

**EDITORIAL TEAM****EDITOR-IN-CHIEF**

MARTIN RAUSSEN  
 Department of Mathematical Sciences,  
 Aalborg University  
 Fredrik Bajers Vej 7G  
 DK-9220 Aalborg, Denmark  
 e-mail: [raussen@math.aau.dk](mailto:raussen@math.aau.dk)

**ASSOCIATE EDITORS**

VASILE BERINDE  
 Department of Mathematics,  
 University of Baia Mare, Romania  
 e-mail: [vberinde@ubm.ro](mailto:vberinde@ubm.ro)  
 KRZYSZTOF CIESIELSKI  
 Mathematics Institute  
 Jagiellonian University  
 Reymonta 4, 30-059 Kraków, Poland  
 e-mail: [ciesiels@im.uj.edu.pl](mailto:ciesiels@im.uj.edu.pl)

STEEN MARKVORSEN  
 Department of Mathematics, Technical  
 University of Denmark, Building 303  
 DK-2800 Kgs. Lyngby, Denmark  
 e-mail: [s.markvorsen@mat.dtu.dk](mailto:s.markvorsen@mat.dtu.dk)

ROBIN WILSON  
 Department of Pure Mathematics  
 The Open University  
 Milton Keynes MK7 6AA, UK  
 e-mail: [r.j.wilson@open.ac.uk](mailto:r.j.wilson@open.ac.uk)

**COPY EDITOR:**

KELLY NEIL  
 School of Mathematics  
 University of Southampton  
 Highfield SO17 1BJ, UK  
 e-mail: [K.A.Neil@maths.soton.ac.uk](mailto:K.A.Neil@maths.soton.ac.uk)

**SPECIALIST EDITORS**

INTERVIEWS  
 Steen Markvorsen [address as above]  
 SOCIETIES

Krzysztof Ciesielski [address as above]  
 EDUCATION

Tony Gardiner  
 University of Birmingham  
 Birmingham B15 2TT, UK  
 e-mail: [a.d.gardiner@bham.ac.uk](mailto:a.d.gardiner@bham.ac.uk)

MATHEMATICAL PROBLEMS  
 Paul Jainta  
 Werkvolkstr. 10  
 D-91126 Schwabach, Germany  
 e-mail: [PaulJainta@aol.com](mailto:PaulJainta@aol.com)

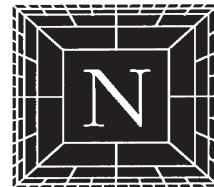
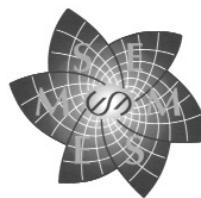
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 June Barrow-Green and Jeremy Gray  
 Open University [address as above]  
 e-mail: [j.e.barrow-green@open.ac.uk](mailto:j.e.barrow-green@open.ac.uk)  
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CONFERENCES  
 Vasile Berinde [address as above]  
 RECENT BOOKS

Ivan Netuka and Vladimír Souček  
 Mathematical Institute  
 Charles University, Sokolovská 83  
 18600 Prague, Czech Republic  
 e-mail: [netuka@karlin.mff.cuni.cz](mailto:netuka@karlin.mff.cuni.cz)  
 and [soucek@karlin.mff.cuni.cz](mailto:soucek@karlin.mff.cuni.cz)

**ADVERTISING OFFICER**

Vivette Girault  
 Laboratoire d'Analyse Numérique  
 Boîte Courrier 187, Université Pierre  
 et Marie Curie, 4 Place Jussieu  
 75252 Paris Cedex 05, France  
 e-mail: [girault@ann.jussieu.fr](mailto:girault@ann.jussieu.fr)

**EUROPEAN MATHEMATICAL SOCIETY****NEWSLETTER No. 52****June 2004**

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**NOTICE FOR MATHEMATICAL SOCIETIES**

*Labels for the next issue will be prepared during the second half of August 2004.  
 Please send your updated lists before then to Ms Tuulikki Mäkeläinen, Department of Mathematics and Statistics, P.O. Box 68 (Gustav Hällströmintie 2B), FI-00014 University of Helsinki, Finland; e-mail: [tuulikki.makelainen@helsinki.fi](mailto:tuulikki.makelainen@helsinki.fi)*

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## EMS Committee

### EXECUTIVE COMMITTEE

#### PRESIDENT

Prof. Sir JOHN KINGMAN (2003-06)  
 Isaac Newton Institute  
 20 Clarkson Road  
 Cambridge CB3 0EH, UK  
 e-mail: [ems president@newton.cam.ac.uk](mailto:ems president@newton.cam.ac.uk)

#### VICE-PRESIDENTS

Prof. LUC LEMAIRE (2003-06)

Department of Mathematics  
 Université Libre de Bruxelles  
 C.P. 218 – Campus Plaine  
 Bld du Triomphe  
 B-1050 Bruxelles, Belgium  
 e-mail: [l lemaire@ulb.ac.be](mailto:l lemaire@ulb.ac.be)

Prof. BODIL BRANNER (2001-04)

Department of Mathematics  
 Technical University of Denmark  
 Building 303  
 DK-2800 Kgs. Lyngby, Denmark  
 e-mail: [b.branner@mat.dtu.dk](mailto:b.branner@mat.dtu.dk)

#### SECRETARY

Prof. HELGE HOLDEN (2003-06)

Department of Mathematical Sciences  
 Norwegian University of Science and Technology  
 Alfred Getz vei 1  
 NO-7491 Trondheim, Norway  
 e-mail: [h.holden@math.ntnu.no](mailto:h.holden@math.ntnu.no)

#### TREASURER

Prof. OLLI MARTIO (2003-06)  
 Department of Mathematics  
 P.O. Box 4, FIN-00014  
 University of Helsinki, Finland  
 e-mail: [olli.martio@helsinki.fi](mailto:olli.martio@helsinki.fi)

#### ORDINARY MEMBERS

Prof. VICTOR BUCHSTABER (2001-04)  
 Steklov Mathematical Institute  
 Russian Academy of Sciences  
 Gubkina St. 8, Moscow 117966, Russia  
 e-mail: [buchstab@mendeleev.ru](mailto:buchstab@mendeleev.ru)

Prof. DOINA CIORANESCU (2003-06)  
 Laboratoire d'Analyse Numérique

Université Paris VI  
 4 Place Jussieu  
 75252 Paris Cedex 05, France  
 e-mail: [cioran@ann.jussieu.fr](mailto:cioran@ann.jussieu.fr)

Prof. PAVEL EXNER (2003-06)

Department of Theoretical Physics, NPI  
 Academy of Sciences  
 25068 Rez – Prague, Czech Republic  
 e-mail: [exner@ujf.cas.cz](mailto:exner@ujf.cas.cz)

Prof. MARTA SANZ-SOLÉ (2001-04)

Facultat de Matemàtiques  
 Universitat de Barcelona  
 Gran Via 585  
 E-08007 Barcelona, Spain

e-mail: [sanz@cerber.mat.ub.es](mailto:sanz@cerber.mat.ub.es)

Prof. MINA TEICHER (2001-04)

Department of Mathematics and  
 Computer Science  
 Bar-Ilan University  
 Ramat-Gan 52900, Israel  
 e-mail: [teicher@macs.biu.ac.il](mailto:teicher@macs.biu.ac.il)

#### EMS SECRETARIAT

Ms. T. MÄKELÄINEN  
 Department of Mathematics and Statistics  
 P.O. Box 68 (Gustav Hällströmintie 2B)  
 FI-00014 University of Helsinki  
 Finland  
 tel: (+358)-9-1912-2883  
 fax: (+358)-9-1912-3213  
 telex: 124690  
 e-mail: [tuulikki.makelainen@helsinki.fi](mailto:tuulikki.makelainen@helsinki.fi)  
 website: <http://www.emis.de>

## EMS Agenda

### 2004

#### 19-24 June

EURESCO Conference.  
 Symmetries and Integrability of Difference Equations:  
 EuroConference on Analytic Linear and Nonlinear Difference Equations and Special Functions  
 Helsinki (Finland).

#### 25 June

EMS Executive Committee meeting at Uppsala (Sweden).  
 Contact: Helge Holden, email: [holden@math.ntnu.no](mailto:holden@math.ntnu.no)

#### 26-27 June

EMS Council meeting at Uppsala (Sweden).  
 Contact: Helge Holden, email: [holden@math.ntnu.no](mailto:holden@math.ntnu.no)  
 or Tuulikki Mäkeläinen email: [tuulikki.makelainen@helsinki.fi](mailto:tuulikki.makelainen@helsinki.fi)

#### 27 June - 2 July

4th European Congress of Mathematics, Stockholm.  
 Web site: <http://www.math.kth.se/4ecm>

#### 4-24 July

EMS Summer School at Cortona (Italy).  
 Evolution equations and applications.

Contact: Graziano Gentili, email: [gentili@unifi.it](mailto:gentili@unifi.it)

#### 15-23 July

EMS Conference at Będlewo (Poland).  
 Analysis on metric measure spaces.

Web site: <http://impan.gov.pl/Bedlewo/>

#### 15 August

Deadline for submission of material for the September issue of the EMS Newsletter.  
 Contact: Martin Raussen, email: [raussen@math.aau.dk](mailto:raussen@math.aau.dk)

#### 30 August - 3 September

EMS Summer School at Universidad de Cantabria, Santander (Spain).  
 Empirical processes: Theory and Statistical Applications.

