The Structure of Locally Finite Two-Connected Graphs

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Submitted: May 15, 1995; Accepted: September 4, 1995.

Abstract

We expand on Tutte's theory of 3-blocks for 2-connected graphs, generalizing it to apply to infinite, locally finite graphs, and giving necessary and sufficient conditions for a labeled tree to be the 3-block tree of a 2-connected graph.

Mathematics Subject Classification: 05C40, 05C38, and 05C05.
Key Words and Phrases: Tutte connectivity, 3-block, 3-block tree

1 Introduction

Connectivity properties of graphs are among the basic aspects of graph theory. Every graph is the disjoint union of its connected components, and every connected graph is the edge disjoint union of its maximal 2-connected subgraphs, encoded in the block-cutpoint tree. A canonical decomposition for finite 2-connected graphs was given by Tutte [11] in the form of the 3-block tree, and generalized to matroids by Cunningham and Edmonds [1]. Such decompositions are important tools in inductive arguments and constructions. Hopcroft and Tarjan [4] gave an important algorithm for computing the 3-block tree of a graph in $O(V+E)$ time, which is comparable to the complexity of computing other non-canonical decompositions, say the ear decomposition, and is also applicable to matroids. Effective decompositon schemes for graphs of connectivity 3 and higher have been given, but none are canonical, and in Section 6 we argue that none will be forthcoming. The uniqueness of Tutte's construction may be exploited to study the symmetry properties of graphs with low connectivity, [8] and [2], particularly in the case of planar graphs [3]. In this paper we will examine the interpretation of Tutte's decomposition and extend the theory to infinite graphs.

2 n-Connectivity

We are concerned with the structure of locally finite graphs of connectivity less than 4, allowing graphs to have loops and multiple edges. If a graph G has at least 3 vertices and is not a triangle, then G is defined to be *n-connected* if G has girth at least n and any two vertices of G are joined by n internally disjoint paths. To avoid uninteresting special cases, the connectivities of the small graphs in Figure 1 are said to be infinite. This definition of n-

[1]Supported by a James Madison University Faculty Summer Research grant.

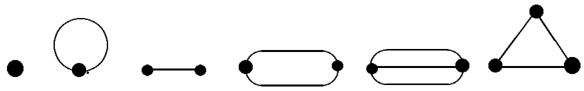

Figure 1: Infinitely connected graphs.

connectivity, also known as *Tutte connectivity,* was introduced in [9], see also [6], and differs for simple graphs from the usual definition of n-connectivity only for $n > 3$, with the exception of the special cases in Figure 1, and has the advantage of being generalizable to matroids, [10], and being invariant under dualization.

We call a graph on two vertices connected by parallel edges a *multilink.* Note, that with the above definition, multilinks on more than three edges are only 2–connected.

For any graph G , there is an equivalence relation defined on the edges of G by setting $e \cong e'$ if e and e' lie on a common cycle, or $e = e'$. The equivalence classes are either single edges or induce maximal 2-connected subgraphs of G, and are called the *blocks* (or 2-*blocks*) of G. The *block–cutpoint tree* is the graph defined on the union of the set of blocks and the set of cutpoints, with a cutpoint adjacent to each of the blocks to which it belongs. It is obvious that this graph is a tree. In general, one might hope that any k–connected graph decomposes as the union of subgraphs which are either $k+1$–connected or have at most k vertices. However, this is not the case, since a 2-connected graph may have no non-trivial 3-connected subgraphs at all, as in Figure 2.

In the interest of analogy, therefore, it is preferable to regard a block–cutpoint tree as an encoding of the instructions for assembling a 1-separable graph from simpler pieces using the operation of vertex–union.

