LAMB'S PLANE PROBLEM IN A THERMO-ELASTIC
MICROPOLAR MEDIUM WITH STRETCH

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ABSTRACT. A study is made of the Lamb plane problem in a thermo-elastic micropolar
medium with the effect of stretch. The problem is solved for an arbitrary, normal
load distribution by using the double Fourier transform. The displacement components,
force stress, couple stress, vector first moment and the temperature field are deter-
mined for a half space subjected to an arbitrary normal load. Two special cases of a
horizontal force and a torque which are oscillating with a frequency ω have been
investigated. It is shown that results of this analysis reduce to those without
stretch.

KEY WORDS AND PHRASES. Lamb's problem, Fourier transform, and micropolar medium.

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1. INTRODUCTION.

Eringen and Suhubi [1] have developed a general theory of linear and nonlinear
micro-elastic continua. This theory contains the Cosserat continuum and the interme-
diate couple stress theories as special cases. In a subsequent paper [2], Eringen
recapitulated his work and renamed his theory as micropolar elasticity. The
micropolar theory essentially deals with such materials whose constituents are
dumbbell type molecules and are allowed to rotate independently without stretch.
Later on, Eringen [3-4] extended his work to include the effect of axial stretch
during the rotation of molecules and developed theories for both micropolar elastic
solids with stretch and micropolar fluids with stretch. The mechanical model
underlying the theory of micropolar elastic solids with stretch can be envisioned as
an elastic medium composed of a large number of short springs. These springs possess
average inertia and can deform in axial directions.

Lamb's problem [5] has been investigated extensively by several researchers in different elastic media with various kinds of loading. In particular, Nowacki and Nowacki [6] have studied the Lamb problem in micropolar elastic media. Recently, Chadha [7] has investigated the same problem in micropolar elastic media, and discussed wave propagation in a semi-infinite micropolar elastic solid due to loading at the plane boundary of semi-half space. Acharya and Sengupta [8] have recently studied Lamb's problem in a thermo-elastic medium under the influence of temperature. They have examined the longitudinal and transverse thermo-elastic wave propagation in a micropolar semi-infinite space bounded by a plane in which a normal loading is applied.

In spite of these studies, no attention is given to Lamb's problem in thermo-micropolar elastic half-space with stretch. The main purpose of this paper is to investigate the problem with the assumption that the heat is radiated from the free plane boundary surface of the semi-infinite space and the maximum temperature difference across the surface is always small. The displacement components, force stress, couple stress, vector first moment and the temperature field are determined for the half-space subjected to an arbitrary normal load. Two special cases of a horizontal force and a torque which are harmonic in time have been discussed. The problem is solved by the double Fourier transform method.

2.   THE FORMULATION OF THE PROBLEM AND THE BOUNDARY CONDITIONS

We consider a homogeneous micropolar elastic semi-infinite space with stretch under the influence of temperature. We assume that there is a uniform stretch in the x-direction only and a loading $g(x,t)$ normal to the free boundary surface $z = 0$. Further, we assume that the micropolar semi-space is free to exchange heat within the region $z > 0$; and prior to the appearance of any disturbance, both media are everywhere at the constant absolute temperature T_0.

We consider the two-dimensional problem so that the displacement and rotation are independent of the y coordinate. Thus we may write $\underline{u} = (u_1, 0, u_3)$ and $\underline{\omega} = (0, \omega_2, 0)$. The displacements are related to the displacement potentials $\phi(x, z, t)$ and $\psi(x, z, t)$ as follows:

$$u_1 = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad u_3 = \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial x} \qquad (2.1ab)$$

so that

$$e = \nabla^2 \phi, \qquad \nabla^2 \psi = \frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x} \qquad (2.2ab)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \qquad \text{and} \qquad e = \frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}. \qquad (2.3ab)$$

