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On the position of the space of representable operators in the space of linear operators[1]

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Abstract. We show results about the existence and the nonexistence of a projection from the space $L(L^1(\lambda), X)$ of all linear and bounded operators from $L^1(\lambda)$ into X onto the subspace $R(L^1(\lambda), X)$ of all representable operators.

Keywords: representable operators, vector measures, L-projections, copies of c_0

Classification: 46B28, 46G10, 47B99, 46B25

Introduction

The problem of the complementability of some space $\mathcal{H}$ of operators in the space $L(X, Y)$ of all linear operators from a Banach space X into a Banach space Y has received much attention since the early sixties (see [Th], [AW], [To], [TW], [Ku], [Ka], [Fa], [Fe1], [Fe2], [E2], [E3], [E5], [EJ], [J], [CC], [BDLR]); particularly studied it has been the case of $\mathcal{H} =$ compact operators, for which the best result known (see [E3], [J]) states that if a copy of c_0 lives in this space, then there does not exist a projection onto it (in passing we observe that in the only two cases known in which c_0 does not embed into $K(X, Y) \neq L(X, Y)$ ([E3], [E6]) there is no hope of finding a norm one projection as proved in the recent paper [EJ]). We observe that the presence of copies of c_0 in the smaller space $\mathcal{H}$ plays an important role for the nonexistence of a projection onto $\mathcal{H}$ even in other cases ([BDLR], [CC], [DD], [E2], [E5]). In particular, from all of the results in the papers quoted above it follows that when X and Y are *classical* Banach spaces, i.e. spaces with dual isometric to some L^p space, no projection from the bigger space $L(X, Y)$ onto a smaller one exists, but in the following case (see [Fa]): *Let* (Ω, Σ, μ) *be a finite measure space; then the space* $R(L^1[0,1], L^1(\mu))$ *of all representable operators is norm one complemented in the space* $L(L^1[0,1], L^1(\mu))$.

In this short note we wish to improve this last result by showing that also for other Banach spaces X the space $R(L^1(\lambda), X), (S, \mathcal{F}, \lambda)$ a finite measure space, is norm one complemented in $L(L^1(\lambda), X)$; we also observe that if $R(L^1(\lambda), X)$ is complemented in $L(L^1(\lambda), X)$, then clearly $R(L^1(\lambda), Y)$ is complemented in $L(L^1(\lambda), Y)$ for any subspace Y complemented in X.

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It is known (see [DU], Chapter III, especially p. 62 and 84) that there is a 1-1 correspondence between the space $L(L^1(\lambda), X)$ (resp. $R(L^1(\lambda), X)$) and a subspace of the space $cabv(\lambda, X)$ (resp. $L^1(\lambda, X)$) of all countably additive vector measures G with bounded variation equipped with variation norm $\|G\|(S)$ (resp. countably additive vector measures with bounded variation having a Bochner density), precisely the subspace of those measures for which there is a constant $C > 0$ such that

