TO DESIGN OF BENDING PROBLEMS WITH BOUNDARY LAYERS FOR ELASTIC PLATES

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Let an elastic plate be isotropic and homogenuous, then equations with respect to a deflection and shearing forces are represented as a split system of a biharmonic equations and two Helmholtz' equations (see e.g. [1]). Consider a case, when boundary conditions of the problem for the deflection and the shearing forces are also split.

Thus, assume that for the deflection we have the following problem

$$D\Delta^2 w(x,y) = f(x,y), \tag{1}$$

$$w(x,y)\Big|_{\partial G} = \frac{\partial}{\partial n} w(x,y)\Big|_{\partial G} = 0, \tag{2}$$

where $(x,y) \in G = (-1,1)\text{x}(-1,1)$, $\Big\{\partial G: \ |x| = 1, \ |y| = 1\Big\}$, D is cylindrical rigidity, and if we change the independent variables x, y, for the shearing forces we have the problems:

$$\varepsilon \Delta Q_\alpha(\xi,\eta) - Q_\alpha(\xi,\eta) = F_\alpha(\xi,\eta), \quad 0 < \xi,\eta < 1, \quad \alpha = 1,2, \tag{3_α}$$

$$\begin{gathered} \partial_1 Q_1(\xi,\eta)\Big|_{\xi=0} = \mu_{01}(\eta), \quad \partial_1 Q_1(\xi,\eta)\Big|_{\xi=1} = \mu_{11}(\eta), \\ Q_1(\xi,0) = \nu_{01}(\xi), \quad Q_1(\xi,1) = \nu_{11}(\xi), \end{gathered} \tag{4_1}$$

$$\begin{gathered} Q_2(0,\eta) = \mu_{02}(\eta), \quad Q_2(1,\eta) = \mu_{12}(\eta), \\ \partial_2 Q_2(\xi,\eta)\Big|_{\eta=0} = \nu_{02}(\xi), \quad \partial_2 Q_2(\xi,\eta)\Big|_{\eta=1} = \nu_{12}(\xi), \end{gathered} \tag{4_2}$$

where $\varepsilon = \dfrac{h^2}{12}(1+2\gamma)$, $2h$ is a thickness of the plate, γ is an arbitrary control parameter, ν is a coefficient of rigidity, $\partial_1 = \partial/\partial\xi, \ \partial_2 = \partial/\partial\eta$.

For solving (1), (2) we use the projective method by [2]. Then the solution of (1), (2) has the following form:

$$w(x,y) = \sum_{i,j=1}^{m} w^{ij} \chi^2 P_i(x) \chi^2 P_j(y),$$

where $\chi^2 P_i(x), \ \chi^2 P_j(y)$ are second order divided differences with respect to indeces from the Legendre polynomials:

For solving (3_α), (4_α) an alternating direction iterative scheme is used, which is a modification of the continual analogue of the alternating direction method [3] for discrete version. This scheme has the following form:

$$(r+1)\overset{(k+\frac{1}{2})}{Q_\alpha}(\xi,\eta_j)+\varepsilon A_1\overset{(k+\frac{1}{2})}{Q_\alpha}(\xi,\eta)\Big|_{\eta=\eta_j}= \\ = r\overset{(k)}{Q_\alpha}(\xi,\eta_j)-\varepsilon A_2\overset{(k)}{Q_\alpha}(\xi,\eta)\Big|_{\eta=\eta_j}-F_\alpha(\xi,\eta_j), \tag{5_α}$$

$$\partial_1\overset{(k+\frac{1}{2})}{Q_1}(\xi,\eta)\Big|_{\xi=0,\,\eta=\eta_j}=\mu_{01}(\eta_j),\quad \partial_1\overset{(k+\frac{1}{2})}{Q_1}(\xi,\eta)\Big|_{\xi=1,\,\eta=\eta_j}=\mu_{11}(\eta_j), \tag{6_1}$$

$$\overset{(k+\frac{1}{2})}{Q_2}(0,\eta_j)=\mu_{02}(\eta_j),\quad \overset{(k+\frac{1}{2})}{Q_2}(1,\eta_j)=\mu_{12}(\eta_j),\quad j=\overline{2,n}, \tag{6_2}$$

$$(r+1)\overset{(k+1)}{Q_\alpha}(\xi_i,\eta)+\varepsilon A_2\overset{(k+1)}{Q_\alpha}(\xi,\eta)\Big|_{\xi=\xi_i}= \\ = r\overset{(k+\frac{1}{2})}{Q_\alpha}(\xi_i,\eta)-\varepsilon A_1\overset{(k+\frac{1}{2})}{Q_\alpha}(\xi,\eta)\Big|_{\xi=\xi_i}-F_\alpha(\xi_i,\eta), \tag{7_α}$$