Let A and B be graphs and suppose there are functions $f_A : e \to A$ and $f_B : e \to B$, where e is a graph consisting of two vertices 1 and 2 joined by an edge which we will also call e. Define the *edge amalgam of A and B over f*—denoted $A +_f B$—to be the graph obtained

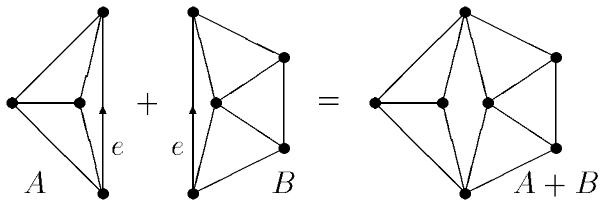

Figure 2: Edge amalgamation.

from the disjoint union of A and B by identifying vertex $f_A(1)$ with $f_B(1)$, vertex $f_A(2)$ with $f_B(2)$, and erasing the edges $f_A(e)$ and $f_B(e)$, see Figure 2. The edge amalgam, also called 2–sum in [6], is analogous to the symmetric difference of sets, and in $A +_f B$ we can regard the graph $B - f_B(e)$ as taking the place of $f_A(e)$ in A, and vice versa. Whenever possible, we will indicate the edge functions f_A and f_B by simply labeling the edges $f_A(e)$ and $f_B(e)$ the same in A and B, in which case we write $A +_e B$ or just $A + B$.

It is clear that A and B are 2-connected if and only if $A +_f B$ is 2-connected. The edge

$f_A(e)$ is called an *amalgamated edge* of A. If X denotes the subgraph of A obtained by erasing the amalgamated edge, X is also a subgraph of $A +_f B$ and we write $A = \overline{X}$.

Edge amalgamation is a commutative operation, so $A+_f B \cong B+_f A$. but it is not in general associative, since $(A +_f B) +_g C$ may be rewritten as $A +_f (B +_g C)$ only if $g_{A+B}(e)$ belongs to B. If both expressions are defined, they are equal, and note that, since the amalgamated edges are erased, it must be true that $f_B(e) \neq g_B(e)$. Since the associativity is conditional, it is not in general possible to simply ignore the parentheses in a long expression, in particular whenever some term has more than two amalgamated edges. A more convenient notation for the result of a sequence of edge amalgamations, therefore, is a labeled tree in which the nodes are labeled with graphs, and the edges are labeled with the two functions indicating which edges are amalgamated in the endpoint graphs. Necessarily, the amalgamating edges at any node of this tree must be distinct. We call such a labeled tree $\mathcal{T}$ an *edge amalgam tree*, and let $G(\mathcal{T})$ denote the graph obtained from the disjoint union of the node labels by amalgamating along the edges determined by the edge functions.

To avoid confusion between $G(\mathcal{T})$ and $\mathcal{T}$, we will hereafter refer to the vertices and edges of an edge amalgam tree as *nodes* and *links*, and denote them with Greek as opposed to Roman letters.

For a finite edge amalgam tree $\mathcal{T}$, it is clear that $G(\mathcal{T})$ is 2-connected if and only if the node graphs are 2-connected, and that $G(\mathcal{T})$ is locally finite if and only if the node graphs are locally finite. For infinite trees, however, neither is the case, as can be seen from Figures 3 and 4. Accordingly, we shall give conditions under which an edge amalgam tree defines a locally finite 2-connected graph.

3 The 3-block tree

A graph G is said to be a 3-*block* if it contains at least three edges and is either a circuit, a finite multilink, or a simple, locally finite 3-connected graph.

Let $\mathcal{T}$ be a countable edge amalgam tree. We call $\mathcal{T}$ a 3-*block tree* if the following conditions are satisfied:

1. If $\{\alpha, \beta\} \in E(\mathcal{T})$ then G_α and G_β are not both circuits, nor are they both multilinks.

2. If $\eta = \{\alpha, \beta\}$ is a link in $\mathcal{T}$ amalgamating an edge e in G_α, then there is a finite subtree $\mathcal{T}' \leq \mathcal{T}$ with $\alpha \in \mathcal{T}'$ and $\beta \notin \mathcal{T}'$, and a path in $G(\mathcal{T}')$ joining the endpoints of e which is made up entirely of edges unamalgamated in $G(\mathcal{T})$.

3. For each vertex v of G_α, α is contained in a finite subtree $\mathcal{T}_v < \mathcal{T}$ such that star(v) in $G(\mathcal{T}_v)$ consists entirely of edges unamalgamated in $\mathcal{T}$.

