in particular the International Congresses of Mathematicians (ICM) were rather select and as a consequence not bigger than allowing group portraits to be taken. But when the ICM was held in Stockholm back in 1962, it was the largest scientific congress that had ever been held in Sweden. Since then the ICM have turned from rather exclusive meetings to large affairs attended by the 'mathematical masses'. This fact, together with the recent acceleration in the number of specialized conferences, often referred to as workshops to emphasize their focused and businesslike character, has sown doubts in the minds of many as to the scientific relevance of such gargantuan meetings, which, cynics protest, are more the occasion for tourism than serious scientific interchange. In such a perspective, the establishment of a European Congress may seem redundant. One may point out though that the situation of mathematics is not unique, but is shared by most academic disciplines, and that it is very important, among other things, to counter the fragmentation of a discipline with opportunities for the contemplation of it as a whole. For this to be fully successful there is both a need for the lectures to be directed to a general mathematical audience and for that audience to seek out lectures not primarily within their own speciality. Popular surveys do not rank highly among the priorities of research mathematicians. They are notoriously difficult to do and the rewards are marginal. A scientific committee selects their choices primarily on the basis of scientific excellence and topicality of the subject matter, not on expository skills; and notwithstanding the desirability of the latter this is basically a sound principle, tampering with which would court disaster and seriously undercut the legitimacy of the whole enterprise. Thus selection as a speaker is seen primarily as recognition of scientific merit and not as an opportunity, as well as an obligation, to communicate effectively.

What is needed is a change of culture, something that cannot be decreed but has to evolve slowly. On what constitutes a good lecture, one can of course argue, there obviously being no specific rules on which everyone can agree. And besides, in all kinds of human interaction rules are simply there to be broken. It suffices to point out that a scientific lecture, and especially a mathematical one, is not necessarily made more accessible by the simple process of 'watering down' (i.e. removing technicalities and making unwarranted simplifications). What is required of a lecture is the conveying of an idea (usually one is enough) and some motivation (which should not be confused with justification often of the type: 'this has applications to physics') and placing the subject matter in a wider mathematical context. Apart from that, a good lecture can contain highly technical material, and there is no reason why the audience should understand most of it as long as they have gotten something to take home with them. It would be very dangerous if mathematicians were to abandon their tradition of honesty and aim for merely the 'flashy'. To reflect on why your work is interesting and to try to truly motivate it, should not be seen as a concession to ignorance, but rather as an additional source of inspiration for your own research.

New features

Obviously this is not the place to comment upon how successfully the various speakers performed their task of communicating to a general mathematical audience. Fortunately, although few lectures can be expected to please everyone, the lecture is rare indeed that does not bring rewards to at least a token number in the audience, and one may argue that this is indeed all that is needed to make it worthwhile. However, there was a feeling among the organizers that something extra was needed to justify yet another general conference in mathematics, as well as serving as a rejuvenation of the concept. Two new features were added. One was to invite scientists in natural science, not only in chemistry and physics, but also mathematical biologists, to give lectures. The other was to latch on to the pre-existing structure of the European Networks and thus give to the ECM a natural European anchoring. The first may be seen as somewhat controversial, wedding mathematics and its ultimate

justification too tightly to applications. Yet, whether we like it or not, practical mathematical applications are what brings in material resources, and for public relations the importance of the willingness of mathematicians to interact externally, and in the process maybe also acquiring greater public visibility, should not be underestimated. From a less opportunistic standpoint, applications should not merely be seen as necessary justifications, but also as sources of inspiration. The second new feature may hopefully ensure that the new tradition of ECM's continues. The funding and organization of a big international congress is indeed a major undertaking and the difficulties involved may be expected to increase with time rather than decrease. The European Union has already invested a formidable apparatus of networks replete with their own special conferences. What would be more natural than a unifying one, which should bring with it a large body of active participants, and hopefully also channels for necessary funding? At Stockholm, admittedly the network lectures played a rather marginal role, but if the idea is accepted and developed in future ECM's they may provide the core activity onto which various extras may be attached as embellishments.