$$\|G(E)\| \leq C\lambda(E) \qquad \forall\, E \in \mathcal{F}. \tag{1}$$

In order to study our problem, it appears thus natural to use some results (see [C], [FRP], [E7], [R]) about the existence of a projection from the space $cabv(\lambda, X)$ onto the subspace $L^1(\lambda, X)$. Since under that correspondence the norms in $L(L^1(\lambda), X)$ and in $cabv(\lambda, X)$ are not equivalent, the mere complementability of $L^1(\lambda, X)$ in $cabv(\lambda, X)$ does not seem sufficient to guarantee the existence of the required projection of $L(L^1(\lambda), X)$ onto $R(L^1(\lambda), X)$; indeed,if G is the representing measure of some $T \in L(L^1(\lambda), X)$ and P is the projection of $cabv(\lambda, X)$ onto $L^1(\lambda, X)$, then PG does not necessarily determine an element in $R(L^1(\lambda), X)$, since if G satisfies (1) it seems that there is no reason why even PG must satisfy a similar condition. However, we shall see that if the space $L^1(\lambda, X)$ is an L-summand (we refer to [HWW] for this well known definition) in the space $cabv(\lambda, X)$, then it is possible to construct simply a norm one projection from $L(L^1(\lambda), X)$ onto $R(L^1(\lambda), X)$. We also observe that if X is a Banach lattice not containing c_0, then $L^1(\lambda, X)$ is complemented in $cabv(\lambda, X)$, as proved in [C], [E7] and [FRP]; even in this case we are able to show a complementability result about $R(L^1(\lambda), X)$ and $L(L^1(\lambda), X)$ is true, actually getting that $R(L^1(\lambda), X)$ is a projection band in $L(L^1(\lambda), X)$. The influence of the results about $cabv(\lambda, X)$ and $L^1(\lambda, X)$ does not stop here; indeed, similarly to the case of the spaces $cabv(\lambda, X)$ and $L^1(\lambda, X)$ (see [DE]), we shall be also able to prove that if the range space X contains a copy of c_0, there is no projection as required. The existing results also show that to get the noncomplementability it is not enough to suppose the mere existence of a copy of c_0 inside $R(L^1(\lambda), X)$, differently from the case, quoted above, of the space of compact operators.

Results

First of all, we introduce the notion of a representable operator.

Definition. *Let $(S, \mathcal{F}, \lambda)$ be a finite measure space and X be a Banach space. A bounded linear operator $T : L^1(\lambda) \to X$ is representable if there exists $g \in L^\infty(\lambda)$ such that*

$$T(f) = \int_S f(s)g(s)d\lambda \qquad \forall\, f \in L^1(\lambda).$$

Now we prove a first complementability result, as announced in the introduction, from the proof of which the relevant role of the existence of an L-projection of $cabv(\lambda, X)$ onto $L^1(\lambda, X)$ appears clear (as underlined in the Introduction).

Theorem 1. *Let $(S, \mathcal{F}, \lambda)$ be a finite measure space and X be a Banach space such that the space $L^1(\lambda, X)$ is complemented in the space $cabv(\lambda, X)$ by an L-projection L. Then $R(L^1(\lambda), X)$ is norm one complemented in $L(L^1(\lambda), X)$.*

PROOF: Let T be an element of $L(L^1(\lambda), X)$ and G the representing vector measure of T; it is well known that $\|G(E)\| \leq \|T\|\lambda(E)$ for all $E \in \mathcal{F}$. We want to show first that $\|LG(E)\| \leq \|T\|\lambda(E)$ for all $E \in \mathcal{F}$. Given $E \in \mathcal{F}$ we have

$$\|G\|(S) = \|G\|(E) + \|G\|(E^c) \leq$$
$$\|LG\|(E) + \|G - LG\|(E) + \|LG\|(E^c) + \|G - LG\|(E^c) =$$
$$\|LG\|(S) + \|G - LG\|(S) = \|G\|(S)$$

from which it follows easily that

$$\|LG(E)\| \leq \|LG\|(E) \leq \|G\|(E) \leq \|T\|\lambda(E) \qquad \forall\, E \in \mathcal{F}.$$

Hence, the measure LG gives rise to an operator LT from $L^1(\lambda)$ into X that is representable ([DU, p. 84]). Furthermore, since it is easily seen that $\|LT\| = \sup\{\|LG(E)\|/\lambda(E) : E \in \mathcal{F}, \lambda(E) \neq 0\}$ (see [DU, p. 84]) we also get $\|LT\| \leq \|T\|$. We are done. □

We observe that the projection L constructed above cannot be an L-projection, since $L(L^1(\lambda), X)$ always has nontrivial M-summands (see [HWW]).