Web site: <http://www.eio.uva.es/ems/>

#### 3-5 September

EMS-Czech Union of Mathematicians and Physicists (Mathematics Research Section).  
 Mathematical Weekend, Prague (Czech Republic).

Web site: <http://mvs.jcmf.cz/emsweekend/>

#### 5-6 September

EMS Executive Committee Meeting in Prague.

### 2005

#### 5-13 February

EMS Summer School at Eilat (Israel).  
 Applications of braid groups and braid monodromy.

#### 25 June - 2 July

EMS Summer School at Pontignano (Italy).  
 Subdivision schemes in geometric modelling, theory and applications.

#### 17-23 July

EMS Summer School and European young statisticians' training camp at Oslo (Norway).

#### 13-23 September

EMS Summer School at Barcelona (Catalunya, Spain).  
 Recent trends of Combinatorics in the mathematical context.

#### 16-18 September

EMS-SCM Joint Mathematical Weekend, Barcelona (Catalunya, Spain).

#### 25 September - 2 October

EMS Summer School and Séminaire Européen de Statistique at Warwick (UK).  
 Statistics in Genetics and Molecular Biology.

#### 12-16 December

EMS-SIAM-UMALCA joint meeting in applied mathematics.  
 Venue: the CMM (Centre for Mathematical Modelling), Santiago de Chile.

### 2007

#### 16-20 July

ICIAM 2007, Zurich (Switzerland).

### Cost of advertisements and inserts in the EMS Newsletter, 2004

(all prices in British pounds)

#### Advertisements

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#### Inserts

**Postage cost:** £14 per gram plus **Insertion cost:** £58 (e.g. a leaflet weighing 8.0 grams will cost  $8 \times £14 + £58 = £170$ ) (weight to nearest 0.1 gram)

# Editorial

**Ari Laptev (Stockholm)**

**Chairman, 4ECM Organizing Committee**

The Fourth European Congress of Mathematics will take place in Stockholm between June 28 and July 2 and is organized by the Royal Institute of Technology (KTH) in collaboration with Stockholm University (SU).

My possible involvement in the organization of this event became apparent in July 2000 at a Conference in Bristol, when I received an e-mail from Anatoly Vershik, who was a member of the EMS Executive Committee (EC) and was participating in the 3ECM in Barcelona. He told me that there were some difficulties in raising funds for organizing the next 4ECM in 2004, which was planned to be held Göteborg. On behalf of the EMS EC he asked me for some help since I was the Vice President of the Swedish Mathematical Society (SMS) at that time. Anatoly found it hard to believe that Sweden did not have the funds to organize the 4ECM. Naturally, I did not have much to say, but in the end of August 2000 there was a meeting at KTH, where it was preliminarily agreed to try to raise funds for the 4ECM in Stockholm.

At that meeting everybody assumed that, as the future President of the SMS, I would take the major responsibility for organizing 4ECM, although this was certainly not my original intention.

In November 2000, Anders Lindquist, chairman of the Mathematics Department, KTH, Ulf Persson and then President of the SMS, joined me in London to attend the EMC Executive Committee meeting. We were met with an element of friendly curiosity. Trying to play a "hard game" our delegation wanted to have as much freedom in organizing 4ECM as possible. To our great surprise we found a lot of understanding among the EMS EC members. In particular, when we suggested Lennart Carleson as a possible President of the 4ECM Scientific Committee, the EC members said that they had the same idea. So the meeting ended on a happy note and in mutual agreement.

Mostly due to Anders Lindquist's contacts we were able to raise some sub-

stantial funds amazingly quickly. Among our major sponsors are the Knut and Alice Wallenberg Foundation, the Swedish Foundation for Strategic Research, the Swedish Research Council, the Nobel Committees for Physics and Chemistry of the Royal Swedish Academy of Sciences and many others.

The Swedish Ministry of Higher Education and the bank of Sweden Tercentenary Foundation have also contributed to 4ECM funds by each providing financial support of 1/2 million SEK.

We are extremely grateful to all the sponsors. Without their support the 4ECM could not have been organized.

The 4ECM Committees worked very well. Lennart Carleson as the President of the Scientific Committee was most efficient in organizing two meetings of his Committee at KTH. At the first meeting the decision regarding the structure of the 4ECM was made and in June 2003, at the Committee's second meeting, all the speakers were chosen, with the exception of those speakers for Science Lectures. The Organizing Committee chose the latter afterwards, in coordination with L. Carleson.

The task of finding a President of the Prize Committee was rather difficult and the EMS Executive Committee was very happy when Nina Uraltseva finally agreed to take on this important responsibility. The Prize Committee was quickly formed and its work culminated at the meeting in Stockholm at the end of February, when 10 EMS Prize Winners were chosen.

Being invited to the meeting of this Committee I can confirm that the main decisions were made surprisingly quickly and in a very friendly way. It is not easy to choose 10 prizewinners from more than 50 young mathematicians nominated for an EMS prize and who almost all deserve such a prize.

On behalf of the 4ECM Organizing Committee I would like to express my gratitude to the members of the Scientific Committee for choosing an extremely interesting programme and to



*Ari Laptev*

the members of the Prize Committee for their hard work in selecting excellent candidates for the EMS prizes.

By now the Organizing Committee has considered two rounds of 4ECM poster applications. There were two deadlines - 15th of February and 20th of April. Altogether 358 posters were accepted. Unfortunately not all of them will be presented at the 4ECM since some of the poster applicants depend on receiving grants in order to attend.

There were more than 400 4ECM grant applications, mostly from Eastern Europe and the Former Soviet Union. Unfortunately our financial possibilities allow us to give only 209 grants, which cover travel, a double hotel room, lunches and the Conference fee. We are very grateful to the EMS EC for helping us to obtain \$25,000 from UNESCO-ROSTE, which allowed us to support participants from Central and Eastern European countries. In total we have more than 1,600,000 SEK for the 4ECM grants, which are approved mostly for young participants.

Although organizing a Congress with more than 1000 participants is hard work, I have found it very rewarding too. It allowed me to start seeing Mathematics much more as a whole subject, with many people trying to create beautiful new structures and find brilliant methods with which to solve old and new exciting problems.

Having only 1½ months remaining I am beginning to feel slightly apprehensive about how it will all go. Almost four years ago I could only guess how much work has to be done and even now I am still not quite sure I understand fully!

# **Evolution Equations and Applications**

**SUMMER SCHOOL of the European Mathematical Society**

**Scuola Matematica Interuniversitaria**

**Supported by the Istituto Nazionale di Alta Matematica and the European Commission**

**Cortona (Palazzone), Italy**

**4th to 24th July 2004**

**The SUMMER SCHOOL will be devoted to evolution problems, including:**

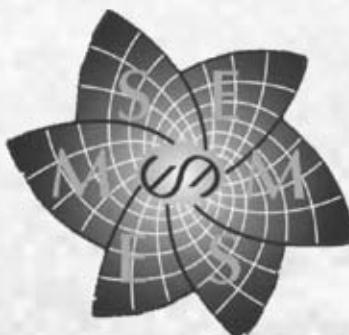
- **semigroups and evolution equations in Banach spaces;**
- **linear and non-linear parabolic type problems using functional analytical methods.**

**The School will provide an occasion to facilitate and stimulate discussion between all participants, in particular, young researchers and lecturers, for whom financial support is available.**

## **Courses**

- **Semigroups for linear evolution equations (Reiner Nagel, University of Tübingen, Germany)**
- **Linear and non linear diffusion problems (Alessandra Lunardi, University of Padova, Italy)**

**Website: <http://www.matapp.unimib.it/smi>**



# EMS Summer Schools

**Luc Lemaire (Brussels) and David Salinger (EMS Publicity Officer)**

Some time ago, we wrote that the European Mathematical Society had been unsuccessful in its application for Framework 6 support of its summer school programme. No sooner had the article appeared, than Vice-President Luc Lemaire was contacted and told that the project had been promoted from the reserve list. The EMS was working in close collaboration with the European Regional Committee of the Bernoulli Society, the Centre de Recerca Matematica (CRM) of Barcelona, the Scuola Matematica Interuniversitaria (SMI) of Firenze, the Mathematical Research and Conference Centre in Będlewo (Poland), the Emmy Noether Institute (German-Israeli research institute located in Israel) and various groups of mathematicians working in European universities. We shall pass over the traumatic hours of form filling and further negotiation, familiar to anyone who gets involved in European grants, but in March the EMS obtained a major grant of the European Commission (Marie Curie Series of Conferences) for the following eight Summer Schools. In the list below, attentive readers will perceive that not all the events happen in summer, but all bar the Będlewo meeting do count as EMS Summer Schools.

4-24 July 2004 at Cortona, Italy: *Evolution equations and applications*;

15-23 July 2004 at Będlewo, Poland: *Analysis on metric measure spaces*;

12-19 December 2004 at Munich, Germany: *The Statistics of Spatio-Temporal Systems*;

5-13 February 2005 at Eilat, Israel: *Applications of braid groups and braid monodromy*;

25 June - 2 July 2005 at Pontignano, Italy: *Subdivision schemes in geometric modelling, theory and applications*;

17-23 July 2005 at Oslo, Norway: *European young statisticians' training camp*;

13-23 September 2005 at Barcelona, Spain: *Recent trends of Combinatorics in the mathematical context*;

25 September - 2 October at Warwick, UK: *Statistics in Genetics and Molecular Biology*.

We also obtained support from Unesco-Roste for the Summer Schools:

- Analysis on metric measure spaces, Będlewo, Poland, 15 to 23 July 2004;
- EMS Course on Empirical Processes: Theory and Statistical Applications in Laredo, from 29 August to 3 September 2004.

