

CONTINUOUS DEPENDENCE OF SOLUTIONS IN MAGNETO-ELASTICITY THEORY

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We prove continuous dependence on the intensity coefficient and continuous dependence on the external data in the theory of magneto-elasticity. We do not require the Lamé coefficients to be positive. We use logarithmic convexity arguments similar to those of Ames and Straughan (1992) in classical thermoelasticity.

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1. Introduction. In recent years, much attention has been directed to the knowledge of existence, uniqueness, and continuous dependence in several thermomechanical situations. We recall the book of Ames and Straughan [2] where the energy method is widely considered as a tool to obtain qualitative properties of solutions. We focus our interest on coupling elastic effects with magnetic effects. A derivation of the equations and recent papers on magneto-thermoelasticity and isothermal magneto-elasticity can be found in [3, 4, 5, 6, 7, 8, 9, 10, 11, 12].

In this paper, we consider the dynamical theory of magneto-elasticity. The system of equations is

$$\rho \mathbf{u}_{,tt} - \mu \Delta \mathbf{u} - (\lambda + \mu) \nabla \operatorname{div} \mathbf{u} - \alpha [\nabla \times \mathbf{h}] \times \mathbf{H} = \rho \mathbf{m}, \quad (1.1)$$

$$\beta \mathbf{h}_{,t} + \nabla \times [\nabla \times \mathbf{h}] - \beta \nabla \times [\mathbf{v} \times \mathbf{H}] = \rho \mathbf{r}, \quad (1.2)$$

$$\operatorname{div} \mathbf{h} = 0, \quad (1.3)$$

where \mathbf{u} denotes the displacement, $\mathbf{v} = \mathbf{u}_t$ is the velocity, and \mathbf{h} the magnetic field. A (known) constant magnetic field is denoted by $\mathbf{H} = (H, 0, 0)$, ρ , α , and β are positive constants, and \mathbf{m} and \mathbf{r} are the supply terms.

Here and from now on, we use summation and differentiation conventions: subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate; summation over repeated subscripts is implied.

The logarithmic convexity method is a very useful source of information about the qualitative behavior of the solutions of several kind of equations and systems (see, e.g., [2]). In particular, the method has been used to analyze the behavior of the solutions in classical thermoelasticity. Ames and Straughan [1]

applied a logarithmic convexity technique to achieve continuous dependence on the supply terms and structural stability on the coupling term for the classical linear theory of thermoelasticity. They did not require the elasticity tensor to be sign-definite. All they needed was that the elasticity coefficients were symmetric.

The aim of this paper is to obtain a continuous dependence result on the intensity of the vector field \mathbf{H} and the supply terms. Our main tool is also the logarithmic convexity method.

In this paper, we restrict our attention to homogeneous and isotropic materials. It is worth recalling that the extension to inhomogeneous and anisotropic materials would be straightforward.

Let B be a bounded domain in the three-dimensional Euclidean space whose boundary ∂B is smooth enough to allow the application of the divergence theorem. We assume that the set of equations (1.1), (1.2), and (1.3) holds in $B \times (0, t_1)$ for a time value $t_1 < \infty$, and we assume the boundary conditions

$$\mathbf{u} = 0, \quad \mathbf{h} \cdot \mathbf{n} = 0, \quad [\nabla \times \mathbf{h}] \times \mathbf{n} = 0, \quad \text{on } \partial B \times (0, \infty), \quad (1.4)$$

for all $t > 0$. Here and from now on, we denote by \mathbf{n} the normal vector to the boundary directed to the exterior. We impose the initial conditions

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{f}(\mathbf{x}), \quad \mathbf{v}(\mathbf{x}, 0) = \mathbf{g}(\mathbf{x}), \quad \mathbf{h}(\mathbf{x}, 0) = \mathbf{h}_0(\mathbf{x}), \quad \text{in } B. \quad (1.5)$$

For later use, we recall that the following inequality

$$\int_B (h_i h_i + h_{i,j} h_{i,j}) dV \leq C \int_B (h_{i,j} - h_{j,i})(h_{i,j} - h_{j,i}) dV \quad (1.6)$$

holds with any vector field (h_i) that satisfies (1.3) and the second and third equalities of (1.4). Here, C is a constant that depends on the domain B .