Aula Magna

After those general preambles it may be appropriate to describe the Fourth European Congress of Mathematicians as an actual event tied to a physical location at a given slot in time. The basic problem of organizing a conference for one thousand odd expected participants is to find a lecture hall big enough. Unlike the case of the Olympics, the erection of new buildings is not an option. Of course big conferences are legion these days, but commercially available space often comes with price-tags not within the capabilities of mathematical meetings. The Royal Swedish Institute of Technology (Kungliga Tekniska Högskolan - KTH) was not able to provide such a hall on its premises, making a direct collaboration with the University of Stockholm a necessity on this basis alone. The fact that there are two separate departments of mathematics in Stockholm has incidentally been a bone of contention for at least fifty years, and may continue to be so for another fifty years, but this is of course only a matter of local interest. The Aula Magna is the official grand lecture hall of the University of Stockholm, and has of course no direct connection, physical or not, to its department of mathematics, nor to that of the KTH, being about equally (physically) distant from both. It is of fairly recent vintage, located on the main campus of the University of Stockholm, easily accessible by the Stockholm Underground ('T-banan', 'T' as in tunnel). Shaped like an amphitheatre, with options of subdivision, the Aula actually boasts a capacity well in excess of the number of actual participants (around 900), which resulted in the somewhat unfortunate impression of not only the lectures but also the opening ceremony being sparsely attended. As expected, the Aula along with its adjacent hallways became the locus of the meeting. Walking along its perimeter you had immediate access to the young assistants donning yellow T-shirts, as well as to the various bookstalls providing not only opportunities for browsing but also seducing discounts. The Springer stall also supplied piles of copies of the Stockholm Intelligencer (still smelling of fresh print), featuring short articles on Sweden and mathematics, including a short but morbid list of distinguished mathematicians who died here. Climbing a few stairs you could also inspect the various poster-sessions. Additional smaller lecture halls were available at the proverbial stone-throws distance, as well as what in recent years has become an absolute necessity for wayward mathematicians - access to e-mail. A fairly spacious room, accessible only by pressing a code at the entrance, was filled with a sufficient number of computers to keep waiting lines rather than tempers short. Furthermore, a small staff of knowledgeable yellow-shirts was always at hand during opening hours. For those able to resist the allure of the screen at lunchtime, there was the temptation of the official cafeteria situated halfway between the two locations (actually accessible from the Aula Magna remaining indoors the entire way, a godsend in the case of inclement weather). It provided the standard Swedish lunch fare to be expected from that kind of self-serve establishment, confidently assured of a captive audience, not only by its isolated existence but also due to pre-paid lunch coupons.

4ecm staff studying new books at the booth of the EMS publishing house

The opening ceremony

The conference got a head start on Sunday afternoon, on June 27, by providing registration outside the Aula. This involved getting a handy black briefcase, doubling as a rucksack, with the logo of the 4ECM sufficiently discreet to encourage post-conference use. It would be tedious to list its contents of 'goodies', but I am sure that most people appreciated the free public transport passes intended to cover the entire period. One of its more trivial items was a coupon to be exchanged for a single glass of wine (of optional colour) to be served at the hour-long reception starting at six o'clock. The next day, Monday June 28, involved the official opening of the meeting at nine thirty in the morning, preceded by an opportunity for last-minute registrations. Back at the ICM of 1962, the old Swedish King Gustavus VI attended, giving out the Fields Medals. Such a spectacle of royalty at a mathematical meeting was, alas, not repeated this time. The presence of the Swedish Majesty back then has been attributed to the above mentioned fact that at the time, it was the largest scientific meeting ever to have taken place in Sweden. No