We follow Eringen [4] and Nowacki [9] to write down the basic field equations in a thermo-micropolar elastic solid medium with stretch and without body forces and body moments. These field equations are

$$(\mu+\alpha)\nabla^2 \underline{u} + (\lambda+\mu-\alpha) \text{ grad div } \underline{u} + 2 \alpha \text{ rot } \underline{\omega} - \nu \text{ grad } \theta = \rho \frac{\partial^2 \underline{u}}{\partial t^2}, \qquad (2.4)$$

$$(\gamma+\varepsilon)\nabla^2 \underline{\omega} + (\gamma+\beta-\varepsilon) \text{ grad div } \underline{\omega} - 4 \alpha \underline{\omega} + 2 \alpha \text{ rot } \underline{u} = J \frac{\partial^2 \underline{\omega}}{\partial t^2}, \qquad (2.5)$$

$$\alpha_0 \ \nabla^2\phi \ - \ \eta_0\phi \ = \ \frac{J}{2}\frac{\partial^2\phi}{\partial t^2} \ , \tag{2.6}$$

where λ, μ, α, β, γ, ε, α_0, η_0 are material constants, ρ is the density of the material, J is the rotational inertia, $\nu = (3\lambda+2\mu) \ \alpha_t$, α_t is the coefficient of linear expansion of the solid, $\theta = T-T_0$ = absolute temperature minus the initial absolute temperature T_0.

Using the values of $\underline{u}$ and $\underline{\omega}$ in equations (2.4)-(2.6) we get

$$(\mu+\alpha) \ \nabla^2 \ u_1 \ + \ (\lambda+\mu-\alpha) \ \frac{\partial e}{\partial x} \ - \ 2\alpha \ \frac{\partial\omega_2}{\partial z} \ - \ \nu \ \frac{\partial\theta}{\partial x} \ = \ \rho \ \frac{\partial^2 u_1}{\partial t^2}, \tag{2.7}$$

$$(\mu+\alpha) \ \nabla^2 \ u_3 \ + \ (\lambda+\mu-\alpha) \ \frac{\partial e}{\partial z} \ + \ 2\alpha \ \frac{\partial\omega_2}{\partial z} \ - \ \nu \ \frac{\partial\theta}{\partial z} \ = \ \rho \ \frac{\partial^2 u_3}{\partial t^2}, \tag{2.8}$$

$$(\gamma+\varepsilon) \ \nabla^2 \ \omega_2 \ - \ 4\alpha\omega_2 \ + \ 2\alpha \ (\frac{\partial u_1}{\partial z} \ - \ \frac{\partial u_3}{\partial x}) \ = \ J \ \frac{\partial^2\omega_2}{\partial t^2}, \tag{2.9}$$

$$\alpha_0 \ \nabla^2\phi \ - \ \eta_0\phi \ = \ \frac{J}{2}\frac{\partial^2\phi}{\partial t^2} \ . \tag{2.10}$$

The temperature field $\theta(x,z,t)$ satisfies Fourier's Law of heat conduction, which in the present case can be written as

$$\kappa\nabla^2\theta \ = \ \rho C \ \frac{\partial^2\theta}{\partial t^2} \ + \ T_0 \ \nu \ \frac{\partial}{\partial t} \ (\nabla^2\phi), \tag{2.11}$$

where κ is thermal conductivity and C is the specific heat at constant strain.

Using (2.1ab)-(2.3ab) and (2.11) in equations (2.7)-(2.10), we obtain

$$(\nabla^2 \ - \ \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2})\phi \ - \ q\theta \ = \ 0, \tag{2.12}$$

$$(\nabla^2 \ - \ \frac{1}{c_3^2}\frac{\partial}{\partial t})\theta \ - \ r \ \frac{\partial}{\partial t} \ (\nabla^2\phi) \ = \ 0, \tag{2.13}$$

$$(\nabla^2 \ - \ \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2})\psi \ - \ p\omega_2 \ = \ 0, \tag{2.14}$$

$$(\nabla^2 \ - \ \gamma_1^2 \ - \ \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2})\omega_2 \ + \ s \ \nabla^2\psi \ = \ 0, \tag{2.15}$$

$$(\nabla^2 \ - \ \gamma_2^2 \ - \ \frac{1}{c_5^2}\frac{\partial^2}{\partial t^2})\Phi \ = \ 0, \tag{2.16}$$

where

$$c_1^2 = \frac{\lambda+2\mu}{\rho}, \ c_2^2 = \frac{\mu+\alpha}{\rho}, \ c_3^2 = \frac{K}{\rho C}, \ c_4^2 = \frac{\gamma+\varepsilon}{J}, \ c_5^2 = \frac{2\alpha_0}{J} \tag{2.17abcde}$$

$$\gamma_1^2 = \frac{4\alpha}{\gamma+\epsilon}, \quad \gamma_2^2 = \frac{\eta_0}{\alpha_0}, \quad p = \frac{2\alpha}{\mu+\alpha}, \quad q = \frac{\nu}{\lambda+2\mu}, \tag{2.18abcd}$$

$$r = \frac{T_0\nu}{\kappa}, \quad s = \frac{2\alpha}{\gamma+\epsilon}. \tag{2.19ab}$$