We impose condition 1 since circuits and multilinks would otherwise have many inequivalent 3-block trees. Condition 2 avoids the situation of Figure 3 in which each amalgamation increases the length of a circuit, resulting in a graph with cut vertices. Condition 3 insures local finiteness, disallowing amalgamations such as in Figure 4. Note that we do not require $\mathcal{T}$ itself to be locally finite; for instance, one may amalgamate a triangle to each of the edges

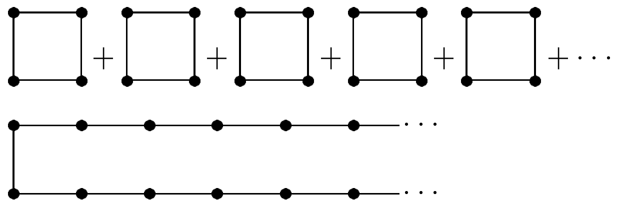

Figure 3:

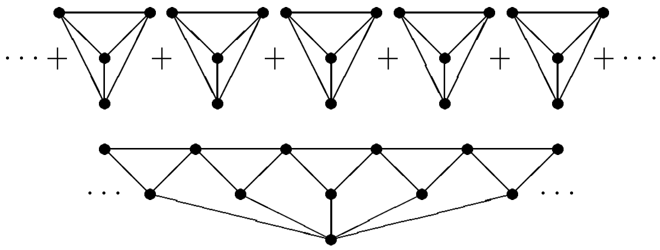

Figure 4:

of an infinite locally finite 3-connected graph (effectively subdividing each edge). In order to describe this graph as $G(\mathcal{T})$, $\mathcal{T}$ must be an infinite star.

Note also that, while the edges of $G(\mathcal{T})$ are partitioned according to the 3-blocks to which they belong, all edges of a particular 3-block may be amalgamated, in which case the 3-block will not correspond to any collection of edges of $G(\mathcal{T})$.

Lemma 1 *If $\mathcal{T}'$ is a finite subtree of $\mathcal{T}$, then $G(\mathcal{T}')$ is homeomorphic to a subgraph of $G(\mathcal{T})$.*

PROOF: Every edge in $G(\mathcal{T}')$ which is not in $G(\mathcal{T})$ is amalgamated, so replace each with the path of unamalgamated edges in $G(\mathcal{T}) - G(\mathcal{T}')$ which is guaranteed to exist by condition 2. □

Lemma 2 *For any 3-block tree $\mathcal{T}$, $G(\mathcal{T})$ is locally finite and 2-connected.*

PROOF: $G(\mathcal{T})$ is locally finite by condition 3.

Clearly, any *finite* 3-block tree represents a 2-connected graph. Let v and w be distinct vertices of $G(\mathcal{T})$, and suppose they correspond to vertices $v' \in G_\alpha$ and $w' \in G_\beta$. Let $\mathcal{T}'$ be any finite subtree of $\mathcal{T}$ containing α and β. Then $G(\mathcal{T}')$ is 2-connected, so there are two internally disjoint paths in $G(\mathcal{T}')$ from v' to w'. Since $G(\mathcal{T}')$ is homeomorphic to a subgraph of $G(\mathcal{T})$, there are two such paths joining v and w in $G(\mathcal{T})$, as well. □

It is our aim to prove

Theorem 1 *Any locally finite 2-connected graph G corresponds to a unique 3-block tree $\mathcal{T}$.*

We will prove Theorem 1 in two steps. In the next section, we will show how to construct a particular 3-block tree for G, and in the one following, we show that this tree is unique.

4 Existence of 3-block trees.

Given a pair of vertices $\{a, b\}$ in a graph G, there is an equivalence relation defined on the edges of G by setting e and e' equivalent if there is a path in G containing both e and e', and which has neither a nor b as an internal vertex. The subgraphs of G which carry the equivalence classes are called the *bridges* of G with respect to $\{a, b\}$. Two distinct bridges clearly can intersect only at a and b, and if G is 2-connected, their intersection is exactly $\{a, b\}$. G may thus be regarded as the union of its bridges along $\{a, b\}$. However, these bridges may not be themselves 2-connected. If a bridge with respect to $\{a, b\}$ in a 2-connected graph G consists of more than a single edge, then adding a new edge connecting a and b to that bridge does give a 2-connected graph, and we call these graphs the *branches* of G with respect to $\{a, b\}$.