Here are the contents of the paper. In Section 2, we prove some lemmas and we state some other preliminaries. In Section 3, we prove the continuous dependence result.

2. Preliminaries. We denote by $(u_i^{(1)}, h_i^{(1)})$ the solution corresponding to the external data $(m_i^{(1)}, r_i^{(1)})$ and intensity $H^{(1)}$. Let $(u_i^{(2)}, h_i^{(2)})$ be the solution corresponding to the external data $(m_i^{(2)}, r_i^{(2)})$ and intensity $H^{(2)}$. We introduce the notation

$$\begin{aligned} w_i &= u_i^{(2)} - u_i^{(1)}, & l_i &= h_i^{(2)} - h_i^{(1)}, & K &= H^{(2)} - H^{(1)}, & \mathbf{K} &= (K, 0, 0), \\ F_i &= \rho(m_i^{(2)} - m_i^{(1)}), & R_i &= \rho(r_i^{(2)} - r_i^{(1)}). \end{aligned} \quad (2.1)$$

It follows that (w_i, l_i) satisfies the system

$$\rho \mathbf{w}_{,tt} - \mu \Delta \mathbf{w} - (\lambda + \mu) \nabla \operatorname{div} \mathbf{w} - \alpha [\nabla \times \mathbf{l}] \times \mathbf{H}^{(1)} - \alpha [\nabla \times \mathbf{h}^{(2)}] \times \mathbf{K} = \mathbf{F}, \quad (2.2)$$

$$\beta \mathbf{l}_{,t} + \nabla \times [\nabla \times \mathbf{l}] - \beta \nabla \times [\dot{\mathbf{w}} \times \mathbf{H}^{(1)}] - \beta \nabla \times [\mathbf{v}^{(2)} \times \mathbf{K}] = \mathbf{R}, \quad (2.3)$$

$$\operatorname{div} \mathbf{l} = 0, \quad (2.4)$$

the boundary conditions

$$\mathbf{w} = 0, \quad \mathbf{l} \cdot \mathbf{n} = 0, \quad [\nabla \times \mathbf{l}] \times \mathbf{n} = 0, \quad \text{on } \partial B \times (0, \infty), \quad (2.5)$$

and the initial conditions

$$\mathbf{w}(\mathbf{x}, 0) = \dot{\mathbf{w}}(0, \mathbf{x}) = \mathbf{l}(\mathbf{x}, 0) = 0, \quad \text{in } B. \quad (2.6)$$

LEMMA 2.1. *Let*

$$V(t) = \int_0^t \int_B [\rho \dot{\mathbf{w}} \dot{\mathbf{w}} + \mu \nabla \mathbf{w} \cdot \nabla \mathbf{w} + (\lambda + \mu) (\operatorname{div} \mathbf{w})^2 + \alpha \mathbf{l} \cdot \mathbf{l}] dV ds. \quad (2.7)$$

Then,

$$\begin{aligned} V(t) = 2 \int_0^t \int_B (t-s) & \left[F_i \dot{w}_i + S_i l_i + \alpha K \left(\dot{w}_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + \dot{w}_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right. \\ & + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) \\ & \left. - \frac{\alpha}{\beta} (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) \right] dV ds, \end{aligned} \quad (2.8)$$

where

$$S_i(t) = \frac{\alpha}{\beta} R_i(t). \quad (2.9)$$

PROOF. Differentiate twice and use the evolution equations and the boundary conditions to obtain

