

# LOCAL PROPERTIES OF MAPS OF THE BALL

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Let  $f$  be an essential map of  $S^{n-1}$  into itself (i.e.,  $f$  is not homotopic to a constant mapping) admitting an extension mapping the closed unit ball  $\bar{B}^n$  into  $\mathbb{R}^n$ . Then, for every interior point  $y$  of  $B^n$ , there exists a point  $x$  in  $f^{-1}(y)$  such that the image of no neighborhood of  $x$  is contained in a coordinate half space with  $y$  on its boundary. Under additional conditions, the image of a neighborhood of  $x$  covers a neighborhood of  $y$ . Differential versions are valid for quasianalytic functions. These results are motivated by game-theoretic considerations.

## 1. Introduction

Let  $\Omega$  be an open subset of  $\mathbb{R}^n$ , and let  $f$  be a real continuous function defined in  $\Omega$ . We say that  $f$  is *one-sided* at a point  $\bar{x} \in \Omega$ , if there exists a neighborhood  $U$  of  $\bar{x}$ , such that either  $f(x) \geq f(\bar{x})$  or  $f(x) \leq f(\bar{x})$  for  $x \in U$ , but  $f$  is not constant on  $U$ . Otherwise (i.e., if either  $f(x) = f(\bar{x})$  near  $\bar{x}$  or  $f(x) - f(\bar{x})$  assumes both positive and negative values at every neighborhood of  $\bar{x}$ )  $f$  is *multisided* at  $\bar{x}$ . If  $f$  is a continuous function  $f : \Omega \rightarrow \mathbb{R}^n$ , then  $f$  is *multisided* at  $\bar{x}$  if every component  $f_i(x)$  is multisided at  $\bar{x}$ .

**THEOREM 1.1.** *Let  $B = B^n$  denote the open unit ball of  $\mathbb{R}^n$ . Let  $f$  be a continuous map of  $\bar{B}$  into  $\mathbb{R}^n$  such that  $f : \partial B (= S^{n-1}) \rightarrow \partial B$ , and  $f|_{\partial B}$  is an essential map. Then, for every  $y \in B$  there exists  $\bar{x} \in f^{-1}(y)$  such that  $f$  is multisided at  $\bar{x}$ .*

Additional assumptions on  $f$  imply that there exist points in  $f^{-1}(y)$  whose neighborhoods are mapped into neighborhoods of  $y$ . (Additional assumptions are needed, see Examples 3.1 and 3.2.)

**THEOREM 1.2.** *Let  $B$  and  $f$  be as in Theorem 1.1. Let  $y \in B$ , such that the inverse image of  $y$  is zero dimensional at every point in the closure of the set*

$\{x : x \in f^{-1}(y), f \text{ is multisided at } x\}$ . Then there exists  $\bar{x} \in f^{-1}(y)$  such that for every open neighborhood  $U$  of  $\bar{x}$ ,  $f(U)$  contains an open neighborhood of  $y$ .

There exists differential (actually, quasianalytic) versions of the preceding concepts. Let  $\mathbf{M} = M_j$  be a logarithmically convex sequence of positive numbers. Let  $V$  be an open subset of  $\mathbb{R}^k$ , where  $k$  is arbitrary. The class  $C_{\mathbf{M}}(V)$  consists of those functions  $f$  in  $C^\infty(V)$  such that for every compact  $K \subset V$  there exists constants  $A$  and  $B$  such that for every multi-index  $\alpha$ , the inequality

$$|\partial^\alpha f(x)| \leq AB^{|\alpha|} M_{|\alpha|} \quad (1.1)$$

holds whenever  $x \in K$ . The well-known Denjoy-Carleman theorem [3] states that the class  $C_{\mathbf{M}}(V)$  is quasianalytic if and only if the series  $\sum_{j=1}^{\infty} M_{j-1}/M_j$  diverges. Following [2], we may consider  $C_{\mathbf{M}}$  manifolds, in particular we consider  $C_{\mathbf{M}}$  curves, that is, curves which possess a parametric representation  $x(t) = (x_1(t), \dots, x_n(t))$ , such that the functions  $x_i(t)$  are in the class  $C_{\mathbf{M}}$ , for all  $1 \leq i \leq n$ .