For information on participation and conditions for financial support, please check on the Society's web site: <http://www.emis.de/etc/ems-summer-schools.html>

## Summer schools for the following years

On June 24, the EU will have decided on some modifications of the rules of applications for Summer Schools support. We shall thus launch a new call for proposals.

## Call for proposals

The EMS will pursue its programme of Summer schools, aiming at running such schools in pure and applied mathematics. The guidelines for such events are that there must be a very strong component of training of young researchers (in the first 10 years of their career) by means of integrated courses and lectures at advanced level. This can be supplemented by conference type research lectures, but the training component is needed. The courses should aim at an international audience (no more than 30% of participants should come from a single state). The EMS

will help with advertisement and organisation, as well as the applications for financial support.

Proposals for future Summer Schools can be sent at any time by e-mail to Luc Lemaire (e-mail: [llemaire@ulb.ac.be](mailto:llemaire@ulb.ac.be))

The deadline for proposals for EMS Summer Schools for 2006, 2007 and 2008 is tentatively fixed on December 31, 2004.

## EMS mathematical weekends

The EMS has launched a new format of joint meetings with its corporate member societies, following the model set out by the Portuguese Mathematical Society in the meeting that took place in Lisbon from September 12 to 14, 2003 (<http://www.math.ist.utl.pt/ems/>).

These "EMS- joint mathematical week-ends" will start on a Friday, and finish on the Sunday, both at lunchtime, so that they can be easily attended during term-time. Each would cover around 4 subjects, chosen by the local organisers to fit the research strengths of the local mathematicians, or new subjects they would want to develop. For each subject, a plenary lecture and two half-days of parallel sessions will be organised. Past experience shows that such an internationalisation of the meetings of national societies helps to substantially increase participation. The EMS will help with scientific organisation, publicity and funding applications. With more than fifty corporate members, the EMS hopes to see regular meetings of this format. Note that mathematics departments or individual members can also plan such meetings.

Two more Week-Ends are planned, one in Prague, Czech Republic, September 3-5, 2004 (<http://mvs.jcmf.cz/emsweekend>), and one in Barcelona, September 16-18, 2005.

## Call for proposals

Proposals for future Joint Mathematical Weekends can be sent at any time by e-mail to Luc Lemaire (e-mail: [llemaire@ulb.ac.be](mailto:llemaire@ulb.ac.be)).

# The Helsinki Meeting of the Executive Committee

*David Salinger, EMS Publicity Officer*

It wasn't cold for Helsinki at the end of February, just a few degrees below zero. The streets were slippery and the sea was frozen, but the Executive Committee met indoors and got on with its business.

The best news was that, after all, we were likely to get funding for the summer schools from the EU's sixth framework programme. Final confirmation arrived after the meeting and the full list is in this issue's agenda. This is the culmination of very hard work, in negotiation and form-filling, in particular by Luc Lemaire and Tuulikki Mäkeläinen. It means also that an additional staff member, Miku Koskenoja, can be employed at the Society's Helsinki office.

Other good news is that the Society admitted as Institutional members the Mathematisches Forschungsinstitut Oberwolfach, Institut Henri Poincaré, Centre Emile Borel, the International Centre for Mathematics, and the Abdus Salam International Centre for Mathematical Physics. The Committee recommended to Council the following societies for corporate membership: the Cyprus, Malta and Romanian Mathematical Societies and the Belgian Statistical Society. It was hoped that other statistical societies would follow that lead; three of the seven forthcoming summer schools were in statistics.

The Society had been asked to suggest names to the European Science Foundation for a panel to consider applications for young investigators' fellowships.

The Committee agreed to recommend to Council that the corporate membership fee should be increased slightly, but that the individual members' rate should remain the same.

The Committee had received no proposals for Diderot forums or EMS lectures. It was recognised that the former were difficult to organize and depended on local enthusiasm. Several different models were suggested to revive the EMS lectures: it was agreed to discuss this further at Council.

There was a strong field of candidates for the EMS prizes, but no nomi-

nations had been received for the Felix Klein prize.

It was agreed to support a mathematical weekend in Prague in September 2004. The offer of a mathematical weekend in Barcelona was welcomed: this is now to take place on 16-18 September 2005.

The President reported that Gérard Huet, Terry Lyons, Simon Tavare and Martin Groetschel had agreed to speak at the European Science Open Forum meeting in Stockholm in August. Luc Lemaire had been invited to give a plenary lecture.

A meeting had been held in Brussels about the proposed European Research Council. If this went ahead, the Society would have to be active to ensure that funding through the Research Council would be mathematics-friendly.

The Committee discussed whether the Society's journal, JEMS, should have a second series devoted to applied mathematics. The chair of the Applied Mathematicss committee would be asked to investigate the viability of such a journal. It was important that JEMS should be recommended to libraries by mathematicians, because,

now that it was being published by our Publishing House, it was no longer included in the Springer package.

An additional 3000 euro was approved to be used for grants to support 4ecm.

The French ministry had produced a report on Zentralblatt, which, while critical of a few aspects, gave overall support. It had recommended the setting up of an advisory board and the Committee endorsed this approach.

Unfortunately, no one on the Committee was able to carry forward the Digital Maths Library initiative.

The EMS publishing house had started producing books and journals. It hoped to move into profit in 2005.

The Newsletter now had a copy editor. Plans were afoot to have parts of the Newsletter in an electronic database.

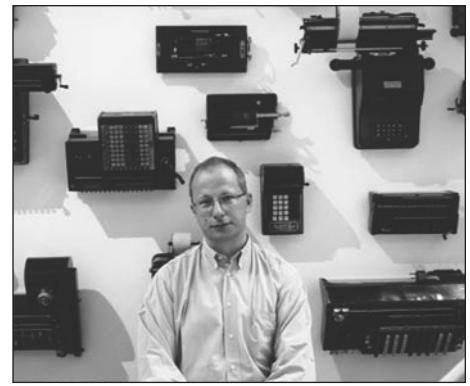
The Committee had received a proposal asking it to support a centre of mathematical excellence in Palestine (Bir-Zeit University). It was agreed to ask the chair of the Developing Countries committee to look into this.

The President thanked Olli Martio and Tuulikki Mäkeläinen for organising the meeting.



# On powers of 2

*Paweł Strzelecki (Warsaw)*



*Paweł Strzelecki  
(Photo "die arge lola", Stuttgart)*

Let's have a look at the following interesting little problem:

**Problem:** Consider a sequence  $(a_n)_{n \in \mathbb{N}}$  consisting of the first digits of the successive powers of 2:

$$1, 2, 4, 8, 1, 3, 6, 1, 2, 5, 1, 2, 4, 8, \dots$$

Will 7 ever appear in this sequence?

This is a translation from Polish of an article that appeared in the Polish journal Delta 7/1994.

This problem, in different versions, is well known in mathematical literature. It can be found in *A walking mathematician*, by Zbigniew Marciniak (Delta 7/1991), or in the famous book *Ordinary differential equations*, by V.I. Arnold. It is usually accompanied by auxiliary facts or suggestions, intended to make a solution more accessible. Nevertheless, I do not know of any publication in which the problem is presented together with a complete solution, including every detail. Let us try to remedy this unfortunate situation. We start with a rather desperate solution requiring just a piece of paper and a pencil, or perhaps some more advanced versions of these tools (a good calculator or a computer?). An assiduous reader will easily verify that

$$2^{46} = 70,368,744,177,664.$$

Going further with this experiment it can be seen that 7 is the first digit of the 56<sup>th</sup>, 66<sup>th</sup>, 76<sup>th</sup>, 86<sup>th</sup> and 96<sup>th</sup> power of 2 (but the first digit of the 106<sup>th</sup> power of 2 is 8, not 7). This method, however, is clearly far from being elegant.

It is time then for a better solution, which might also lead to new conclusions. First of all, we should try to realize what the statement, that 7 is the first digit of a number  $2^n$ , actually means.

The answer is simple: 7 is the first digit of  $2^n$  if and only if for some natural

$k$  we have

$$7 \cdot 10^k < 2^n < 8 \cdot 10^k$$

We can get a simpler description of this condition if we take the decimal logarithms of both sides; this yields

$$k + \log(7) < n \log(2) < k + \log(8).$$

Since decimal logarithms of 7 and 8 lie between 0 and 1, we conclude that  $k$  is simply the integer part of the number  $n \log(2)$ , which leads to the following inequalities:

$$\log(7) < n \log(2) - [n \log(2)] < \log(8).$$

And now it suffices to bring together some known facts.

**Lemma 1.** The number  $\log(2)$  is irrational.

**Lemma 2.** If a number  $x$  is irrational and  $c(n) := nx - [nx]$ , then for any  $a$  and  $b$  in  $[0,1]$  infinitely many members of the sequence  $c(n)$  lie within the interval  $(a, b)$ .

