Journal of Integer Sequences, Vol. 15 (2012),
Article 12.9.8

The History of the Primality of One: A Selection of Sources

Chris K. Caldwell, Angela Reddick,[1] and Yeng Xiong[1]
Department of Mathematics and Statistics
University of Tennessee at Martin
Martin, TN 38238, USA
caldwell@utm.edu
yenxion@ut.utm.edu

Wilfrid Keller
Universität Hamburg
Hamburg
Germany
keller@rrz.uni-hamburg.de

Abstract

The way mathematicians have viewed the number one (unity, the monad) has changed throughout the years. Most of the early Greeks did not view one as a number, but rather as the origin, or generator, of number. Around the time of Simon Stevin (1548–1620), one (and zero) were first widely viewed as numbers. This created a period of confusion about whether or not the number one was prime. In this dynamic survey, we collect a cornucopia of sources which deal directly with the question "what is the smallest prime?" The goal is to create a source book for studying the history of the definition of prime, especially as applied to the number one.

1 Introduction

It seems that a question like "what is the first prime?" would have the simple and obvious answer "two." This is the most common answer throughout history, and the only accepted

[1] An undergraduate student at the time we began this project. Partially supported by a University of Tennessee at Martin Undergraduate Research Grant.

answer among mathematicians today, but it was not always the only answer. Some ancient Greeks defined the primes as a subset of the odd numbers, so started the sequence of primes with three. Others (including John Pell, John Wallis and Edward Waring) started the sequence of primes with the number one. We summarize this history in our companion article [19]. There we point out, for example, that from the ancient Greeks to the time of Stevin one was not even considered a number, so no one would ask if it was prime. The goal of this work is more simple: to collect a list of references helpful in addressing questions about the smallest primes in general, and about the primality of unity in particular.

When selecting sources, we sought all that made the author's view clear. This is often difficult because of language and typographical barriers. It is also difficult because few addressed the question explicitly. For example, Gauss does not even define prime in his pivotal *Disquisitiones Arithmeticae* [46], but his view on the primality of one can be implied from his statement of the fundamental theorem of arithmetic.

We must not confuse what most authors wrote with what they believed to be correct. Definitions often depend on context. Those who teach undergraduate mathematics courses will be very familiar with the example of $\log x$. This expression often represents the common (base 10) logarithm in a pre-calculus course, the multivalued inverse of the exponential in a complex variables course, and the single-valued natural (base e) logarithm in a real analysis course. These differences are not matters of belief, just of tradition and context. So it is not surprising that V.-A. Lebesgue and G. H. Hardy seem ambivalent to the primality of one in the list below.

Finally, we appreciate that the Journal of Integer Sequences has allowed this work to be stored as a dynamic survey. This means that it can be edited and updated as time continues. We would be glad to hear of any significant additions or corrections (especially when you can share images or scans of the original texts!).

1.1 Notes about the table of sources

- When possible, we tried to reproduce the language, spelling and typography[2] of the original sources. These could help the reader better understand the quote.
- The table's first column, titled 'one,' contains 'yes' when the author mentioned one as a prime number.
- When ellipses are in the original quotes, we will use '. . . '. If we are using ellipses to denote the omission of part of a quote, we will use '[. . .]'.
- Any date before 1200 is an approximation.

[2]However, we made only the most minimal effort to preserve line breaks and white-space. For example, some early publications placed blank spaces both before *and* after punctuation marks such as a colon; yet we normally used the modern spacing and put this space only after the colon.

2 Sources

one	who / year	quote (or comment)
no	Plato 400BCE	Tarán writes [128, p. 276]: "The Greeks generally, Plato and Aristotle included, considered two to be the first prime number (cf. Plato, *Republic* 524 D 7, *Parmenides* 143 C-144 A, pp. 14-15 *supra*, Aristotle, *Physics* 207 B 5-8, 220 A 27, *Metaphysics* 1016 B 17-20, 1021 A 12-13, 1052 B 20-24, 1053 A 27-30, 1057 A 3-6, 1088 A 4-8, Euclid, *Elem.* VlI, Defs. 1-2); and so for them one is not a number (Aristotle is explicit about this and refers to it as a generally accepted notion [cf. p. 20, note 95 and p. 35 with note 175]; for some late thinkers who treat one as an odd number cf. Cherniss, *Plutarch's Moralia*, vol. XIII, I, p. 269, n. *d*). Nor did the early Pythagoreans consider one to be a number, since in all probability they subscribed to the widespread notion that number is a collection of units (cf. Heath, *Euclid's Elements*, II, p. 280; Cherniss, *Crit. Pres. Philos.,* pp. 387 and 389)."
yes	Speusippus 350BCE	Tarán writes [128, p. 276]: "Speusippus, then, is exceptional among pre-Hellenistic thinkers in that he considers one to be the first prime number. And Heath, *Hist. Gr. Math.*, I, pp. 69-70, followed by Ross, *Aristotle's Physics*, p. 604, and others, is mistaken when he contends that Chrysippus, who is said to have defined one as $\pi\lambda\tilde{\eta}\theta o\varsigma$ $\breve{\epsilon}\nu$ (cf. Iamblichus, *In Nicom. Introd. Arith.*, p. II, 8-9 [Pistelli]), was the first to treat one as a number (cf. further p. 38f. with note 189 *supra*)."
no	Aristotle 350BCE	Heath says [63, p. 73] that "Aristotle speaks of the dyad as 'the only even number which is prime' (Arist. Topics, Θ. 2, 157 a 39). Also Tarán [128, p. 20] states Aristotle explicitly argues one is not a number (Metaphysics 1088 A 6-8), saying "Aristotle never considers one to be a number and for him the first number is two." See also [128, p. 276].
no	Euclid 300BCE	Heath notes [63, p. 69]: "Euclid implies [one is not a number] when he says that a unit is that by virtue of which each of existing things is called one, while a number is 'the multitude made up of units,' [. . .]." On page 73, Heath mentions that Euclid includes two among the primes.

one	who / year	quote (or comment)
no	Theon of Smyrna 100BCE	Smith writes [124, p. 20]: "Aristotle, Euclid, and Theon of Smyrna defined a prime number as a number 'measured by no number but an unit alone,' with slight variations of wording. Since unity was not considered as a number, it was frequently not mentioned. Iamblichus says that a prime number is also called 'odd times odd,' which of course is not our idea of such a number. Other names were used, such as 'euthymetric' and 'rectilinear,' but they made little impression upon standard writers. The name 'prime number' contested for supremacy with 'incomposite number' in the Middle Ages, Fibonacci (1202) using the latter but saying that others preferred the former." Heath states [63, p. 73] that Theon of Smyrna sees two as "odd-*like* without being prime" and cites "Theon of Smyrna, p. 24. 7."
no	Nicomachus 100	"The Unit then is perfect potentially but not actually, for taking it into the sum as the first of the line I inspect it according to the formula to see what sort it is, and I find it to be prime and incomposite; for in very truth, not by participation like the others, but it is first of every number and the only incomposite" [69, p. 20]. Smith writes [124, p. 27]: "It is not probable that Nicomachus (*c.* 100) intended to exclude unity from the number field in general, but only from the domain of polygonal numbers. It may have been a misinterpretation of the passage of Nicomachus that led Boethius to add the great authority of his name to the view that one is not a number." Tarán notes [128, p. 276]: "For, if we started the number series with three (as some Neopythagoreans did [cf. e.g. Nicomachus, *Intr. Arith.* I, II], who consider prime number to be a property of odd number only [cf. Tarán, *Asclepius on Nicomachus*, pp. 77-78, on I, νη and ξα, with references]), then there would be in ten three prime numbers (3, 5, 7) and five composite ones (4, 6, 8, 9, 10)." Heath notes that Nicomachus defines primes and composites as subdivisions of the odds [63, p. 73], so two is not prime. Also "According to Nicomachus 3 is the first prime number [...]" [39, p. 285].
—	Iamblichus 300	Heath also notes [63, p. 73] that Iamblichus defines primes and composites as subdivisions of the odds, so two is not prime.

one	who / year	quote (or comment)
no	Martianus Capella 400	Stahl and Johnson [90, pp. 285–286] translate Martianus Capella as follows: “[743] We have briefly discussed the numbers comprising the first series, the deities assigned to them, and the virtues of each number. I shall now briefly indicate the nature of number itself, what relations numbers bear to each other, and what forms they represent. A number is a collection of monads or a multiple proceeding from a monad and returning to a monad. There are four classes of integers: the first is called ‘even times even’; the second ‘odd times even’; the third ‘even times odd’; and the fourth ‘odd times odd’; these I shall discuss later. [744] Numbers are called prime which can be divided by no number; they are seen to be not ‘divisible’ by the monad but ‘composed’ of it: take, for example, the numbers five, seven, eleven, thirteen, seventeen, and others like them. No number can divide these numbers into integers. So they are called ‘prime,’ since they arise from no number and are not divisible into equal portions. Arising in themselves, they beget other numbers from themselves, since even numbers are begotten from odd numbers, but an odd number cannot be begotten from even numbers. Therefore prime numbers must of necessity be regarded as beautiful. [745] Let us consider all numbers of the first series according to the above classifications: the monad is not a number; the dyad is an even number; the triad is a prime number, both in order and in properties; the tetrad belongs in the even times even class; the pentad is prime; the hexad belongs to the odd times even or even times odd (hence it is called perfect); the heptad is prime; the octad belongs to [. . .]” [The numbers [743], [744] and [745] are in the quoted text, numbering the paragraphs.]
no	Boethius 500	Masi notes [92, pp. 89–95] that Boethius (like Nicomachus), defines prime as a subdivision of the odds, and starts his list of examples at three.
no	Cassiodorus 550	A prime number, notes [53, p. 5], “is one which can be divided by unity alone; for example, 3, 5, 7, 11, 13, 17, and the like.” For him, prime is a subset of odd; perfect, abundant and deficient are all subsets of even [20, pp. 181–182].

one	who / year	quote (or comment)
no	Isidore of Seville 636	In "Etymologiarum sive Originum, Liber III: De mathematica" Isidore says (Grant's translation[3] [53, pp. 4–5]): "Number is a multitude made up of units. For one is the seed of number but not number. [...] Number is divided into even and odd. Even number is divided into the following: evenly even, evenly uneven, unevenly even and unevenly uneven. Odd number is divided into the following: prime and incomposite, composite, and a third intermediate class *(mediocris)* which in a certain way is prime and incomposite but in another way secondary and composite. [...] Simple [or prime] numbers are those which have no other part [or factor] except unity alone, as three has only a third, five only a fifth, seven only a seventh, for these have only one factor."
no	Al-Khwarizmi 825	"Boetius (AD 475–524/525), who wrote the most influential book of mathematics during the Middle Ages, *De Institutione Arithmetica Libri Duo*, following a personal restrictive interpretation of Nichomachus and affirmed that one is not a number. Even Arab mathematicians (e.g. Abu Ja'far Mohammed ibn Musa Al-Khowarizmi, *c.* AD 825) excluded unity from the number field. Rabbi ben Ezra (*c.* 1095–*ca.* 1167), instead in his *Sefer ha-Echad* (Book of Unity) argued that one should be looked upon as a number. Only during the 16th century did authors begin to raise the question as to whether this exclusion of unity from the number field was not a trivial dispute (Petrus Ramus, 1515–1572), but Simon Stevin (*c.* 1548–*c.* 1620) argued that a part is of the same nature as the whole, and hence, that unity is a number." [110, p. 812].
no	al-Kindī 850	After considering and rejecting the possibility of one being a number al-Kindī writes [67, p. 102]: "Since, therefore, it is clear that one is not a number, the definition said of number shall then encompass /number fully, *viz.*, that it is a magnitude (composed of) onenesses, a totality of onenesses, and a collection of onenesses. Two is, then, the first number." (He did see the number two "as prime, if in a qualified way" [67, p. 181].)