A scalar function  $f \in C_{\mathbf{M}}(\Omega)$  is  $C_{\mathbf{M}}$  one-sided at a point  $\bar{x} \in \Omega$  if

- (i) there exists a  $C_{\mathbf{M}}$  curve  $x(t)$  such that  $x(0) = \bar{x}$ , and  $f(x(t))$  is not locally constant at  $t = 0$ ,
- (ii) for every  $C_{\mathbf{M}}$  curve  $x(t)$  such that  $x(0) = \bar{x}$ , either
  - (1) the first nonvanishing derivative of  $f(x(t))$  (with respect to  $t$  at  $t = 0$ ) is of even order and is positive or
  - (2) the first nonvanishing derivative of  $f(x(t))$  is of even order and is negative or
  - (3)  $f(x(t)) \equiv f(\bar{x})$ .

If  $f \rightarrow \mathbb{R}^n$ ,  $f \in C_{\mathbf{M}}$ , and the function  $f_i(x)$  is  $C_{\mathbf{M}}$  one-sided at  $\bar{x}$  for no  $1 \leq i \leq n$  is, we say that  $f$  is  $C_{\mathbf{M}}$  multisided at  $\bar{x}$ .

The relation between ordinary one-sidedness and  $C_{\mathbf{M}}$  one-sidedness is spelled out in the following proposition.

**PROPOSITION 1.3.** *Let  $f$  be a scalar function,  $f \in C_{\mathbf{M}}(\Omega)$ . Then,  $f$  is one-sided at  $\bar{x} \in \Omega$  if and only if  $f$  is  $C_{\mathbf{M}}$  one-sided at  $\bar{x}$ .*

The proof of [Proposition 1.3](#) depends on the desingularization results of Bierstone and Milman [2]. The proposition implies immediately the following version of [Theorem 1.1](#).

**THEOREM 1.4.** *Let  $B$  and  $f$  be as in [Theorem 1.1](#), and let  $f \in C_{\mathbf{M}}(B)$ . Then there exists  $\bar{x} \in f^{-1}(y)$  such that  $f$  is  $C_{\mathbf{M}}$  multisided at  $\bar{x}$ .*

Note that the boundary behavior of  $f$  along with quasianalyticity imply that no component  $f_i$  of  $f$  is locally constant at any point. As seen in [Example 3.3](#), we do not have a  $C_{\mathbf{M}}$  strengthening of [Theorem 1.2](#).

Observe that [Proposition 1.3](#) and [Theorem 1.4](#) are valid if, instead of assuming that  $M$  is a Denjoy-Carleman class, we assume that  $M = C(\Omega)$ , where the collection of algebras  $C(U)$  ( $U$  runs over all open subsets of  $\mathbb{R}^m$ ,  $m$  arbitrary)

satisfies conditions (3.1), (3.2), (3.3), (3.4), (3.5), and (3.6) of [2]. As a matter of fact, only these assumptions are needed for the validity of the desingularization results in [2].

The motivation for the study of multisidedness comes from recent results on set-valued maps of the closed simplex  $\Delta$  (see [4]), results which have game-theoretic applications. For example, it is shown that under certain conditions, if  $F$  is a set-valued map from  $\Delta$  to the convex subsets of  $\Delta$ , then for every  $y \in \text{int}(\Delta)$ , there exists  $x$ , such that  $y$  is in the relative interior of  $F(x)$  (see [4, Theorem 1]). If in addition, the closure of the set  $\{x : y \in \text{relint}(F(x))\}$  is zero dimensional, then there exists  $\bar{x} \in \text{int}(\Delta)$  such that  $y \in \text{int}(F(\bar{x}))$  (see [4, Theorem 2]). The purpose of this paper is to find a single-valued analog of the results of [4]. The proofs of Theorems 1.1 and 1.2 here are similar in spirit to the proofs of Theorems 1 and 2 in [4], respectively. In [4], the maps are not deformed continuously (as in the present paper); instead, certain neighborhoods of the origin are removed from the values of the (set-valued) maps. The weaker conclusion in Theorem 1.1 as well as the slightly weaker zero-dimensionality assumption in Theorem 2 of [4] are compensated by a certain finiteness assumption made in [4].

Note that, Theorems 1.1 and 1.4 continue to hold, if we replace the standard base of  $\mathbb{R}^n$  by any other basis  $\mathbf{v}_1, \dots, \mathbf{v}_n$ , write  $f$  as  $\sum_{i=1}^n \hat{f}_i(x)\mathbf{v}_i$ , and define one-sidedness and multisidedness using the functions  $\hat{f}_i$ . (This was pointed out by I. Yomdin (personal communication).)