It now becomes important to find examples of spaces X for which $L^1(\lambda, X)$ is an L-summand in $cabv(\lambda, X)$. We can (partially) answer this question with the following

Proposition 2. *Let X be a Banach space such that $L^1(\lambda, X)$ is an L-summand in the bidual. Then $L^1(\lambda, X)$ is an L-summand in $cabv(\lambda, X)$.*

PROOF: In the recent papers [E7], [R] it is remarked that $cabv(\lambda, X)$ is isometric to a closed subspace of $(L^1(\lambda, X))^{**}$; hence the restriction of the L-projection of $(L^1(\lambda, X))^{**}$ onto $L^1(\lambda, X)$ works well to reach our target.

So if

(i) $X = L^1(\mu)$,
(ii) X is a predual of a W^*-algebra,
(iii) X is a nicely placed subspace of $L^1(\mu)$,
(iv) X is isometric to a quotient space Y/Z where both $L^1(\lambda, Y)$ and $L^1(\lambda, Z)$ are L-summands in their respective biduals (for instance we can choose $Y = L^1(\mu)$ and Z a reflexive subspace of it),

then we can apply our Theorem 1 as a consequence of results in [E7], [HWW], [R]. We note that the case (i) gives the old result by Fakhouri ([Fa]) quoted in the Introduction, but our proof seems to be simpler. □

In the next result we present some other cases in which Theorem 1 is applicable even if we do not know if the considered quotient space satisfies or not the hypothesis of Proposition 2.

Proposition 3. *Let X be a Banach space such that $L^1(\lambda, X)$ is an L-summand in the space $cabv(\lambda, X)$ and Z be a closed subspace of X having the Radon-Nikodym Property such that the map $\tilde{Q}$ from $cabv(\lambda, X)$ into $cabv(\lambda, X/Z)$ defined by $[\tilde{Q}(\nu)](E) = Q[\nu(E)]$, $E \in \mathcal{F}$ (Q denotes the quotient map of X onto X/Z), is also a quotient map (see* [E7] *for results implying the validity of this assumption). Then, $L^1(\lambda, X/Z)$ is an L-summand in $cabv(\lambda, X/Z)$.*

PROOF: Denote by L the existing L-projection of $cabv(\lambda, X)$ onto $L^1(\lambda, X)$. In [E7] it is proved that the map $\tilde{L} : cabv(\lambda, X/Z) \to L^1(\lambda, X/Z)$ defined by

$$\tilde{L}(\tilde{\nu}) = \tilde{Q}[L(\nu)] \qquad \forall\, \tilde{\nu} \in cabv(\lambda, X/Z), \nu \in cabv(\lambda, X), \tilde{Q}(\nu) = \tilde{\nu}$$

actually is a projection onto $L^1(\lambda, X/Z)$. Now, we show it is an L-projection. Let us suppose there is $\tilde{\nu}_0 \in cabv(\lambda, X/Z)$ for which

$$h = \|\tilde{L}(\tilde{\nu}_0)\|(S) + \|\tilde{\nu}_0 - \tilde{L}(\tilde{\nu}_0)\|(S) - \|\tilde{\nu}_0\|(S) > 0.$$

Choose $\nu_0 \in cabv(\lambda, X)$ such that $\tilde{Q}(\nu_0) = \tilde{\nu}_0$ and $\|\nu_0\|(S) < \|\tilde{\nu}_0\|(S) + h$. We get

$$\|\nu_0\|(S) < \|\tilde{\nu}_0\|(S) + h = \|\tilde{L}(\tilde{\nu}_0)\|(S) + \|\tilde{\nu}_0 - \tilde{L}(\tilde{\nu}_0)\|(S) \leq \\ \|L(\nu_0)\|(S) + \|\nu_0 - L(\nu_0)\|(S) = \|\nu_0\|(S)$$

from which our claim follows. □

For instance, Proposition 3 can be applied with $X = L^1$ and $Z = H_0^1$.