such distinction could be given to the 4ECM, so neither the old King's grandson, the present King Charles XVI, nor the great-granddaughter, the crown princess Victoria, were able to squeeze the events into their busy schedules. (One should not overestimate the love the general public, including that of royalty, feels for mathematics) Instead two academic notables, Bremer and Franke, provided the required official lustre. The latter, Sigbrit Franke, the chancellor of the university system in Sweden, also served the function of awarding the EMS prizes to the young mathematicians, to which we will return shortly. Both Bremer and Franke, not surprisingly, made a special point in referring to Sofia Kovalevskaya in their short speeches. Kovalevskaya, as can have escaped the notice of few mathematicians, was the first ever woman professor in mathematics, holding her position at the precursor of Stockholm University at the end of the 19th century, and ever since then being a mathematical role model for half the population of the world. Kingman, in his capacity as the President of the EMS, gave the mathematical speech with commendable aplomb, stressing the importance of mathematics, and urging young mathematicians to go for the hard problems, irrespective of whether pure or applied, because you can never tell. He also encouraged the new generation by reminding them that they are as great and imaginative as the great ones of the past, because what puzzled our ancestors does not necessarily puzzle us. The chair of the Prize Committee, Nina Uraltseva from St. Petersburg, who complained that she was a bit too short for the microphone, explained that the work of the prize committee had been very hard. There were fifty nominees, out of which ten had to be selected. To those who did not make it, there is only one thing to say - work harder and try to outdo those who were selected. This is not the place to give a list of the winners, along with short descriptions of their accomplishments. It suffices to add that in addition to the traditional EMS prizes, a so called BIT prize was awarded for work in numerical analysis. No official ceremony is complete without some kind of entertainment. The musical interlude in this case was provided by a small Swedish ensemble specializing in Elizabethan music but also performing, fittingly, some classical Swedish songs. Dressed in period costumes, they did their thing, singing with their own instrumental support, concluding their act by strutting around the stage pilfering the belongings of those unfortunate enough to have the privilege of sitting on the first row.

The week in review

The conference was kicked off by the first plenary lecturer, Oded Shramm, associated with Microsoft Research. An appropriate beginning in view of the fact that President Laptev, earlier in the ceremony, got stuck on his power-point presentation, cursing modern computer technology. Schramm, however, did not address such wordly issues of practicality, but expounded on conformally invariant random processes instead, albeit with many a computer visualization. And then there was time for lunch, and in spite of the customary denial of the existence of such entities, free to all participants. The afternoon was devoted to parallel sessions, four in fact, involving twelve lectures in total. The day was capped off by an EMS reception at the old location for the department of mathematics at KTH, a building commonly referred to as Sing-sing, due to its intimidating lay-out. The reception wisely took place on the ground floor, the limited space of which quickly got extremely crowded. One surmises that afterwards there were only empty wine-bottles among the left-overs. The next day started out with presentations of the prize winners and their work (it should be noted that some of them had also, independently one assumes, been invited as regular speakers as well.) Those were followed by invited lectures and then in the afternoon there were three Science lectures. The last one was that of R.Ernst, a Nobel Prize winner in Chemistry, giving a survey from Fourier to Medical imaging, constituting no doubt a very instructive lecture to the mathematicians, giving them, among other things, a sense of the somewhat alien culture of big science. Tuesday was capped off by a visit to the Town Hall of Stockholm replete with a buffet courtesy of the city of Stockholm. The Town Hall, designed by the Swedish architect Ragnar Östberg and built in the twenties, is one of the most commonly pictorially reproduced landmarks of the city, located at the edge of an island, and commanding a presence on the main waterway. Its design was inspired by the palace of the dodges in Venice and its main feature is the great banquet hall inside, somewhat puzzlingly known as the Blue Hall (guides of the building are just delighted to explain to you the historical reason why), which every December is host to the Nobel banquet following the prize awards at the Concert Hall. This time however, the setting was somewhat less sumptuous. Two grand buffets (presumably identical) were displayed on two long tables on one side. Food was plentiful, but to savour it you needed to do the customary juggle of balancing your wineglass, your plate, and your knife and fork with just two hands, while standing firmly on your feet and trying to make coherent conversation. Afterwards Kingman rose to the occasion thanking the hosts referring to the great success of the conference (so far). He concluded by delivering a splendid celebration of the importance of mathematics, reminding everyone that while back in 1900, some of the scientific accomplishments honoured at this very place involved no mathematics, nowadays this would be almost impossible, and that all scientists should acknowledge this fact. In fact, he reminded us all that the entire human race will benefit from the development of mathematics. In order to gently usher out the mathematical crowd from the premises, a guided tour was offered at the end providing a natural conclusion. For those of us who afterwards lingered on by the waterfront, we may late forget the glorious view made sublime (as one used to say) by the slowly setting sun. The traditional association of Stockholm with Venice, made particularly palpable by the Town Hall, is not plucked out of thin air, but rests on water. Wednesday was taken over by plenary talks in the morning and three additional science lectures in the afternoon. The Austrian Nowak expounded on mathematical biology with special emphasis on evolutionary processes, and Berry presented a cascade of computer generated pictures relating to optics and concomitant singularities. Thursday was only half a day, with invited lectures in the morning and scheduled excursions in the afternoon. In the evening, the French ambassador gave a reception for the notables of the EMS, the organizers and last but not least the young prize-winners. France as a country and culture should be commended for the official respect it accords mathematics and for always recognising mathematical achievement. I fear that the 4ECM may very well have received more publicity in France than in Sweden. Friday was the closing up, with parallel network lectures in the morning and a series of plenary talks in the afternoon, the last fittingly delivered by a local speaker, Johan Håstad at KTH, talking on the difficulty of proving the generally believed NP≠P.