$$\begin{aligned} \frac{d^2 V}{dt^2} &= 2 \int_B [\mu w_{i,j} \dot{w}_{i,j} + (\lambda + \mu) w_{i,i} \dot{w}_{j,j} + \rho \dot{w}_i \ddot{w}_i + \alpha l_i \dot{l}_i] dV \\ &= 2 \int_B [\mu w_{i,j} \dot{w}_{i,j} + (\lambda + \mu) w_{i,i} \dot{w}_{j,j}] dV \\ &\quad - 2 \int_B [\mu w_{i,j} \dot{w}_{i,j} + (\lambda + \mu) w_{i,i} \dot{w}_{j,j} - \alpha H^1 (\dot{w}_2 (l_{2,1} - l_{1,2}) + \dot{w}_3 (l_{3,1} - l_{1,3}))] dV \\ &\quad + 2 \int_B \left[F_i \dot{w}_i + \alpha K \left(\dot{w}_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + \dot{w}_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right] dV \\ &\quad - 2 \int_B [\alpha H^1 (\dot{w}_2 (l_{2,1} - l_{1,2}) + \dot{w}_3 (l_{3,1} - l_{1,3}))] dV \end{aligned}$$

$$\begin{aligned}
& + 2 \int_B \left[S_i l_i + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) \right. \\
& \quad \left. - \frac{\alpha}{\beta} (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) \right] dV \\
& = 2 \int_B \left[F_i \dot{w}_i + S_i l_i + \alpha K \left(\dot{w}_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + \dot{w}_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right. \\
& \quad \left. + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) - \frac{\alpha}{\beta} (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) \right] dV.
\end{aligned} \tag{2.10}$$

The lemma follows after two quadratures and from the equality

$$\int_0^t \left(\int_0^s f(\tau) d\tau \right) ds = \int_0^t (t-s) f(s) ds, \tag{2.11}$$

which is satisfied by every function $f(s)$. \square

It will be useful to introduce the notation

$$P_i(t) = \int_0^t l_i(s) ds. \tag{2.12}$$

To obtain our results, we define the function

$$H(t) = \int_0^t \int_B \left(\rho w_i w_i + \frac{\alpha}{\beta} (t-s) (P_{i,j} - P_{j,i}) (P_{i,j} - P_{j,i}) \right) dV ds. \tag{2.13}$$

It is clear that

$$\frac{dH}{dt} = 2 \int_0^t \int_B \left(\rho w_i \dot{w}_i + \frac{\alpha}{2\beta} (P_{i,j} - P_{j,i}) (P_{i,j} - P_{j,i}) \right) dV ds. \tag{2.14}$$

The second derivative of the function H is given in the next lemma.

LEMMA 2.2. *The second derivative of the function H is*

$$\begin{aligned}
\frac{d^2 H}{dt^2} & = 4 \int_0^t \int_B \left[\rho w_i \dot{w}_i + \frac{\alpha}{\beta} (t-s) (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) \right] dV ds \\
& \quad - 4 \int_0^t \int_B (t-s) \left[(F_i \dot{w}_i + S_i l_i) + \alpha K \left(\dot{w}_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + \dot{w}_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right. \\
& \quad \left. + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) \right] dV ds \\
& \quad + 2 \int_B \left[(F_i w_i + Q_i l_i) + \alpha K \left(w_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + w_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right] dV \\
& \quad + 2 \int_B \left[K \alpha \left(l_1 \left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + l_2 u_{2,1}^{(2)} + l_3 u_{3,1}^{(2)} \right) \right. \\
& \quad \left. - K \alpha (l_1 (-f_{3,3} - f_{2,2}) + l_2 f_{2,1} + l_3 f_{3,1}) \right] dV,
\end{aligned} \tag{2.15}$$

where

$$Q_i(t) = \int_0^t S_i(s) ds. \quad (2.16)$$

PROOF. A direct differentiation gives

$$\frac{d^2 H}{dt^2} = 2 \int_0^t \int_B \rho \dot{w}_i \ddot{w}_i dV ds + 2 \int_0^t \int_B \left[\rho w_i \ddot{w}_i + \frac{\alpha}{\beta} (P_{i,j} - P_{j,i}) (l_{i,j} - l_{j,i}) \right] dV ds. \quad (2.17)$$

