Window Observers for Linear Systems

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Given a linear system $\dot{x} = Ax + Bu$ with output $y = Cx$ and a window function $\omega(t)$, i.e., $\forall t$, $\omega(t) \in \{0,1\}$, and assuming that the window function is Lebesgue measurable, we refer to the following observer, $\dot{\hat{x}} = A\hat{x} + Bu + \omega(t)LC(x - \hat{x})$ as a window observer. The stability issue is treated in this paper. It is proven that for linear time-invariant systems, the window observer can be stabilized by an appropriate design under a very mild condition on the window functions, albeit for linear time-varying system, some regularity of the window functions is required to achieve observer designs with the asymptotic stability. The corresponding design methods are developed. An example is included to illustrate the possible applications.

Keywords: Observer; Stability; Windowing function

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1. INTRODUCTION

In many engineering applications of state observers, engineers may encounter such situations that the measured output is only available at "observing windows". This restriction may rise either because of the limited availability of the sensors or the need to reduce the number of sensors. Examples may include distributed parameter systems with scanners as output devices and three-phase electrical drive systems with only one current sensor. By parallelism, we may also encounter

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this problem in controller design. An appropriate example is the power factor correction controller in power systems. Their operation may be intermittent. The example of power correction controller design illustrates well that uncertain window patterns may occur in practical system design, although, in general, the window patterns are more or less regular.

In light of these engineering applications, we are well motivated to consider the corresponding theoretical problem: how do we design appropriate observers or controllers to guarantee the stability of the systems? Is there any method to estimate the rate of convergence? This paper handles these problems. The developed design principle is applied to observer design for automotive alternator control systems. We call them as window observers.

By window observer, we refer to the following variable structure observer: when the output is available, we have exactly the conventional Luenberger observer. However, when the output is unavailable, the observer runs in open loop mode. This kind of observing scheme was initiated as a very specific engineering practice as presented in Utkin [3, 4], where window observers were designed to observe the automobile alternator back EMF. In these works, it is found that because of limited availability of (current) sensors and the switching nature of electrical power systems, windowing the observing action is inevitable. In fact, various window observers are designed and applied to a Ford project with good performance, please refer to Utkin [3, 4] for details. In light of these successes, the general scheme of window observers is proposed in this paper along with the treatment of the stability issue. General design rules are stated and proven.

First, we will formulate the problem rigorously and state our main results in the following section. In Section 3, we prove Theorem 1. The prerequisites are treated in Section 3.1, the proof of the theorem is presented in Section 3.2. To include the treatment of linear time-varying systems, we require some kind of regularity of the window functions here. However, we can go much more further with linear time-invariant systems. In this case, we can deal with general Lebesgue measurable window functions. The corresponding result is proven in Section 4. In Section 5, we present an interesting example that is related to automobile alternator control system design.

2. PROBLEM STATEMENT AND MAIN RESULTS

2.1. Problem Statement

Suppose that the system

$$(\Xi)\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \text{ with } x \in R^n, \ u \in R^m, \ y \in R^r,$$

is observable with matrix A being constant or time-varying but bounded. Then conventional observer can be designed to estimate the state variables. However, in this paper, we confine our discussion to those cases where the output is only available at certain observing windows in the time domain.

ASSUMPTIONS

(A1) *If $\{T_n\}_{n=0}^{\infty}$ is an increasing sequence in the time domain $(0, \infty)$, consider the window function $\omega(t) = \sum_n \chi_{[T_n, T_n + D_n(T_{n+1} - T_n)]}$, where we use $\chi_{[T_n, T_n + D_n(T_{n+1} - T_n)]}$ to denote the characteristic function of the interval $[T_n, \ T_n + D_n(T_{n+1} - T_n)]$, where $0 < D_n < 1$. We refer to these window functions as regular.*

(A2) *More generally, we will consider time-invariant systems with a window function $\omega(t)$ being only Lebesgue measurable, and for any finite interval $[0, T]$, there exists $0 < \beta < 1$, such that the Lebesgue measure of the observing windows $\mu\{[0, T] \cap \{t | \omega(t) = 1\}\} > \beta T$.*

A window observer is obtained by windowing the conventional Luenberger observer:

$$\dot{\hat{x}} = A\hat{x} + Bu + \omega(t)L(y - C\hat{x}). \tag{1}$$

Our results reveal that under Assumptions (A1) or (A2), we can always stabilize these window observers by appropriate choice of the feedback matrix L.