[3] There is a wonderful 1493 version of this text online at http://tudigit.ulb.tu-darmstadt.de/show/inc-v-1/0039.

one	who / year	quote (or comment)
no	Hugh of St. Victor 1120	"Arithmetic has for its subject equal, or even, number and unequal, or odd, number. Equal number is of three kinds: equally equal, equally unequal, and unequally equal. Unequal number, too, has three varieties: the first consists of numbers which are prime and incomposite; the second consists of numbers which are secondary and composite; the third consists of numbers which, when considered in themselves, are secondary and composite, but which, when one compares them with other numbers [to find a common factor or denominator], are prime and incomposite." [53, p. 56]
—	Rabbi ben Ezra 1140	Smith notes [124, p. 27]: "One writer, Rabbi ben Ezra (*c.* 1140), seems, however, to have approached the modern idea. In his *Sefer ha-Echad (Book on Unity)* there are several passages in which he argues that one should be looked upon as a number." On the other hand, M. Friedländer [44, p. 658] notes: "The book [Rabbi ben Ezra's *Arithmetic*] opens with a parallelism between the Universe and the numbers; there we have nine spheres and a being that is the beginning and source of all the spheres, and at the same time separate and different from the spheres. Similarly there are nine numbers, and a unit that is the foundation of all numbers but is itself no number."
—	Fibonacci 1202	Smith quotes Fibonacci's *Liber Abaci* I, 30, as follows [124, p. 20]: "Nvmerorum quidam sunt incompositi, et sunt illi qui in arismetrica et in geometria primi appellantur. [. . .] Arabes ipsos hasam appellant. Greci coris canon, nos autem sine regulis eos appellamus." Besides Fibonacci's preferred 'incomposite,' 'simple number' also seems common in the later periods. See also the 1857 copy of *Liber Abaci* [13, p. 30].
no	Prosdocimo 1483	Smith writes [123, pp. 13–14]: "This rare work was written for the Latin schools, and is a good example, the first to appear in print, of the non-commercial algorisms of the fifteenth century. It follows 'Bohectius' (Boethius) in defining number and in considering unity as not itself a number, as is seen in the facsimile of the first page."

one	who / year	quote (or comment)
—	——	L. L. Jackson [68, p. 30], writing about the teaching of mathematics in the sixteenth century, notes: “This difference of opinion as to the nature of unity was not new in the sixteenth century. The definition had puzzled the wise men of antiquity. Many Greek, Arabian, and Hindu writers had excluded unity from the list of numbers. But, perhaps, the chief reason for the general rejection of unity as a number by the arithmeticians of the Renaissance was the misinterpretation of Boethius’s arithmetic. Nicomachus (c. 100 A.D.) in his *Αριθμητικης βιβλια δυο* had said that unity was not a polygonal number and Boethius’s translation was supposed to say that unity was not a number. Even as late as 1634 Stevinus found it necessary to correct this popular error and explained it thus: $3-1=2$, hence 1 is a number.”
no	John of Holywood 1488	“Therfor sithen þe ledynge of vnyte in hym-self ones or twies nought comethe but vnytes, Seithe Boice in Arsemetrike, that vnyte potencially is al nombre, and none in act. And vndirstonde wele also that betwix euery.” The editor noted beside this section that “Unity is not a number” [126, p. 47].
no	P. Ciruelo 1526	Ciruelo states [26, p. 15] that primes are a subset of the odds: “Numeri imparis tres ſunt ſpecies immediate quæ ſunt, primus, ſecundus, & ad alterum primus. Numerus impar primus eſt qui ſola vnitate parte aliquota metiri poteſt, vt. 3. 5. 7. idemꝗ incompoſitus nominatur, & ratio vtriuſꝗ denominationis eſt eadem : quia numeri imparis nulla poteſt eſſe pars aliquota præter vnitatem, niſi illa etiam ſit numerus impar.” This source did not have page numbers, but this quote is on the 15th page.
no	J. Köbel 1537	Menninger [96, p. 20] quotes: “Wherefrom thou understandest that 1 is no number / but it is a generatrix / beginning / and foundation of all other numbers.” He also gives the original: “*Darauss verstehstu das I. kein zal ist / sonder es ist ein gebererin / anfang / vnd fundament aller anderer zalen.*”
no	G. Zarlino 1561	Gioseffo Zarlino’s influential music theory text [142, p. 22] says: “*Li numeri Primi & incompoſti ſono quelli, i quali non poſſono eſſer numerati o diuiſi da altro numero, che dall’ vnità; come 2. 3. 5. 7. 11. 13. 17. 19. & altri ſimili.*”

one	who / year	quote (or comment)
—	S. Stevin 1585	Menninger writes [96, p. 20]: "Stevin, the man who first introduced the algorism of decimal fractions, was probably the first mathematician expressly to assert (in 1585) the numerical nature of One". However, there are others. First might be Speusippus (ca. 365BCE) [128, pp. 264, 276], but these exceptions are rare and had little effect on common thought. Speusippus viewed one as prime.
no	P. A. Cataldi 1603	Cataldi's treatise on perfect numbers [21, pp. 28–40] contains a factor table to 750 and a list of primes below 750 (from 2 to 741).
no	C. Clavius 1611	Clavius, commenting on Euclid, wrote [27, p. 307]: "PRIMVS numerus eſt, quem vnitas ſola metitur. *QVOD ſi numerum quempiam nullus numerus, ſed ſola vnitas metiatur, it a vt neg, pariter par, neg, pariter impar, neque impariter impar poßit dui, appellabitur numerus primus; quales ſunt omnes iſti 2. 3. 5. 7. 11. 13. 17. 19. 23. 29. 31. &c. Nam eos ſola vnitas metitur.*"
no	D. Henrion 1615	Henrion [37, p. 207], expounding on Euclid's definition, wrote: "11. Nombre premier, eſt celuy qui eſt meſuré par la ſeule vnité. *C'eſt à dire, que ſi vn nombre n'eſt meſuré par aucun autre nombre, mais ſeulement par l'vnité, il eſt nombre premier, & tels ſont tous ceux-cy* 2.3.5.7.11.13.17.19.23.29.31. *&c. Car la ſeule vnité meſure iceux.*" This was very slightly reworded in a later 1676 edition [38, p. 381] (after his death): "11. Nombre premier, eſt celuy qui eſt meſuré par la ſeule unité. *C'eſt-à-dire, que ſi un nombre n'eſt meſuré par aucun autre nombre, mais ſeulement par ſoy même, & l'unité, il eſt nombre premier, & tels ſont tous ceux-cy* 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, *&c. Car chacun d'iceux n'eſt meſuré par aucun autre nombre, mais par la ſeule unité."* (Note that Denis Henrion and Pierre Hérigone are *both* pseudonyms for the Baron Clément Cyriaque de Mangin (1580–1643), see the entry "Pierre Hérigone" in MacTutor [105].)
no	M. Mersenne 1625	*"Les nombres premiers entr'eus ſont ceus qui ont la ſeule vnité pour leur meſure commune : & les nombres compoſez ſont ceux qui ſont meſurez par quelque nombre, qui leur ſert de meſure commune.*

one	who / year	quote (or comment)
		Ce Thorême comprend la 13. & 14. definition du 7, & n'a befoin que d'explication: ie di donc premierementque le nõbre premier n'a autre mefure que l'vnité, tel qu'eft, 2, 3, 5 &c. vous treuuerez les autres nombres premiers par l'ordre naturel des nõbres *impairs*, fi vous en oftez tous les nõbres qui font éloignez par 3. nombres du 3, & par cinq nombres du 5, & par 7, nombres du 7, & ainfi des autres, [...]" [97, pp. 298–299]. In another text Mersenne wrote: "[...] il faut multiplier tous les nombres premiers moindres que 10, a scauoir 2, 3, 5, 7." [32, p. 23]
no	A. Metius 1640	"Numeri confiderantur aut abfolutè pe fe : aut inter fe relativè. Numerus abfolutè pe fe confideratus, eft aut per fe Primus, aut Compofitus. Numerus per fe primus eft, quem præter unitatem nullus alius numerus metitur, *quales funt 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, &c. namque eos fola unitas dividit, ut nihil fuperfit.*" [98, pp. 43–44]
no	M. Bettini 1642	Bettini [12, p. 36] writes about Euclid's definition: "Qui lib. 7 def. 11. fic: *Primus numerus eft, quem vnitas fola metitur*, quales funt 2. 3. 5. 7. 11. 13. 17. 19. 23. 29. 31. &c. primos numeros & inuenire, & infinitos effe docet lib. 9. propof. 20."
no	Léon de Saint-Jean 1657	"Sunt infuper numeri *Primi*, qui fola vnitate, nec alio præter vnitatem numero, menfurantur. Dicitur autem numerus vnus alterum menfurare, qui multoties repetitus alterum ita explet ; vt nihil fuperfluat, aut defit. Itaque vocantur numeri primi ac *Simplices*, quales funt 2. 3. 5. 7. 11. 1. 3.[*sic*] 17. 19. 23 &c." [83, p. 581]
no	F. v. Schooten 1657	Schooten includes a table of primes [120, pp. 393–403] below 10,000 entitled "Sectio V. *Syllabus numerorum primorum, qui continentur in decem prioribus chiliadibus.*" This list of primes begins with two (p. 394).
yes	T. Brancker & D. Pell 1668	The introduction to Brancker's table of primes [112, p. 201] describes the table and one of its most common uses: "It may be of great ufe fometimes to have *a complete and orderly enumeration of all incompofits between 0, and 100,000, without any mixture of Compofits*; thus 1. 2. 3. 5. 7. 11. 13. &c, leaving out 9, 21 and all other compofits. [...] If to *each of thefe primes* you fet the Briggian Logarithm, you may find the Logarithms for *all of the reft of the numbers* in the firft 100 Chilliads, by addition of the logarithms of their incompofite Factors."

one	who / year	quote (or comment)
		Maseres reprints the appendix from *Teutsche Algebra* which contains the tables (see [91, Preface p. vii, 353]) as pages 353 to 416 of his text. Bullynck states that "Pell was involved in reading, correcting and supplementing the translation [Brancker's translation of Rahn's *Teutsche Algebra*]; in the end he replaced almost half of Rahn's text with his own [...] Brancker calculated the factor table afresh up to 100,000, following Pell's directions. After the English translation in 1668, the book would be generally known as Pell's *Algebra*, and the *Table of Incomposits* as Pell's *Table*, although [Balthasar] Keller and Brancker, independently, had calculated the table, and Rahn wrote the original work" [16, p. 143].
no	S. Morland 1673	"A prime *number* is that which is meaſured onely by an Unite. That is to ſay 2, 5, 7, 11, 13, *&c* are *prime numbers*, becauſe neither of them can poſſibly be divided into equal parts by any thing leſs then an Unite." [103, p. 25] [Surely the exclusion of 3 from his list of primes was an accident.]
no	J. Moxon 1679	Moxon wrote the first English language dictionary of mathematics (which defines number on page 97, primes on page 118, and unity on page 162). "**Prime**, or **First Number**, Is defined by *Euclid* to be that which onely Unity doth meaſure, as 2, 3, 5, 7, 11, 13, 17, 19, 33 [*ſic*], 29, 31, &c. for onely Unity can meaſure theſe." [104, p. 118] "**Number**, Is commonly defined to be, *A Collection of Units*, or *Multitude compoſed of Units*; ſo that *One* cannot be properly termed a *Number*, but the begining of *Number*: Yet I confeſs this (though generally received) to ſome ſeems queſtionable, for againſt it thus one might argue: A Part is of the ſame matter of which is its Whole; An Unit is part of a multitude of Units; Therefore an Unit is of the ſame matter with a multitude of Units: But the matter and ſubſtance of Units is Number; Therefore the matter of an Unit is Number. Or thus, A Number being given, If from the ſame we ſubtract 0, (no Number) the Number given doth remain: Let 3 be the Number given, and from the ſame be taken 1, or an Unit, (which, as theſe will ſay, is no Number) then the Number given doth remain, that is to ſay, 3, which to ſay, iſ abſurd. But this by the by, and with ſubmiſſion to better Judgments." [104, p. 97]