Now, we make some calculations to determine the evolution of the second integral. If we multiply (2.2) by w_i , and integrate over B , we obtain

$$\begin{aligned} & \int_B \rho w_i \ddot{w}_i dV \\ &= - \int_B \left[\mu w_{i,j} w_{i,j} + (\lambda + \mu) w_{i,i} w_{j,j} - \alpha H^1 (w_2 (l_{2,1} - l_{1,2}) + w_3 (l_{3,1} - l_{1,3})) \right] dV \\ & \quad + \int_B \left[F_i w_i + \alpha K \left(w_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + w_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right] dV. \end{aligned} \quad (2.18)$$

If we integrate (2.3) with respect to the time parameter, multiply it by l_i , and integrate over B , we obtain

$$\begin{aligned} & \frac{\alpha}{\beta} \int_B l_i l_i dV \\ &= - \int_B \left[\alpha H^1 (w_2 (l_{2,1} - l_{1,2}) + w_3 (l_{3,1} - l_{1,3})) - \frac{\alpha}{\beta} (P_{i,j} - P_{j,i}) (l_{i,j} - l_{j,i}) \right] dV \\ & \quad + \int_B \left[Q_i l_i + K \alpha \left(l_1 \left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + l_2 u_{2,1}^{(2)} + l_3 u_{3,1}^{(2)} \right) \right. \\ & \quad \left. - K \alpha (l_1 (-f_{3,3} - f_{2,2}) + l_2 f_{2,1} + l_3 f_{3,1}) \right] dV. \end{aligned} \quad (2.19)$$

It follows that

$$\begin{aligned} & \int_B \left(\rho w_i \ddot{w}_i + \frac{\alpha}{\beta} (P_{i,j} - P_{j,i}) (l_{i,j} - l_{j,i}) \right) dV \\ &= - \int_B \left[\mu w_{i,j} w_{i,j} + (\lambda + \mu) w_{i,i} w_{j,j} + \frac{\alpha}{\beta} l_i l_i \right] dV \\ & \quad + \int_B \left[(F_i w_i + Q_i l_i) + \alpha K \left(w_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + w_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right] dV \quad (2.20) \\ & \quad + \int_B \left[K \alpha \left(l_1 \left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + l_2 u_{2,1}^{(2)} + l_3 u_{3,1}^{(2)} \right) \right. \\ & \quad \left. - K \alpha (l_1 (-f_{3,3} - f_{2,2}) + l_2 f_{2,1} + l_3 f_{3,1}) \right] dV. \end{aligned}$$

Then, we obtain

$$\begin{aligned}
\frac{d^2 H}{dt^2} &= 4 \int_0^t \int_B \rho \dot{w}_i \dot{w}_i dV ds \\
&\quad - 2 \int_0^t \int_B \left[\mu w_{i,j} w_{i,j} + (\lambda + \mu) w_{i,i} w_{j,j} + \frac{\alpha}{\beta} l_i l_i + \rho \dot{w}_i \dot{w}_i \right] dV ds \\
&\quad + 2 \int_B \left[(F_i w_i + Q_i l_i) + \alpha K \left(w_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + w_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right] dV \\
&\quad + 2 \int_B \left[K \alpha \left(l_1 \left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + l_2 u_{2,1}^{(2)} + l_3 u_{3,1}^{(2)} \right) \right. \\
&\quad \left. - K \alpha (l_1 (-f_{3,3} - f_{2,2}) + l_2 f_{2,1} + l_3 f_{3,1}) \right] dV.
\end{aligned} \tag{2.21}$$

[Lemma 2.2](#) is a consequence of [Lemma 2.1](#) and equality (2.21). \square

We now state a lemma concerning the behaviour of the magnetic field, which will also be used in the next section.

LEMMA 2.3. *There exist three positive constants A , B^* , and C^* such that*

$$\int_0^t \int_B l_i l_i dV ds \leq \int_0^{t_1} \int_B [A S_i S_i + B^* \rho \dot{w}_i \dot{w}_i + C^* K^2] dV ds, \tag{2.22}$$

for $t \leq t_1$.