2.2. Main Results

We state our results in the following two theorems, each corresponding to one of the assumptions stated above.

THEOREM 1 *Suppose the plant (Ξ) satisfies our Assumption (A1), then,*

1. *For a linear time-invariant plant, there exists a constant feedback matrix L which asymptotically stabilize the window observer (1) with predefined convergence rate.*
2. *For a linear time-varying plant, there exists a time-varying feedback matrix L which asymptotically stabilize the window observer (1) with predefined convergence rate.*

THEOREM 2 *Suppose the plant (Ξ) satisfies our Assumption (A2), and is time-invariant, then there exists a constant feedback matrix L which asymptotically stabilize window observer (1) with predefined convergence rate.*

3. WINDOW OBSERVERS WITH REGULAR WINDOW FUNCTIONS

In this section, we deal with general linear system, but with a regular window function, *i.e.*, the window function satisfies our Assumption (A1).

3.1. Lemma and Some Prerequisites

To prove the theorem, the following lemma concerning the behavior of conventional Luenberger observers is needed.

LEMMA 1 *Suppose that the plant (Ξ) is observable. Given any $T > 0$, $0 < \alpha < 1$, there exists a feedback matrix L, such that, for the error system $\dot{e} = Ae - LCe$ with $e(t) = x(t) - \hat{x}(t)$, $\forall\ e(0) \in R^n$, $\|e(T)\| \leq \alpha\|e(0)\|$.*

Instead of proving this lemma directly, its dual control problem is addressed below. We will first present the proof for the case of time-invariant systems.

Proof of Lemma 1 when system is time-invariant Given time-invariant system $\dot{x} = Ax + Bu$, without loss of generality, suppose that dimension of control is 1. Otherwise, we can select $0 \neq b \in \text{range}(B)$ and F, such that pair $(A + BF, b)$ is controllable, Wonham [2].

Then, our system can be transformed into controllability canonical form, so we further assume that the system matrices are

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & \dots \\ 0 & 0 & 1 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ a_1 & a_2 & a_3 & \dots & a_n \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \\ 1 \end{bmatrix}.$$

By feedback control, assign distinct descending eigenvalues at $\{\lambda_i\}_{i=1}^n$, then

$$x = V \begin{bmatrix} e^{\lambda_1 t} & 0 & \dots & 0 \\ 0 & e^{\lambda_2 t} & \dots & 0 \\ 0 & \dots & e^{\lambda_{n-1} t} & 0 \\ 0 & \dots & 0 & e^{\lambda_n t} \end{bmatrix} V^{-1} x(0),$$

$$\text{where } V = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \lambda_1 & \lambda_2 & \dots & \lambda_n \\ \dots & \dots & \dots & \dots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \dots & \lambda_n^{n-1} \end{bmatrix},$$

the Van Dermond matrix.

Because $\det(V) = \Pi_{i>j}(\lambda_i - \lambda_j) \geq (\min_{i>j}(|\lambda_i - \lambda_j|))^{((n^2-n)/2)}$, we may assign the eigenvalues in such a fashion that $b \leq \min_{i \neq j}(|\lambda_i - \lambda_j|) \leq \max_{i \neq j}(|\lambda_i - \lambda_j|) \leq a$, where a, b are positive constants.

Then when all eigenvalues are made negative with sufficiently high magnitudes, the contracting property of the mapping, $\Phi: x(0) \longrightarrow x(T)$ is guaranteed and $\|\Phi\| \leq \alpha < 1$. This proves Lemma 1 for linear time-invariant plant.