Note that, unlike the situation in Sard's theorem, Theorem 1.1 applies to all points  $y \in B$ .

## 2. Proofs

Let  $f$  satisfy the assumptions of Theorem 1.1, and let  $y$  be an arbitrary point of  $B$ . We may assume, without loss of generality (to simplify notation a bit), that  $y = 0$ . Let  $\bar{x}$  be a point in  $D_i = f_i^{-1}(0) \cap B$ . Let  $C_i^+$  ( $C_i^-$ ) denote the set of points  $\bar{x} \in D_i$  for which there exists a neighborhood  $U$  of  $\bar{x}$  in which  $f_i(x) \geq 0$  ( $f_i(x) \leq 0$ ), but  $f_i(x)$  is not a constant in  $U$  (i.e.,  $f_i(x)$  does not vanish identically in  $U$ ). Set  $N_i = f_i^{-1}(-\infty, 0)$  and  $P_i = f_i^{-1}(0, \infty)$ . For any set  $A$ , let  $\rho(x, A)$  denote the distance of  $x$  from  $A$  ( $= \inf_{y \in A} |x - y|$ ). We now deform the map  $f$  as follows: for every  $t \geq 0$ , set

$$f_t(x) = (f_1(x, t), \dots, f_n(x, t)), \quad (2.1)$$

where

$$f_i(x, t) = f_i(x) + t\rho(x, \partial B)[\rho(x, N_i) - \rho(x, P_i)] \quad (2.2)$$

for  $1 \leq i \leq n$ .

Observe that  $f_t(x) \equiv f(x)$ , if  $x \in \partial B$ . If  $x \in C_i^+ \cup P_i$ , then  $\rho(x, \partial B) > 0$  and  $\rho(x, P_i) = 0$ . Moreover,  $f_i$  is nonnegative in a full neighborhood of  $x$ . Hence,  $\rho(x, N_i) > 0$ . Thus,  $f_i(x, t) > f_i(x) \geq 0$  if  $t > 0$ . Similarly, if  $x \in C_i^- \cup N_i$ , then  $f_i(x, t) < 0$  (for positive  $t$ ). It follows that  $f_t(x) = 0$  is possible for  $t > 0$  only if  $x \in f^{-1}(0) \setminus \bigcup_{i=1}^n (C_i^+ \cup C_i^-)$ . But the restriction of  $f_t(x)$  to the boundary  $\partial B$  is essential for positive  $t$  as well, so that the set  $f_t^{-1}(0)$  is nonempty. Hence, there exists  $\bar{x} \in f^{-1}(0) \setminus \bigcup_{i=1}^n (C_i^+ \cup C_i^-)$ . This means that for every  $1 \leq i \leq n$ ,  $f_i(\bar{x}) = 0$ , but  $\bar{x}$  is neither in  $C_i^+$  nor in  $C_i^-$ . This means that either there exists a neighborhood of  $\bar{x}$ , where  $f_i$  vanishes identically, or in every neighborhood of  $\bar{x}$   $f_i$  assumes both positive and negative values. Thus,  $f$  is multisided at  $\bar{x}$ , proving [Theorem 1.1](#).

In proving [Theorem 1.2](#), we continue to use the notations introduced in the proof of [Theorem 1.1](#). Assume, without loss of generality, that the set  $f^{-1}(0)$  is zero dimensional at each point of the closure of the set  $F = \{x : x \in f^{-1}(0), f \text{ is multisided at } x\}$ . Note that  $F = f^{-1}(0) \setminus \bigcup_{i=1}^n (C_i^+ \cup C_i^-)$ , and that the deformations defined by (2.1) and (2.2) eliminate all zeros of  $f$  outside  $F$  without introducing new ones. It is possible, however, that  $f_t(x)$  will no longer vanish (for  $t > 0$ ) on all of  $F$ . We modify the deformation by setting

$$g_t(x) = (g_1(x, t), \dots, g_n(x, t)), \quad (2.3)$$

where

$$g_i(x, t) = f_i(x) + t\rho(x, \partial B)\rho(x, F)[\rho(x, N_i) - \rho(x, P_i)] \quad (2.4)$$

for  $1 \leq i \leq n$ .

It is easily seen that  $g_t(x) \neq 0$  if  $t > 0$  and  $x \notin \bar{F}$ . The assumptions on boundary behavior imply that for all  $t \geq 0$ , we have

$$d(g_t, B, 0) \neq 0. \quad (2.5)$$

(We use standard notation for degree.)