Before we prove these two lemmas let's have a look at their consequences. First, by applying Lemma 2 so that  $x = \log(2)$ ,  $a = \log(7)$ ,  $b = \log(8)$ , we get that 7 is the first digit of infinitely many powers of 2. If we apply Lemma 2 again, to get the numbers  $x =$

$\log(2)$ ,  $a = \log(77) - 1$ ,  $b = \log(78) - 1$ , then because of the equalities  $1 = [\log 77] = [\log 78]$ , we conclude that the digit 7 can even appear twice, in the first two places of the decimal notation of a power of 2. Using an analogous argument we readily discover that any finite sequence of digits can appear at the beginning of a decimal notation of a power of 2, like 1995 or 1234 or 567890 etc. At the end of this article we exhibit a table of important historical dates compared with the corresponding powers of 2 - to satisfy the sceptics. We can even deduce more:

**Corollary.** If an integer  $p > 1$  is not an integer power of 10, then any sequence of digits can appear at the beginning of the decimal notation of some  $n^{\text{th}}$  power of  $p$  for some  $n$ .

To prove the corollary, just observe that  $\log(p)$  is irrational and then repeat the same argument as above.

But then, why does 7 not appear among the first members of the sequence we introduced at the very beginning? Why does this deceitful sequence pretend to be periodical? The reason is simple. The number  $\log(2) = 0.3010299956\dots$  can be very well approximated by the rational number 0.3, and for all rational  $x$  the sequence  $c(n) = nx - [nx]$  is periodical. This is why after seeing the first initial mem-

## FEATURE

bers of the sequence  $a(n)$  we come to an erroneous conclusion that the sequence has period 10 and 7 is not a member of it, while 8 appears quite often.

In 1910 Waclaw Sierpinski, Hermann Weyl and P. Bohl proved independently of each other that for an irrational  $x$  the sequence  $c(n) = nx - \lfloor nx \rfloor$  is homogeneously distributed over the interval  $[0,1]$ . More precisely, if we take arbitrary  $a$  and  $b$  (with  $a < b$ ) in  $[0,1]$ , and let  $k(n, a, b)$  denote the number of elements of the set

$$\{c(i) : 1 \leq i \leq n, c(i) \in (a, b)\},$$

then

$$\lim_{n \rightarrow \infty} \frac{k(n, a, b)}{n} = b - a$$

In more illustrative terms, the theorem

can be rephrased as follows: If we walk along a circumference of perimeter 1 for long enough and make steps of an irrational length, then we shall step on each hole with a frequency which is directly proportional to the size of the hole.

Let's translate this fact into our language of powers of 2. Let  $a(7, n)$  and  $a(8, n)$  be the number of sevens and eights, correspondingly, among the first  $n$  members of the sequence  $a(n)$ . By the last formula, we have

$$\lim_{n \rightarrow \infty} \frac{a(7, n)}{n} = \log(8) - \log(7),$$

$$\lim_{n \rightarrow \infty} \frac{a(8, n)}{n} = \log(9) - \log(8),$$

so consequently

## A calendar involving powers of 2

EVENT	Year	Power of 2
Foundation of the Polish State	966	$2^{568} = 9.66... \times 10^{170}$
The King of Poland Jan III beats the Turcs at Vienna	1683	$2^{709} = 1.683... \times 10^{212}$
The first edition of Newton's <i>Principia</i>	1687	$2^{6143} = 1.687... \times 10^{1849}$
The Battle of Waterloo	1815	$2^{931} = 1.815... \times 10^{280}$
Foundation of the Warsaw University	1816	$2^{7339} = 1.816... \times 10^{2209}$
Birth of Stefan Banach	1892	$2^{8133} = 1.892... \times 10^{2448}$
Beginning of World War II	1939	$2^{5522} = 1.939... \times 10^{1662}$
End of World War II	1945	$2^{1931} = 1.945... \times 10^{581}$
First published issue of "Delta"	1974	$2^{1549} = 1.974... \times 10^{1081}$

## Appendix

### Proof of Lemma 1

If the equality  $\log 2 = \frac{r}{m}$  held true for some natural numbers  $r$  and  $m$ , then by the definition of logarithm,  $10^{\frac{r}{m}} = 2$  and consequently  $10^r = 2^m$ . This, however, is a contradiction, since 5 is a factor of any power of 10, whereas it is certainly not a factor of a power of 2.

### Proof of Lemma 2

First note that all the members of the sequence  $(c_n)_{n \in \mathbb{N}}$  are different. Indeed, if  $c_k = c_m$  for some  $k, m \in \mathbb{N}$  with  $k > m$ , then  $(k - m)x = \lfloor kx \rfloor - \lfloor mx \rfloor$ . However, the product of a positive natural number and an irrational number cannot be an integer.

Now, let  $n$  be a natural number such that  $\frac{1}{n} < b - a$ . The numbers  $c_1, c_2, c_3, \dots, c_{n+1}$  being pairwise different with  $c_i \in [0, 1]$  for  $i = 1, 2, 3, \dots, n+1$ , we can use the pigeon-hole principle to infer that for some  $s$  and  $t$ , both  $s$  and  $s+t$  are included between 1 and  $n+1$  and satisfy the inequality

$$(1) \quad 0 < \varepsilon := |c_s - c_{s+t}| \leq \frac{1}{n} < b - a.$$

From now on we will refer to a very use-

ful representation of points. Imagine the real axis is wound around a circle  $T$  with radius 1 and a distinguished point 0. For each pair of numbers  $a, b \in [0, 1]$  the arc of  $T$  that corresponds to the interval  $[a, b]$  on the real axis will be also denoted by  $(a, b)$ . Let  $f : T \rightarrow T$  represent the anticlockwise rotation by an angle of  $2\pi x$  radians. Now, instead of considering the numbers  $c_n$  as elements of the interval  $[0, 1]$ , let's think of the images of the distinguished point 0 under successive iterations of the function  $f$  on  $T$ . In fact, a moment of reflection should make it clear that if

$$b_n = f^n(0) = \overbrace{f \circ \dots \circ f}^n(0),$$

then the length of the arc  $(0, b_n)$  is equal to  $c_n$ . Thus, due to equation (1), the length of the arc between the points  $b_s$  and  $b_{s+t}$  is less than  $b - a$ . This means that the  $t^{\text{th}}$  iteration of  $f$  is a rotation by an angle of  $2\pi\varepsilon$  radians; the direction of this rotation has no significance for our purposes.

Now, this clearly implies that there are infinitely many points  $b_t, b_{2t}, b_{3t}, \dots$  that belong to the arc  $(a, b)$ . Thus if you start at

$$\lim_{n \rightarrow \infty} \frac{a(7, n)}{a(8, n)} = \frac{\log(8) - \log(7)}{\log(9) - \log(8)} = 1.1337... > 1.$$

This means that looking at sufficiently long initial fragments of the sequence  $a(n)$ , we will see slightly more sevens than eights.

The result of Bohl, Sierpiński and Weyl which we have previously mentioned and our little fact on sevens and eights are actually simple consequences of a very general and deep ergodic theorem due to G.D.Birkhoff (1931), but that's another story.

A problem for the reader: For what  $n$  does the number  $2^n$  have four consecutive sevens at the beginning? What about five sevens? How do you estimate from above, the least  $n$  such that the decimal notation of  $2^n$  begins with 1995 consecutive sevens?



the point 0 and follow along the circle  $T$  for an infinitely long time in the same direction making steps of length  $\varepsilon$ , then you step infinitely many times over the arc  $(a, b)$ . Indeed, its length  $b - a$  is bigger than the length of your step. This completes the proof.

Paweł Strzelecki [P.Strzelecki@mimuw.edu.pl] studied mathematics in Warsaw and graduated in 1987. He finished his PhD-thesis under the supervision of Bogdan Bojarski in 1993. Since that time he has been working as an assistant professor at the Institute of Mathematics at Warsaw University where he now also serves as the deputy director for teaching affairs. In 1994-1999 he served as the mathematics editor for the Polish popular mathematical-physical-astronomical monthly "Delta". In 1993-1998 he presented mathematics in a Polish TV program for young people. He was an Alexander von Humboldt fellow in Bonn (1999 – 2001 and 2002 – 2003). He likes nonlinear elliptic PDEs, clever hard-analytic proofs, compensated compactness and Sobolev spaces, bright mountain mornings, bright people and good food (no matter from which corner of the world). He is happily married to a non-mathematician.

# A forgotten mathematician

**Robbert Fokkink (Delft, the Netherlands)**



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Not so long ago, I was listening to a lecture in which a fine result was casually mentioned. It was a classical embedding theorem of Haefliger, which roughly says that you can embed a space  $X$  in  $\mathbf{R}^n$  if the reflection  $(x, y) \rightarrow (y, x)$  on  $X \times X$  is a conjugate of the antipodal map  $x \rightarrow -x$  on  $S^{n-1}$ . Some research into the literature on this area quickly showed that E.R. van Kampen was the first to come up with such embedding theorems, sometime in the early thirties. Very little about his work can be found in mathematical history books and his collected works were not to be found in the library. This is remarkable, because E.R. van Kampen is a much cited mathematician, who even had a component of the AMS classification named after him (20F06). So who was he?

## The life of E.R. van Kampen

Egbertus Rudolf van Kampen, known as Egbert, was born on 28 May 1908 in Berchem as the youngest in a family of three children. Egbert's parents had moved from the Netherlands to Belgium a couple of years before, when his father became an accountant at the Minerva car factory in Antwerp.