Remark The above proof can not be generalized for time-varying systems. In the next subsection, we present a second design method based on optimal control theory. This method can be extended to handle the time-varying situation, the resultant feedback design is also time-varying.

3.1.1. A Second Proof of the Lemma Based on the Optimal Control Theory

In the following, we provide a second proof of the lemma for time-invariant systems. This approach can handle the time-varying case.

To start with, select a cost functional $J = (1/2)x(T)^T Sx(T) + (1/2) \int_0^T u^T R u \, dt$, with S, R being positive definite matrices.

By variational method, the following two-point boundary value problem is derived, Bryson [5]:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A & -BR^{-1}B^T \\ 0 & -A^T \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix}, \quad \text{with } x(0) \text{ given and } \lambda(T) = Sx(T).$$

Solve the second equation, we get $\lambda(t) = e^{-A^T(t-T)}Sx(T)$.

Substitute it into the first equation and solve $x(T)$,

$$x(T) = e^{AT}x(0) - \int_0^T e^{A(T-\tau)}BR^{-1}B^T e^{A^T(T-\tau)}d\tau \cdot Sx(T), i.e.,$$

$$x(T) = \left(I + \int_0^T e^{A(T-\tau)}BR^{-1}B^T e^{A^T(T-\tau)}d\tau \cdot S \right)^{-1} e^{AT}x(0).$$

The inverse in the above expression exists for sufficiently large S and R^{-1} because of the controllability of the plant, Wonham [2]. This can be easily understood through the matrix identity $(I - \Psi)^{-1} = I + \Psi + \Psi^2 + \cdots$ whenever $\|\Psi\| < 1$.

It's obvious that with increasing S or decreasing R, the norm of the mapping: $\Phi: x(0) \longrightarrow x(T)$ can be designed with a norm $\|\Phi\| = \alpha < 1$ as specified. The feedback matrix is available by solving the corresponding Riccati equation.

This finishes the second proof of Lemma 1, and this approach provides a method to handle the time-varying situation. For linear time-varying system, we only need to substitute $e^{A(T-\tau)}$ with the fundamental matrix "$\Phi(T, \tau)$" in the above expressions.

3.1.2. An Important Observation

We need some information concerning the divergence rate of the solution of our window observer when it is running in open loop mode.

Suppose a plant with equation $\dot{x} = f(x,t)$, has an equilibrium point $x = 0$, its right-hand side function is Lipschitz continuous in x, i.e.,

$$|f(x,t) - f(y,t)| \le \hat{L}|x - y|, \forall x, y \in R^n,$$

with $\hat{L}$ being a Lipschitz constant.

Then, on any finite time interval $[T_1, T_2]$, we can get an upper bound of the norm of the state *via* its initial value by use of the Bellman–Gronwall lemma.

On the finite interval $[T_1, T_2]$,

$$\max_{s \in [T_2, T_1]} (\|x(s) - x(T_1)\|) \le \hat{L}(T_2 - T_1)\|x(T_1)\|e^{\hat{L}(T_2 - T_1)},$$

Given the plant (Ξ), the above inequality is true with the global Liptschitz constant

$$\hat{L} = \max_t \|A(t)\| \tag{2}$$

3.2. Proof of Theorem 1

We can make use of the fact (2) and the lemma presented in Section 3.1 to complete the proof of Theorem 1.

As in Lemma 1, denote the error as $e(t) = x(t) - \hat{x}(t)$. On interval $[T_n + D_n(T_{n+1} - T_n), T_{n+1}]$, by applying the above estimation, there exists a constant $\hat{L}_n$, such that

$$\|e(T_{n+1})\| \le \hat{L}_n \|e(T_n + D_n(T_{n+1} - T_n))\|,$$

where $\hat{L}_n$ is in the form of $\hat{L}(1 - D_n)(T_{n+1} - T_n)e^{\hat{L}(1 - D_n)(T_{n+1} - T_n)} + 1$.