We next eliminate ϕ or θ from equations (2.12)-(2.13), and ψ or ω_2 from (2.14)-(2.15) to obtain the following partial differential equations:

$$\left[(\nabla^2 - \frac{1}{c_1^2}\frac{\partial^2}{\partial t^2})(\nabla^2 - \frac{1}{c_3^2}\frac{\partial}{\partial t}) - \tau^2\frac{\partial}{\partial t}\nabla^2\right](\phi,\theta) = 0, \tag{2.20}$$

$$\left[(\nabla^2 - \frac{1}{c_2^2}\frac{\partial^2}{\partial t^2})(\nabla^2 - \gamma_1^2 - \frac{1}{c_4^2}\frac{\partial^2}{\partial t^2}) + \zeta^2\nabla^2\right](\psi,\omega_2) = 0, \tag{2.21}$$

where $\tau^2 = qr$, $\zeta^2 = ps$.

Following Eringen [4] and Nowacki [9] the stress tensor σ_{ji} and the couple stress tensor μ_{ji} are given by

$$\sigma_{ji} = (\lambda u_{k,k}-\nu\theta)\delta_{ij} + (\mu-\alpha)(u_{i,j} + u_{j,i}) + 2\alpha(u_{i,j} - \epsilon_{kji}\omega_k), \tag{2.22}$$

$$\mu_{ji} = \beta_0\epsilon_{kji}\phi_{,k} + \beta\omega_{k,k}\delta_{ij} + (\gamma-\epsilon)\omega_{j,i} + (\gamma+\epsilon)\omega_{i,j}, \tag{2.23}$$

$$\beta_j = \alpha_0\phi_{,j} + \frac{1}{3}\beta_0\epsilon_{kji}\omega_{k,i}, \tag{2.24}$$

where ϵ_{jki} is unit antisymmetric tensor, β_j is the vector first moment and $i,j,k=1,2,3$. These expressions in the present case reduce to the form

$$\sigma_{33} = 2\mu(\frac{\partial^2\phi}{\partial z^2} - \frac{\partial^2\psi}{\partial x\partial z}) + \lambda\nabla^2\phi - \nu\theta, \tag{2.25}$$

$$\sigma_{31} = \mu\left[2\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial z^2} - \frac{\partial^2\psi}{\partial x^2}\right] + \alpha(\nabla^2\psi - 2\omega_2), \tag{2.26}$$

$$\mu_{32} = (\gamma+\epsilon)\frac{\partial\omega_2}{\partial z} - \beta_0\frac{\partial\phi}{\partial x}, \tag{2.27}$$

$$\beta_3 = \alpha_0\frac{\partial\phi}{\partial z} + \frac{1}{3}\beta_0\frac{\partial\omega_2}{\partial x}. \tag{2.28}$$

3.   BOUNDARY CONDITIONS.

In view of the normal loading of magnitude $g(x,t)$ applied on $z = 0$, the boundary conditions are given by

$$\sigma_{33} = -g(x,t), \quad \sigma_{31} = 0, \quad \mu_{32} = 0, \quad \beta_3 = 0, \text{ at } z = 0. \tag{3.1abcd}$$

In view of the assumption that the temperature difference across the free surface is always small, the linearized form of the radiation condition is valid on the boundary $z = 0$ so that

$$\frac{\partial\theta}{\partial z} + h\theta = 0 \qquad \text{on } z = 0. \tag{3.2}$$

Further, if we assume that the loading function $g(x,t)$ is bounded and finite on $z = 0$, then $\phi, \psi, \theta, \omega_2$ and Φ vanish at infinity.