The children spend the war years 1914-18 with family in Amsterdam. When the war is over, the family move to The Hague, where Egbert completes high school, Eerste Christelijke H.B.S. At his final examination in 1924, he is mentioned in the national press. The *Telegraaf* and the *Amsterdammer* report of a young mathematician, a



Egbert van Kampen (l) with his sister Elizabeth and brother Laurens. Like Egbert, Elizabeth studied mathematics in Leiden. Laurens became a professor of economy at the Vrije Universiteit in Amsterdam. Both lived to the age of 96.

Photo: Archief Mieke van der Veen

unique talent in Europe, who has obtained his teaching qualifications for mathematics, but nevertheless was not exempted from this subject. His study of mathematics and physics continues in Leiden, where he probably did get the necessary exemptions as he completes the studies very quickly. In 1927, at the age of only nineteen, he travels to Göttingen. There he meets Alexander and van der Waerden and is given the subject for his Ph.D. thesis: find a computable, topological definition of the notion "variety". Egbert solves the problem by showing that the local homology group is a topological invariant. According to Van Kampen, a variety is a simplicial complex with the right local homology. Armed with this definition, he generalizes the Alexander duality theorem from spheres to

general varieties and expands Veblen's theory of intersections and intersection multiplicities. In 1929, he obtains his Ph.D. with the thesis *Die kombinatorische Topologie und die Dualitätssätze*. The thesis, directed by Van der Woude, is missing from the University Library in Leiden.

In the summer of 1928, Van Kampen spends five months in Hamburg, where he meets Emil Artin. Around that time, he is recruited by *Johns Hopkins* University, but because he is still too young to enter the United States without the accompaniment of his parents, he first becomes an assistant to Schouten in Delft. He writes a number of papers with Schouten on tensor analysis, which is Schouten's specialty. [4, 6, 9]

In 1931, Van Kampen departs to Johns Hopkins. In his autobiography, Mark Kac gives an anecdote about Van Kampen's arrival in the USA: "University professors, clergymen and those of several other occupations could emigrate outside the national quotas and Van Kampen had one of these highly coveted non-quota visas. There was a story that at the immigration desk he was asked a routine question and instead of answering 'professor' or 'teacher' or even 'mathematician', he answered 'topologist'. The immigration official could not find 'topologist' on the list of occupations qualifying for a non-quota visa and he naturally asked what a topologist does. As Van Kampen tried to explain, the immigration official gained the impression that he was dealing with a nut. He detained Van Kampen for further examination and Johns Hopkins authorities had to step in to extricate him. Van Kampen never confirmed or denied this story - but it was completely in character".

In his first year in the U.S., Van Kampen writes a paper on the fundamental group, with which he truly establishes a name for himself. In 1933, he spends a year at the Institute for Advanced Study, and in 1935 he gives a talk on topological groups at the International Congress at Moscow.

In his mid-twenties, Egbert van Kampen seems to be at the beginning of a brilliant career, but in reality he has already done his most important work. He remains particularly productive and, together with Aurel Wintner, fills the American Journal of Mathematics.

At the end of the thirties, in a letter home, Van Kampen begins to complain of headaches. He sees a specialist who diagnoses a neck problem. As a result he starts physiotherapy but it is later determined that the problem is more serious, a melanoma that originated from a birthmark near his left eye. In April 1941, it is operated on and in the autumn Egbert takes up his courses again, full of good hope for his recovery. This however is not to last and he once more suffers from violent headaches and becomes deaf in his left ear. In December 1941 he is once more admitted to the hospital where his health deteriorates quickly. In January 1942 a second operation follows in vain. On February 10, Egbert falls into a coma. He dies the next day, only 33 years old.



Van Kampen (l) with Ehrenfest, Rotterdam circa 1927

## The work of E.R. van Kampen

Egbert van Kampen has worked on many subjects: topology, group theory, tensor analysis, harmonic analysis and probability theory. His most well-known articles are on topology and group theory. He was thought of as somewhat of an oracle, as part of his work was only discovered when others started realizing it again.

### The first publication in 1928

Van Kampen's first article [1] appears in the *Hamburger Abhandlungen* in 1928. It consists of ten sentences and one illustration and is exactly what one would expect from a brilliant young person: original and illegible.

It concerns two curves  $\alpha$  and  $\beta$  that are attached to a surface  $V$ , of which it is questioned whether they can be separated from each other, without the endpoints leaving the surface.

In 1925, in an article with the same title, Emil Artin had claimed the following: if  $\alpha$  and  $\beta$  are linked, then either  $\alpha$  or  $\beta$  has a knot. The illustration in Van Kampen's article shows that this statement is incorrect. It is clear that  $\alpha$  and  $\beta$  do not have a knot, but that they are linked anyway. Van Kampen shows this by computing that the fundamental group of the complement of  $\alpha \cup \beta$  in half-space is not free. He first draws an auxiliary line  $\gamma$  through the

endpoints of  $\alpha$  and  $\beta$ , which, for that matter, is not strictly necessary. With some effort one can then see that the complement of  $\alpha \cup \beta \cup \gamma$  in  $\mathbb{R}^3$  has the same fundamental group as the complement of  $\alpha \cup \beta$  in half-space. This group has three normal subgroups of index 2 and Van Kampen notes that one of those has the group  $\mathbb{Z}^3 \oplus \mathbb{Z}/3\mathbb{Z}$  as quotient. It follows from this that the fundamental group can not be free. The theory that Van Kampen is expanding is at that time brand new and not entirely worked out yet. The computations that he needed to do were no small matter. Perhaps that is why he left out all the details. Nowadays the computation is much simpler, not in the least because of the work of Van Kampen himself.

### Van Kampen's embedding obstruction

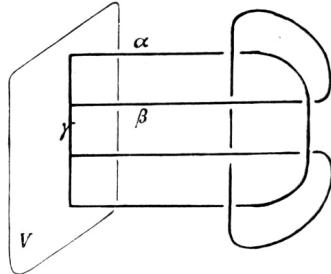
In 1932, Van Kampen publishes an article concerning embeddings of complexes in Euclidean spaces [7]. Again, Artin's article from 1925 provides the inspiration. Van Kampen studies an embedding problem that will only be definitively solved much later: under which conditions does an  $n$ -dimensional complex fit in  $\mathbb{R}^{2n}$ ? In 1930, Kuratowski had solved this for  $n = 1$ . Perhaps Van Kampen was not aware of that result, because he does not mention Kuratowski and he disposes of the case  $n = 1$  as being simple.

## Zur Isotopie zweidimensionaler Flächen im $R_4$ .

Von E. R. VAN KAMPEN.

In seiner gleichnamigen Arbeit hat E. ARTIN<sup>1)</sup> auf einige Beispiele von verknoteten und verketteten Kugeln im  $R_4$  hingewiesen. Aus heuristischen Gründen kommt er dort zur Vermutung, daß auf diese Art keine Verkettung von zwei unverknoteten Kugeln herzustellen ist.

Das folgende Beispiel zeigt, daß dies doch möglich ist.



Man lasse die beiden Kurven  $\alpha$  und  $\beta$  um die Ebene  $V$  im  $R_4$  rotieren, dann entstehen zwei offenbar unverknotete Kugeln. Um zu beweisen, daß sie verkettet sind, braucht man nur zu zeigen, daß die Gruppe  $G$  der zwei Kugeln nicht frei ist mit zwei Erzeugenden. Diese Gruppe ist aber, wie E. ARTIN gezeigt hat, gleich der Gruppe des Gebildes  $V + \alpha + \beta$  oder auch von  $\gamma + \alpha + \beta$ , die einfach zu bestimmen ist.

Man berechne nun nach dem Verfahren von K. REIDEMEISTER<sup>2)</sup> die invarianten Untergruppen vom Index 2 von  $G$ . Es gibt deren drei, wovon eine offenbar nicht frei ist (wie das der Fall sein würde, wenn  $G$  frei wäre), da die Invarianten seiner Faktorkommutatorgruppe  $(0, 0, 0, 3)$  sind. Diese invariante Untergruppe ist die Fundamentalgruppe des zweifach überlagerten, längs  $\alpha + \beta + \gamma$  verzweigten,  $R_3$ .

Es ist natürlich klar, daß auf diese Weise unendlich viele verschiedene Verkettungen zu konstruieren sind.

<sup>1)</sup> Diese Zeitschrift, Bd. IV, S. 174.

<sup>2)</sup> Diese Zeitschrift, Bd. V, S. 7; vgl. auch O. SCHREIER, Bd. V, S. 172.

The first article (1928) of Van Kampen in the *Hamburger Abhandlungen*

For a map  $f: G \hookrightarrow \mathbf{R}^2$  with finitely many double points, Van Kampen defines a vector  $\mathbf{v}$  with coordinates in  $\mathbf{Z}/2\mathbf{Z}$ . The number of coordinates of  $\mathbf{v}$  is equal to the number of pairs  $\{I, J\}$  of disjunct edges of the graph. The  $\{I, J\}$ th coordinate of  $\mathbf{v}$  determines the parity of  $f(I) \cap f(J)$ . Van Kampen then defines vectors  $w(p, E)$  in the same space  $(\mathbf{Z}/2\mathbf{Z})^N$  as  $\mathbf{v}$ , where  $p$  is a vertex and  $E$  is an edge that does not contain  $p$ . The  $\{I, J\}$ th coordinate of  $w(p, E)$  equals 1 if and only if  $p \in I$  and  $E = J$ , or vice-versa. Let  $L$  be the linear space generated by the  $w(p, E)$ . Van Kampen's embedding criterion is then that  $G$  can be embedded if and only if  $\mathbf{v} \in L$ . For the map of the graph above,  $\mathbf{v}$  has a single 1 and zeros everywhere else. For all vectors  $w(p, E)$  the sum of the coordinates is even, so in this case  $\mathbf{v} \notin L$ .