The *deletion of e from G*, denoted $G - e$, is the graph obtained from G by deleting the edge e, but not its endpoints. The *contraction of e in G*, denoted $G \cdot e$, is obtained from $G - e$ by identifying the endpoints of e.

Let G be a locally finite 2-connected graph. Choose an arbitrary edge e of G, and let a and b be its endpoints. We will construct a 3-block tree $\mathcal{T}_e$ with $G(\mathcal{T}_e) = G$.

We call e *undeletable* if $G - e$ is 1-separable, *incontractible* if $G \cdot e$ is 1-separable, and *ordinary* otherwise. An edge e cannot be both undeletable and incontractible. since the first requires that every cycle containing the endpoints of e also contain e, while the second insures two vertices u and v so that every path from u to v passes through one of the endpoints of e, so neither the two disjoint paths from u to v could contain e. Note that if $\mathcal{T}$ is a 3-block tree, then condition 3, and the fact that edge amalgamation does not destroy cutpoints, guarantees that if the edge $e \in G(\mathcal{T})$ belongs to the 3-block G_α, then its type—undeletable, incontractible or ordinary—in $G(\mathcal{T})$ is the same as its type in G_α.

We begin the construction of $\mathcal{T}_e$ by defining $\mathcal{T}_0$ to consist of a single vertex α_0 labeled "G," with distinguished edge e. Then $\mathcal{T}_0$ is an edge amalgam tree (though *not* a 3-block tree) and $G = G(\mathcal{T}_0)$.

To define $\mathcal{T}_1$, we perform what we will call a *simple expansion of the vertex α_0 at the edge e;* that is, we will replace α_0 with a star which is an edge–amalgam tree for G_{v_0} (though, again, not a 3-block tree.)

First, we define the graph B_e—the 3-block of G containing e. This will be the label of the central node of $\mathcal{T}_1$. We consider three cases, depending on the type of the edge e.

e undeletable. In this case, $G - e$ is not 2-connected, and its block–cutpoint tree is a non-trivial path $A_1, v_1, A_2, \ldots, v_{k-1}, A_k$, where the vertices A_i correspond to blocks of $G - e$ and the v_i are the cut vertices of $G - e$. Set $v_0 = a$ and $v_k = b$. Let B_e be the $(k+1)$–cycle with vertices $v_0, v_1, \ldots, v_k$, and let $\overline{e_i}$ denote the edge of B_e joining v_i and v_{i+1} (subscripts modulo n.) If some A_i consists of more than a single edge, we define $\overline{A_i}$ to be the graph obtained from A_i by adding an edge $\overline{e_i}$ joining v_i and v_{i+1}. G is then obtained by amalgamating each $\overline{A_i}$ to B_e along $\overline{e_i}$, identifying any unamalgamated edges $\overline{e_i}$ with the corresponding edge in G (see Figure 5.)

Remark 1. If A_i is not a single edge then it is 2-connected (since it is a block of $G - e$), so the edge $\overline{e_i}$ is not undeletable in $\overline{A_i}$.

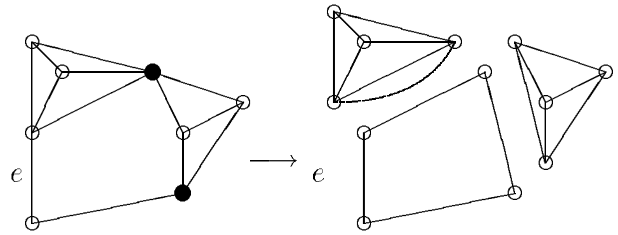

Figure 5: Simple expansion at an undeletable edge.

e incontractible. Let $A_1, \ldots, A_k$ denote the bridges of the endpoints of e; there are only finitely many of them, since G is locally finite. Then $k \geq 3$ and one of the A_i is the edge e itself. If A_i consists of more than a single edge, let $\overline{A_i}$ denote the graph A_i with a new edge $\overline{e_i}$ joining a and b. If A_i is a single edge, set $\overline{e_i} = A_i$. Let B_e denote the multilink consisting of e together with all the edges "$\overline{e_i}$." G is again the amalgam of B_e with the graphs $\overline{A_i}$ along the edges $\overline{e_i}$ (see Figure 6.)