On the interval $[T_n, T_n + D_n(T_{n+1} - T_n)]$, we can choose a constant α_n such that $\alpha_n \cdot \hat{L}_n \le 1 - \varepsilon$, where $0 < \varepsilon < 1$. The corresponding observer feedback matrix can be found by the eigenvalue placement method.

The resultant observer is asymptotically stable. Because we have got a contracting sequence $\{x(T_n)\}_{n=1}^{\infty}$ and further more, $\forall n$, the maximum estimation error on interval $[T_n + D_n(T_{n+1} - T_n), T_{n+1}]$ is bounded by $C_1 \|e(T_n)\|$, where C_1 is a constant independent on our design. Q.E.D.

4. WINDOW OBSERVERS WITH LEBESGUE MEASURABLE WINDOW FUNCTIONS

When the plant is linear time-invariant, we can generalize Theorem 1 to the result stated in Theorem 2 with the window functions being only Lebesgue measurable. To prove Theorem 2, some prerequisites are needed.

4.1. Some Prerequisites

We need to approximate the measurable window functions with regular ones. To start with, we first take a closer look at the behavior of the solutions with regular window functions. The result is expressed in Lemma 2 concerning the conventional Luenberger observer as follows.

LEMMA 2 *Suppose the plant* (Ξ) *is observable,* $\{\lambda_i\}_{i=1}^{n}$ *are the eigenvalues of the error system* $\dot{e} = Ae - LCe$, *and are assigned in conformity with the rule* "$b \leq min_{i \neq j}(|\lambda_i - \lambda_j|) \leq max_{i \neq j}(|\lambda_i - \lambda_j|) \leq a$". *Then, there exists a constant* C_2, *and a polynomial* $p(\lambda_1, \lambda_2, \ldots, \lambda_n)$, *such that for function* $U(t) = e^T e$, *at any interval* $[t_1, t_2]$, $U(t_2) \leq C_2 p(\lambda_1, \lambda_2, \ldots, \lambda_n) U(t_1) e^{-2\lambda_{min}(t_2 - t_1)}$, *where* $\lambda_{min} = min\{-\lambda_i\}_{i=1}^{n}$.

Proof of Lemma 2 Directly follows from the proof of Lemma 1.

We also need a general result on the continuous dependence of ODE solutions on their right hand side functions. Please refer to Gamkrelidze [1] for more detailed description.

Suppose we are dealing with system $(\Sigma): \dot{x} = A(t)x$ where $x \in R^n$. To impose conditions on the right hand side functions, the following $(n \times n)$ matrix-valued function space E is introduced in Gamkrelidze [1]. Let $\Im$ be an interval of the time axis.

$A(t) \in E \Leftrightarrow A(t)$ is summable on $\Im$, *i.e.*, $\int_{\Im} \|A(t)\| dt < \infty$.

A norm $\|\cdot\|_w$ is introduced in space E, $\forall A(t) \in E$, $\|A(t)\|_w = max_{t', t'' \in \Im} |\int_{t'}^{t''} A(t) dt|$.

A family of matrices $D \subset E$ is called integrally uniformly bounded if the set of values $\{\int_{\Im} \|A(t)\| dt < \infty: A(t) \in D\}$ is bounded.

The continuous dependence of solutions of the system (Σ) on the right-hand side functions within an integrally uniform bounded set

$D \subset E$ is well known and can be found in Chapter 5 of Gamkrelidze [1].

4.2. Proof of Theorem 2

Under Assumption (A2), with any measurable window function $\omega(t)$, design the observer (1) in such a way that the eigenvalues of $A - LC$ are assigned as $\{\lambda_i\}_{i=1}^n$ by the rule developed in Lemma 1. The error system of the corresponding window observer is $\dot{e} = Ae - \omega(t)LCe$. Let $e(t)$ be a solution, and define $V(t) = e^T e$.