In the higher dimensional case  $n \geq 3$ , the edges become  $n$ -dimensional cells and the vertices become  $(n - 1)$ -

dimensional cells. The definition of the vector  $\mathbf{v}$  uses the "intersection number", the theory of which Van Kampen had set up in his thesis. He first notes that the obstruction against an embedding can only lie in the  $n$ -dimensional cells. Next he deduces a criterion in four steps. Lemma 1 states that double points can be removed pairwise, so only the parity matters. Lemma 2 states that double points on adjacent edges are no problem, so only disjunct subgraphs matter. Lemma 3 states that for every vector in  $\mathbf{v} + L$  there is a map  $f: K \hookrightarrow \mathbf{R}^{2n}$  giving that vector. Lemma 4 conversely states that every  $f: K \hookrightarrow \mathbf{R}^{2n}$  gives a vector in  $\mathbf{v} + L$ . The complete proof is barely one page long. As an added bonus, Van Kampen shows that every  $n$ -dimensional variety can be embedded in  $\mathbf{R}^{2n}$ , a result that is now known as Whitney's embedding theorem.

It is an amazing result with only one flaw: the proof of Lemma 1 is wrong.

Van Kampen's proof only holds for  $n \geq 3$ , but his criterion is best illustrated using the "simple" case  $n = 1$ . In that case the complex is a graph  $G$ . For example, draw 5 points in the plane and join each of them to all the others. The result is a depiction of the complete graph with 5 nodes in the plane. Some connecting lines may intersect each other, so the map is not an embedding. You might try to make it into an embedding by trial and error, by removing intersecting edges and replacing them by new edges. For the graph with 5 nodes, any such attempt ultimately ends unsuccessfully. In Figure 1, for example, there is only one intersection point left, which can not be removed. Van Kampen shows that this trial and error takes place within a cohomology class and that the graph can be embedded precisely when that class is zero. At this time he uses other terminology, because in 1932, cohomology theory has not really been developed yet.

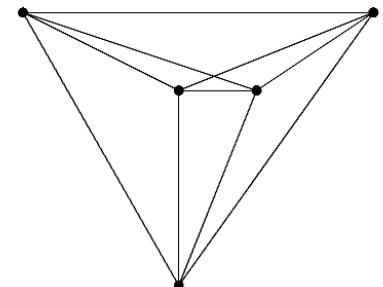


Figure 1 The complete graph with 5 nodes

Afterwards, Van Kampen sends a hasty rectification indicating a partial correction. He writes that it is "unnecessary to neglect orientation", by which he probably means that the coefficients must be taken in  $\mathbf{Z}$  instead of in  $\mathbf{Z}/2\mathbf{Z}$ .

After this Van Kampen no longer publishes about embeddings, even when Whitney develops a complete theory a couple of years later. Only in 1957 does Arnold Shapiro show that Van Kampen's embedding criterion follows from the results of Whitney.

As noted before, Van Kampen's proof only holds for dimension  $n \geq 3$ . It is therefore surprising when in 1991, Sarkaria shows that the criterion is also correct for  $n = 1$ . In 1994, Freedman, Krushkal, and Teichner show that the criterion does not hold for  $n = 2$ . Krushkal and Teichner also introduce the name "Van Kampen's embedding obstruction", some sixty years after the publication of the original article.



Group photo of staff and post docs at the Johns Hopkins campus in 1933, with f.l.t.r.: Egbert van Kampen, George Schweigert, Beatrice Aitchison, William Morrill, Oscar Zariski, Charles Wheeler and John Williamson.

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### Three consecutive articles on the fundamental group

At the beginning of the thirties, Oscar Zariski, arguably the most famous mathematician from Hopkins, works on the fundamental group of the complement of an algebraic curve. An algebraic curve can be knotted in the projective space, which, as in the case of a usual curve in  $\mathbf{R}^3$ , can be read off the fundamental group of the complement. Zariski determined the generators of the fundamental group and found relations between them, but it was not clear to him then whether or not there were more relations. Zariski presented the problem to Van Kampen who then solved it and wrote three consecutive papers [10, 11, 12] on the problem in the American Journal of Mathematics, the internal publication of Johns Hopkins. All three articles have become standard references, although they are rarely mentioned together. The algebraic geometers refer to the first article, the topologists to the second, and the group theorists to the third. This is remarkable as in fact only the first article stands alone, and even then only barely.

The articles have been written in an uninvoluted tone. The first article states: *As the resulting proof seemed too algebraic for this simple and nearly topological question, Dr. Zariski asked me to publish a topological proof which is contained in this paper.* Part two states: *The opportunity of simplifying the treatment of a fundamental group by means of this theorem has been overlooked several times [...] for this reason we do not think it superfluous to devote a separate paper to it.* Part three states: *It is this 2-dimensional method of the proof and not any original result which justifies the publication of this paper.*

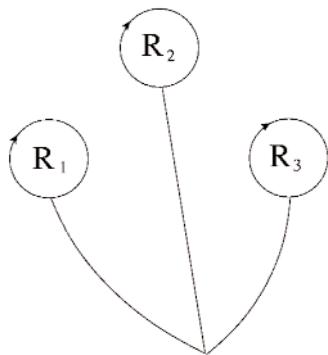
The first part of the three articles concerns Zariski's problem, which Van Kampen solves by skillfully cutting the space in two pieces  $U$  and  $V$  which each have a free fundamental group. He then divides every homotopy into small pieces that lie either completely in  $U$  or completely in  $V$ . As the fundamental group of both  $U$  and  $V$  is free, the only freedom of choice is for the pieces of homotopy that lie in the intersection  $U \cap V$ . This freedom turns out to exactly correspond with the relations of Zariski. Therefore there are no other relations. This result is

now known as the Zariski-van Kampen theorem.

The second article continues with the question of how one computes the fundamental group of a reunion  $U \cup V$  from the groups of  $U$  and  $V$ . In its modern form this result is formulated by means of coverings, but Van Kampen turns his attention to the cut that divides the space in two or more pieces. He is not entirely satisfied with his result and writes: *a path is shown to a general theorem, of which the general formulation would be more confusing than helpful, so that it is suppressed.* This suppressed general theorem probably concerns a cut that is as general as possible, which divides the space into an arbitrary number of pieces. It is Corollary 2 of this article, in which a simply connected set cuts the space into two pieces, that will later be called the Seifert-van Kampen Theorem. This theorem was in fact first proved by Seifert, another Ph.D. student of Van der Waerden.

An annoying technical detail that Van Kampen comes across is that when he divides a homotopy, he must continue to link everything to the base

point. All small pieces of homotopy must be linked to it. In the algebraic translation, the pieces of homotopy become relations, and the links to the base point become conjugations. In the third part of the trio of articles, Van Kampen makes a virtue of necessity by noting that the inverse translation gives a graphic representation of group relations. He translates words in groups into bouquets of curves, which each represent the conjugacy class of a relation. With this manner of thinking he simplifies the proof of a result of Schreier. This third article is only discovered in 1966, when Roger Lyndon and Carl Weinbaum use the diagrams to study relations with "small cancellation". The bouquets of curves are now called van Kampen diagrams.



The typical form of a Van Kampen diagram: relations on stalks. Rewriting a word corresponds to gluing the bouquet together.

## Pontrjagin-van Kampen duality and later work

In 1933, Van Kampen spends a year in Princeton, at which time Pontrjagin proves that compact Abelian groups are dual to discrete Abelian groups. Pontrjagin assumes the groups to be separable, because in 1933, the Haar measure has only been constructed for such groups. Van Neumann remarks that duality also holds without separability, if instead of the Haar measure, one uses Von Neumann's theory for almost-periodic functions. Van Kampen takes this up and writes a survey article on the duality of groups[16]. Since then this duality is also called the Pontrjagin-van Kampen duality.

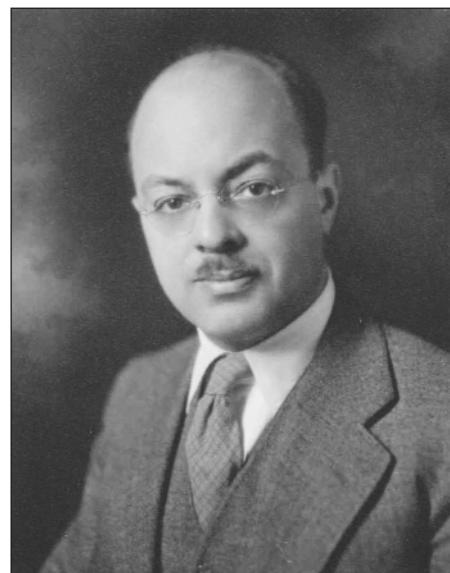
On this topic, Van Kampen is quite lavish and writes sixteen pages, beginning with the definition of a topological group and ending with the duality of locally compact Abelian groups without the restriction of separability. The article is well readable and writ-

ten for the general public, in contrast to Van Kampen's previous work, which gave the impression of being merely a reminder for the author himself.