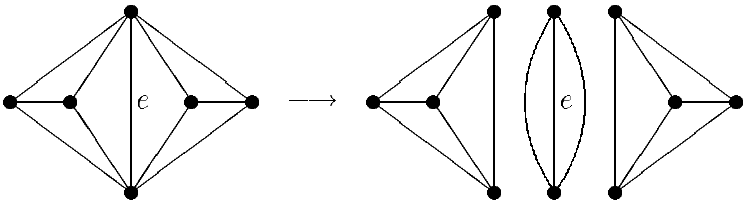

Figure 6: Simple expansion at an incontractible edge.

Remark 2. Since A_i has a single bridge with respect to $\{a, b\}$, $\overline{e_i}$ is not incontractible in $\overline{A_i}$.

e ordinary. We first define a partial order on the collection of all subgraphs $Y_i \subseteq G$ for which $G = \overline{X_i} + \overline{Y_i}$ for some X_i properly containing e. Specifically, we set $Y_j \preceq Y_i$ if $Y_i = \overline{Z} + \overline{Y_j}$ for some subgraph Z. Let $\{A_i\}$ denote the collection of maximal elements with respect to this partial order, and let a_i and b_i be the vertices of attachment of A_i.

Lemma 3 *Two maximal elements A_i and A_j intersect in at most one common vertex of attachment.*

Proof: Let $G = \overline{X_i} + \overline{A_i} = \overline{X_j} + \overline{A_j}$. If A_i and A_j had *both* vertices of attachment in common, then we could write $G = \overline{X_i \cap X_j} + \overline{A_i \cup A_j}$, and $A_i \cup A_j = \overline{A_i} + \overline{A_j}$, contrary to the maximality of A_i and A_j.

Suppose A_i and A_j have an interior vertex v in common. Then there are two internally disjoint paths joining v and e, each passing from A_i to X_i and also from A_j to X_j. By the above, not both a_i and b_i can belong to A_i nor to A_j. Choose notation so that the paths are labeled as in Figure 7. Thus $b_j \in A_i$ and $b_i \in A_j$, and every path from any internal vertex of A_i to e which does not pass through a_i must pass through b_i, which

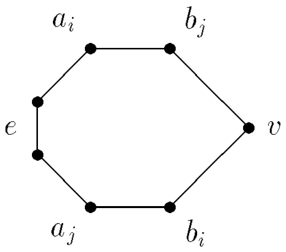

Figure 7:

is in A_j. Thus, the path must continue through a_j, since if it continued through b_j, it would still be within A_i. But then $\{a_i, a_j\}$ separates $A_i \cup A_j$ from e, violating the maximality of A_i, so A_i and A_j have no interior vertex in common. $\Box$

In this case, we form B_e by replacing each A_i with a new edge $\overline{e_i}$ joining a_i and b_i, and we form $\overline{A_i}$ by adding the edge $\overline{e_i}$ to A_i. It follows from the maximality of the A_i that B_e is simple (since otherwise a multilink could be split off of B_e) and that B_e is 3-connected (since if there were a two-cutset, one of its bridges could be split off of B_e.) Further, B_e is locally finite since no vertex of B_e has larger valence in B_e than it has in G. Once again, G is the amalgam of B_e with the $\{\overline{A_i}\}$ (see Figure 8.)

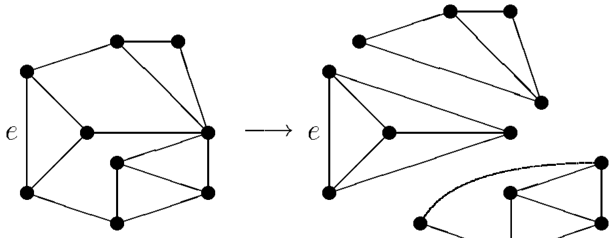

Figure 8: Simple expansion at an ordinary edge.