one	who / year	quote (or comment)
no	V. Giordano 1680	"*Tuttii numeri, che non poſſono eſſere miſurati giuſtamente da altri numeri, cioe che non ſono numeri parimente pari, ne parimente diſpari, ne meno diſparimente diſpari, mà che poſſono eſſere miſurati ſolamente dall'vnità, ſi dicono numeri primi: come ſono i ſeguenti 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 &c.*" [48, p. 310]
no	G. Clerke 1682	"Docuit *Euclides*, lib. 7. Definit. 11. numerum illum eſſe primum quem unitas ſola metitur, hoc eſt, dividit, ita, 2 3 5 7 11 13 17 19 23 29 31, &c. ſunt omnes primi: [...]" [28, p. 39].
yes	J. Wallis 1685	Wallis' "A Discourse of Combinations, Alternations, and Aliquot Parts" is reprinted as pp. 269–352 of [91]. Here he makes the following definitions [91, p. 292] (see also [133, p. 496]): "6. It is manifeſt that the Number 1, hath no Aliquot Part, and but one Diviſor, that is 1. Becauſe there is no Number leſs than itſelf that may be a part of it : But it meaſures itſelf ; and therefore is its own Diviſor. 7. Any other Prime Number hath one Aliquot Part, and Two Diviſors. For a *Prime Number*, we call, ſuch as is meaſured (beſide itſelf) by no other Number but an Unit. As 2, 3, 5, 7, 11, &c. Each of which are meaſured by 1, and by itſelf; but not by any other Number. And hath therefore 2 Diviſors, and 1 Aliquot Part; but no more. 8. Every *Power* of a *Prime Number* (other than of 1, which here is underſtood to be excluded,) hath ſo many Aliquot Parts as are the dimenſions of ſuch Power; and one Diviſor more than ſo. [...]"
no	T. Corneille 1685	Corneille, in his encyclopedic dictionary, defined [30, p. 110]: "NOMBRE. ſ.m. *Pluſieurs unitez conſiderées enſemble* [...] *Nombre premier*, Celuy que la ſeule unité meſure; comme 2. 3. 5. 7. 11. qu'on ne ſçauroit meſurer par aucun autre nombre, [...]".
yes	J. Prestet 1689	"Je nommerai *nombres ſimples* ou *premiers*, ceux qu'on ne peut diviſer au juſte ou ſans reſte par aucun autre entier que par eux-mêmes ou par l'unité; comme chacun des dix 1, 2, 3, 5, 7, 11, 13, 17, 19, 23." [111, p. 141]
no	C. F. M. Dechales 1690	Expounding on Euclid's book 7, Dechales writes [33, p. 169]: "1. Unitas eſt ſecundùm quam unumquodque dicitur unum. Nempe ab unitate dicitur unus homo, unus leo, unus lapis. Hæc definitio dat primam tantum unitatis cognitionem, quod in præſenti materia ſufficit, unitatem enim per ſe melius cognoſcimus, quàm ex quacumque definitione.

one	who / year	quote (or comment)
		2. Numerus eſt ex unitatibus compoſita multitudo. *Unde tot habet partes quot unitates, denominationemque habet ex multitudine unitatum. Ex quo ſequitur omnes numeros inter ſe commenſurabiles eſſe, cum eos unitas metiatur.* 11. Primus numerus abſolutè dicitur is quem ſola unitas metitur, *ut 2, 3. 5. 7. 11. 13, quia nullam habent partem aliquotam unitate majorem.*"
no	A. Arnauld 1690	"On dit qu'un nombre eſt nombre premier, quand il n'a de meſure que l'unité & ſoy-même, (ce qui ſe ſous-entend ſans qu'on le diſe.) Comme 2. 3. 5. 7. 11. 13, &c." [3, p. 98].
no	J. Ozanam 1691	Ozanam, essentially an expositor, defines [107, p. 27]: "Le *Nombre Premier* eſt celuy qui n'eſt meſuré par aucun nombre que par l'unité: comme 2, 3, 5, 7, 11, 17, 19, &c. On le nomme auſſi *Nombre lineaire*, & encore *Nombre incompoſé*, pour le differencier du *Nombre compoſé*." [Where is 13?]
—	——	Ağargün and Özkan, in "A historical survey of the fundamental theorem of arithmetic" [1] address the development of the fundamental theorem of arithmetic and affirm with C. Goldstein [51] that up to the 17th century mathematicians were not interested in the prime factorization integers for its own sake, but as a means of finding divisors. Note how this may alter the way you view the primality of one.
no	E. Phillips 1720	"**Prime**, **Simple**, or **Incompoſit Number**, (in *Arithm.*) is a Number, which can only be meaſur'd or divided by it ſelf, or by Unity, without leaving any Remainder; as 2, 3, 5, 7, 11, 13, *&c.* are Prime Numbers. **Compoſite** or **Compound Number**, is that which may be divided by ſome Number, leſs than the Compoſite it ſelf, but greater than Unity; as 4,6, 8, 9, 10, &c." [109, p. 460]. (This book does not have page numbers but this is on the 460th page.)
no	"Shuli Jingyun" c. 1720	Denis Roegel [116] reconstructed the tables from the Siku Quanshu (c. 1782) *which are supposedly* copies of those from the Shuli Jingyun (1713-1723) [116]. The list of primes begins $2, 3, 5, 7, \ldots$ [4]

[4] Original image http://www.archive.org/stream/06076320.cn#page/n66/mode/2up.

one	who / year	quote (or comment)
yes	J. Harris 1723	John Harris' dictionary [62] defines: "INCOMPOSITE *Numbers*, are the ſame with thoſe *Euclid* calls *Prime Numbers*. In Dr. *Pell's* Edition of of *Brancker's Algebra*, there is a Table, as it's there called, of *Incompoſite Numbers*, leſs than 100000; tho' it contains far more *Compoſite* than *Incompoſite Numbers* [. . .] 'Tis true that 2 and 5 are Incompoſite Numbers, as well as 1 and 3; but they are not put into the Tables, becauſe no other Incompoſite Numbers can terminate in them: [. . .]."
no	F. Brunot 1723	"*Le Nombre entier* ſignifie une ou pluſieurs unitez de même genre lorſque l'on n'y conſidere aucune partie." [15, p. 2] "*Le Nombre premier, ſimple*, ou *qui n'eſt pas compoſé*, eſt celui qui n'a aucunes parties aliquotes que l'unité, comme 2, 3, 5, 7, 11, 13, &c." [15, p. 3]
no	J. Cortès 1724	Cortès states that he follows Euclid on a previous page. "El Numero primero ſe dize aquel que de ſola la unidad puede ſer medido, y no de otro numero, como 2. 3. 5. 7. 11. 13. y otros de eſta manera." [31, p. 7]
no	E. Stone 1726	Edmund Stone's mathematical dictionary [127, p. 293] states: "PRIME NUMBERS, in Arithmetick, are thoſe made only by Addition, or the Collection of Unites, and not by Multiplication : So an Unite only can meaſure it; as 2, 3, 4, 5, *&c.* and is by ſome call'd a *Simple*, and by others an *Uncompound Number*." (This book does not contain any page numbers but this is on the 293rd page.)
no	E. Chambers 1728	"PRIME *Number*, in Arithmetic, a Number which can only be meaſur'd by Unity; or whereof 1 is the only aliquot part. See NUMBER. Such are 5, 7, 11, 13, *&c.*" [23, p. 871] "'Tis diſputed among Mathematicians, whether or no *Unity* be a *Number*.—The generality of Authors hold the Negative; and make *Unity* to be only inceptive of Number, or the Principle thereof; as a Point is of Magnitude, and *Uniſon* of Concord. *Stevinus* is very angry with the Maintainers of this Opinion : and yet, if Number be defin'd a Multitude of *Unites* join'd together, as many Authors define it, 'tis evident *Unity* is not a Number." [23, p. 323]

one	who / year	quote (or comment)
no	J. Kirkby 1735	“53. An *Even Number* is that which is meaſured by 2. 54. An *Odd Number* is one more than an even Number. 55. A *Prime* or *Incompoſite Number* is that which no Number meaſures but Unity, as 3, 5, 7, 11, 13, 17, 19.” [70, p. 7]
no	C. R. Reyneau 1739	“On remarquera ſur les nombres que leurs diviſeurs premiers ne ſont pas toujours de ſuite les nombres premiers 2, 3, 5, 7, 11, *&c.*” [115, p. 248]
yes	C. Goldbach 1742	Goldbach’s letter to Euler [50] (with what Euler would modify to the “Goldbach Conjecture”) uses 1 as a prime in sums such as: $4 = \begin{cases} 1+1+1+1 \\ 1+1+2 \\ 1+3, \end{cases} \quad 5 = \begin{cases} 2+3 \\ 1+1+3 \\ 1+1+1+2 \\ 1+1+1+1+1, \end{cases} \quad 6 = \begin{cases} 1+5 \\ 1+2+3 \\ 1+1+1+3 \\ 1+1+1+1+2 \\ 1+1+1+1+1+1. \end{cases}$
yes	G. S. Krüger 1746	Krüger’s list of primes [74, p. 839] (calculated by Peter Jaeger [40, footnote, p. 15]) starts with 1 and ends with 100,999.
yes	M. L. Willig 1759	Willig’s factor list (from 1 to 10,000) [141, p. 831] starts: **1** | Primz. **2** | Primz.
yes	N. de la Caille 1762	“Numerus, qui nullius alterius, quam unitatis, eſt multiplus, dicitur *numerus primus*. Horum numerorum amplæ tablæ apud varios ſcriptores extant; en eos, qui centenario ſunt inferiores: 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,” [18, p. 13].
no	L. Euler 1770	Euler writes [40, pp. 14–16]: “But, on the other hand, the numbers 2, 3, 5, 7, 11, 13, 17, &c. cannot be represented in the same manner by factors, unless for that purpose we make use of unity, and represent 2, for instance, by 1×2. But the numbers which are multiplied by 1 remaining the same, it is not proper to reckon unity as a factor. All numbers, therefore, such as 2, 3, 5, 7, 11, 13, 17, &c. which cannot be represented by factors, are called *simple*, or *prime numbers*; whereas others, as 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, &c. which may be represented by factors are called *composite numbers*.

one	who / year	quote (or comment)
		Simple or prime numbers deserve therefore particular attention, since they do not result from the multiplication of two or more numbers. It is also particularly worthy of observation, that if we write these numbers in succession as they follow each other, thus, $$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, \&c.$$ we can trace no regular order; there increments being sometimes greater, sometimes less; and hitherto no one has been able to discover whether they follow any certain law or not."
yes	J. H. Lambert 1770	Table VI, Numeri Primi, begins with 1, 2, 3, 5, 7, 11, . . . ; repeated in the Latin version (same table number and page) [75, p. 73].
no	S. Horsley 1772	"Hence it follows, that all the Prime numbers, except the number 2, are included in the ſeries of odd numbers, in their natural order, infinitely extended; that is, in the ſeries 3. 5. 7. 9. 11. 13. 15. [. . .]" [66, p. 332].
yes	A. Felkel 1776	Felkel's Table A [41] (at the front of his factor table) lists the primes from 1 to 20353.
yes	E. Waring 1782	Waring writes [134, p. 379] (translated to English [136, p. 362b]): "1. Omnis par numerus conſtat e duobus primis numeris, & omnis impair numerus vel eſt primus numerus, vel conſtat e tribus primis numeris, &c." (Every even number is the sum of two primes; every odd the sum of three.) "3. Haud dantur tres primi numeri in arithmeticâ progreſſione, quorum communis differentia haud diviſibilis ſit per numerum 6; ni 3 ſit primus terminus arithmeticae ſeriei, in quo caſu poſſunt eſſe tres & haud plures termini ejuſdem arithmeticæ ſeriei primi numeri, & quorum communis differentia haud diviſibilis ſit per 6: hic excipiantur duæ arithmeticæ ſeries 1, 2, 3 & 1, 3, 5, 7." (Here he is explaining there are only two arithmetic sequences of primes which do not have a common difference divisible by 6.) Also [135, p. 391] "[. . .] adding the prime numbers 1, 2, 3, 5, 7, 11, 13, 19, &c. [. . .]" [Where is 17?]
yes	A. G. Rosell 1785	"De este modo, 1, 2, 3, 5, 7, 11, 13, &c. son números primeros, y 4, 6, 8, 9, 10, &c. números compuestos." [117, p. 39]

one	who / year	quote (or comment)
yes	A. Bürja 1786	“Eine Primzahl oder einfache Zahl nennet man diejenige die durch keine andere, ſondern nur allein durch die Einheit und durch ſich ſelbſt gemeſſen wird. Z. E. 1, 2, 3, 5, 7, 11, 13, 17 ſind Primzahlen. Daß aber jede Zahl durch die Einheit und durch ſich ſelbſt gemeſſen wird, bedarf keines Beweiſes.” [17, p. 45]
no	F. Meinert 1789	“So ſind 2, 3, 5, 7, 11, 13 ꝛc. Primzahlen; 4, 6, 8, ꝛc. aber zuſammengeſetzte Zahlen.” [94, p. 69]
no	C. F. Gauss 1801	Gauss states and proves (for the first time) the uniqueness case of the fundamental theorem of arithmetic: “A composite number can be resolved into prime factors in only one way” [46]. Euler (1770) assumed and Legendre (1798) proved the existence part of this theorem [1]. (Preset (1689) used, and al-Fārisī (ca. 1320) may have also proved, the existence part of this theorem.) Gauss’ table had 168 primes below 1000 in [47, p. 436] (including 1 as prime would give 169).
yes	A. M. Chmel 1807	“Numerus integer praeter se ipſum et unitatem nullum alium diviſorem (menſuram) hacens, dicitur *ſimplex*, vel *numerus primus*, (Primzahl). Numerns autem talis, qui praeter ſe ipſum et 1, adhuc unum vel plures diviſores habet, vocatur *compoſitus*. *Coroll*. 1. Numeri *primi* ſunt: 1, 2, 3, 5, 7, 11, 13, 17, 19 etc. *Compoſiti*: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 etc.” [24, p. 65].
no	G. S. Klügel 1808	“**Primzahl**, einfache Zahl, (**numerus primus**) iſt eine ſolche, welche keine ganze Zahlen zu Factoren hat oder, welche nur von der Einheit allein gemeſſen wird, wie die Zahlen 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 u. ſ. f.” [71, p. 892].
no	P. Barlow 1811	Barlow writes [7, p. 54]: “[...] we have 2 3 5 7 ... 97, which are all the prime numbers under 100.” Also, in 1847: “A *prime number* is that which cannot be produced by the multiplication of any integral factors, or that cannot be divided into any equal integral parts greater than unity.” [8, p. 642].
yes	J. G. Garnier 1818	“*Lambert*, et tout récemment l’astronome *Burkardt* ont donné des tables très-étendues de nombres premiers qui servent à la décomposition d’un nombre en ses facteurs nombres premiers.” [45, p. 86] This is followed with a table of primes, starting at 1, extending to 500.