This marks the beginning of a new period. After 1935, Van Kampen leaves the area of algebraic topology for what it is and mostly studies almost-periodic functions and differential equations. It is no coincidence that this is the area of interest of Aurel Wintner, a colleague at Johns Hopkins. Egbert van Kampen has always allowed himself to be inspired by the people around him. At the end of his life, when the young Mark Kac comes to Baltimore, Van Kampen even writes a number of articles on probability theory. His last article, of considerable size, deals with product measures and convolutions. It appears in the middle of 1940 [49].

## Conclusion

At the age of only sixteen, Egbert Van Kampen appeared in the newspaper as an unprecedented mathematical talent. He was a renowned mathematician but nevertheless, his work had to wait a long time for recognition. Perhaps this was due to his inaccessible manner of writing, a proof is frequently over before you notice it. Perhaps it was due to his discretion. In an American newspaper a colleague noted, "He never spoke very much about his accomplishments". While Van Kampen's fame is based on the work of his beginning years, after 1933 he wrote a further forty articles. Who knows what beautiful things still await us.



Egbert van Kampen, 1940.  
Photo: Archief Mieke van den Veen

## Word of thanks

This overview would not have been possible without the photographs and newspaper cut-outs of Mieke van der Veen, daughter of Elizabeth van Kampen. I would like to thank her for the authorization to use this material and for the care with which she saved it. Boudewijn van Kampen, also a family member, provided the photograph of Egbert van Kampen and Paul Ehrenfest. Egbert van Kampen's great-great-grandfather was Jacobus van der Blij, and he was also the great-grandfather of the retired professor of mathematics van der Blij from Utrecht.

The anecdote about the visum of Van Kampen comes from the memoirs of Mark Kac, *Enigmas of chance*, London, Harper and Row, 1985. More anecdotes can be found in these memoirs, around page 85. The autobiography of Kac was brought to my attention by Cor Kraaijkamp.

Slava Krushkal helped me with the details of the Van Kampen embedding obstruction, Mark de Bock of the municipality of Antwerp and Albert Schiltmeijer from Amsterdam sent me data concerning the Van Kampen family. Jim Stimpert of Johns Hopkins Library sent information concerning Van Kampen's publication.

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Robbert Fokkink [R.J.Fokkink@ewi.tudelft.nl] studied mathematics in Amsterdam and Delft. His PhD thesis from 1991 is on topological dynamics. From 1992 to 1999 he worked at Delft Hydraulics. He is currently a member of the probability group at Delft University.

# Quantum Cryptography

*Nuno Crato (Lisbon)*

"Quantum cryptography" is the final of a three-article sequence written by Nuno Crato and published at the Portuguese newspaper *Expresso* in September 2001. These three articles were submitted to the Public Awareness of Mathematics 2003 competition and won its First Prize.

Crato's articles were published in the widely read magazine *Revista*, which is included in the weekly newspaper *Expresso*. Readership for this newspaper is about half a million people, which is a large number for Portugal. In fact, *Expresso* is the most sold newspaper in the country.

The first and second articles referred briefly to the history of cryptography, discussed the asymmetric key solution and the RSA method. They stressed the importance of mathematical concepts for safe communication and showed today's relevance of mathematical cryptography. These two articles discussed concepts that were novel for the general reader, but that are sufficiently well known for most mathematicians. The third article discussed the emergent research on quantum cryptography, explaining one way by which the uncertainty of quantum world can be used to provide safe communications. It is this third article we reproduce here.

In the next issue of this Newsletter, we will publish an interview with Nuno Crato by José Francisco Rodrigues. This conversation between the two Portuguese mathematicians addresses subjects such as the importance of mathematics popularization, the difficulties and caveats of this work, and its relevance for the public and the public status of Mathematics.

#### QUANTUM CRYPTOGRAPHY

**It sounds like science fiction, but it is a reality: the most bizarre properties of subatomic particles allow us to create unbreakable ciphers**

The security of bank transactions, electronic commerce, and email communication is based on the safest cryptography systems created by man. But "safest"

does not mean "impossible to break". The safety of one of the most reliable modern cryptography systems - the so called RSA, which we have been reporting on in this *Revista* - is based on the difficulty of finding the prime factors of very large numbers. There are no algorithms known to date that enable their factorization in a reasonable amount of time, even if we use the most powerful computers available. However, if a mathematician discovers a process that enables a speedy factorization, or if a new generation of computers such as quantum computers are commercialized, the world of communication as we know it can be easily jeopardized. In the event that one of these revolutionary technologic advances becomes widespread, electronic commerce would cease to be secure, the military would have to reinvent its communication systems, and bank institutions would have to take a step back and make transactions at a very slow pace. It would be the collapse of informational technology in our present day society.

Not surprisingly, people are looking



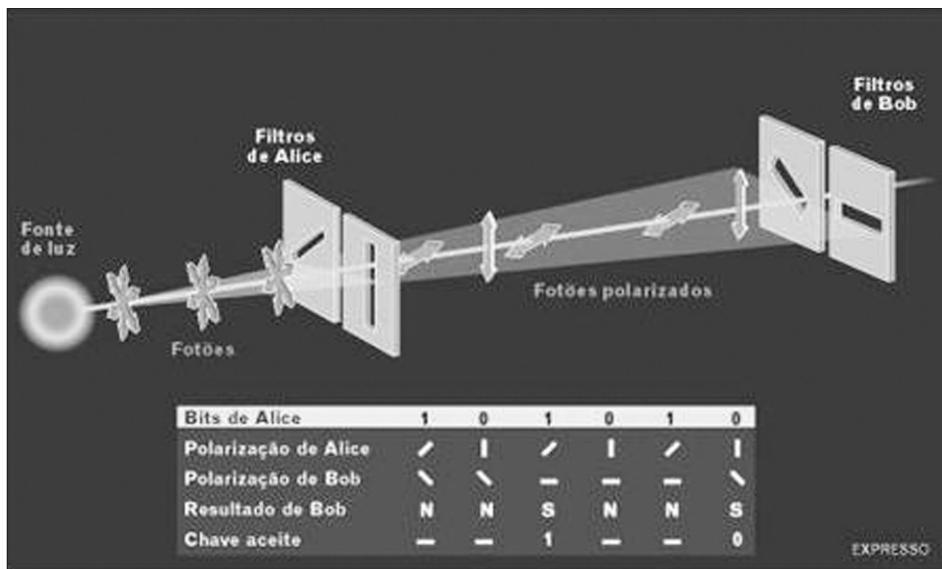
*A new book explaining how quantum mechanics can be used to conceive an innovative computing system*



for a new cryptography system that will be absolutely secure. Before RSA can fail us, mathematicians, physicists, and computer scientists hope to develop a new procedure that will be virtually unbreakable. This might well be possible, because the new system scientists envision to create is based on the most profound laws of matter, those that govern the uncertainty of the quantum world. The impossibility of getting to know *a priori* the behavior of elementary particles is what will guarantee communication security.

The idea has been brewing in the minds of scientists for some time now. Charles Bennet, a computer scientist at the IBM Watson Research Center has been one of those looking for a solution to this problem. Finally, in the 80's he and his colleague Gilles Brassard managed to conceptualize a quantum cryptography system. For a long time their ideas were only dreams. During the last two years, however, technological and scientific advances have made it possible to build prototypes of quantum systems that seem to be absolutely unbreakable.

To discuss issues related to ciphered messages, experts usually refer to three different fictitious characters: Alice, Bob, and Eve. The first represents the sender, the second the receiver, and the third the intruder who attempts to break the secret communication. At the core of the process proposed by Charles Bennet and his collaborators, there is a random code key as long as the message itself. This key is simply a binary number, a sequence of zeros and ones led by a one. Alice begins by transforming the text she wants to send Bob by translating it into another binary number. Then, she adds the key number to her message number. She sends the result to Bob, who has the



key that Alice used. By subtracting the key from the message received, he gets the original message. To read it, he only has to transform the sequence of zeros and ones into a sequence of letters, but this is a routine procedure any computer is capable of doing.

For this system to be unbreakable, it is important that the key is a random sequence of zeros and ones and that it is used only once. This usually means that those numbers have to be generated in advance and that Alice has to send them to Bob. This is where problems usually arise. If Alice and Bob never meet face to face, as it happens in e-commerce, they have to exchange the key through some communication channel. How can they do that? Well, they could agree on another key, but this again does not solve the problem, as to do this Alice and Bob would have to meet or trust a messenger... Since Eve is always peering over them, absolute security seems impossible.

This is where the quantum world comes into play by the hands of Charles Bennett and other computer scientists. It is a strange world, with rules difficult to understand on the basis of our everyday experiences. One of those rules is uncertainty. And this uncertainty is not based on our insufficient knowledge; rather it is intrinsic to the very life of particles. How could this be used for safe communication?

In order to create the random code key, Alice begins by sending Bob a sequence of light particles, that is, a sequence of photons. She has two polarizers in her system, one vertically oriented and another tilted at 45°, as we can see in the figure. To set up the key, she randomly alternates the polarizers and, for example, makes zero correspond to a

vertically polarized photon and makes one correspond to a 45° polarized photon. Bob has two other polarizers, one of them horizontally oriented and another one at -45°. When he receives Alice's photons, he makes them pass through one of his polarizers, alternating the two in a completely random fashion.