$\mathcal{T}_1$ is a star whose central vertex β is labeled "B_e," and whose pendant vertices $\{\alpha_i\}$ are labeled with the various $\overline{A_i}$. Given a link $\eta = \{\beta, \alpha_i\}$ in $\mathcal{T}_1$, define the edges η_β and η_{α_i} to be the edges $\overline{e_i}$ in B_e and $\overline{A_i}$, respectively, with the obvious orientations. Clearly, $G = G(\mathcal{T}_1)$.

Next, suppose $\mathcal{T}_n$ has been defined and construct $\mathcal{T}_{n+1}$ by performing, for each pendant node α of $\mathcal{T}_n$ which is not a 3-block, a simple expansion of α at the edge η_α (where η is the unique edge of $\mathcal{T}_n$ incident to α).

Then $G = G(\mathcal{T}_{n+1})$, and each nonpendant node of $\mathcal{T}_{n+1}$ is labeled with a 3-block. Moreover, by remarks 1 and 2 above, no two adjacent nodes of $\mathcal{T}_{n+1}$ are labeled with either links or circuits.

Let $\mathcal{T}_e = \lim \mathcal{T}_n$.

Let $\mathcal{T}_n^0$ denote the edge amalgam tree obtained from $\mathcal{T}_n$ by removing the pendant nodes which are not 3-blocks and let $G^0(\mathcal{T}_n)$ be the graph obtained from $G(\mathcal{T}_n^0)$ by removing the edge η_α for each pendant link $\eta \in \mathcal{T}_n$ with endpoint $\alpha \in \mathcal{T}_n^0$ (in effect, we remove from $G(\mathcal{T}_n^0)$ all those edges which are amalgamated in $G(\mathcal{T}_e)$).

It is clear that the $G^0(\mathcal{T}_n)$ constitute an increasing sequence of subgraphs of $G(\mathcal{T}_e)$ and that $\lim G^0(\mathcal{T}_n) = G(\mathcal{T}_e)$. It is also clear that the $G^0(\mathcal{T}_n)$ are subgraphs of G; thus, to prove that $G = G(\mathcal{T}_e)$, it suffices to show

Lemma 4 *Each finite subgraph of G is contained in $G^0(\mathcal{T}_n)$ for some n.*

Proof: It suffices to show that each edge of G belongs to one of the graphs G_α for some node $\alpha \in \mathcal{T}$, for then we may choose n so large that α is a node of $\mathcal{T}_n^0$.

Let e' be any edge of G. If $e' \in B_e$, then $e' \in G^0(\mathcal{T}_1)$, so suppose $e' \notin B_e$. Let d be the length of a shortest circuit in G containing both e and e'. Let α be the node of $\mathcal{T}_1$ with $e' \in G_\alpha$, and let $e'' = \eta_\alpha$, where η is the link of $\mathcal{T}_1$ connecting α to the central vertex. Then the length of a shortest circuit in G_α containing both e' and e'' is either d (if B_e is a multilink) or strictly less than d (otherwise). Since two multilinks cannot be adjacent in any $\mathcal{T}_n$, it follows by induction that e' belongs to G_γ for some node $\gamma \in \mathcal{T}_n^0$ for some n, and since e' is not amalgamated, it belongs to $G^0(\mathcal{T}_{n+1})$. $\square$

So we have that $G = G(\mathcal{T}_e)$, and by the lemma, $\mathcal{T}_e$ satisfies conditions 2 and 3, and so $\mathcal{T}_e$ is a 3-block tree representing G.

5 Invariance of $\mathcal{T}_e$

Proposition 1 *Let G be a locally finite 2-connected graph, and let e be an edge of G. If $\mathcal{T}$ is any 3-block tree for G, then $\mathcal{T} = \mathcal{T}_e$.*

Proof: We will show that the 3-block of $\mathcal{T}$ which contains the edge e is equal to B_e in $\mathcal{T}_e$. Once this is established, then the subgraphs A_i are determined, and an induction shows that, for all k, the subtree of $\mathcal{T}$ consisting of all nodes a distance k from the node labeled B_e is identical with the corresponding subtree of $\mathcal{T}_e$, and the result follows.

Let α be the node of $\mathcal{T}$ whose graph G_α contains e.