one	who / year	quote (or comment)
yes	O. Gregory 1825	Gregory's low level "Mathematics for practical men" states (English [54, pp. 44–45], German [55, pp. 40–42]): "1. A *unit*, or *unity*, is the representation of any thing considered individually, without regard to the parts of which it is composed. 2. An *integer* is either a unit or an assemblage of units: and a *fraction* is any part or parts of a unit. [...] 4. One number is said to *measure* another, when it divides it without leaving any remainder. [...] 8. A *prime number*, is that which can only be measured by 1, or unity." On the next page he lists the first twenty primes starting with 1.
yes	A. M. Legendre 1830	Presenting Euclid's argument there are infinitely many primes, he begins: "Car si la suite des nombres premiers 1.2.3.5.7.11, etc. était finie, et que p fût le dernier ou le plus grand de tous, [...]" [81, p. 14]. (See also the quote for de Mondesir in 1877.)
yes	F. Minsinger 1832	Minsinger's school book [101, pp. 36–37] lists the 169 primes below 1000: from 1 to 997.
no	M. Ohm 1834	"**Note: Die erstern Primzahlen sind der Reihe nach: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29,** [...]" [106, p. 140]. (Martin Ohm is a mathematician, the physicist's Georg Ohm's younger brother.)
no	A. Reynaud 1835	"*Un nombre est dit* PREMIER, *lorsqu'il n'est divisible que par lui-même et par l'unité.* [...] On trouve de cette manière que les nombres premiers sont, 2, 3, 5, 7, [...] 607, etc." [114, pp. 48–49].
no	F. Lieber et al. 1840	Encyclopædia Americana [84, p. 334] states: "PRIME NUMBERS are those which have no divisors, or which cannot be divided into any number of equal integral parts, less than the number of units of which they are composed; such as 2, 3, 5, 7, 11, 13, 17, &c."
no	R. C. Smith 1842	This low level school book states [125, p. 118]: "A Prime Number is one that is divisible only by itself or unity, as 2, 3, 5, 7, 11, 13, 17, &c."
no	J. Ozanam et al. 1844	Ozanam's *Récréations* (1694) was reworked by Jean-Etienne Montucla in 1778: "who so greatly enlarged and improved the 'Recreations' of Ozanam, that he may be said to have made the work his own" [108, p. vi]. The 3rd edition of Charles Hutton's English translation [108, p. 16] states:

one	who / year	quote (or comment)
		"A *prime* number is that which has no other divisor but unity." The table of primes from 1 to 10,000 (on the same page) starts at 2.
no	C. Beck 1845	"Tous les nombres premiers depuis 1 jusqu'à 1000 sont contenus dans le tableau suivant: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, [...] 997." [9, p. 57]
yes	J. B. Weigl 1848	Weigl's school book [138, p. 28] writes: "**Die ersten Primzahlen sind: 1, 2, 3, 5, 7, 11, 13, 15** [*sic*] **...**".
no?	J. Thomson 1849	"All whole numbers are either prime or composite; a *prime* number being that which is not produced by the multiplication of other integers, while a *composite* one is the product of two or more such factors. Thus 2, 3, 5, 7, 11, &c. are primes; while 4, 6, 8, 9, 10, &c. are composite." [129, p. 63]
yes	E. Hinkley 1853	This odd low-level text is based on the author's tables of primes and factorizations to 20,000; supplemented with Brancker's table from 20,000 to 100,000. The preface [65, p. 3] states: "This is the first book, made or published in the country, devoted exclusively to the subjects of *prime numbers* and *prime factors*." Then on page 7: "The numbers 1, 2 and 3, are evidently prime numbers."
no	P. L. Chebyshev 1854	Chebyshev's *Collected Works* [131, p. 51] reprints his *Mémoire sur les nombres premiers* from 1854 which states: "Ce sont les questions sur la valeur numérique des séries, dont les termes sont des fonctions des nombres premiers 2, 3, 5, 7, 11, 13, 17, etc."
no	C. J. Hargreave 1854	Glaisher, in his "Factor Table for the Fourth Million" (1879), discusses counts of primes by others [49, pp. 34–35] as follows. (For Hargreave, Glaisher cites [61, pp. 114–122].) "The results obtained by these writers do not agree. Thus in the case of 1,000,000 the number of primes is determined by Hargreave at 78,494, by Meissel at 78,498, and by Piarron de Mondesir at 78,490. The true number, excluding unity, as counted from the Tables is 78,498, agreeing with Meissel's result. Hargreave and Meissel exclude unity in their determinations, but M. de Mondesir includes it. [...] Legendre counted the number of primes in the first million as 78,493, which, as he included unity, is in error by 6 (see p. 30)."

one	who / year	quote (or comment)
yes	A. Comte 1854	The philosopher and non-mathematician J. S. Mill [99, p. 196] writes: "But M. Comte's puerile predilection for prime numbers almost passes belief. His reason is that they are the type of irreductibility: each of them is a kind of ultimate arithmetical fact. This, to any one who knows M. Comte in his later aspects, is amply sufficient. Nothing can exceed his delight in anything which says to the human mind, Thus far shalt thou go and no farther. If prime numbers are precious, doubly prime numbers are doubly so; meaning those which are not only themselves prime numbers, but the number which marks their place in the series of prime numbers is a prime number. Still greater is the dignity of trebly prime numbers; when the number marking the place of this second number is also prime. The number thirteen fulfils these conditions: it is a prime number, it is the seventh prime number, and seven is the fifth prime number. Accordingly he has an outrageous partiality to the number thirteen. Though one of the most inconvenient of all small numbers, he insists on introducing it everywhere." There is an example of this in Comte's *System of Positive Polity* [29, p. 420].
no	V.-A. Lebesgue 1856	"[...] on représentera la suite complète des nombres premiers par $p_0 = 2$, $p_1 = 3$, $p_2 = 5, \ldots, p_{i-1}, p_i, p_{i+1}$." [77, p. 130] Note that Victor-Amédée Lebesgue is a number theorist. He is unrelated to Henri Lebesgue (1875–1941) who worked with integration and measure theory.
yes	V.-A. Lebesgue 1859	"[...] les nombres premiers 1, 2, 3, 5, 7, 11, 13, [...]" [78, p. 5]
yes	V.-A. Lebesgue 1862	"On entend par *diviseur* d'un nombre n tout nombre qui s'y trouve contenu une ou plusiers fois exactement; quel que soit n, les nombres 1 et n en sont diviseurs. Le nombre n est *premier* lorsqu'il n'a que ces deux diviseurs; il est *composé* dans le cas contraire. Les nombres 1, 2, 3, 5, 7, 11, 13, 17, 19,... sont premiers;" [79, p. 10]
no	L. Dirichlet 1863	Dedekind compiled Dirichlet's lectures, *Vorlesugen über Zahlentheorie* [34, p. 12], a few years after Dedekind died. He wrote:

one	who / year	quote (or comment)
		“Da jede Zahl sowohl durch die Einheit, als auch durch sich selbst theilbar ist, so hat jede Zahl – die Einheit selbst ausgenommen – mindestens zwei (positive) Divisoren. Jede Zahl nun, welche keine anderen als diese beiden Divisoren besitzt, heisst eine *Primzahl (numerus primus)*; es ist zweckmässig, die Einheit nicht zu den Primzahlen zu rechnen, weil manche Sätze über Primzahlen nicht für die Zahl 1 gültig bleiben.” This last part is “It is convenient not to include unity among the primes, because many theorems about prime numbers do not hold for the number 1” [36, p. 8]. The parenthetical “(positive)” was not in the 1863 edition, but added by the 1879 edition [35, p. 12]. Nice quote: “Thus in a certain sense the prime numbers are the material from which all other numbers may be built” [36, p. 9].
yes	J. Bertrand 1863	Joseph Bertrand defines primes (definition 109, [11, p. 86]) by: “Un nombre entier est dit premier lorsqu’il n’a pas d’autres diviseurs entiers que lui-même et l’unité. Exemples. 2, 3, 5, 7, sont des nombres premiers, 9 n’est pas premier, car il est divisible par 3.” Despite this example, Table I is titled “Contenant tous les nombres premiers depuis 1 jusq’à 9907”, and starts, just like it says, at 1 (p. 342).
no	V.-A. Lebesgue 1864	The “TABLEAU des nombres premiers impairs, inférieurs à 5500” lists 24 odd prime less than 100, starting at 3 [80, p. 12].
yes	J. Ray 1866	Joseph Ray’s elementary algebra text states [113, p. 50]: “A **Prime Number** is one which has no divisor except itself and unity. A **Composite Number** is one which has one or more divisors besides itself and unity. Hence, All numbers are either prime or composite; and every composite number is the product of two or more prime numbers. The prime numbers are 1, 2, 3, 5, 7, 11, 13, 17, etc.”
—	C. Aschenborn 1867	Aschenborn’s arithmetic text for artillery and engineering school [4, p. 86] explains how to determine the least common multiple of two numbers and then states: “Man thut dies nach der Reihe mit den Primzahlen 2, 3, 5, ... , bis je 2 der gegebenen, resp. verkleinerten Zahlen relative Primzahlen sind.”

one	who / year	quote (or comment)
no	E. Meissel 1870	"Es sei $p_1 = 2$; $p_2 = 3$; $p_3 = 5$; … p_n die n^{te} Primzahl; […]" [95, p. 636].
yes	A. J. Manchester 1870	A lesson [87, p. 131] in a periodical for Rhode Island school teachers instructs: "A *prime* number can be divided by no whole number, except 1 (one) and itself without a remainder. 2, 5, 17, 29, 47, 13 are prime numbers. A composite number can be divided by some whole number besides itself, and 1 (one) without a remainder. 10, 21, 49, 51, 87, 39, 46 are composite numbers. *Teacher.* Name all of the prime numbers from 1 to 50. *Pupil.* 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47." And on page 133: "*Teacher.* Name the composite numbers from 1 to 150 that, at first sight, seem to be prime. *Pupil.* 39, 51, 57, 67, 89, 91, 93, 111, 117, 119, 123, 129, 133, 141, 143, 147." [Surely "67, 89" was meant to be "69, 87."]
yes	E. Brooks 1873	Edward Brooks, in his schoolbook [14, p. 58] which is mostly questions and few answers, wrote: "Numbers which can be produced by multiplying together other numbers, each of which is greater than a unit, are called *composite numbers.* Numbers which cannot be produced by multiplying together two or more numbers, each of which is greater than a unit, are called *prime numbers.*"
no	G. M'Arthur 1875	In Encyclopædia Britannica, 9th ed., the entry for Arithmetic [89, p. 528] states: "A *prime number* is a number which no other, except unity, divides without a remainder; as 2, 3, 5, 7, 11, 13, 17, &c." Later an example: "The *prime factors* of a number are the prime numbers of which it is the continued product. Thus, 2, 3, 7 are the prime factors of 42; 2, 2, 3, 5, of 60."
yes	J. Glaisher 1876	"M. Glaisher, en comptant 1 et 2 comme premiers, a trouvé les valeurs suivantes: […]" [85, p. 232]. This is "Mister Glaisher, by counting 1 and 2 as first, has found the following values: […]". Also, in an appendix of his "Factor table for the Fourth Million" (1879), Glaisher gives a list [49, p. 48] of primes from 1 to 30,341; a list which literally begins with unity.