4. SOLUTION OF THE PROBLEM.

We solve the above equations (2.16), (2.20) and (2.21) by using the double Fourier transform defined as follows:

$$\overline{f}(k,z,n) = \frac{1}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,z,t) \; e^{i(kx+nt)} \; dxdt, \tag{4.1}$$

where the inverse transform is given by

$$f(x,z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \overline{f}(k,z,n) \; e^{-i(kx+nt)} \; dkdn. \tag{4.2}$$

Thus the equations reduce to the form

$$(\frac{d^2}{dz^2} - \lambda_1^2)(\frac{d^2}{dz^2} - \lambda_2^2)(\overline{\phi},\overline{\theta}) = 0, \tag{4.3}$$

$$(\frac{d^2}{dz^2} - \lambda_3^2)(\frac{d^2}{dz^2} - \lambda_4^2)(\overline{\psi},\overline{\omega}_2) = 0, \tag{4.4}$$

$$(\frac{d^2}{dz^2} - \lambda_5^2)\Phi = 0, \tag{4.5}$$

where

$$\lambda_1^2 + \lambda_2^2 = (2k^2 - \frac{n^2}{c_1^2} - \frac{in}{c_3^2} - in\tau^2), \tag{4.6}$$

$$\lambda_1^2 \lambda_2^2 = (\frac{n^2}{c_1^2} - k^2)(\frac{in}{c_3^2} - k^2) - in\tau^2 k^2), \tag{4.7}$$

$$\lambda_3^2 + \lambda_4^2 = (2k^2 - \frac{n^2}{c_2^2} - \frac{n^2}{c_4^2} + \gamma_1^2 - k^2), \tag{4.8}$$

$$\lambda_3^2 \lambda_4^2 = (\frac{n^2}{c_2^2} - k^2)(\frac{n^2}{c_4^2} - k^2 - \gamma_1^2) - k^2\zeta^2, \tag{4.9}$$

$$\lambda_5^2 = k^2 + \gamma_2^2 - \frac{n^2}{c_5^2}. \tag{4.10}$$

In view of the boundary conditions at infinity, the bounded solutions of (4.3)-(4.5) assume the form

$$\overline{\phi} = Ae^{-\lambda_1 z} + Be^{-\lambda_2 z}, \tag{4.11}$$

$$\overline{\theta} = A_1 e^{-\lambda_1 z} + B_1 e^{-\lambda_2 z}, \tag{4.12}$$

$$\overline{\psi} = Ce^{-\lambda_3 z} + De^{-\lambda_4 z}, \tag{4.13}$$

$$\overline{\omega}_2 = C_1 e^{-\lambda_3 z} + D_1 e^{-\lambda_4 z}, \tag{4.14}$$

$$\overline{\Phi} = E \, e^{-\lambda_5 z}, \tag{4.15}$$

where

$$A_1 = \alpha_1 A, \; B_1 = \alpha_2 B, \; C_1 = \alpha_3 C, \; D_1 = \alpha_4 D, \tag{4.16abcd}$$

and

$$\alpha_j = \begin{bmatrix} \frac{1}{q} (\lambda_j^2 + \frac{n^2}{c_1^2} - k^2), & \text{for } j=1,2. \\[3mm] \frac{1}{p} (\lambda_j^2 + \frac{n^2}{c_2^2} - k^2), & \text{for } j=3,4. \end{bmatrix} \qquad (4.17ab)$$

It is assumed that $\text{Re}(\lambda_j) \geq 0$, $j=1,2,3,4,5$.

Applying the Fourier transform (4.1) to (3.1abcd)-(3.2) and using (2.25)-(2.28), it turns out that

$$2\mu\{\frac{d^2\bar{\phi}}{dz^2} + ik\frac{d\bar{\psi}}{dz}\} + \lambda(\frac{d^2}{dz^2} - k^2)\,\bar{\phi} - \nu\bar{\theta} = -\bar{g}\,(k,n), \qquad (4.18)$$

$$\mu\{k^2\bar{\psi} + \frac{d^2\bar{\psi}}{dz^2} - 2ik\frac{d\bar{\psi}}{dz}\} + \alpha(\frac{d^2\bar{\psi}}{dz^2} - k^2\bar{\psi} - 2\bar{\omega}_2) = 0, \qquad (4.19)$$

$$(\gamma+\epsilon)\frac{d\bar{\omega}_2}{dz} + i\beta_0 k\,\bar{\Phi} = 0, \qquad (4.20)$$

$$\alpha_0 \frac{d\bar{\Phi}}{dz} - \frac{1}{3} i\beta_0 k\,\bar{\omega}_2 = 0, \qquad (4.21)$$

$$\frac{d\bar{\theta}}{dz} + h\,\bar{\theta} = 0, \qquad (4.22)$$

where $\bar{g}(k,n)$ is the double Fourier transform of $g(x,t)$.