By assumption, the set  $f^{-1}(0)$  is zero dimensional at every point of the compact set  $\bar{F}$ . This implies that for every  $\epsilon > 0$ , the set  $\bar{F}$  may be covered by a finite number of disjoint open sets whose diameter is less than  $\epsilon$ , and whose boundaries do not meet  $f^{-1}(0)$ . Let  $\{D_{i,m}\}_{i=1}^{P_m}$  denote such a collection of sets with  $\text{diam}(D_{i,m}) < 1/m$ ,  $\bar{F} \subseteq \bigcup_{i=1}^{P_m} D_{i,m}$ ,  $D_{i,m} \cap D_{j,m} = \emptyset$  for  $i \neq j$ , and  $\partial D_{i,m} \cap f^{-1}(0) = \emptyset$ . By construction, if  $t \geq 0$ , then  $g_t(x) \neq 0$  for all  $x \in \partial D_{i,m}$ ,  $1 \leq i \leq P_m$ . Hence,  $d(g_t, D_{i,m}, 0)$  is well defined for all  $t \geq 0$  and

$$\sum_{i=1}^{P_m} d(g_t, D_{i,m}, 0) = d(g_t, B, 0) \quad (2.6)$$

for all positive  $t$ . It follows from (2.5) and (2.6) that there exists  $i_0 = i_0(m)$  such that  $d(g_{1/m}, D_{i_0(m), m}, 0) \neq 0$ . By compactness, there exist a point  $\bar{x} \in \bar{F}$  and a sequence  $D_{i_0(m), m}$  of neighborhoods (with  $D_{i_0(m), m} \cap \bar{F}$  compact) such that  $\bar{x} = \bigcap_{m=1}^{\infty} D_{i_0(m), m}$ . By construction, the degree  $d(g_t, D_{i_0(m), m}, 0)$  is independent of  $t \geq 0$ . For every open neighborhood  $U$  of  $\bar{x}$ , there exists a number  $M$  such that the ball around  $\bar{x}$  with radius  $1/M$  is contained in  $U$ , so that  $D_{i_0(m), m} \subset U$  if  $m > 2M$ . It follows that  $d(f, D_{i_0(m), m}, 0) = d(g_{1/m}, D_{i_0(m), m}, 0) \neq 0$ . Hence,  $f(U)$  contains a full neighborhood of 0 and [Theorem 1.2](#) follows.

The proof of [Proposition 1.3](#) relies on the results of [2]. We may assume, without loss of generality, that  $\bar{x} = 0$  and  $f(0) = 0$ . Let  $h(x)$  be a scalar  $C_M$  function in a neighborhood  $U$  of the origin, with  $h(0) = 0$  such that  $h$  does not vanish identically in  $U$  and  $h(x) \geq 0$  in  $U$ . Let  $x(t)$  be an arbitrary  $C_M$  curve in  $U$  such that  $x(0) = 0$ . If there exists a sequence  $t_n \rightarrow 0$ ,  $t_n \neq 0$ , such that  $h(x(t_n)) = 0$  for all  $n$ , then the quasianalyticity of  $h$  and  $x$  implies that  $h(x(t))$  vanishes identically. Otherwise, the function  $h(x(t))$  is strictly positive for small  $t \neq 0$ . Not all derivatives of  $h(x(t))$ , with respect to  $t$ , vanish at  $t = 0$ ; the first nonzero one must be of even order and positive.

It remains to show that, there exist  $C_M$  curves  $x(t)$  such that  $h(x(t))$  does not vanish identically. This follows from the ‘‘curve selection lemma.’’ We prove the existence of such curves following Atiyah (see [1, page 147]) and Bierstone-Milman [2], and noting that there exists an  $n$ -dimensional  $C_M$  manifold  $U'$  and a proper  $C_M$  map  $\phi$  from  $U'$  onto a neighborhood of the origin (contained in  $U$ ) such that  $\phi^{-1}$  is one to one on  $\{x : h(x) \neq 0\}$ , and such that  $h \circ \phi$  is locally a monomial times an invertible factor (in suitable coordinates). In particular, we may choose a (connected) neighborhood  $W$  in  $U'$  and  $C_M$  coordinates  $w_1, \dots, w_n$  in  $W$  such that  $\phi(0) = 0$  and

$$h(\phi(w)) = g(w) \prod_{i=1}^n w_i^{k_i} \quad (2.7)$$

for  $w \in W$ , where the  $C_M$  function  $g$  never vanishes in  $W$ , and  $k_i$  is a nonnegative integer for  $1 \leq i \leq n$ . By assumption,  $g > 0$  in  $W$  and the numbers  $k_i$  are even. Define a  $C_M$  curve by  $x(t) = \phi(t, \dots, t)$ . Then,

$$h(x(t)) = g(t, \dots, t) t^{\sum_{i=1}^n k_i} \quad (2.8)$$

Hence  $h$  is  $C_M$  one-sided at 0.