In the papers [C], [E7] and [FRP] it is shown that $L^1(\lambda, X)$ is a projection band inside $cabv(\lambda, X)$, when X is a Banach lattice not containing copies of c_0 (in $cabv(\lambda, X)$, the positive elements are those measures taking $\mathcal{F}$ into the positive cone of X); we are also able to use this result to get one more complementability result of $R(L^1(\lambda), X)$ inside $L(L^1(\lambda), X)$ that surely is not a consequence of the previous ones, because the projection of $cabv(\lambda, X)$ onto $L^1(\lambda, X)$ constructed in [C],[E7] and [FRP] is not an L-projection. Actually, we shall prove more than the mere complementability (and in this way we make more precise the result from the paper [Fa] quoted at the beginning).

Theorem 4. *Let X be a Banach lattice not containing copies of c_0. Then $R(L^1(\lambda), X)$ is a projection band in $L(L^1(\lambda), X)$.*

PROOF: We fix some notation: if $T \in L(L^1(\lambda), X)$, then G_T will denote its representing measure (possessing a Radon-Nikodym derivative g_T in case $T \in R(L^1(\lambda), X)$) and if $G \in cabv(\lambda, X)$ is a representing measure of some operator in $L(L^1(\lambda), X)$ we shall denote it by T_G. First of all we observe that, under our hypotheses, $L(L^1(\lambda), X)$ is a Banach lattice (see [MN, Theorem 1.5.11]) under the *natural* order, i.e. $T \leq H$ if and only if $H - T \geq 0$; hence $T \leq H$ if and only if $G_T \leq G_H$. We start showing that if T is in $R(L^1(\lambda), X)$, then even T^+, T^-, $|T|$ are in $R(L^1(\lambda), X)$; to this aim we observe that, for $E \in \mathcal{F}$

$$G_{T^+}(E) = T^+(\chi_E) = \sup\{T(f) : 0 \leq f \leq \chi_E\} =$$
$$\sup\left\{\int_S g_T(s)f(s)d\lambda : 0 \leq f \leq \chi_E\right\} \leq \sup\left\{\int_S |g_T(s)|f(s)d\lambda : 0 \leq f \leq \chi_E\right\} =$$
$$\int_E |g_T(s)|d\lambda,$$

where we used the fact that $L^1(\lambda, X)$ is a Banach lattice and that the integral is a positive operator. Since the measure $\int_\cdot |g_T(s)|d\lambda$ is in $L^1(\lambda, X)$ and $L^1(\lambda, X)$ is an ideal in $cabv(\lambda, X)$ (see the paper [C] for instance), we get that $T^+ \in R(L^1(\lambda), X)$; similarly we can prove that $T^- \in R(L^1(\lambda), X)$ and so also $|T| \in R(L^1(\lambda), X)$. Once we have got this, it is very simple to prove that $R(L^1(\lambda), X)$ is an ideal in $L(L^1(\lambda), X)$. Let now T be an element in the band generated by $R(L^1(\lambda), X)$; there is a net $(T_\alpha) \subset R(L^1(\lambda), X)$ and a decreasing net $(H_\alpha) \subset L(L^1(\lambda), X)$ such that

$$|T_\alpha - T| \leq H_\alpha, \qquad H_\alpha \downarrow 0.$$

We have the following chain of inequalities, valid for all $E \in \Sigma$,

$$(G_{T_\alpha} - G_T)^+(E) = \sup\{(G_{T_\alpha} - G_T)(B) : B \in \mathcal{F}, B \subset E\} =$$
$$\sup\{(T_\alpha - T)(\chi_B) : B \in \mathcal{F}, B \subset E\} \leq \sup\{(T_\alpha - T)(f) : 0 \leq f \leq \chi_E\} =$$
$$(T_\alpha - T)^+(\chi_E) \leq |T_\alpha - T|(\chi_E) \leq H_\alpha(\chi_E) = G_{H_\alpha}(E).$$