For a measurable function $\omega(t)$, we can approximate it by a series of regular window functions $\{\omega_k(t)\}_{k=1}^{\infty}$, i.e., $\omega_k(t)$ satisfies our Assumption (A1) and $\lim_{k\to\infty} \omega_k(t) = \omega(t)$ by measure.

Because of the uniform boundedness of all the functions involved in the limit process, $\lim_{k\to\infty}(Ae - \omega_k(t)LCe) = Ae - \omega(t)LCe$ in function space E with any designed feedback matrix L.

Denote the solution of $\dot{e} = Ae - \omega_k(t)LCe$ as $e_k(t)$, and assume identical initial conditions for all $e_k(t)$ with $k = 1, 2, \ldots$. By continuous dependence of the solutions of the error system on the right-hand side functions, at any finite interval $[0, T]$, $\lim_{k\to\infty} e_k(t) = e(t)$, so, $\lim_{k\to\infty} V_k = V = e^T e$.

Now we can use our Lemma 2 to estimate the behavior of V_k, hence that of V.

Suppose, $\omega_k(t) = \sum_n \chi_{[T_n, T_n + D_n(T_{n+1} - T_n)]}$, then by Lemma 2, on any interval

$$[T_n, T_n + D_n(T_{n+1} - T_n)], V_k(T_n + D_n(T_{n+1} - T_n))$$
$$\leq Cp(\lambda_1, \lambda_2, \ldots, \lambda_n)V_k(T_n)e^{-2\lambda_{\min}D_n(T_{n+1}-T_n)}.$$

Whereas on interval $[T_n + D_n(T_{n+1} - T_n), T_{n+1}]$, denote $\hat{L} = \|A\|$, by observation (2) in Section 3.1, there exits a small quantity ε, such that

$$V_k(T_{n+1}) \leq V_k(T_n + D_n(T_{n+1} - T_n))e^{2(\hat{L}+\varepsilon)(1-D_n)(T_{n+1}-T_n)},$$

then

$$V_k(T_{n+1}) \leq Cp(\lambda_1, \lambda_2, \ldots, \lambda_n)$$
$$V_k(T_n)e^{-2\lambda_{\min}D_n(T_{n+1}-T_n)+2(\hat{L}+\varepsilon)(1-D_n)(T_{n+1}-T_n)}.$$

By iterating this estimation, we can conclude that on interval $[0, T_n]$,

$$V_k(T_n) \leq Cp(\lambda_1, \lambda_2, \ldots, \lambda_n) V_k(0)$$
$$e^{-2\lambda_{\min}\mu(\{t|\omega_k(t)=1\}\cap[0,T_n])+2(\hat{L}+\varepsilon)\mu(\{t|\omega_k(t)=0\}\cap[0,T_n])},$$

where μ is Lebesgue measure.

So, for $V = e^T e$, in the interval $[0, T]$, $V(T) \leq Cp(\lambda_1, \lambda_2, \ldots, \lambda_n)$ $V(0)e^{-2\lambda_{\min}\beta T + 2(\hat{L}+\varepsilon)(1-\beta)T}$.

We have proven that by proper placement of the eigenvalues, the global asymptotic stability of the window observers is guaranteed with predefined convergence rate.   Q.E.D.

5. AN EXAMPLE-AUTOMOTIVE ALTERNATOR BACKEMF OBSERVATION

The concept of window observers was originated from some early work on observing automotive back EMF. The need of a window observer is caused by only measuring the link current, which coincides with one of the phase currents only within some time windows corresponding to the switching logic of the converter. In this example, an intermediate third order observer is designed to observing the back EMF and speed of the alternator, Utkin [3, 4].

Consider a three-phase AC synchronous generator, which is used in the automotive electrical power supply system. The system setup is shown in Figure 1.