Wrapping up

The concluding ceremony was, as such things tend to be, rather short. Kingman expressed the pride of the EMS to be associated with the ECM and thanked the organizers, and especially its President. There was a lot of applause. Then there was a ref-

erence to the upcoming centenary birthday of Henri Cartan, who had an honorary role in the first ECM which took place in Paris 1992, and to whom the entire congress relayed its congratulations. Laptev then referred to the statistics of the event, with the final tally of 930 members, over three hundred posters, and sixty three scheduled talks, of which only one had to be cancelled. As expected he could not refrain from congratulating himself on having arranged such nice weather for the entire duration. No mean feet indeed, and a definite contribution to the general pleasure. Finally the congress was treated to an invitation to the fifth ECM to take place in Amsterdam in 2008. After the usual problems with an unresponsive machine, the audience was eventually exposed to a power-point show by the representative of the next ECM, promising future delights. By the time the audience emerged out of the Aula Magna for the last time, bookstalls were being dismantled, posters taken down, and the last remaining conference briefcases sold off at very reasonable prices. Clearly within hours, no physical traces would be left of the congress. On the other hand, one would hope that mental traces, preferably of the positive and edifying kind, will remain for a very long time.

Ulf Persson [ulfp@math.chalmers.se] has been professor at Chalmers University of Technology in Gothenburg (Göteborg) since 1989. He earned his Ph.D at Harvard 1975 as a student of David Mumford. Works in Algebraic Geometry, especially compact complex surfaces. Hobbies include mathematical pictures programmed directly in PostScript. Has served as the President of the Swedish Mathematical Society and has been actively engaged in public debate on mathematical education. Publishes regularly reviews on scientific-philosophical matters in the Swedish press and is the editor of the newsletter of the aforementioned Swedish Mathematical Society.

Starting the race towards 5ecm at Amsterdam in 2008. From left to right: H.J.J. te Riele,J.O.O. Wiegerinck, C.M. Ran (http://sta.science.uva.nl/ brandts/5ECM/)

EMS Prizes

The EMS prizes are awarded by the European Mathematical Society in recognition of distinguished contributions in Mathematics by young researchers not older than 35 years. The prizes are presented every four years at the European Congresses of Mathematics.

The Prize Committee is appointed by the EMS and consists of a number of recognized mathematicians from a wide variety of fields. The prizes were first awarded in Paris in 1992 and then in Budapest in 1996 and in Barcelona in 2000.

Each prize winner 2004 received 5,000 Euro.