Similarly, we can get that, for all $E \in \Sigma$,

$$(G_{T_\alpha} - G_T)^-(E) \leq G_{H_\alpha}(E).$$

If we are able to show that $G_{H_\alpha} \downarrow 0$, we shall have that $G_T \in L^1(\lambda, X)$ because $L^1(\lambda, X)$ is a band in $cabv(\lambda, X)$, a fact implying that $T \in R(L^1(\lambda), X)$. But this is quite clear because (H_α) is decreasing and so $H_\alpha \leq H_\beta$ for $\alpha \geq \beta$ from which $G_{H_\alpha} \leq G_{H_\beta}$ follows. Furthermore, using also a result due to Riesz-Kantorovich (see [AB, Theorem 1.13]) it is easy to see that $\inf G_{H_\alpha} = 0$. Hence, $R(L^1(\lambda), X)$ is a band in $L(L^1(\lambda), X)$. It remains just to show that $R(L^1(\lambda), X)$ is a projection band in $L(L^1(\lambda), X)$. So let us take $T \geq 0, T \in L(L^1(\lambda), X)$; we consider the set $Z = [0, T] \cap R(L^1(\lambda), X)$ and we observe that each element of Z has a representing measure contained in the set $Y = [0, G_T] \cap L^1(\lambda, X)$; conversely, each element in Y determines an element in Z, because if $G \in Y$ we have $0 \leq G \leq G_T$ from which clearly follows, for all $E \in \Sigma$

$$\|G(E)\| \leq \|G_T(E)\| \leq \|T\|\lambda(E)$$

and so $T_G \in Z$. Let $G_0 = \sup Y$; such a supremum must exist since $L^1(\lambda, X)$ is a projection band in $cabv(\lambda, X)$ (hence $G_0 \le G_T$ and $G_0 \in L^1(\lambda, X)$); it is not difficult to show that $T_{G_0} = \sup Z$, a fact that concludes our proof. $\square$

We observe that similar arguments to those used in Theorem 5 in [E7] allow us to show that suitable quotients of spaces X's for which the complementability occurs also enjoy the same property; we do not prove here this result, but we simply state it as

Proposition 5. *Suppose X is such that $R(L^1(\lambda), X)$ is complemented into $L(L^1(\lambda), X)$. Suppose Z is a closed subspace of X with the Radon-Nikodym property. Define a map $\tilde{Q} : L(L^1(\lambda), X) \to L(L^1(\lambda), X/Z)$ by putting $[\tilde{Q}(T)](f) = Q[T(f)]$ (Q is the quotient map of X onto X/Z) for all $f \in L^1(\lambda)$. If $\tilde{Q}$ is a quotient map, thus $R(L^1(\lambda), X/Z)$ is complemented into $L(L^1(\lambda), X/Z)$.*

This Proposition 5 allows us to improve the results about the complementability of $R(L^1(\lambda), X)$ inside $L(L^1(\lambda), X)$ that are consequences of Proposition 3; however we must underline that, in our opinion, Proposition 3 is of an independent interest, because of the nature of the projection from $cabv(\lambda, X/Z)$ onto $L^1(\lambda, X/Z)$ obtained there; in particular, we observe that Proposition 3 allows us to present some more occurrence in which the Lebesgue Decomposition Theorem (see [DU]) can be improved (see Remark 2 in [E7]).

The assumption of surjectivity considered in Proposition 5 (see also Proposition 3) cannot be dropped at all, since if $X = l_1$ and $X/Z = c_0$, we have that $R(L^1(\lambda), X) = L(L^1(\lambda), X)$, but that $R(L^1(\lambda), X/Z)$ is not complemented (see the following Theorem 6) in $L(L^1(\lambda), X/Z)$.

As remarked in the Introduction there is some case in which the projection from $L(L^1(\lambda), X)$ onto $R(L^1(\lambda), X)$ cannot be found; this happens, for instance, when X contains a copy of c_0 as it happens in the case of the spaces $cabv(\lambda, X)$ and $L^1(\lambda, X)$ (see [DE]).