PROOF. In view of (2.3), we have

$$\begin{aligned}
\int_0^t \int_B l_i l_i dV ds &= \frac{1}{\beta} \int_0^t \int_B \beta l_i l_i dV ds \\
&= -\frac{1}{\beta} \int_0^t \int_B \frac{\partial}{\partial s} [(t-s) \beta l_i l_i] dV ds \\
&\quad + \frac{2}{\beta} \int_0^t \int_B (t-s) \beta l_i \frac{\partial l_i}{\partial s} dV ds \\
&= \frac{2}{\beta} \int_0^t \int_B (t-s) \left[R_i l_i - (l_{i,j} - l_{j,i})(l_{i,j} - l_{j,i}) \right. \\
&\quad \left. - \beta H^{(1)} (\dot{w}_2 (l_{2,1} - l_{1,2}) + \dot{w}_3 (l_{3,1} - l_{1,3})) \right. \\
&\quad \left. - \beta K (\dot{u}_2^{(2)} (l_{2,1} - l_{1,2}) + \dot{u}_3^{(2)} (l_{3,1} - l_{1,3})) \right] dV ds.
\end{aligned} \tag{2.23}$$

The use of the arithmetic-geometric mean inequality leads to the following estimates:

$$\begin{aligned}
 \int_0^t \int_B (t-s) R_i l_i dV ds &\leq \frac{\epsilon_1}{2} \int_0^t \int_B t_1 R_i R_i dV ds + \frac{1}{2\epsilon_1} \int_0^t \int_B t_1 l_i l_i dV ds, \\
 \int_0^t \int_B (t-s) (\dot{w}_2(l_{2,1} - l_{1,2}) + \dot{w}_3(l_{3,1} - l_{1,3})) dV ds \\
 &\leq \frac{\epsilon_2}{2} \int_0^t \int_B t_1 (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) dV ds + \frac{1}{2\epsilon_2} \int_0^t \int_B t_1 \dot{w}_i \dot{w}_i dV ds, \quad (2.24) \\
 \int_0^t \int_B (t-s) K (\dot{u}_2^{(2)}(l_{2,1} - l_{1,2}) + \dot{u}_3^{(2)}(l_{3,1} - l_{1,3})) dV ds \\
 &\leq \frac{\epsilon_3}{2} \int_0^t \int_B t_1 K^2 dV ds + \frac{1}{2\epsilon_3} \int_0^t \int_B t_1 \dot{u}_i^{(2)} (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) dV ds,
 \end{aligned}$$

where ϵ_1 , ϵ_2 , and ϵ_3 are arbitrary positive constants.

If we assume that $\dot{u}_i^{(2)}$ is uniformly bounded on the interval $[0, t_1]$, we can make a suitable choice of the parameters ϵ_i ($i = 1, 2, 3$) to obtain the estimate (2.22), where A , B^* , and C^* can be easily computed. \square

3. Continuous dependence. In this section, we obtain continuous dependence and structural stability results. We assume that the functions

$$\sup_B |h_{i,j}^{(2)}|^2, \quad \sup_B |\dot{u}_{i,j}^{(2)}|^2, \quad \sup_B |u_{i,j}^{(2)} - f_{i,j}|^2, \quad (3.1)$$

are uniformly bounded by a constant M .

Here, we introduce a family of functions

$$H_\omega(t) = H(t) + \omega, \quad (3.2)$$

where ω is an arbitrary positive constant.

LEMMA 3.1. *Let*

$$\omega = \int_0^{t_1} \int_B (F_i F_i + 2K^2 + S_i S_i + Q_i Q_i) dV ds. \quad (3.3)$$

Then, there exists a positive constant ξ such that

$$H_\omega \frac{d^2 H_\omega}{dt^2} - \left(\frac{dH_\omega}{dt} \right)^2 \geq -\xi H_\omega^2, \quad (3.4)$$

for $t \leq t_1$.