Prize committee
Enrico Arbarello, - Rome
Victor Buchstaber, - Moscow
John Coates,- Cambridge, UK
Jacek Graczyk, - Orsay
Bertil Gustafsson, - Uppsala
Stefan Hildebrandt, - Bonn
Jean-François Le Gall, - Paris
Vladimir Lin, - Haifa
Leonid Polterovich, - Tel Aviv
Domokos Szasz, - Budapest
Dimitri Yafaev, - Rennes
Eduard Zehnder, - Zürich

EMS Prize winners 2004 with the president of the Prize committee, Nina Uraltseva. Sitting from the left: Xavier Tolsa, Paul Biràn, Sylvia Serfaty, Stefano Bianchini, Otmar Venjakob. Standing from the left: Franck Barthe, Warwick Tucker, Nina Uraltseva (President of the Prize committee), Elon Lindenstrauss, Andrei Oukonkov, Stanislav Smirnov.

EMS Prizes 2004 with citations

Franck Barthe, *Institut de Mathématiques: Laboratoire de Statistique et Probabilités, Toulouse, France*
Barthe pioneered the use of measure-transportation techniques (due to Kantorovich, Brenier, Caffarelli, Mc Cann and others) in geometric inequalities of harmonic and functional analysis with striking applications to geometry of convex bodies. His major achievement is an inverse form of classical Brascamp-Lieb inequalities. Further contributions include discovery of a functional form of isoperimetric inequalities and a recent solution (with Artstein, Ball and Naor) of a long-standing Shannon's problem on entropy production in random systems.

Stefano Bianchini, *Instituto per le Applicazioni del Calcolo "M. Picone", Rome, Italy*
Stefano Bianchini has introduced an entirely new perspective to the theory of discontinuous solutions of one-dimensional hyperbolic conservation laws, representing solutions as local superposition of travelling waves and introducing innovative Glimm functionals. His ideas have led to the solution of the long standing problem of stability and convergence of vanishing viscosity approximations. In his best individual achievement, published in 2003 in Arch.Ration. Mech. Anal., he shows convergence of semidiscrete upwind schemes for general hyperbolic systems. In the technically demanding proof the travelling waves are constructed as solutions of a functional equation, appling center manifold theory in an infinite dimensional space.

Paul Biràn, *School of Mathematical Sciences, Tel-Aviv University, Israel*
Paul Biràn has made fundamental and influential contributions to symplectic topology as well as to algebraic geometry and Hamiltonian systems. His work is characterised by new depths in the interactions between complex algebraic geometry and symplectic topology. One of the earlier contributions is his surprising solution of the symplectic packing problem, completing work of Gromov, McDuff and Polterovich, showing that

compact symplectic manifolds can be packed by symplectic images of equally sized Euclidean balls without wasting volume if the number of balls is not too small. Among the corollaries of his proof, Biràn obtains new estimates in the Nagata problem. A powerful tool in symplectic topology is Biràn's decomposition of symplectic manifolds into a disc bundle over a symplectic submanifold and a Lagrangian skeleton. Applications of this discovery range from the phenomenon of Lagrangian barriers to surprising novel results on topology of Lagrangian submanifolds. Paul Biràn not only proves deep results, he also discovers new phenomena and invents powerful techniques important for the future development of the field of symplectic geometry.

Elon Lindenstrauss, *Clay Mathematics Institute, Massachusetts and Courant Institute of Mathematical Sciences, New York, USA*
Elon Lindestrauss has done deep and highly original work at the interface of ergodic theory and number theory. Although he has worked widely in ergodic theory, his recent proof of the quantum unique ergodicity conjecture for arithmetic hyperbolic surfaces breaks fertile new ground, with great promise for future applications to number theory.

Already, in joint work with Katok and Einsiedler, he has used some of the ideas in this work to prove the celebrated conjecture of Littlewood on simultaneous diophantine approximation for all pairs of real numbers lying outside a set of Hausdorff dimension zero. This goes far beyond what was known earlier about Littlewood's conjecture, and spectacularly confirms the high promise of themethods of ergodic theory in studying previously intractable problems of diophantine approximation.