Let  $h$  be  $C_M$  one-sided such that for every  $C_M$  curve  $x(t)$  with  $x(0) = 0$ , either the first nonvanishing derivative of  $h(x(t))$  is positive and of even order, or  $h(x(t)) = 0$ . We will show that  $h$  is one-sided. If  $h(x)$  assumes negative values in every neighborhood of 0, then either (i)  $g(0) > 0$  and the numbers  $k_i$  are not all even, or (ii)  $g(0) < 0$  and all the numbers  $k_i$  are even. In case (i), suppose that  $k_1$  is odd. Considering the  $C_M$  curve  $x(t) = \phi(t, 0, \dots, 0)$ , we see that  $h$  is not

$C_M$  one-sided. In case (ii), consider the curve  $x(t) = \phi(t, \dots, t)$ . Then (2.8) holds with  $g(0) < 0$ , contradicting the assumption. It follows that there exists a neighborhood of 0, where  $h(x)$  is nonnegative. By assumption,  $h(x)$  is not identically equal to zero. Hence  $h$  is one-sided. The case where the first nonvanishing derivative of  $h(x(t))$  is negative, is similar.

*Remark 2.1.* If  $f \in C_M$  is a vector-valued function such that all components  $f_i$  are one-sided, then there exists a  $C_M$  curve passing through  $\bar{x}$ , such that for each  $1 \leq i \leq n$  not all derivatives of  $f_i(x(t))$  vanish at  $t = 0$ . (Set  $h = \prod_{i=1}^n f_i$ .)

### 3. Examples

*Example 3.1.* The following function  $f$  is multisided at the origin and no  $f_i$  is locally constant, but no neighborhood of the origin is mapped onto a full neighborhood of the origin. Set  $f_1 = x_1$ ,  $f_2 = x_1 x_2$ . If  $(y_1, y_2) = (f_1(x_1, x_2), f_2(x_1, x_2))$  and  $y_1 = 0$ , then  $y_2 = 0$ , so that no point  $(0, y_2)$  with  $y_2 \neq 0$  is in the image of  $f$ . However, every other line through the origin has points of the image of  $f$  on both sides, arbitrarily close to the origin.

*Example 3.2.* The assumptions of [Theorem 1.1](#) do not imply the conclusion of [Theorem 1.2](#). This can be seen by considering the simple one-dimensional function  $f(x) = 0$  for  $|x| < 1/2$ ,  $f(x) = 2x - 1$  for  $1/2 \leq x \leq 1$ , and  $f(x) = 2x + 1$  for  $-1 \leq x \leq -1/2$ . Then,  $f$  is multisided at  $|x| < 1/2$ —a one-dimensional set, and the conclusion of [Theorem 1.2](#) fails for  $y = 0$ .

*Example 3.3.* The inverse image of a point may consist of a closed analytic curve even if  $f$  is analytic and the assumptions of [Theorem 1.2](#) are satisfied. Let  $1/2 > \delta > 0$  and set

$$f_1 = \frac{x_1 [(\delta - x_1)^2 + x_2^2 - \delta^2]^2}{(1 - 2\delta x_1)^2}, \quad f_2 = \frac{x_2 [(\delta - x_1)^2 + x_2^2 - \delta^2]^2}{(1 - 2\delta x_1)^2}. \quad (3.1)$$

Then  $f^{-1}(0)$  is a circle of radius  $\delta$  centered at  $(\delta, 0)$ . The function is one-sided at every point of the circle except for the origin. The conclusion of [Theorem 1.2](#) holds at the origin. Nevertheless, there exists a curve through the origin on which both  $f_1$  and  $f_2$  vanish. Hence, [Theorem 1.2](#) cannot be strengthened (even in the analytic case) so as to assert that  $f$  has a nonvanishing derivative (of a sufficiently high order) on every analytic curve through  $\bar{x}$ .

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