one	who / year	quote (or comment)
no	K. Weierstrass 1876	"Dies führt zu dem Begriff der Primzahlen. Nimmt man die Primzahlen sämmtlich als positiv an, so kann man jede Zahl als Product von Primzahlen und einer Einheit $+1$ oder -1 darstellen, und zwar auf eine einzige Weise. Der Begriff der Primzahlen kann im Gebiete der complexen ganzen Zahlen, die aus den vier Einheiten 1, -1, i, $-i$ durch Addition zusammengesetzt sind, aufrecht erhalten werden. Denn jede Zahl $a+bi$ lässt sich auf eine einzige Weise durch ein Product von primären Primzahlen und einer der vier Einheiten ausdrücken." [137, p. 391]
yes	P. de Mondesir 1877	Glaisher, in his "Factor Table for the Fourth Million" (1879), discusses counts of primes by others [49, pp. 34–35] as follows. (For Piarron de Mondesir, Glaisher cites [102].) "The results obtained by these writers do not agree. Thus in the case of 1,000,000 the number of primes is determined by Hargreave at 78,494, by Meissel at 78,498, and by Piarron de Mondesir at 78,490. The true number, excluding unity, as counted from the Tables is 78,498, agreeing with Meissel's result. Hargreave and Meissel exclude unity in their determinations, but M. de Mondesir includes it. [...] Legendre counted the number of primes in the first million as 78,493, which, as he included unity, is in error by 6 (see p. 30). M. de Mondesir finds the number of primes inferior to 100,000 (including unity) to be 9,593, and remarks that Legendre gave 9,592. The former value is the correct one."
no	H. Scheffler 1880	"Hiernach sind die reellen Primzahlen 2, 3, 5, 7 ..., welche früher dafür gehalten wurden, sämmtlich gemeine reelle Primzahlen [...]" [119, p. 79].
no	G. Wertheim 1887	"Wir wollen die Anzahl der Zahlen des Gebiets von 1 bis n, welche durch keine der i ersten Primzahlen $p_1 = 2, p_2 = 3, p_3 = 5, \ldots, p_i$ theilbar sind, durch $\varphi(n,i)$ bezeichnen." [140, p. 20]
no	P. L. Chebyshev 1889	"*Einfach* heisst eine Zahl, welche nur durch *Eins* und durch sich selbst theilbar ist; eine solche wird auch *Primzahl* genannt. *Eine zusammengesetzte Zahl* nennt man dagegen eine solche, welche durch eine andere Zahl, die grösser als *Eins* ist, ohne Rest getheilt werden kann. So sind 2, 3, 5, 7, 11, und viele andere *Primzahlen*, hingegen 4, 6, 8, 9, 10 und andere dergleichen *zusammengesetzte Zahlen*." [130, pp. 2–3]

one	who / year	quote (or comment)
yes	A. Cayley 1890	Encyclopædia Britannica, 9th ed., entry for number [22, p. 615]: "In the ordinary theory we have, in the first instance, positive integer numbers, the unit or unity 1, and the other numbers 2, 3, 4, 5, &c. [...]." "A number such as 2, 3, 5, 7, 11, &c., which is not a product of numbers, is said to be a prime number; and a number which is not prime is said to be composite. A number other than zero is thus either prime or composite; [...]." "Some of these, 1, 2, 3, 5, 7, &c. are prime, others, $4, = 2^2, 6, = 2.3$, &c., are composite; and we have the fundamental theorem that a composite number is expressible, and that in one way only, as a product of prime factors, $N = a^\alpha b^\beta c^\gamma \ldots (a, b, c, \ldots$ primes other than 1; $\alpha, \beta, \gamma, \ldots$ positive integers)."
no	E. Lucas 1891	"Il y a donc deux espèces d'entiers positifs, les nombres premiers et les nombres composés; mais on doit observer que l'unité ne rentre dans aucune de ces deux espèces et, dans la plupart des cas, il ne convient pas de considérer l'unité comme un nombre premier, parce que les propriétés des nombres premiers ne s'appliquent pas toujours au nombre 1." In a footnote he gives the example "Ainsi le nombre 1 est premier à lui-même, tandis qu'un nombre premier p n'est pas premier à lui-même; [...]" [86, p. 350].
yes	W. Milne 1892	This low-level school book is mostly questions, few answers. "Thus 1, 3, 5, 7, 11, 13, etc., are prime numbers." [100, p. 92]. On page 95, 1 is not listed as a prime factor of 1008.
yes	R. Fricke 1892	"Man bezeichne nun die Primzahlen 1, 2, 3, 5, ..." [42, p. 592].
no	J. P. Gram 1893	Gram reports $\pi(100000) = 9592$, which is true if 1 is omitted from the primes. (He uses θ instead of π.) [52, p. 312].
no	P. Bachmann 1894	"Denkt man sich sodann alle Primzahlen bis zu einer bestimmten Primzahl p hin, 2, 3, 5, 7, $\ldots p_0$, p, so sei [...]" [5, p. 135].
yes	R. Fricke & F. Klein 1897	"Die der Primzahl x voraufgehenden Primzahlen seien $1, 2, 3, 5, \ldots, \lambda$, so dass l ein Multiplum des Productes $2 \cdot 3 \cdot 5 \cdots \lambda$ ist." [43, p. 609]
yes	L. Kronecker 1901	"[...] daß die 16 Primzahlen 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 kleiner sind als 50." [73, p. 303]

one	who / year	quote (or comment)
yes	G. Chrystal 1904	"It is also obvious that every integer (other than unity) has at least two divisors, namely, unity and itself; if it has more, it is called a *composite integer*, if it has no more, a *prime integer*. For example, 1, 2, 3, 5, 7, 11, 13, ... are all prime integers, whereas 4, 6, 8, 9, 10, 12, 14 are composite." [25, p. 38]
yes	G. H. Hardy 1908+	Hardy's first-year university textbook [56] states that 1 is prime in at least two places. First, while discussing Euclid's proof that there are infinitely many primes, Hardy notes [56, p. 122]: "If there are only a finite number of primes let them be 1, 2, 3, 5, 7, 11, ... N." This was unchanged for the first six editions of his text 1908, 1914, 1921 [57, pp. 143–4], 1925, 1928 and 1933. (See the Hardy 1938 entry.) Next, he writes [56, p. 147]: "The decimal .111 010 100 010 10 ..., in which the nth figure is 1 if n is prime, and zero otherwise, represents an irrational number." This example remained the same in all 10 editions (e.g., [57, p. 174], and "the revised 10th edition" 2008 [60, p. 151]). He also has the ambiguous statement ([56, p. 48], [57, p. 56]): "Let y be defined as *the largest prime factor of* x (cf. Exs. VII. 6). Then y is defined only for integral values of x. When $x = \pm\ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, \ldots$ $y = \quad 1, 2, 3, 2, 5, 3, 7, 2, 3, \ \ 5, 11, \ \ 3, 13, \ldots$ The graph consists of a number of isolated points." This is essentially unchanged in the revised 10th edition [60, p. 151]; but whether or not 1 is considered prime, it is reasonable to accept 1 as the largest prime factor of 1. (Certainly 1 is the largest prime power dividing 1.)
no	E. Landau 1909	"Unter einer Primzahl versteht man eine positive ganze Zahl, welche von 1 verschieden und nur durch 1 und durch sich selbst teilbar ist. Die Reihe der Primzahlen beginnt mit 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, ..." [76, p. 3].

one	who / year	quote (or comment)
no	W. F. Sheppard 1910	Encyclopædia Britannica's (11th ed.) entry for 'arithmetic' [122, p. 531] states: "A number (other than 1) which has no factor except itself is called a *prime number*, or, more briefly, a *prime*. Thus 2, 3, 5, 7 and 11 are primes, for each of these occurs twice only in the table. A number (other than 1) which is not a prime number is called a *composite* number." "The number 1 is usually included amongst the primes; but, if this is done, the last paragraph [talking about the fundamental theorem of arithmetic] requires modification, since 144 could be expressed as $1.\ 2^4.\ 3^2$, or as $1^2.\ 2^4.\ 3^2$, or as $1^p.\ 2^4.\ 3^2$, where p might be anything."
no	G. B. Mathews 1910	Encyclopædia Britannica's (11th ed.) entry for 'number' [93, p. 851] reads: "The first noteworthy classification of the natural numbers is into those which are prime and those which are composite. A prime number is one which is not exactly divisible by any number except itself and 1; all others are composite." That definition is ambiguous, but later on the same page to he clearly is excluding unity from the primes: "Every number may be uniquely expressed as a product of prime factors. Hence if $n = p^\alpha q^\beta r^\gamma \ldots$ is the representation of any number n as the product of powers of different primes, the divisors of n are the terms of the product $(1+p+p^2+\ldots+p^\alpha)(1+q+\ldots+q^\beta)(1+r+\ldots+r^\gamma)\ldots$ their number is $(\alpha+1)(\beta+1)(\gamma+1)\ldots$, and their sum is $\Pi(p^{\alpha+1}-1)\div\Pi(p-1)$." The same article [93, p. 863] later states: "Similar difficulties are encountered when we examine Mersenne's numbers, which are those of the form 2^p-1, with p a prime; the known cases for which a Mersenne number is prime correspond to $p = 2, 3, 5, 7, 13, 17, 19, 31, 61$." If 1 was prime, then so would be 2^1-1.
no	H. v. Mangoldt 1912	"Ein anderes Beispiel ist die Reihe $2;\ 3;\ 5;\ 7;\ 11;\ \cdots$ der Primzahlen." [88, p. 176]

one	who / year	quote (or comment)
yes	D. N. Lehmer 1914	Lehmer begins the introduction to his *List of Prime Numbers From* 1 *to* 10,006,721 as follows [82]: "A prime number is defined as one that is exactly divisible by no other number than itself and unity. The number 1 itself is to be considered as a prime according to this definition and has been listed as such in the table. Some mathematicians [a footnote here cites E. Landau [76]], however, prefer to exclude unity from the list of primes, thus obtaining a slight simplification in the statement of certain theorems. The same reasons would apply to exclude the number 2, which is the only even prime, and which appears as an exception in the statement of many theorems also. The number 1 is certainly not composite in the same sense as the number 6, and if it is ruled out of the list of primes it is necessary to create a particular class for this number alone."
no	E. Hecke 1923	"Wenn es außer der trivialen Zerfällung in ganzzahlige Faktoren, bei der ein Faktor ± 1 und der andere $\pm a$ ist, keine andere gibt, so nennen wir a eine **Primzahl**. Solche gibt es, wie $\pm 2, \pm 3, \pm 5 \ldots$. Die Einheiten ± 1 wollen wir nicht zu den Primzahl rechnen." [64, p. 5]
no	G. H. Hardy 1929	"More amusing examples are (c) $0.01101010001010\cdots$ (in which the 1's have prime rank) and (d) $0.23571113171923\cdots$ (formed by writing down the prime numbers in order)." [58, p. 784] Example (c) is in all the editions of his *A Course of Pure Mathematics* where he included 1 as prime (so there it starts 0.111). This was never corrected in that text (see the Hardy 1908 entry). Here he begins Euclid's proof that there are infinitely primes as follows [58, p. 802]: "If the theorem is false, we may denote the primes by $2, 3, 5, \cdots, P$, and all numbers are divisible by one of these."
no	G. H. Hardy 1938	In the seventh edition of his text *A Course of Pure Mathematics*, Hardy starts Euclid's proof of the infinitude of primes as follows [59, p. 125]: "Let $2, 3, 5, \ldots, p_N$ be all the primes up to p_N, ..." This is a change from the the first six editions where unity was prime (see the Hardy 1908 entry). This new wording was used from the 7th edition (1938) through the revised 10th edition (2008).
yes	M. Kraitchik 1942	Kraitchik's recreational mathematics text [72, p. 78] says "For example, there are 26 prime numbers between 0 and 100, only 21 between 100 and 200, and no more than 4 between 10^{12} and $10^{12} + 100$."