Andrei Okounkov, *Princeton University, USA*
Andrei Okounkov contributed greatly to the field of asymptotic combinatorics. An extremely versatile mathematician, he found a wide array of applications of his methods. His early results include a proof of a conjecture of Olshanski on the representations theory of groups with infinite-dimensional duals. Okounkov gave the first proof of the celebrated Baik-Deift-Johansson conjecture, which states that the asymptotics of random partitions distributed according to the Plancherel measure coincides with that of the eigenvalues of large Hermitian matrices. An important and influential result of Okounkov is a formula he found in joint work with Borodin, which expresses a general Toeplitz determinant as the Fredholm determinant of the product of two associated Hankel operators. The new techniques of working with random partitions invented and successfully developed by Okounkov lead to a striking array of applications in a wide variety of fields: topology of module spaces, ergodic theory, the theory of random surfaces and algebraic geometry.

Sylvia Serfaty, *Courant Institute of Mathematical Sciences, New York, USA*
Sylvia Serfaty was the first to make a systematic and impressive asymptotic analysis for the case of large parameters in Theory of Ginzburg-Landau equation. She established precisely the values of the first, second and third (with E.Sandier) critical fields for nucleation of one stable vortex, vortex fluids and surface superconductivity. In micromagnetics, her work with F. Alouges and T. Rivière breaks new ground on singularly perturbed variational problems and provides the first explanation for the internal structure of cross-tie walls.

Stanislav Smirnov, *KTH, Sweden and Geneva University, Switzerland*
Stanislav Smirnov's most striking result is the proof of existence and conformal invariance of the scaling limit of crossing probabilities for critical percolation on the triangular lattice. This gives a formula for the limiting values of crossing probabilities, breakthrough in the field, which has allowed for the verification of many conjectures of physicists, concerning power laws and critical values of exponents. Stanislav Smirnov also made several essential contributions to complex dynamics, around the geometry of Julia sets and the thermodynamic formalism.

Xavier Tolsa, *ICREA and Universitat Autònoma de Barcelona, Spain*
Xavier Tolsa has made fundamental contributions to Harmonic and Complex Analysis. His most outstanding work solves Vitushkin's problem about semi-additivity of analytic capacity. The problem was raised in 1967 by Vitushkin in his famous paper on rational approximation in the plane. Tolsa's result has important consequences for a classical (100 years old) problem of Painlevé about a geometric characterization of planar compact sets are removable in the class of bounded analytic functions. Answering affirmatively Melnikov's conjecture, Tolsa provides a solution of the Painlevé problem in terms of the Menger curvature. Xavier Tolsa has also published many important and influential resultsrelated to Calderón-Zygmund theory and rational approximation in the plane.

Warwick Tucker, *Uppsala University, Sweden*
Warwick Tucker has given a rigorous proof that the Lorenz attractor exists for the parameter values provided by Lorenz. This was a long standing challenge to the dynamical system community, and was included by Smale in his list of problems for the new millennium. The proof uses computer estimates with rigorous bounds based on higher dimensional interval arithmetics. In later work, Warwick Tucker has made further significant contributions to the development and application of this area.

Otmar Venjakob, *Mathematisches Institut: Universität Heidelberg, Germany*
Otmar Venjakob has made a number of important discoveries in both the algebraic and arithmetic aspects of non-commutative Iwasawa theory, especially on problems which appeared intractable from the point of view of the classical commutative theory. In arithmetic geometry, Iwasawa theory is the only general technique known for studying the mysterious relations between exact arithmetic formulae and special values of L-functions, as typified by the conjecture of Birch and Swinnerton-Dyer. Venjakob's work applies quite generally to towers of number fields whose Galois group is an arbitrary compact p-adic Lie group (which is not, in general, commutative), and has done much to show that a rich theory is waiting to be developed. His most important results include the proof of a good dimension theory for modules over Iwasawa algebras, and the proof of the first case of a structure theory for modules over these algebras. With Hachimori he discovered the first examples of arithmetic Iwasawa modules which are completely faithful, as well as proving a remarkable asymptotic upper bound for the rank of the Mordell Weil group of elliptic curves in certain towers of number fields over Q whose Galois group is a p-adic Lie group of dimension 2. Very recently, he found the key to the problem of defining, in non-commutative Iwasawa theory, the analogue of the characteristic series of modules over Iwasawa algebras.