PROOF. From the definition of H_ω and [Lemma 2.2](#), it follows that

$$\begin{aligned}
 H_\omega \frac{d^2 H_\omega}{dt^2} - \left(\frac{dH_\omega}{dt} \right)^2 &= 4N^2 + 4\omega \int_0^t \int_B \left(\rho \dot{w}_i \dot{w}_i + (t-s) \frac{\alpha}{\beta} (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) \right) dV ds \\
 &\quad - H_\omega \left(4 \int_0^t \int_B (t-s) \left[(F_i \dot{w}_i + S_i l_i) + \alpha K \left(\dot{w}_2 (h_{2,1}^{(2)} - h_{1,2}^{(2)}) + \dot{w}_3 (h_{3,1}^{(2)} - h_{1,3}^{(2)}) \right) \right. \right. \\
 &\quad \left. \left. + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) \right] dV ds \right. \\
 &\quad \left. + 2 \int_0^t \int_B \left[(F_i w_i + Q_i l_i) + \alpha K \left(w_2 (h_{2,1}^{(2)} - h_{1,2}^{(2)}) + w_3 (h_{3,1}^{(2)} - h_{1,3}^{(2)}) \right) \right] dV \right. \\
 &\quad \left. + 2 \int_B \left[K \alpha \left(l_1 \left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + l_2 u_{2,1}^{(2)} + l_3 u_{3,1}^{(2)} \right) \right. \right. \\
 &\quad \left. \left. - K \alpha (l_1 (-f_{3,3} - f_{2,2}) + l_2 f_{2,1} + l_3 f_{3,1}) \right] dV ds \right), \tag{3.5}
 \end{aligned}$$

where

$$\begin{aligned}
 N^2 &= I_1 I_2 - I_3^2, \\
 I_1 &= \int_0^t \int_B \left(\rho w_i w_i + (t-s) \frac{\alpha}{\beta} (P_{i,j} - P_{j,i}) (P_{i,j} - P_{j,i}) \right) dV ds, \\
 I_2 &= \int_0^t \int_B \left[\rho \dot{w}_i \dot{w}_i + (t-s) \frac{\alpha}{\beta} (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) \right] dV ds, \\
 I_3 &= \int_0^t \int_B \left(\rho w_i \dot{w}_i + (t-s) \frac{\alpha}{\beta} (P_{i,j} - P_{j,i}) (l_{i,j} - l_{j,i}) \right) dV ds. \tag{3.6}
 \end{aligned}$$

In view of the Schwarz inequality, we have $N^2 \geq 0$.

Now, we estimate some integrals which are on the right-hand side of equality (3.5). After some uses of the Hölder inequality and inequality (1.6), we can obtain the existence of constants a_i such that

$$\begin{aligned}
 \int_0^t \int_B (t-s) F_i \dot{w}_i dV ds &\leq a_1 \left(\int_0^t \int_B \rho \dot{w}_i \dot{w}_i dV ds \right)^{1/2} \left(\int_0^t \int_B F_i F_i dV ds \right)^{1/2}, \\
 \int_0^t \int_B (t-s) S_i l_i dV ds &\leq a_2 \left(\int_0^t \int_B (t-s) (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) dV ds \right)^{1/2} \\
 &\quad \times \left(\int_0^t \int_B S_i S_i dV ds \right)^{1/2}, \\
 \int_0^t \int_B (t-s) \alpha K \left(\dot{w}_2 (h_{2,1}^{(2)} - h_{1,2}^{(2)}) + \dot{w}_3 (h_{3,1}^{(2)} - h_{1,3}^{(2)}) \right) dV ds \\
 &\leq a_3 \left(\int_0^t \int_B \rho \dot{w}_i \dot{w}_i dV ds \right)^{1/2} \left(\int_0^t \int_B K^2 dV ds \right)^{1/2},
 \end{aligned}$$

$$\begin{aligned}
& \int_0^t \int_B (t-s) K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) dV ds \\
& \leq a_4 \left(\int_0^t \int_B (t-s) (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i}) dV ds \right)^{1/2} \\
& \quad \times \left(\int_0^t \int_B K^2 dV ds \right)^{1/2}.
\end{aligned} \tag{3.7}$$

From (3.7), it follows that

$$\begin{aligned}
& \int_0^t \int_B (t-s) \left[(F_i \dot{w}_i + S_i l_i) + \alpha K \left(\dot{w}_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + \dot{w}_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right. \\
& \quad \left. + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) \right] dV ds \\
& \leq D \left(\int_0^t \int_B (\rho \dot{w}_i \dot{w}_i dV ds + (t-s) (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i})) dV ds \right)^{1/2} \\
& \quad \times \left(\int_0^t \int_B (F_i F_i + S_i S_i + K^2) dV ds \right)^{1/2},
\end{aligned} \tag{3.8}$$

where D is an easily computable constant that depends on the constitutive coefficients, the initial conditions, the time t_1 , and the domain.