one	who / year	quote (or comment)
no	B. L. van der Waerden 1949	"An element $p \neq 0$ which admits only trivial factorizations of the kind $p = ab$, where a or b is a unit, is called an *indecomposable element* or a *prime element*. (In the case of integers we say: *prime number*;* in the case of polynomials: *irreducible polynomial*.)" The footnote on 'prime number' is: "By prime numbers we usually understand only the positive prime numbers $\neq 1$, such as 2, 3, 5, 7, 11, ..." [132, p. 59].
yes	A. H. Beiler 1964	Beiler, a well-known expositor, wrote about the "ubiquitous primes" [10, p. 211]: "From the humble 2, the only even prime, and 1, the smallest of the odd primes, they rise in an unending succession aloof and irrefrangible." [See also pp. 212–13, 223.]
no	J. Shallit 1975	Jeffrey Shallit [121], as a student, wrote an interesting note about the prime factorization of one suggesting that its prime factorization should be regarded as the empty list.
yes	C. Sagan 1997	The aliens in Carl Sagan's novel *Contact* [118, p. 76] transmit the first 261 primes starting with one: 1, 2, 3, 5, 7, ..., to signal their existence.
yes	M. Weik 2000	The *Computer Science and Communications Dictionary* states [139, p. 1326]: "**prime number:** A whole number that has no whole number divisors except 1 and itself, i.e., that when divided by a whole number other than 1 and itself will always produce a mixed number, i.e., a whole number and a fraction. *Note:* The first few prime numbers are 1, 2, 3, 5, 7, 11, 13, [*sic*] 19, and 23. Even numbers, except 2, products of two or more whole numbers, 0, mixed numbers, and repeating numbers such as 7777 ... or 3333 ..., are not prime numbers." [Isn't 11 a repeating number?]
yes	J. B. Andreasen et al. 2010	The CliffsNotes preparation guide [2, p. 342] for an Elementary Education (K–6) teacher certification test gives the following definition: "**prime number:** A number with exactly two whole number factors (1 and the number itself). The first few prime numbers are 1, 2, 3, 5, 7, 11, 13, and 17." [Hopefully this is a typographical error as unity does not have "*exactly two* whole number factors."]

one	who / year	quote (or comment)
yes	Carnegie Library of Pittsburgh 2011	The *Handy Science Answer Book* [6, p. 13] states: "A prime number is one that is evenly divisible only by itself and 1. The integers 1, 2, 3, 5, 7, 11, 13, 17, and 19 are prime numbers. [...] the largest known (and fortieth) prime number [*sic*]: $2^{20996011} - 1$. [...] Mersenne primes occur where 2^{n-1} [*sic*] is prime."

3 Acknowledgements

We would like to thank Professor David Broadhurst of Open University, UK, for help in finding some of these sources.

References

[1] A. G. Ağargün and E. M. Özkan, A historical survey of the fundamental theorem of arithmetic, *Historia Math.* **28** (2001), 207–214. Available from http://www.sciencedirect.com/science/journal/03150860/28/3.

[2] J. B. Andreasen, L.-A. T. Spalding, and E. Ortiz, *FTCE: Elementary Education K-6*, Cliffs Notes, Wiley, Hoboken, NJ, 2010.

[3] A. Arnauld, *Nouveaux Elemens de Geometrie*, Guillaume Desprez, Paris, 2nd edition, 1683. Available from http://books.google.com/books?id=0WMVAAAAQAAJ.

[4] K. H. M. Aschenborn, *Lehrbuch der Arithmetik mit Einschluß der Algebra und der niederen Analysis*, Verlag der Königlichen Geheimen Ober-Hofbuchdruckerei (R. v. Decker), Berlin, 2nd edition, 1867. Available from http://books.google.com/books?id=3Rw7AQAAIAAJ.

[5] P. Bachmann, *Zahlentheorie: Versuch einer Gesammtdarstellung dieser Wissenschaft in ihren Haupttheilen,* Theil II: *Die analytische Zahlentheorie*, B. G. Teubner, Leipzig, 1894. Available from http://archive.org/details/dieanalytischeza00bachuoft.

[6] N. E. Balaban and J. E. Bobick, editors, *The Handy Science Answer Book,* compiled by the Science and Technology Department of the Carnegie Library of Pittsburgh, Visible Ink Press, 4th edition, 2011.

[7] P. Barlow, *An Elementary Investigation of the Theory of Numbers*, J. Johnson, London, 1811. Available from http://ebooks.library.cornell.edu/cgi/t/text/text-idx?c=math;idno=05210001.

[8] P. Barlow, *The Encyclopædia of Pure Mathematics,* Section V: *Theory of Numbers*, J. J. Griffin, London, 1847. Available from http://books.google.com/books?id=3fIUAQAAMAAJ.

[9] C. Beck, *Traité d'arithmétique,* 1re partie*: Nombres abstraits*, Librairie Polytechnique d'A. Decq, Bruxelles, 1845. Available from http://books.google.com/books?id=1wOOAAAAQAAJ.

[10] A. H. Beiler, *Recreations in the Theory of Numbers*, Dover, New York, 1964.

[11] J. Bertrand, *Traité d'arithmétique*, Hachette, Paris, 3rd edition, 1863. Available from http://books.google.com/books?id=mf9BAAAAcAAJ.

[12] M. Bettini, *Apiaria universæ philosophicæ mathematicæ, in quibus paradoxa,* Vol. II: *Apiarium undecimum, in quo arithmetica paradoxa et arcana*, Jo. Baptiste Ferroni, Bononia (Bologna), 1642. Available from http://books.google.com/books?id=iGBDAAAAcAAJ.

[13] B. Boncompagni, editor, *Scritti di Leonardo Pisano,* Vol. I: *Liber abbaci*, Tipografia delle Scienze Matematiche e Fisiche, Roma, 1857. Available from http://books.google.com/books?id=gvRFAAAAcAAJ.

[14] E. Brooks, *The New Normal Mental Arithmetic: A Thorough and Complete Course, by Analysis and Induction*, Christopher Sower, Philadelphia, 1873. Available from http://books.google.com/books?id=VRGwHNTbcu8C.

[15] F. Brunot, *Traité de la science des nombres, divisé en deux livres, Où l'on donne des principes d'arithmetique et d'algebre*, Quillau & Desaint, Paris, 1723. Available from http://books.google.co.uk/books?id=TK02AAAAMAAJ.

[16] M. Bullynck, Factor tables 1657–1817, with notes on the birth of number theory, *Rev. Histoire Math.* **16** (2010), 133–216. Preprint available from http://www.kuttaka.org/FactorTables.pdf.

[17] A. Bürja, *Der selbstlernende Algebrist, oder deutliche Anweisung zur ganzen Rechenkunst,* Theil I, Lagarde und Friedrich, Berlin & Libau, 1786. Available from http://books.google.co.uk/books?id=8a02AAAAMAAJ.

[18] N. L. de la Caille, *Lectiones elementares mathematicæ, seu Elementa algebræ, et geometriæ,* translated by K. Scherffer, printed by J. T. Trattner, Vienna, Praga & Tergeste (Trieste), 1762. Available from http://books.google.co.uk/books?id=MrhJAAAAMAAJ.

[19] C. Caldwell and Y. Xiong, What is the smallest prime? In preparation, 2012.

[20] Cassiodorus Senator, *An Introduction to Divine and Human Readings,* translated with an introduction and notes by L. W. Jones, Octagon Books, New York, 1966, reprinted from the 1946 Columbia University Press version.

[21] P. A. Cataldi, *Trattato de' numeri perfetti*, Heredi di Giovanni Rossi, Bologna, 1603. Available from http://mathematica.sns.it/media/volumi/69/.

[22] A. Cayley, Numbers, in *The Encyclopædia Britannica: A Dictionary of Arts, Sciences, and General Literature,* edited by T. S. Baynes, Vol. 17, pp. 614–624, The Henry G. Allen Company, New York, 9th edition, 1890. Available at http://books.google.com/books?id=mtFGAQAAIAAJ.

[23] E. Chambers, *Cyclopædia: or, An Universal Dictionary of Arts and Sciences; containing the Definitions of the Terms, and Accounts of the Things signify'd thereby,* Vol. I, printed for J. and J. Knapton et al., London, 1728. Available from http://digital.library.wisc.edu/1711.dl/HistSciTech.Cyclopaedia01.

[24] A. M. Chmel, *Institutiones mathematicae,* Vol. I: *Prolegomena matheseos universae et arithmeticam tam elementarem, quam universalem seu algebram*, C. Haslinger, Lincia (Linz), 1807. Available from http://books.google.co.uk/books?id=_tA2AAAAMAAJ.

[25] G. Chrystal, *Algebra: An Elementary Text-Book,* Part I, Adam and Charles Black, London, 5th edition, 1904. Available fom http://djm.cc/library/Algebra_Elementary_Text-Book_Part_I_Chrystal_edited.pdf.

[26] P. Ciruelo, *Cursus quatuor Mathematicarum Artium Liberalium*, Miguel de Eguía, Alcalá de Henares, 1526. Available from http://books.google.com/books?id=ivzSmHbr_40C.

[27] C. Clavius, *Opera mathematica,* Vol. I, Antonius Hierat, Moguntia (Mainz), 1611. Available from http://books.google.com/books?id=lGBDAAAAcAAJ.

[28] G. Clerke, *Oughtredus explicatus, sive Commentarius in ejus clavem mathematicam*, printed for Richard Davis, London, 1682. Available from http://books.google.com/books?id=IgQOAAAAQAAJ.

[29] A. Comte, *System of Positive Polity,* Vol. 4: *Theory of the Future of Man,* translated by R. Congreve, Longmans, Green and Co., 1877. Available from http://books.google.com/books?id=A3e8_x9I90MC.

[30] T. Corneille, *Dictionnaire universel des termes des arts et des sciences,* Vol. II, Veuve de Jean Baptiste Coignard, Paris, 1694. Available from http://books.google.com/books?id=EIg7MNSeVyIC,.

[31] J. Cortès, *Arithmetica practica muy util, y necessaria para todo genero de Tratantes y Mercaderes*, Herederos de Diego de Larumbe, Zaragoza, 1724. Available from http://books.google.co.uk/books?id=4Y_4r-X8zhMC.

[32] C. De Waard and A. Beaulieu, editors, *Correspondance du P. Marin Mersenne, religieux minime,* Vol. XV: 1647, Editions du Centre National de la Recherche Scientifique, Paris, 1983.

[33] C. F. M. Dechales, *Cursus seu mundus mathematicus,* Vol. I, Anissonios, J. Posuel & C. Rigaud, Lugdunum (Lyon), 1690. Available from http://books.google.com/books?id=83pnrQrhB_IC.

[34] P. G. L. Dirichlet, *Vorlesungen über Zahlentheorie,* edited and supplemented by R. Dedekind, Friedrich Vieweg & Sohn, Braunschweig, 1863. Available from http://books.google.com/books?id=PycPAAAAQAAJ.

[35] P. G. L. Dirichlet, *Vorlesungen über Zahlentheorie,* edited and supplemented by R. Dedekind, Friedrich Vieweg & Sohn, Braunschweig, 3rd edition, 1879. Available from http://books.google.com/books?id=npsAAAAAMAAJ.

[36] P. G. L. Dirichlet, *Lectures on Number Theory,* with supplements by R. Dedekind, translated by J. Stillwell, American Mathematical Society, 1999. Partially available from http://books.google.com/books?id=8h8aWmnp1k8C&printsec=.

[37] Euclid, *Les quinze livres des elements d'Euclide,* translated from Latin to French by D. Henrion, Jean Anthoine Joallin, Paris, 1615. Available from http://books.google.com/books?id=CSuKSyi39okC.

[38] Euclid, *Les quinze livres des elements d'Euclide,* Vol. I, translated and annotated by D. Henrion, Jean Lucas, Rouen, 1676. Available from by http://books.google.com/books?id=Rm0-AAAAcAAJ.

[39] Euclid, *The Thirteen Books of the Elements,* Vol. 2: *Books* III–IX*,* translated and introduced by T. L. Heath, Dover, New York, 1956.

[40] L. Euler, *Elements of Algebra, Translated From the French; With the Additions of La Grange and the Notes of the French Translator,* Vol. I: *Containing the Analysis of Determinate Quantities*, J. Johnson & Co., London, 2nd edition, 1810.