Substitution of (4.11) - (4.15) into (4.18) - (4.22) yields

$$q_1 A + q_2 B + q_3 C + q_4 D = -\bar{g}(k,n), \qquad (4.23)$$

$$p_1 A + p_2 B + p_3 C + p_4 D = 0, \qquad (4.24)$$

$$r_3 C + r_4 D + r_5 E = 0, \qquad (4.25)$$

$$t_3 C + t_4 D + t_5 E = 0, \qquad (4.26)$$

$$s_1 A + s_2 B = 0, \qquad (4.27)$$

where

$$q_j = \begin{bmatrix} \lambda_j^2(\lambda + 2\mu) - k^2\lambda - \alpha_j\nu), & j = 1,2, \\[3mm] -2i\,\mu k\lambda_j, & j = 3,4, \end{bmatrix} \qquad (4.28ab)$$

$$p_j = \begin{bmatrix} -2i\,\mu k\lambda_j, & j = 1,2, \\[3mm] \mu(k^2 + \lambda_j^2) + \alpha(\lambda_j^2 - k^2 - 2\alpha_j), & j = 3,4, \end{bmatrix} \qquad (4.29ab)$$

$$r_j = \begin{bmatrix} (\gamma+\varepsilon)\ \alpha_j\lambda_j, & j = 3,4, \\ \\ -i\beta_0 k, & j = 5, \end{bmatrix} \tag{4.30ab}$$

$$s_j = (\lambda_j - h)\ \alpha_j, \qquad j = 1,2, \tag{4.31}$$

$$t_j = \begin{bmatrix} i\beta_0 k\ \alpha_j, & j = 3,4, \\ \\ 3\alpha_0\lambda_j, & j = 5. \end{bmatrix} \tag{4.32ab}$$

Solving equations (4.23)-(4.27) for A,B,C,D and E we obtain

$$A = \frac{\Delta_1}{\Delta}\ \bar{g}\ (k,n), \quad B = \frac{\Delta_2}{\Delta}\ \bar{g}(k,n), \quad C = \frac{\Delta_3}{\Delta}\ \bar{g}(k,n). \tag{4.33abc}$$

$$D = \frac{\Delta_4}{\Delta}\ \bar{g}(k,n), \quad E = \frac{\Delta_5}{\Delta}\ \bar{g}(k,n), \tag{4.34ab}$$

where

$$\Delta = m_1(p_3 m_5 - q_3 m_4) - m_2(p_4 m_5 - q_4 m_4), \tag{4.35}$$

$$\Delta_1 = s_2(p_3 m_1 - p_4 m_2), \quad \Delta_2 = s_1(p_4 m_2 - p_3 m_1), \tag{4.36ab}$$

$$\Delta_3 = m_1 m_4, \quad \Delta_4 = -m_2 m_4, \quad \Delta_5 = m_3 m_4, \tag{4.37abc}$$

and

$$m_1 = (r_4 t_5 - r_5 t_4), \quad m_2 = (r_3 t_5 - r_5 t_3), \quad m_3 = (r_3 t_4 - r_4 t_3), \tag{4.38abc}$$

$$m_4 = (s_1 p_2 - s_2 p_1), \quad m_5 = (s_1 q_2 - s_2 q_1). \tag{4.39ab}$$

Using the Fourier inverse transformation (4.2) in (4.11) - (4.15) we obtain

$$\phi = \frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (Ae^{-\lambda_1 z} + Be^{-\lambda_2 z})e^{-i(kx + nt)}dkdn, \tag{4.40}$$

$$\theta = \frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\alpha_1 Ae^{-\lambda_1 z} + \alpha_2 Be^{-\lambda_2 z})e^{-i(kx + nt)}dkdn, \tag{4.41}$$

$$\psi = \frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (C\ e^{-\lambda_3 z} + D\ e^{-\lambda_4 z})e^{-i(kx + nt)}dkdn, \tag{4.42}$$

$$\omega_2 = \frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\alpha_3 C\ e^{-\lambda_3 z} + \alpha_4 D\ e^{-\lambda_4 z})e^{-i(kx + nt)}dkdn, \tag{4.43}$$

$$\phi = \frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\ e^{-\lambda_5 z}\ e^{-i(kx + nt)}dkdn, \tag{4.44}$$