Suppose that the rotor provides a constant magnetizing flux and the generator shaft speed ρ is constant. Then it's reasonable to assume that the back EMF v_a, v_b, v_c are sinusoidal, we may use a simple model of back EMF in series with a phase inductor L and phase resistor R to represent each phase of the generator. Let the kth ($k = 1, 2, 3$) bridge switching function be $d_k \in \{0.5, 0.5\}$, with $d_k = 0.5$ representing the closing of the upper switch, and $d_k = -0.5$ the closing of the lower switch, then, the phase current i_1 is governed by, Wu [6],

$$\dot{Li}_1 = -\frac{R}{L}(Li_1) - \frac{V_o}{3}(2d_1 - d_2 - d_3) + v_a, \tag{3}$$
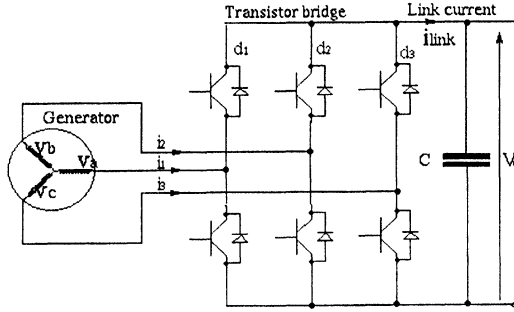
FIGURE 1 System configuration of the automotive alternator.

Under the assumption that the shaft speed ρ is constant and that the back EMF is sinusoidal with the same frequency (2-pole machine), the Eq. (3) is complemented to form the following state space model.

Define a state vector $y^T = [Li_1 \quad v_a \quad \dot{v}_a \quad \rho^2]$. A state space model is formulated as follows:

$$\begin{cases} \dot{y}_1 = -\frac{R}{L}y_1 - \frac{V_o}{3}(2d_1 - d_2 - d_3) + y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = -y_2 y_4 \\ \dot{y}_4 \approx 0 \end{cases}$$

where y_1, V_o, d_1, d_2, d_3 are available.

For this plant, we have proposed the following observer to estimate the back EMF $y_2 = v_a$ and $y_4 = \rho^2$ when y_1 is measured.

$$\begin{cases} \dot{\hat{y}}_1 = -\frac{R}{L}\hat{y}_1 - \frac{V_o}{3}(2d_1 - d_2 - d_3) + \hat{y}_2 + M_0 \ \mathrm{sign}(y_1 - \hat{y}_1) \\ \dot{\hat{y}}_2 = \hat{y}_3 + M_0 \ \mathrm{sign}(y_1 - \hat{y}_1) \\ \dot{\hat{y}}_3 = -\hat{y}_2\hat{y}_4 + M_2 M_0 \ \mathrm{sign}(y_1 - \hat{y}_1) - \hat{y}_4 \ \mathrm{sign}(y_1 - \hat{y}_1) \\ \dot{\hat{y}}_4 = M_0 \ \mathrm{sign}(y_1 - \hat{y}_1) \end{cases} \qquad (4)$$

where we have used $\mathrm{sign}(x)$ to denote the sign of a variable x.

However, in automotive alternator control systems, due to cost and package restrictions, it is strongly desired that the observer should only utilize the link current i_{link} measurement. According to the operation of six-step synchronous rectification, Mohan [8], in the observer (4), the output y_1 is only available inside the windows where the phase 1 current, (y_1/L), is identical to the link current, i_{link}.

Window observers with window function $2d_1 - d_2 - d_3 - \text{sign}(d_1)$ can be used in the design.

A full description of the whole design deserves a separate paper. We simplify the example by decouple the fast and slow dynamics of (4) and neglect the fast one – the sliding mode as explained in the following.