$$\overset{(k+1)}{Q_1}(\xi_i,0)=\nu_{01}(\xi_i),\quad \overset{(k+1)}{Q_1}(\xi_i,1)=\nu_{11}(\xi_i), \tag{8_1}$$

$$\partial_2\overset{(k+\frac{1}{2})}{Q_2}(\xi,\eta)\Big|_{\xi=\xi_i,\eta=0}=\nu_{02}(\xi_i),\quad \partial_2\overset{(k+\frac{1}{2})}{Q_2}(\xi,\eta)\Big|_{\xi=\xi_i,\eta=1}=\nu_{12}(\xi_i),\quad i=\overline{2,n}, \tag{8_2}$$

where $A_1=-\partial^2/\partial\xi^2$, $A_2=-\partial^2/\partial\eta^2$, r is an iterative parameter.

The problems (5_α), (6_α) and (7_α), (8_α) are the problems with boundary layer for second order ordinary differential equations. For solving these problems we apply a generalized finite difference method and (Q)-formulas [3]. The iterative schemes have the form:

$$(r+1)(B_{1,h}\overset{(k+\frac{1}{2})}{Q_\alpha})_{ij}+\varepsilon(A_{\alpha,01,h}\overset{(k+\frac{1}{2})}{Q_\alpha})_{ij}=r(B_{1,h}\overset{(k)}{Q_\alpha})_{ij}- \\ -\varepsilon(B_{1,h}B_{2,h}^{-1}A_{\alpha,02,h}\overset{(k)}{Q_\alpha})_{ij}-\varepsilon(B_{1,h}B_{2,h}^{-1}F_\alpha^2)_{ij}-(B_{1,h}F_\alpha)_{ij}+F^1_{\alpha,ij},\ i=\overline{n_1,n}, \tag{9_α}$$

$$\partial_1\overset{(k+\frac{1}{2})}{Q_{1,1j}}=\mu_{01}(\eta_j),\quad \partial_1\overset{(k+\frac{1}{2})}{Q_{1,n+1,j}}=\mu_{11}(\eta_j), \tag{10_1}$$

$$\overset{(k+1)}{Q_{2,1j}}=\mu_{02}(\eta_j),\quad \overset{(k+1)}{Q_{2,n+1,j}}=\mu_{12}(\eta_j),\ \ j=\overline{2,n}, \tag{10_2}$$

$$(r+1)(B_{2,h}\overset{(k+1)}{Q_\alpha})_{ij}+\varepsilon(A_{\alpha,02,h}\overset{(k+1)}{Q_\alpha})_{ij}=r(B_{2,h}\overset{(k+\frac{1}{2})}{Q_\alpha})_{ij}- \\ -\varepsilon(B_{2,h}B_{1,h}^{-1}A_{\alpha,01,h}\overset{(k+\frac{1}{2})}{Q_\alpha})_{ij}-\varepsilon(B_{2,h}B_{1,h}^{-1}F_\alpha^1)_{ij}-(B_{2,h}F_\alpha)_{ij}+F^2_{\alpha,ij},\ \ j=\overline{n_2,n}, \tag{11_α}$$

$$\overset{(k+1)}{Q_{1,i1}}=\nu_{01}(\xi_i),\quad \overset{(k+1)}{Q_{1,i,n+1}}=\nu_{11}(\xi_i), \tag{12_1}$$

$$\partial_2\overset{(k+\frac{1}{2})}{Q_{2,i1}}=\nu_{02}(\xi_i),\quad \partial_2\overset{(k+\frac{1}{2})}{Q_{2,i,n+1}}=\nu_{12}(\xi_i), \tag{12_2}$$

$$i = \overline{2, n}.$$

here $A_{\alpha,01,h},\ A_{\alpha,02,h},\ B_{1,h},\ B_{2,h},\ F^1_{\alpha,ij},\ F^2_{\alpha,ij}$ $(F^1_{1,ij} = 0,\ F^2_{2,ij} = 0)$ are defined by the formulas of the generalized finite difference method and (Q) formulas; $n_1 = 1$, $n_2 = 2$ for $\alpha = 1$ and $n_1 = 2$ $n_2 = 1$ for $\alpha = 2$.

Theorem. *If the operators $B^{-1}_{1,h}A_{\alpha,01,h},\ B^{-1}_{2,h}A_{\alpha,02,h}$ are positive definite, then the iterative processes (9_α), (10_α) and (11_α), (12_α) converge for $r \geq 0$ and any $\overset{(0)}{Q}_\alpha,\ A_2\overset{(0)}{Q}_\alpha$.*

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