[41] A. Felkel, *Tafel aller einfachen Factoren der durch* 2*,* 3*,* 5 *nicht theilbaren Zahlen von* 1 *bis* 10 000 000*,* Theil I: *Enthaltend die Factoren von* 1 *bis* 144000, von Ghelen, Wien, 1776. Available from http://www.europeana.eu/portal/record/09428/E1B1958C0CE15249884EE1D17D5F510E996B9E33.html.

[42] R. Fricke, Zur gruppentheoretischen Grundlegung der automorphen Functionen, *Math. Ann.* **42** (1893), 564–594. Available from http://gdz.sub.uni-goettingen.de/dms/load/img/?PPN=PPN235181684_0042.

[43] R. Fricke and F. Klein, *Vorlesungen über die Theorie der automorphen Functionen,* Vol. I: *Die gruppentheoretischen Grundlagen*, B. G. Teubner, Leipzig, 1897. Available from http://books.google.com/books?id=H5kLAAAAYAAJ.

[44] M. Friedländer, Review of Abraham Ibn Ezra's arithmetic, *Jewish Quarterly Review* **8** (1896), 657–662. Available from http://www.jstor.org/stable/1450203.

[45] J.-G. Garnier, *Traité d'arithmétique*, J.-N. Houdin, Gand (Gent), 4th edition, 1818. Available from http://books.google.co.uk/books?id=PAYOAAAAQAAJ.

[46] C. F. Gauss, *Disquisitiones arithmeticae*, V. Fleischer, Leipzig, 1801. English translation by A. A. Clarke in 1966 for Yale University Press, revised by W. C. Waterhouse in 1986 for Springer-Verlag.

[47] C. F. Gauss, *Werke,* Vol. II, edited by E. Schering, Königliche Gesellschaft der Wissenschaften, Göttingen, 1863. Available from http://books.google.com/books?id=fDMAAAAAQAAJ.

[48] V. Giordano, *Euclide restituto overo Gli antichi elementi geometrici ristaurati e facilitati*, Angelo Bernabò, Roma, 1680. Available from http://books.google.com/books?id=u1QxAQAAMAAJ.

[49] J. Glaisher, *Factor Table for the Fourth Million, Containing the Least Factor of Every Number Not Divisible by* 2, 3, *or* 5 *between* 3,000,000 *and* 4,000,000, Taylor and Francis, London, 1879.

[50] C. Goldbach, Letter to Euler dated 7 June 1742, in *Correspondance mathématique et physique de quelques célèbres géomètres du* XVIII*ème siècle,* edited by P.-H. Fuss, Vol. I, pp. 125–129, Académie Impériale des Sciences, St.-Pétersbourg, 1843. Available from http://eulerarchive.maa.org/correspondence/letters/OO0765.pdf, image of original letter at http://upload.wikimedia.org/wikipedia/commons/1/18/Letter_Goldbaxh-Euler.jpg.

[51] C. Goldstein, On a seventeenth century version of the "fundamental theorem of arithmetic", *Historia Math.* **19** (1992), 177–187. Available from http://www.sciencedirect.com/science/journal/03150860/19/2.

[52] J.-P. Gram, Rapport sur quelques calculs entrepris par M. Bertelsen et concernant les nombres premiers, *Acta Math.* **17** (1893), 301–314. Available from http://archive.org/details/actamathematica21lefgoog.

[53] E. Grant, *A Source Book in Medieval Science*, Harvard University Press, 1974.

[54] O. Gregory, *Mathematics for Practical Men: Being a Common-Place Book of Principles, Theorems, Rules, and Tables in Various Departments of Pure and Mixed Mathematics*, Baldwin, Cradock, & Joy, London, 1825. Available from http://books.google.co.uk/books?id=1o8-AAAAYAAJ.

[55] O. Gregory, *Mathematik für Praktiker, oder Sammlung von Grund- und Lehrsätzen, Regeln und Tafeln aus den verschiedenen Theilen der reinen und angewandten Mathematik,* translated by M. W. Drobisch, Baumgärtner's Buchhandlung, Leipzig, 1828. Available from http://books.google.co.uk/books?id=g5lEAAAAcAAJ.

[56] G. H. Hardy, *A Course of Pure Mathematics*, Cambridge University Press, 1st edition, 1908. Available from http://quod.lib.umich.edu/cgi/t/text/text-idx?c=umhistmath;idno=ACM1516.0001.001.

[57] G. H. Hardy, *A Course of Pure Mathematics*, Cambridge University Press, 3rd edition, 1921. Available from http://www.gutenberg.org/files/38769/38769-pdf.pdf.

[58] G. H. Hardy, An introduction to number theory, *Bull. Amer. Math. Soc.* **35** (1929), 778–818. Available from http://projecteuclid.org/DPubS?service=UI&version=1.0&verb=Display&handle=euclid.bams/1183493592.

[59] G. H. Hardy, *A Course of Pure Mathematics*, Cambridge University Press, 7th edition, 1938.

[60] G. H. Hardy, *A Course of Pure Mathematics*, Cambridge University Press, 10th edition reissued with a foreword by T. W. Körner (Centenary edition), 2008.

[61] C. J. Hargreave, On the law of prime numbers, *Philosophical Magazine* **4**(viii) (1854), 114–122.

[62] J. Harris, *Lexicon Technicum: Or, An Universal English Dictionary of Arts and Sciences: Explaining Not Only the Terms of Art, But the Arts Themselves,* Vol. II, printed for D. Brown et al., London, 2nd edition, 1723. Available from http://books.google.com/books?id=BWFEAAAAcAAJ&vq.

[63] T. L. Heath, *A History of Greek Mathematics,* Vol. I*: From Thales to Euclid*, Dover, New York, 1981.

[64] E. Hecke, *Vorlesungen über die Theorie der algebraischen Zahlen*, Akademische Verlagsgesellschaft, Leipzig, 1923, 2nd edition by Chelsea, New York, 1970, reprinted by the American Mathematical Society, 2000. Available from http://books.google.com/books?id=MteTdxHj2ekC.

[65] E. Hinkley, *Tables of the Prime Numbers, and Prime Factors of the Composite Numbers, from* 1 *to* 100,000*; With the Methods of Their Construction, and Examples of Their Use*, printed for the author, Baltimore, 1853. Available from http://openlibrary.org/books/OL24168026M.

[66] S. Horsley, *Κόσκινον Ερατοσθένους*, or, The Sieve of Eratosthenes. Being an account of his method of finding all the Prime Numbers, *Philos. Trans. R. Soc. Lond.* **62** (1772), 327–347. Available from http://www.jstor.org/stable/106053.

[67] A. L. Ivry, *Al-Kindī's Metaphysics: A Translation of Ya'qub ibn Isaḥāq al-Kindī's Treatise "On First Philosophy" (fī al-Falsafah al-Ūlā)*, Studies in Islamic Philosophy and Science, State University of New York Press, 1974. Available from http://www.muslimphilosophy.com/books/kindi-met.pdf.

[68] L. L. Jackson, *The Educational Significance of Sixteenth Century Arithmetic From the Point of View of the Present Time,* Vol. 8 of *Contributions to Education*, Teachers College, Columbia University, 1906. Available from http://books.google.com/books?id=65YVAAAAIAAJ.

[69] G. Johnson, *The Arithmetical Philosophy of Nicomachus of Gerasa*, The New Era Printing Company, Lancaster, PA, 1916. Available from http://books.google.com/books?id=FIpNAAAAMAAJ.

[70] J. Kirkby, *Arithmetical Institutions Containing a Compleat System of Arithmetic: Natural, Logarithmical, and Algebraical in All Their Branches*, printed for B. Motte, C. Bathurst, and J. Clark, London, 1735. Available from http://books.google.com/books?id=WyIOAAAAQAAJ.

[71] G. S. Klügel, *Mathematisches Wörterbuch oder Erklärung der Begriffe, Lehrsätze, Aufgaben und Methoden der Mathematik,* Abtheilung I: *Die reine Mathematik,* Vol. 3 (*von K bis P*), E. B. Schwickert, Leipzig, 1808. Available from http://books.google.com/books?id=9jgDAAAAQAAJ.

[72] M. Kraitchik, *Mathematical Recreations*, W. W. Norton, New York, 1942, reprinted by Dover, 2006. Partially available from http://books.google.com/books?id=iCq9rJbxcSEC.

[73] L. Kronecker, *Vorlesungen über Mathematik,* Part II: *Allgemeine Arithmetik,* Vol. 1: *Zahlentheorie*, B. G. Teubner, Leipzig, 1901. Available from http://archive.org/details/vorlesungenberm00krongoog.

[74] J. G. Krüger, *Gedancken von der Algebra, nebst den Primzahlen von* 1 *bis* 1000000, Lüderwalds Buchhandlung, Halle, 1746. Available from http://books.google.com/books?id=2Lg2AAAAMAAJ.

[75] J. H. Lambert, *Supplementa tabularum logarithmicarum et trigonometricarum,* edited by A. Felkel, Typographia regiæ academiæ scientiarum, Olisipone (Lisboa), 1798. Available from http://books.google.com/books?id=dbM2AAAAMAAJ.

[76] E. Landau, *Handbuch der Lehre von der Verteilung der Primzahlen,* Vol. I, B. G. Teubner, Leipzig & Berlin, 1909. Available from http://books.google.com/books?id=81m4AAAAIAAJ.

[77] V. A. Lebesgue, Remarques diverses sur les nombres premiers, *Nouvelles Annales de Mathématiques,* Sér. 1, **15** (1856), 130–134. Available from http://www.numdam.org/item?id=NAM_1856_1_15__130_0.

[78] V. A. Lebesgue, *Exercices d'analyse numérique*, Leiber et Faraguet, Paris, 1859. Available from http://books.google.com/books?id=ea8WAAAAQAAJ.

[79] V. A. Lebesgue, *Introduction à la théorie des nombres*, Mallet-Bachelier, Paris, 1862. Available from http://books.google.com/books?id=fq8WAAAAQAAJ.

[80] V. A. Lebesgue, *Tables diverses pour la décomposition des nombres en leurs facteurs premiers,* computations by J. Hoüel, Gauthier-Villars, Paris, 1864. Available from http://books.google.com/books?id=pSXOAAAAMAAJ.

[81] A.-M. Legendre, *Théorie des nombres,* Vol. I, Firmin Didot Frères, Paris, 3rd edition, 1830. Available from http://books.google.com/books?id=BTIVAAAAQAAJ.

[82] D. N. Lehmer, *List of Prime Numbers from* 1 *to* 10,006,721, Carnegie Institution of Washington, 1914.

[83] Léon de Saint-Jean, *Studium sapientiæ universalis*, Jacobus Quesnel, Paris, 1657. Available from http://books.google.com/books?id=FmFEAAAAcAAJ.

[84] F. Lieber and E. Wigglesworth, editors, *Encyclopædia Americana: A Popular Dictionary of Arts, Sciences, Literature, History, Politics and Biography,* Vol. 10, new edition, Thomas, Cowperthwait, & Co., Philadelphia, 1840. Available from http://books.google.com/books?id=WJhRAAAAYAAJ.

[85] E. Lucas, Théorie des fonctions numériques simplement périodiques, *Amer. J. Math.* **1** (1878), 184–240. Available from http://www.jstor.org/stable/2369311.

[86] E. Lucas, *Théorie des nombres,* Vol. I, Gauthier-Villars, Paris, 1891. Aavailable from http://archive.org/details/thoriedesnombre00lucagoog.

[87] A. J. Manchester, Questions for written examinations, *The Rhode Island Schoolmaster* **16** (1870), 130–133. Available from http://books.google.com/books?id=lvABAAAAYAAJ.

[88] H. v. Mangoldt, *Einführung in die höhere Mathematik,* Vol. 2: *Differentialrechnung*, S. Hirzel, Leipzig, 1912. Available from http://books.google.com/books?id=ybRXAAAAYAAJ.

[89] G. M'Arthur, Arithmetic, in *The Encyclopædia Britannica: A Dictionary of Arts, Sciences, and General Literature,* edited by T. S. Baynes, Vol. 2, pp. 524–536, Adam and Charles Black, Edinburgh, 9th edition, 1898. Available at http://books.google.com/books?id=LdxTAAAAYAAJ.

[90] Martianus Capella, *The Marriage of Philology and Mercury,* translated by W. H. Stahl and R. Johnson, with E. L. Burge, Vol. II of *Martianus Capella and the Seven Liberal Arts*, Columbia University Press, 1992. Partially available from http://books.google.com/books?id=nZ-Z9eI6dXwC&printsec=.