Thus, using (4.40)-(4.44) we can obtain the displacement components, force stress, couple stress tensor, vector first moments, and the temperature field in the integral form

$$u_1 = -\frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_1^*(z,k,n)\ \bar{g}\ (k,n)e^{-i(kx + nt)}dkdn, \tag{4.45}$$

$$u_3 = -\frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_3^*(z,k,n)\ \bar{g}\ (k,n)e^{-i(kx + nt)}dkdn, \tag{4.46}$$

$$\omega_2 = \frac{1}{2\pi}\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \omega_2^*(z,k,n)\ \bar{g}\ (k,n)e^{-i(kx + nt)}dkdn, \tag{4.47}$$

$$\sigma_{33} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_{33}^{*}(z,k,n) \, \bar{g}(k,n) e^{-i(kx + nt)} dk dn, \tag{4.48}$$

$$\sigma_{31} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_{31}^{*}(z,k,n) \, \bar{g}(k,n) e^{-i(kx + nt)} dk dn, \tag{4.49}$$

$$\mu_{32} = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mu_{32}^{*}(z,k,n) \, \bar{g}(k,n) e^{-i(kx + nt)} dk dn, \tag{4.50}$$

$$\beta_3 \quad -\frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \beta_3^{*}(z,k,n) \, \bar{g}(k,n) e^{-i(kx + nt)} dk dn, \tag{4.51}$$

$$\Theta = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Theta^{*}(z,k,n) \, \bar{g}(k,n) e^{-i(kx + nt)} dk dn, \tag{4.52}$$

where

$$u_1^{*}(z,k,n) = \frac{1}{\Delta} \left[ik(\Delta_1 e^{-\lambda_1 z} + \Delta_2 e^{-\lambda_2 z}) + \lambda_3 \Delta_3 e^{-\lambda_3 z} + \lambda_4 \Delta_4 e^{-\lambda_4 z} \right], \tag{4.53}$$

$$u_3^{*}(z,k,n) = \frac{1}{\Delta} \left[\lambda_1 \Delta_1 e^{-\lambda_1 z} + \lambda_2 \Delta_2 e^{-\lambda_2 z} - ik(\Delta_3 e^{-\lambda_3 z} + \Delta_4 e^{-\lambda_4 z}) \right], \tag{4.54}$$

$$\omega_2^{*}(z,k,n) = \frac{1}{\Delta} \left[\alpha_3 \Delta_3 e^{-\lambda_3 z} + \alpha_4 \Delta_4 e^{-\lambda_4 z} \right], \tag{4.55}$$

$$\sigma_{33}^{*}(z,k,n) = \frac{1}{\Delta} \left[\{(\lambda + 2\mu)\lambda_1^2 - k^2\lambda - \nu\alpha_1\}\Delta_1 e^{-\lambda_1 z} \right.$$
$$+ \{(\lambda + 2\mu)\lambda_2^2 - k^2\lambda - \nu\alpha_2\}\Delta_2 e^{-\lambda_2 z} \tag{4.56}$$
$$\left. -2i\mu k(\lambda_3 \Delta_3 e^{-\lambda_3 z} + \lambda_4 \Delta_4 e^{-\lambda_4 z}) \right],$$

$$\sigma_{31}^{*}(z,k,n) = \frac{1}{\Delta} \left[2i\mu k(\lambda_1 \Delta_1 e^{-\lambda_1 z} + \lambda_2 \Delta_2 e^{-\lambda_2 z}) \right.$$
$$+ \{\mu(k^2 + \lambda_3^2) + \alpha(\lambda_3^2 - k^2 - 2\alpha_3)\}\Delta_3 e^{-\lambda_3 z} \tag{4.57}$$
$$\left. + \{\mu(k^2 + \lambda_4^2) + \alpha(\lambda_4^2 - k^2 - 2\alpha_4)\}\Delta_4 e^{-\lambda_4 z} \right],$$

$$\mu_{32}^{*}(z,k,n) = \frac{1}{\Delta} \left[(\gamma+\varepsilon)(\lambda_3 \alpha_3 \Delta_3 e^{-\lambda_3 z} + \alpha_4 \lambda_4 \Delta_4 e^{-\lambda_4 z}) - i\beta_0 k \Delta_5 e^{-\lambda_5 z} \right], \tag{4.58}$$

$$\beta_3^{*}(z,k,n) = \frac{1}{\Delta} \left[i\beta_0 k(\alpha_3 \Delta_3 e^{-\lambda_3 z} + \alpha_4 \Delta_4 e^{-\lambda_4 z}) + 3\alpha_0 \lambda_5 \Delta_5 e^{-\lambda_5 z} \right], \tag{4.59}$$

$$\theta^{*}(z,k,n) = \frac{1}{\Delta} \left[\alpha_1 \Delta_1 e^{-\lambda_1 z} \cdot \alpha_2 \Delta_2 e^{-\lambda_2 z} \right]. \tag{4.60}$$

