ON A VIEW OF MATHEMATICAL OPTICS

By

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On a View of Mathematical Optics. By William R. Hamilton, *Royal Astronomer of Ireland, &c.*

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The Memoirs on Systems of Rays, which have been presented by me to the Royal Irish Academy, and of which some have been published in the XVth and XVIth volumes of the *Transactions* of that Academy, contain a view of mathematical optics, which appears to me to be analogous to the view taken by Descartes of algebraical geometry, and likely to lead those who shall adopt it to analogous changes of method. It has been thought desirable, by the Mathematical Committee of the British Association for the Advancement of Science, that a short statement of this view of optics should be given in the forthcoming publication of that body. Such a statement, therefore, I shall now offer, as briefly as I can; endeavouring only to communicate the view itself, and abstaining from giving any account of the results to which it has conducted me.

The *general problem* that I have proposed to myself in optics, is to *investigate the mathematical consequences of the law of least action*: a general law of vision, in which are included, as it is well known, all the particular conditions of reflexion and refraction, gradual and sudden, ordinary and extraordinary. And the *central idea* from which my whole *method* flows, is the idea of *one radical or characteristic relation for each optical system of rays*, that is, for each combination of straight or bent, or curved paths, along which light is supposed to be propagated according to the law of least action. This characteristic relation, being different for different systems, and being such that the mathematical properties of the system can all be deduced from it, in the same manner as the method invented by Descartes for the algebraical solution of geometrical problems, flows all from the central idea of one radical relation, for each plane curve, or curved surface, in the form of which relation are included all the properties of the curve or the surface. In the radical relation thus contemplated by Descartes, in his view of algebraical geometry, the related things are elements of position of a variable point which has for locus a curve or a surface; and the number of these related elements is either two or three. In the relation contemplated by me, in my view of algebraical optics, the related things are, in general, in number, eight: of which, six are elements of position of two variable points of space, considered as visually connected; the seventh is an index of colour; and the eighth, which I call the CHARACTERISTIC FUNCTION,—because I find that in the manner of its dependence on the seven foregoing are involved all the properties of the system,—is the *action* between the two variable points; the word *action* being used here, in the same sense as in that known law of vision which has been already mentioned. I have assigned, for the variation of this characteristic function, corresponding to any infinitesimal variations in the positions on which it depends, a fundamental formula; and I consider as *reducible to the study of this one characteristic function, by the means of this one fundamental formula, all the problems of mathematical optics*, respecting all imaginable combinations of mirrors, lenses, crystals and atmospheres. And though, among these problems of mathematical optics, it is not here intended to include investigations respecting the *phænomena of interference*, yet it is easy to perceive, from the nature of the quantity which I have called the characteristic function, and which in the hypothesis of undulations is *the time of propagation of light from one variable point to another*, that the study of this function must be useful in such investigations also. My own researches, however, have been hitherto chiefly directed to the consequences of the law of least action, and to the properties of optical systems, and systems of rays in general. And having stated, in the foregoing remarks, the *view* that has guided these researches, I must refer, for the *results*, to the volumes already mentioned, of the Royal Irish Academy, and to the XVIIth volume, not yet published, in which a third supplement to my Essay on the Theory of Systems of Rays has been ordered by the Academy to be printed.