The arithmetic-geometric mean inequality implies that

$$\begin{aligned}
& 4H_\omega \int_0^t \int_B (t-s) \left[(F_i \dot{w}_i + S_i l_i) + \alpha K \left(\dot{w}_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + \dot{w}_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right. \\
& \quad \left. + K \alpha \left(l_1 \left(-\dot{u}_{2,2}^{(2)} - \dot{u}_{3,3}^{(2)} \right) + l_2 \dot{u}_{2,1}^{(2)} + l_3 \dot{u}_{3,1}^{(2)} \right) \right] dV ds \\
& \leq D^2 H_\omega^2 + 4 \left(\int_0^t \int_B (\rho \dot{w}_i \dot{w}_i dV ds + (t-s) (l_{i,j} - l_{j,i}) (l_{i,j} - l_{j,i})) dV ds \right) \\
& \quad \times \left(\int_0^t \int_B (F_i F_i + S_i S_i + K^2) dV ds \right).
\end{aligned} \tag{3.9}$$

Similarly, we can obtain several constants b_i such that

$$\int_0^t \int_B F_i w_i dV ds \leq b_1 \left(\int_0^t \int_B \rho w_i w_i dV ds \right)^{1/2} \left(\int_0^t \int_B F_i F_i dV ds \right)^{1/2}, \tag{3.10}$$

$$\begin{aligned}
& \int_0^t \int_B Q_i l_i dV ds \leq \left(\int_0^t \int_B l_i l_i dV ds \right)^{1/2} \left(\int_0^t \int_B Q_i Q_i dV ds \right)^{1/2}, \\
& \int_0^t \int_B \alpha K \left(w_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + w_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) dV ds
\end{aligned} \tag{3.11}$$

$$\leq b_3 \left(\int_0^t \int_B \rho w_i w_i dV ds \right)^{1/2} \left(\int_0^t \int_B K^2 dV ds \right)^{1/2},$$

$$\begin{aligned}
& \int_0^t \int_B \left[K \alpha \left(l_1 \left(\left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) - (-f_{3,3} - f_{2,2}) \right) \right. \right. \\
& \quad \left. \left. + l_2 \left(u_{2,1}^{(2)} - f_{2,1} \right) + l_3 \left(u_{3,1}^{(2)} - f_{3,1} \right) \right) \right] dV ds
\end{aligned} \tag{3.12}$$

$$\leq b_4 \left(\int_0^t \int_B l_i l_i dV ds \right)^{1/2} \left(\int_0^t \int_B K^2 dV ds \right)^{1/2}.$$

Thus,

$$\begin{aligned} 2H_\omega \int_0^t \int_B \left(F_i w_i + \alpha K \left(w_2 \left(h_{2,1}^{(2)} - h_{1,2}^{(2)} \right) + w_3 \left(h_{3,1}^{(2)} - h_{1,3}^{(2)} \right) \right) \right) dV ds \\ \leq EH_\omega^2 + H_\omega \left(\int_0^t \int_B (F_i F_i + K^2) dV ds \right). \end{aligned} \quad (3.13)$$

In (3.13), E is a constant that can be computed in terms of the constitutive coefficients, the initial conditions, the time t_1 , and the domain.