[91] F. Maseres, editor, *The Doctrine of Permutations and Combinations, Being an Essential and Fundamental Part of the Doctrine of Chances; Together With Some Other Mathematical Tracts*, B. and J. White, London, 1795. Available from http://books.google.com/books?id=BO5LAAAAMAAJ.

[92] M. Masi, *Boethian Number Theory: A Translation of the De Institutione Aritmetica,* Vol. 6 of *Studies in Classical Antiquity*, Rodopi, Amsterdam, 1983. Partially available at http://books.google.com/books?id=StVUEsGK2RkC.

[93] G. B. Mathews, Number, in *The Encyclopædia Britannica: A Dictionary of Arts, Sciences, Literature and General Information,* edited by H. Chisholm, Vol. 19, pp. 847–863, Cambridge University Press, New York, 11th edition, 1911. Available at http://archive.org/details/encyclopaediabri19chisrich.

[94] F. Meinert, *Lehrbuch der Mathematik,* Theil 1: *Gemeine und allgemeine Arithmetik*, Hemmerde und Schwetschke, Halle, 1789. Available from http://books.google.com/books?id=3q9EAAAAcAAJ.

[95] E. Meissel, Ueber die Bestimmung der Primzahlenmenge innerhalb gegebener Grenzen, *Math. Ann.* **2** (1870), 636–642. Available from http://gdz.sub.uni-goettingen.de/dms/load/img/?PPN=PPN235181684_0002.

[96] K. Menninger, *Number Words and Number Symbols: A Cultural History of Numbers,* reprinted from 1969 translation by P. Broneer, Dover, 1992. Available from http://books.google.com/books?id=BFJHzSIj2u0C.

[97] M. Mersenne, *La Vérité des sciences: Contre les sceptiques ou pyrrhoniens*, Toussainct du Bray, Paris, 1625. Available from http://books.google.com/books?id=FF44pHy9x_sC.

[98] A. Metius, *Arithmetica et Geometria*, B. & A. Elsevirius, Lugdunum Batavorum (Leiden), 1640. Available from http://books.google.com/books?id=ZdFGAAAAcAAJ.

[99] J. S. Mill, *Auguste Comte and Positivism,* reprinted from the Westminster Review, N. Trübner, London, 1865. Available from http://books.google.com/books?id=HvQGAAAAcAAJ.

[100] W. J. Milne, *Standard Arithmetic*, American Book Company, New York, 1892. Available from http://books.google.com/books?id=ppnxMePtA2oC.

[101] F. Minsinger, *Lehrbuch der Arithmetik und Algebra*, Karl Kollmann, Augsburg, 1832. Available from http://books.google.com/books?id=drEoAAAAcAAJ.

[102] P. de Mondesir, Sur les nombres premiers: Formules pour le calcul exact de la totalité des nombres premiers compris entre 0 et un nombre pair quelconque $2n$, *Compte rendu de la 6^e session de l'Association Française pour l'Avancement des Sciences* (1878), 79–92.

[103] S. Morland, *The Description and Use of Two Arithmetick Instruments. Together with a Short Treatise, Explaining and Demonstrating the Ordinary Operations of Arithmetick*, Moses Pitt, London, 1673.

[104] J. Moxon, *Mathematicks Made Easie: Or, a Mathematical Dictionary, Explaining the Terms of Art, and Difficult Phrases Used in Arithmetick, Geometry, Astronomy, Astrology, and Other Mathematical Sciences*, printed for J. Moxon, London, 1679.

[105] J. J. O'Connor and E. F. Robertson, MacTutor History of Mathematics archive, University of St Andrews, Scotland, http://www-history.mcs.st-andrews.ac.uk.

[106] M. Ohm, *Die reine Elementar-Mathematik,* Vol. 1*: Die Arithmetik bis zu den höhern Gleichungen*, Jonas Verlags-Buchandlung, Berlin, 2nd edition, 1834. Available from http://books.google.com/books?id=xPkUAQAAMAAJ.

[107] J. Ozanam, *Dictionaire mathematique ou Idee generale des mathematiques*, Huguetan, Amsterdam, 1691. Available from http://books.google.com/books?id=SoU-AAAAcAAJ.

[108] J. Ozanam and J.-E. Montucla, *Recreations in Science and Natural Philosophy,* translated by C. Hutton, Thomas Tegg, London, 3rd edition, 1844. Available from http://ebooks.library.cornell.edu/cgi/t/text/text-idx?c=math;idno=07160001.

[109] E. Phillips, *The New World of Words: or, Universal English Dictionary,* edited by J. Kersey, printed for J. Philips, H. Rhodes, and J. Taylor, London, 7th edition, 1720. Available from http://books.google.com/books?id=tp4NAAAAQAAJ.

[110] L. Pogliani, M. Randić, and N. Trinajstić, About one—an inquiry about the meanings and uses of the number one, *Int. J. Math. Educ. Sci. Technol.* **31** (2000), 811–824.

[111] J. Prestet, *Nouveaux elemens des mathematiques ou Principes generaux de toutes les sciences qui ont les grandeurs pour objet,* Vol. I, André Pralard, Paris, 2nd edition, 1689. Available from http://books.google.com/books?id=bnIheoK3D-EC.

[112] J. H. Rahn, *An Introduction to Algebra, Translated out of the High-Dutch into English by Thomas Brancker. Much Altered and Augmented by D[r]. P[ell]. Also a Table of Odd Numbers less than One Hundred Thousand, shewing those that are Incomposit*, printed for Moses Pitt, London, 1668.

[113] J. Ray, *Primary Elements of Algebra, for Common Schools and Academies*, Van Antwerp, Brag & Co., Cincinnati & New York, 2nd edition, 1866. Available from http://www.archive.org/details/primaryxxelement00rayjrich.

[114] A. A. L. Reynaud, *Traité d'arithmétique*, Bachelier, Paris, 18th edition, 1835. Available from http://books.google.com/books?id=6LWE9SsKTYgC.

[115] C.-R. Reyneau, *La Science du calcul des grandeurs en general, ou Les Elemens des mathematiques*, François Pitteri, Venise, 1739. Available from http://books.google.com/books?id=68BNAAAAMAAJ.

[116] D. Roegel, A reconstruction of the tables of the Shuli Jingyun (1713–1723), Research report, LORIA, 2011. Available from http://hal.inria.fr/hal-00654450/en.

[117] A. G. Rosell, *Instituciones matemáticas,* Vol. I: *La aritmetica propia y los principios del algebra*, Imprenta Real, Madrid, 1785. Available from http://books.google.co.uk/books?id=_t70xeaCHK4C.

[118] C. Sagan, *Contact*, Pocket Books, 1997.

[119] H. Scheffler, *Die polydimensionalen Grössen und die vollkommenen Primzahlen*, Friedrich Vieweg & Sohn, Braunschweig, 1880. Available from http://books.google.com/books?id=9u0GAAAAYAAJ.

[120] F. v. Schooten, *Exercitationum mathematicarum,* Vol. I: *Propositionum arithmeticarum et geometricarum centuriam*, J. Elsevirius, Lugdunum Batavorum (Leiden), 1657. Available from http://books.google.com/books?id=-aBGAAAAcAAJ.

[121] J. Shallit, The prime factorization of 1, *SIGAPL APL Quote Quad* **6**(4) (1976), 36–37.

[122] W. F. Sheppard, Arithmetic, in *The Encyclopædia Britannica: A Dictionary of Arts, Sciences, Literature and General Information,* edited by H. Chisholm, Vol. 2, pp. 523–542, Cambridge University Press, New York, 11th edition, 1910. Available at http://archive.org/details/encyclopaediabri02chisrich.

[123] D. E. Smith, *Rara Arithmetica: A Catalogue of the Arithmetics Written Before the Year* MDCI *With Description of Those in the Library of George Arthur Plimpton of New York*, Ginn & Company, Boston & London, 1908. Available from http://books.google.com/books?id=N44AAAAAMAAJ.

[124] D. E. Smith, *History of Mathematics,* Vol. II: *Special Topics of Elementary Mathematics*, Dover, 1958. Partially available from http://books.google.com/books?id=uTytJGnTf1kC.

[125] R. C. Smith, *Arithmetic on the Productive System; Accompanied by a Key and Cubical Blocks*, John Paine, Hartford, 1842. Available from http://books.google.com/books?id=2CFRAAAAYAAJ.

[126] R. Steele, editor, *The Earliest Arithmetics in English*, H. Milford, London, 1922. Available from http://ia600202.us.archive.org/15/items/earliestarithmet00steeuoft/earliestarithmet00steeuoft.pdf.

[127] E. Stone, *A New Mathematical Dictionary*, printed for J. Senex et al., London, 1726. Available from http://books.google.com/books?id=JWxbAAAAQAAJ.

[128] L. Tarán, *Speusippus of Athens: A Critical Study with a Collection of the Related Texts and Commentary*, Vol. 39 of *Philosophia Antiqua*, E. J. Brill, Leiden, 1981. Partially available from http://books.google.com/books?id=cUPXqSb7V1wC.

[129] J. Thomson, *An Introduction to the Differential and Integral Calculus: with an Appendix, Illustrative of the Theory of Curves and Other Subjects*, Simms and M'Intyre, London, 2nd edition, 1849. Available from http://ebooks.library.cornell.edu/cgi/t/text/text-idx?c=math;idno=01890002.

[130] P. L. Tschebyscheff, *Theorie der Congruenzen (Elemente der Zahlentheorie),* edited and translated by H. H. Schapira, Mayer & Müller, Berlin, 1889. Available from http://books.google.com/books?id=hUxtAAAAMAAJ.

[131] P. L. Tchebychef, *Œuvres,* Vol. I, published by A. A. Markoff and N. Sonin, Commissionaires de l'Académie Impériale des Sciences, St.-Pétersbourg, 1899. Available from http://books.google.com/books?id=TGBtAAAAMAAJ.

[132] B. L. van der Waerden, *Modern Algebra,* Vol. 1, translated by F. Blum, Frederick Ungar, New York, 1949.

[133] J. Wallis, *Opera mathematica,* Vol. 2: *De algebra tractatus, historicus & practicus. Cum variis appendicibus*, E Theatro Sheldoniano, Oxonia (Oxford), 1693. Available from http://books.google.com/books?id=EuzpN1t5SOcC.

[134] E. Waring, *Meditationes Algebraicæ*, J. Archdeacon, Cantabrigia (Cambridge), 1782. Available from http://books.google.com/books?id=1MNbAAAAQAAJ.

[135] E. Waring, Some properties of the sum of the divisors of numbers, *Philos. Trans. R. Soc. Lond.* **78** (1788), 388–394. Available from http://www.jstor.org/stable/106667.

[136] D. Weeks, *Meditationes Algebraicæ: An English translation of the work of Edward Waring*, American Mathematical Society, 1991.

[137] K. Weierstrass, *Mathematische Werke,* Vol. 4: *Vorlesungen über die Theorie der Abelschen Transcendenten,* edited by G. Hettner and J. Knoblauch, Mayer & Müller, Berlin, 1902. Available from http://books.google.com/books?id=EFptAAAAMAAJ.

[138] J. B. Weigl, *Lehrbuch der Rechenkunst zum Gebrauche an lateinischen und Gewerbsschulen und zum Selbstunterrichte*, J. E. v. Seidel, Sulzbach, 6th edition, 1848. Available from http://books.google.com/books?id=ZWIoAAAAcAAJ.

[139] M. Weik, *Computer Science and Communications Dictionary,* Vol. II, Kluwer Academic Publishers, 2000.

[140] G. Wertheim, *Elemente der Zahlentheorie*, B. G. Teubner, 1887. Available from http://books.google.com/books?id=k70KAAAAYAAJ.

[141] M. L. Willig, *Gründliche Vorstellung der Reesischen allgemeinen Regel nebst deren Anwendung auf die üblichsten Rechnungsarten*, G. L. Förster, Bremen & Göttingen, 1759. Available from http://www.archive.org/details/michellorenzwil00reesgoog.

[142] G. Zarlino, *Le istitutioni harmoniche*, Francesco Senese, Venetia, 1561. Available from http://books.google.com/books?id=WsVCAAAAcAAJ.

2010 *Mathematics Subject Classification*: Primary 11A41; Secondary 11A51, 01A55.
Keywords: prime number, unity, history of mathematics.

(Concerned with sequences A000027, A000040, and A008578.)

Received September 27 2012; revised version received November 18 2012. Published in *Journal of Integer Sequences*, December 27 2012.

Return to Journal of Integer Sequences home page.