As developed in Utkin [3, 4], in the window version of the observer (4), sliding mode occurs on surface $s = y_1 - \hat{y}_1$ very quickly, then y_2 is obtained in the corresponding windows as $x_1 = \hat{y}_2 + (M_0 \text{sign}(y_1 - \hat{y}_1))_{eq}$, where $(M\text{sign}(y_1 - \hat{y}))_{eq}$ denotes the equivalent control value of the feedback term. So the low frequency component of $x_1(t)$ is nearly sinusoidal within any observing window. In the following, we neglect the effects of the fast dynamics, and simply assume that $x_1(t) = y_2(t)(2d_1 - d_2 - d_3 - \text{sign}(d_1))$, so that x_1 also satisfies the following dynamic equations in these observing windows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\rho^2 x_1 \\ \dot{\rho} = 0, \end{cases} \tag{5}$$

There have some tradeoffs to get the low frequency component of x_1. A low pass filter is needed. However, a low pass filter introduces some phase delay, which is undesirable. In the following, we describe a window observer, which constitutes the slow motion of observer (4), to observe the speed ρ and at the same time add more filtering action to the low pass filtered version of x_1.

With reference to the model (5), define a state variable $z^T = [x_1 \quad \dot{x}_1 \quad \rho^2]$ design the window observer as follows.

$$\begin{cases} \dot{z}_1 = z_2 - (2d_1 - d_2 - d_3 - \text{sign}(d_1))l_1(z_1 - x_1) \\ \dot{z}_2 = -z_3 x_1 - (2d_1 - d_2 - d_3 - \text{sign}(d_1))l_2(z_1 - x_1) \\ \dot{z}_3 = -(2d_1 - d_2 - d_3 - \text{sign}(d_1))l_3(z_1 - x_1). \end{cases}$$

The error system is $\dot{e} = Ae - (2d_1 - d_2 - d_3 - \text{sign}(d_1))LCe$ with $e = z_1 - x_1$, and

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -x_1 \\ 0 & 0 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}, \quad \text{and } C = [1 \quad 0 \quad 0].$$
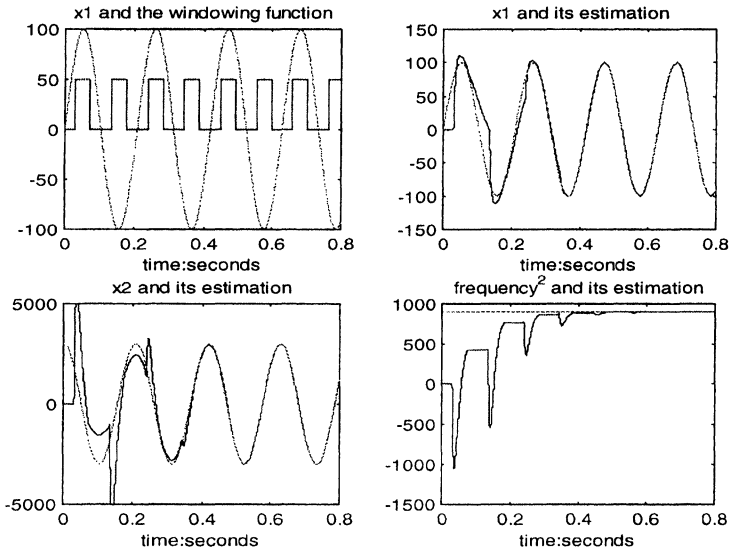
FIGURE 2 Simulation of the example window observer presented in Section 5.

We simulate the availability of a constant feedback matrix design as an example. In the simulation, we further neglect the "negative" window (where the window function $2d_1 - d_2 - d_3 - \text{sign}(d_1)$ is negative).

Inside these observing windows, we can choose a nominal time-invariant system with $A_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -x_{10} \\ 0 & 0 & 0 \end{bmatrix}$, where x_{10} being any value of x_1 within the window.

Simulations are made and the results are presented in Figure 2 to conclude this example.

6. CONCLUSION AND FUTURE DIRECTIONS

In this paper we have introduced the concept of window observers, and have discussed the stability issue. For linear observable system, design rules are developed and stability of the resultant observers is rigorously proven. The complications of linear time-varying system or even nonlinear system and the availability of constant feedback gain

design are briefly discussed in the Section 5 with the aid of an interesting application of an automotive alternator back EMF observer. These may represent some of the future directions. For more interesting applications, readers can consult some recent papers, Utkin [3, 4].

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As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods**: artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning

- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management

- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

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