In view of the estimates (2.22), (3.10), and (3.13), we can see that

$$\begin{aligned} 2H_\omega \int_0^t \int_B Q_i l_i dV ds \\ \leq 2H_\omega \left(\int_0^t \int_B (AS_i S_i + C^* K^2) dV ds \right)^{1/2} \left(\int_0^t \int_B Q_i Q_i dV ds \right)^{1/2} \\ + 2H_\omega \left(B^* \int_0^t \int_B \rho \dot{w}_i \dot{w}_i dV ds \right)^{1/2} \left(\int_0^t \int_B Q_i Q_i dV ds \right)^{1/2}, \\ 2H_\omega \int_0^t \int_B \left[K \alpha \left(l_1 \left(\left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + (f_{3,3} + f_{2,2}) \right) \right. \right. \\ \left. \left. + l_2 \left(u_{2,1}^{(2)} - f_{2,1} \right) + l_3 \left(u_{3,1}^{(2)} - f_{3,1} \right) \right) \right] dV ds \\ \leq N^* H_\omega \left(\int_0^t \int_B (AS_i S_i + C^* K^2) dV ds \right)^{1/2} \left(\int_0^t \int_B K^2 dV ds \right)^{1/2} \\ + N^* H_\omega \left(B^* \int_0^t \int_B \rho \dot{w}_i \dot{w}_i dV ds \right)^{1/2} \left(\int_0^t \int_B K^2 dV ds \right)^{1/2}. \end{aligned} \quad (3.14)$$

Again, N^* is an easily computable positive constant. If we use the arithmetic-geometric mean inequality, we obtain

$$\begin{aligned} 2H_\omega \int_0^t \int_B \left(Q_i l_i + \left[K \alpha \left(l_1 \left(\left(-u_{2,2}^{(2)} - u_{3,3}^{(2)} \right) + (f_{3,3} + f_{2,2}) \right) \right. \right. \right. \\ \left. \left. + l_2 \left(u_{2,1}^{(2)} - f_{2,1} \right) + l_3 \left(u_{3,1}^{(2)} - f_{3,1} \right) \right) \right] \right) dV ds \\ \leq 4F^2 H_\omega^2 + \left(\int_0^t \int_B ((C^* + 1)K^2 + AS_i S_i + Q_i Q_i) dV ds \right) \\ + \frac{B^* F^2}{2} H_\omega^2 + 4 \left(\int_0^t \int_B \rho \dot{w}_i \dot{w}_i dV ds \right) \left(\int_0^t \int_B (K^2 + Q_i Q_i) dV ds \right), \end{aligned} \quad (3.15)$$

where F can be computed in terms of the constitutive coefficients, the initial conditions, the time t_1 , and the domain.

From (3.5), (3.9), (3.13), and (3.15), we conclude that we can explicitly determine a constant ξ satisfying (3.4). \square

THEOREM 3.2. *Let (w_i, l_i) be a solution of the problem determined by system (2.2), (2.3) with initial conditions (2.4), and boundary conditions (2.5). Then, there*

exists a positive constant M^* such that

$$H_\omega(t) \leq M^* \left(\int_0^{t_1} \int_B (F_i F_i + 2K^2 + S_i S_i + Q_i Q_i) dV ds \right)^{1-t/t_1}, \quad (3.16)$$

for all $t \leq t_1$, where ω is given in (3.3).

PROOF. If we define the function

$$P(t) = \ln \left[H_\omega(t) \exp \left(\frac{1}{2} \xi t^2 \right) \right], \quad (3.17)$$

then

$$\frac{d^2 P}{dt^2} = H_\omega^{-2} \left(H_\omega \frac{d^2 H_\omega}{dt^2} - \left(\frac{dH_\omega}{dt} \right)^2 + \xi H_\omega^2 \right). \quad (3.18)$$

Thus, according to (3.4),

$$\frac{d^2 P}{dt^2} \geq 0. \quad (3.19)$$

Jensen's inequality gives

$$H_\omega(t) \leq [H_\omega(0)]^{1-t/t_1} [H_\omega(t_1)]^{t/t_1} \exp \left[\frac{1}{2} \xi t(t_1 - t) \right], \quad (3.20)$$

for $t \in [0, t_1]$. The theorem is proved taking

$$M^* = \max(1, H_\omega(t_1)) \exp \left[\frac{1}{8} \xi t_1^2 \right]. \quad (3.21)$$

□

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