5. PARTICULAR CASES:

(i) We consider a time periodic concentrated force acting at the origin in the direction of x-axis so that the loading function assumes the form

$$g(x,t) = F\delta(x) e^{-i\omega t}, \tag{5.1}$$

where F is the magnitude of the force, $\delta(x)$ is the Dirac function of distribution and ω is the frequency.

The double Fourier transform of g(x,t) is

$$\bar{g}(k,\eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F\delta(x)e^{-i\omega t}e^{i(kx+\eta t)}dxdt,$$

$$= \frac{F}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} e^{i(\eta-\omega)t}dt,$$

$$= \sqrt{(2\pi)}\ F\delta(\eta-\omega). \tag{5.2}$$

Thus from (4.45)-(4.60) with (5.2), we obtain

$$u_1 = -\frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [u_1^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}\ dk, \tag{5.3}$$

$$u_3 = -\frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [u_3^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}\ dk, \tag{5.4}$$

$$\omega_2 = \frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [\omega_2^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.5}$$

$$\sigma_{33} = \frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [\sigma_{33}^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.6}$$

$$\sigma_{31} = \frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [\sigma_{31}^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.7}$$

$$\mu_{32} = \frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [\mu_{32}^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.8}$$

$$\beta_3 = e - \frac{1}{3} \frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [\beta_3^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.9}$$

$$\theta = \frac{F}{\sqrt{(2\pi)}} e^{-i\omega t} \int_{-\infty}^{\infty} [\theta^*(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk. \tag{5.10}$$

If we neglect the stretch effect, we recover the corresponding expressions for the displacements, stresses, and the temperature field in the form

$$u_1 = -\frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_1(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.11}$$

$$u_3 = -\frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_2(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.12}$$

$$\omega_2 = \frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.13}$$

$$\sigma_{33} = \frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.14}$$

$$\sigma_{31} = \frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.15}$$

$$\sigma_{32} = -\frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_1(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.16}$$

$$\theta = \frac{F\ e^{-i\omega t}}{\sqrt{(2\pi)}} \int_{-\infty}^{\infty} [M_3(z,k,\eta)]_{\eta=\omega}\ e^{-ikx}dk, \tag{5.17}$$

where

$$M_1(z,k,n) = \frac{1}{\Delta^*} \ [ik(\Delta_1^* e^{-\lambda_1 z} + \Delta_2^* e^{-\lambda_2 z}) + \lambda_3 \Delta_3^* e^{-\lambda_3 z} + \lambda_4 \Delta_4^* e^{-\lambda_4 z}], \tag{5.18}$$

$$M_2(z,k,n) = \frac{1}{\Delta^*} \ [\lambda_1 \Delta_1^* e^{-\lambda_1 z} + \lambda_2 \Delta_2^* e^{-\lambda_2 z} - ik \ (\Delta_3^* e^{-\lambda_3 z} + \Delta_4^* e^{-\lambda_4 z})], \tag{5.19}$$

$$M_3(z,k,n) = \frac{1}{\Delta^*} \ [\alpha_3 \Delta_3^* e^{-\lambda_3 z} + \alpha_4 \Delta_4^* e^{-\lambda_4 z}], \tag{5.20}$$

$$M_4(z,k,n) = \frac{1}{\Delta^*} \ [\{\lambda+2\mu)\lambda_1^2 - k^2\lambda - \nu\alpha_1\}\Delta_1^* e^{-\lambda_2 z}$$

$$+ \{(\lambda + 2\mu)\lambda_2^2 - k^2\lambda - \nu\alpha_2\} \Delta_2^* e^{-\lambda_2 z}$$

$$- 2i\mu k(\lambda_3 \Delta_3^* e^{-\lambda_3 z} + \lambda_4 \Delta_4^* e^{-\lambda_4 z})], \tag{5.21}$$