If e is undeletable, then G_α is a cycle C, and the internal vertices of $C - e$ correspond to cutpoints of $G - e$. Let e' be an amalgamated edge along this cycle. Then $e' = \eta_\alpha$ for some link η with endpoint α. Let β be the other endpoint of η. Let $\mathcal{T}'$ denote the component of $\mathcal{T} - \eta$ containing β, and let $e'' = \eta_\beta$. Then e'' is not undeletable in $G(\mathcal{T}')$, since otherwise, G_β would be a cycle, as well. Therefore, every cutpoint of $G - e$ is a vertex of C, and $G_\alpha = C = B_e$.

If e is incontractible, then G_α is a multilink, and each component of $\mathcal{T} - \alpha$ corresponds to a union of some of the bridges of the endpoints of e. If some such component, say $\mathcal{T}'$, corresponds to more than one branch, then its amalgamated edge e' will be incontractible in $G(\mathcal{T}')$, and so the 3-block G_β containing it is a multilink. But β is adjacent to α in $\mathcal{T}$, so this is impossible. Therefore, each component of $\mathcal{T} - \alpha$ corresponds to exactly one branch, and so $G_\alpha = B_e$.

If G_α is simple, locally finite and 3-connected, then e is ordinary in G. Thus, B_e is also simple, locally finite and 3-connected. We must show that $B_e = G_\alpha$.

Let η be a link in $\mathcal{T}$ joining α with, say, β, and let $\mathcal{T}'$ be the component of $\mathcal{T} - \alpha$ containing β. Then it is clear that the subgraph $G(\mathcal{T}') - \eta_\beta$ of $G(\mathcal{T})$ is maximal with respect to the partial order defined earlier, and the same is true for every link of $\mathcal{T}$ incident with α. Since these maximal elements are uniquely determined once e has been chosen, it follows that $G_\alpha = B_e$. $\square$

6 Higher connectivity

The decomposition of a 2-connected graph into its 3-block tree is distinguished from other decompositions of 2-connected graphs by the fact that the 3-block tree is uniquely defined. Thus any symmetry exhibited by the graph $\mathcal{T}(G)$ will be reflected in the tree $\mathcal{T}$. Specifically, any automorphism of $G(\mathcal{T})$ induces an automorphism of $\mathcal{T}$. This is also true in the case of the block-cutpoint tree, however, analogous decompositions of graphs of higher connectivity, see [7], are not unique.

Theorem 2 *Let G be a simple 1-connected vertex transitive graph. Then either G is 2-connected or its block-cutpoint tree is infinite.*

PROOF: Let $\mathcal{T}$ be the block-cutpoint tree. If $\mathcal{T}$ is finite, then it has a center, which is an edge or a vertex. The center cannot be an edge, since the cutpoint corresponding to one of its endpoints would be distinguished in G. Similarly, the center cannot be a node corresponding to a cutpoint, so the center is a node corresponding to a block. Moreover, the central block must contain every vertex of G, so any other block can at most be a loop, and since G is simple, there is only one block. $\square$

A similar result is also true for 2-connected graphs.

Theorem 3 *Let G be a simple 2-connected vertex transitive graph. Then either G is either a cycle, 3-connected or its 3-block tree is infinite.*

PROOF: Let $\mathcal{T}$ be the 3-block tree. If $\mathcal{T}$ is finite, then it has a center, which is an edge or a vertex. The center cannot be a link of $\mathcal{T}$, since the two endpoints of the amalgamated edge corresponding to it would be distinguished, and G is not a multilink.

Thus the center is a node, and that node cannot represent a multilink, since, again, its endpoints would be distinguished in G. Every vertex of G must be in the 3-block of the central node, hence every other 3-block is a mulitlink. Since G is simple, $\mathcal{T}$ consists of exactly one 3-block, hence G is a cycle or 3-connected. $\square$

The pattern of these two proofs indicates the "meta-result" that there cannot be a canonical tree decomposition of 3-connected graphs in analogy with two and one connectivity, since we know there exists infinitely many finite 3-connected vertex transitive graphs which are not 4-connected, e.g. the Cayley graphs of finite groups with three generators, and these graphs would have to be indecomposable.

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