Theorem 6. *Let X contain a copy of c_0. Then $R(L^1(\lambda), X)$ is uncomplemented in $L(L^1(\lambda), X)$.*

Proof: We first construct a complemented copy of c_0 in $R(L^1(\lambda), X)$ following a general procedure described in [E4]. Let us denote by (x_n) the copy of the unit vector basis of c_0 in X and by (r_n) the sequence of Rademacher functions in $(L^1(\lambda))^*$. The sequence $(r_n \otimes x_n)$ is easily seen to be a copy of the unit vector basis of c_0 in $R(L^1(\lambda), X)$ (see for instance [E3]). If (x_n^*) is a bounded sequence in X^* such that $x_m(x_n^*) = \delta_{mn}$, the sequence $(r_n \otimes x_n^*)$ is easily seen to be a weak*-null sequence in $(R(L^1(\lambda), X))^*$ (here we consider (r_n) as a sequence in $L^1(\lambda)$ and we use its weak convergence to θ as well as the fact that each representable operator is a Dunford-Pettis operator, [DU]). Hence, we can suppose ([E1], [S]) that $(r_n \otimes x_n)$ spans a complemented copy K of c_0 in $R(L^1(\lambda), X)$; it is now a standard fact ([Ka]) that it is possible to construct a linear map from l^∞ into $L(L^1(\lambda), X)$ that is an isomorphism onto some subspace H of $L(L^1(\lambda), X)$, with

H containing K. These facts, as observed in ([Ka], [E2], [E3], [E5], [J]), imply the nonexistence of a projection from $L(L^1(\lambda), X)$ onto $R(L^1(\lambda), X)$. We are done. $\square$

Remark 1. We observe that in order to get the conclusion of Theorem 5 it suffices to suppose that $R(L^1(\lambda), X)$ contains a complemented copy of c_0; we did that assuming that X contains a copy of c_0. This is the only possibility we have; indeed, it is well known that $R(L^1(\lambda), X)$ is isometrically isomorphic to $L^\infty(\lambda, X)$ (see [DU]) and so it is enough to use a result by Diaz ([D]) stating that if c_0 embeds complementably in $L^\infty(\lambda, X)$, then X necessarily contains a copy of c_0 to show the necessity of our assumption.

Remark 2. The proof of Theorem 5 actually shows that any subspace $\mathcal{H}$ of the space of all Dunford-Pettis operators from $L^1(\lambda)$ into X is uncomplemented in $L(L^1(\lambda), X)$, whenever X contains a copy of c_0, provided $\mathcal{H}$ contains finite dimensional operators.

Remark 3. We observe that on the contrary to the case of the nonexistence of a projection onto the space of compact operators mentioned at the beginning, it is possible for $R(L^1(\lambda), X)$ to contain a copy of c_0 and to be complemented in $L(L^1(\lambda), X)$ at the same time; for instance, in the case of $X = L^1(\mu)$ it is well known that c_0 lives in $R(L^1(\lambda), X)$ (see [Fe2], [E3]), even if $R(L^1(\lambda), X)$ is complemented in $L(L^1(\lambda), X)$; under this point of view $R(L^1(\lambda), X)$ behaves differently from the space of compact operators.

The present, in some sense surprising, results suggest the following final comment: the problem of the complementability of $R(L^1(\lambda), X)$ in $L(L^1(\lambda), X)$ is quite different from that of the complementability of other spaces of operators in $L(L^1(\lambda), X)$ (even if in both cases the considered norms are the supnorm) and is quite close to (or at least heavily depending upon) the problem of the complementability of $L^1(\lambda, X)$ in $cabv(\lambda, X)$ (even if the considered norms are different). We think it could be interesting to continue to investigate to which extent this dependence is valid.

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