$$M_5(z,k,n) = \frac{1}{\Delta^*} \ [2i\mu k(\lambda_1 \Delta_1^* e^{-\lambda_1 z} + \lambda_2 \Delta_2^* e^{-\lambda_2 z})$$

$$+ \{\mu(k^2+ \lambda_3^2) + \alpha(\lambda_3^2- k^2- 2\alpha_3)\}\Delta_3^* e^{-\lambda_3 z}$$

$$+ \{\mu(k^2+ \lambda_4^2) + \alpha(\lambda_4^2- k^2- 2\alpha_4)\}\Delta_4^* e^{-\lambda_4 z}], \tag{5.22}$$

$$M_6(z,k,n) = \frac{1}{\Delta^*} \ [(\gamma + \epsilon)(\lambda_3\alpha_3 \Delta_3^* e^{-\lambda_3 z} + \lambda_4 \alpha_4 \Delta_4^* e^{-\lambda_4 z})], \tag{5.23}$$

$$M_7(z,k,n) = \frac{1}{\Delta^*} \ [\alpha_1 \Delta_1^* e^{-\lambda_1 z} + \alpha_2 \Delta_2^* e^{-\lambda_2 z}], \tag{5.24}$$

and

$$\Delta^* = (p_3 r_4 - p_4 r_3)(s_2 q_1 - s_1 q_2) - (q_3 r_4 - q_4 r_3)(s_2 p_1 - s_1 p_2), \tag{5.25}$$

$$\Delta_1^* = - s_2(p_3 r_4 - p_4 r_3), \tag{5.26}$$

$$\Delta_2^* = \ s_1(p_3 r_4 - p_4 r_3), \tag{5.27}$$

$$\Delta_3^* = - r_4(s_1 p_2 - s_2 p_1), \tag{5.28}$$

$$\Delta_4^* = \ r_3(s_1 p_2 - s_2 p_1) . \tag{5.29}$$

These results agree with those obtained by Acharya and Sengupta [8].

(ii) In this case we consider a torque with its axis parallel to the z-axis so that g(x,t) can be written as

$$g(x,t) = G[\delta(x-a)-\delta(x+a)]e^{-i\omega t} \tag{5.30}$$

where G is the magnitude of the force.

The double Fourier transformation of (5.30) gives

$$\bar{g}(k,n) = \frac{G}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [\delta(x-a)-\delta(x+a)]e^{i(kx+nt)}e^{-i\omega t} dx dt,$$

$$= 2i \ \sqrt{(2\pi)} \ G \sin (ka)\delta(n-\omega). \tag{5.31}$$

Then from (4.45) - (4.60) with (5.31), we obtain

$$u_1 = - iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [u_1^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.32)$$

$$u_3 = - iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [u_3^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.33)$$

$$\omega_2 = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [\omega_2^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.34)$$

$$\sigma_{33} = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [\sigma_{33}^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.35)$$

$$\sigma_{31} = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [\sigma_{31}^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.36)$$

$$\mu_{32} = - iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [\mu_{32}^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.37)$$

$$\beta_3 = - \frac{iG}{3} \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [\beta_3^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.38)$$

$$\theta = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [\theta^*(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk. \qquad (5.39)$$

In the absence of the stretch effect, we obtain the corresponding expressions for the displacements, stresses and the temperature field in the form

$$u_1 = - iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_1(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.40)$$

$$u_3 = - iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_2(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.41)$$

$$\omega_2 = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_3(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.42)$$

$$\sigma_{33} = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_4(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.43)$$

$$\sigma_{31} = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_5(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.44)$$

$$\mu_{32} = - iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_6(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk, \qquad (5.45)$$

$$\theta = iG \sqrt{(2/\pi)}e^{-i\omega t}\int_{-\infty}^{\infty} [M_7(z,k,n)]_{n=\omega}\sin(ka)e^{-ikx}dk. \qquad (5.46)$$

These results also agree with the corresponding results without stretch.

6. CONCLUSION.

The displacement field, force stress, couple stress, temperature field and vector first moment have been obtained. It is noted that the displacement field, force stress, couple stress and temperature field involve the parameters α_0, β_0 and η_0 of the micropolar elastic media with stretch. In addition to the displacements, force stress, couple stress, and temperature field, vector first moment β_j has been determined which vanishes in the case of thermo-micropolar elasticity. Some numerical calculation for specific models of physical interest will be carried out and will be communicated in a subsequent paper.

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