The photons that Alice sends and Bob receives may or may not pass through his device, depending on how the polarizers are oriented. If Alice sends a photon polarized vertically and Bob attempts to make it pass through his horizontal polarizer, the particle is stopped and does not pass. If Alice sends Bob a photon polarized at 45° and Bob receives it with the polarizer at -45°, the particle is also stopped and does not go through. Polarizers perpendicular to the photon polarization do not let the particles pass.

Surprises arise when Alice sends a vertically polarized photon and Bob receives it with the diagonal polarizer or when Alice sends a diagonally polarized photon and Bob receives it with the horizontal polarizer - that is, when Alice and Bob's polarizers make a 45° angle. In this case, the uncertainty principle of quantum mechanics is set in motion: half of the particles go through Bob's polarizer and the other half are stopped. And this happens without any prior knowledge of which particles are going to go through and which particles are going to be stopped.

At this stage, only Alice knows the polarization of the photons she sent. And only Bob knows which ones finished their trip. This way, Bob discovers the polarization established by Alice for the photons that passed through. If a photon passed through a diagonal filter, Alice must have polarized it vertically and it can be assigned the value zero. If a photon

passed through a horizontal filter, Alice must have polarized it diagonally and it can be assigned the value one. Those photons that were retained remain a mystery to Bob.

But now Alice needs to know which photons went through Bob's filters. To know this, Alice and Bob can communicate through a less secure system and they can even be heard by Eve. Even if Eve finds out which particles reached Bob, she still cannot figure out which filter they passed through. Thus, the key is established solely on the basis of the photons that completed the voyage. Now, Alice and Bob can communicate with complete security. The uncertainty of the quantum world gives them the certainty that their code cannot be broken.

Maybe we are close to implementing this cryptography system. Only a few years ago it all sounded like science fiction, but recent technological breakthroughs made it possible to create prototype computation models with viable applications of these ideas. As we can anticipate, the technological challenges are huge. How do we transmit light a photon at a time? How do we make sure these particles reach their destination? Nonetheless, these problems are being solved little by little. We are already able to use quantum cryptography through fiber cables and through air for a few kilometers. Thus, we may not be far from being able to protect our secrets by making them travel a particle at a time - at the speed of light.

*Nuno Crato [nocrato@iseg.utl.pt] is Associate Professor with habilitation at the Mathematics Department of Instituto Superior de Economia e Gestão, Lisbon Technical University. He received his Ph.D. from the University of Delaware and worked previously in the U.S. for a number of years. His research interests lie in Stochastic Processes and Time Series Analysis, with various applications, from finance and economics to climate models. He is a member of the board of the Portuguese Mathematical Society (SPM). In parallel with his research and teaching activity, he is a science writer. He has a column at the Portuguese newspaper Expresso and has worked for a few TV science documentaries. He is co-author of Trânsitos de Vénus (2004), Eclipses! (1999) and other science books. The European Mathematical Society awarded him the First Prize in mathematics popularization writing in 2003.*

# Household names in Swedish mathematics

## Some who should be and some others

**Kjell-Ove Widman (Institut Mittag-Leffler, Stockholm)**



For natural reasons, this causerie on Swedish mathematicians – just in time for the 4th European Congress of Mathematics in Stockholm - contains very little in the way of flesh on its bones. The canonical, meaty source is Gårding, 1998, from which I have used much material. To avoid delicate questions concerning my colleagues, I have decided not to mention the names of any mathematicians under the traditional Swedish retirement age of 65. This of course, for the most part, leaves contemporary mathematics out of the picture, but that is the way it has to be.

### The Mittag-Leffler Era

On an international scale, Swedish mathematics begins with **Gösta Mittag-Leffler** (1846-1927). He learned analysis, includ-



**Gösta Mittag-Leffler**

ing complex analysis, in Uppsala in the 1860's, from his teachers H. Daug and G. Dillner. However, the knowledge gained would hardly have sufficed for a research career had he not gone to Hermite, and in particular to Weierstrass, to learn more. In fact a glance at Mittag-Leffler's thesis does not convince one that he had mastered the definition of the derivative of a complex-valued function of a complex variable. "Die Strenge des Weierstrass" was not lost on him, however, and he started producing mathematics papers which

were well up to the standard of the day. Not wanting to stay in Germany, and finding no suitable job in Sweden, he applied for, and was appointed to, a professorship in Helsinki, where he had one student with a name that has remained in the books: Hjalmar Mellin (1854-1933), inventor of the transform bearing his name. As a professor in Stockholm from 1881 to 1911, he made a successful career as a researcher, teacher, thesis adviser, and founder of the **Acta Mathematica**, while at the same time managing to amass a substantial fortune as a businessman. He willed his house and fortune to the Royal Swedish Academy of Sciences, making the future Institute Mittag-Leffler possible. A lasting contribution to history was his successful effort to bring **Sonya Kovalevsky** to Sweden.

Several of Mittag-Leffler's students became well known and respected mathematicians in their day. **Ivar Fredholm** (1866-1927, professor in Stockholm) is best known for his work on integral equations and spectral theory, with a theorem whose generalization to abstract spaces goes under the name of "Fredholm's alternative". His main interest was mathematical physics and partial differential equations, where he constructed fundamental solutions for various classes of equations in two and three variables. These studies were continued by Fredholm's student, **Nils Zeilon** (1886-1957, professor in Lund), who managed to considerably enlarge the classes of equations for which a fundamental solution could be shown to exist. This line of investigation was later carried to a logical end in the thesis of Hörmander.

**Ivar Bendixson** (1861-1935, professor in Stockholm), learned set theory from the papers by Cantor, solicited by Mittag-Leffler for the *Acta*, and dynamical systems from the papers of Poincaré. The result was the Cantor-Bendixson theorem in set theory and the Poincaré-Bendixson theorem on the existence of closed cycles of ODEs in two dimensions.

**Edvard Phragmén** (1863-1937, professor in Stockholm), is of course best known for the Phragmén-Lindelöf theorem, but

also for his discovery of the famous error in Poincaré's first paper in the *Acta*. Having succeeded Sonya Kovalevsky as a professor, Phragmén left mathematics for insurance and politics after about ten years.

Decidedly less known is **Helge von Koch** (1870-1924, professor in Stockholm), a precocious mathematician whose work on infinite determinants and on continuous non-differentiable functions have receded into the darkness of history.

Two of Mittag-Leffler's students gave up pure mathematics for other fields of endeavour: **Gustav Cassel** (1866-1945) decided mathematics was too difficult and went on to become the father of the Stockholm school of economics, with Nobel Prize winners Bertil Ohlin and Gunnar Myrdal as prime exponents.

Number theory, more specifically analytic number theory, had been taken up both by Phragmén and von Koch in the early years of the century, inspired by the successes of Hadamard and de la Vallée Poussin. **Harald Cramér** (1893-1985, professor in Stockholm), probably inspired and tutored by Marcel Riesz, wrote a thesis on the subject in 1917, and continued to produce good papers for several years. According to legend however, he fell out with Mittag-Leffler, who promised that as long as he, Mittag-Leffler, had any influence, Cramér would never get a professorship. This can arguably be considered a stroke of luck, both for Swedish mathematics and for Cramér himself, since Cramér then turned to probability and mathematical statistics and founded a thriving Swedish school in the area. His magnificent monograph from 1946, *Methods of Mathematical Statistics*, has rightly been considered a standard reference in the area.

**Marcel Riesz** (1886-1969, brother of Fredrik Riesz), came to Sweden to assist Mittag-Leffler in the editing of the *Acta*, but stayed on at the University of Stockholm as a docent until 1927, when he became professor in Lund. Riesz introduced new areas of analysis in Sweden, and had a profound influence on Swedish mathematics. One of Riesz's most famous

## SOCIETIES

theorems, generalized by one of his students and known as the *Riesz-Thorin interpolation theorem*, gave rise to a whole new subject. Other claims to Riesz's fame are his theorem on conjugate functions and his work on Riesz potentials. The latter work was used in potential theory and in connection with the wave operator and the propagation of singularities. Riesz's personality was decidedly un-Swedish. He liked to talk at length, and was a continuous source of inspiration to his students through his indefatigable enthusiasm for mathematics.

### Björling and Holmgren descendants

Before continuing with the students of Marcel Riesz, we must go back to another germ of modern Swedish mathematics, namely to **Carl Björling** (1839-1910, professor in Lund). Inspired by the Danish mathematician Zeuthen, and in competition with his colleague Anders Bäcklund, he took up algebraic geometry. This was a subject much in vogue at the time due mainly to the work of Italian mathematicians<sup>11</sup>. Among them was Pasquale del Pezzo, Duca di Cajanello and brother-in-law of Gösta Mittag-Leffler.

Björling had a student by the name of **Anders Wiman** (1865-1957), who later became professor in Uppsala and had two famous students, Fritz Carlsson and Arne Beurling. Wiman had started out in the footsteps of his teacher Björling, but in Uppsala he also worked in analysis, in particular in analytic functions.

Björling's colleague, **Anders Bäcklund** (1845-1922), also dabbled in algebraic geometry. By a quirk of history, Bäcklund found a new field of interest when Sophus Lie was among the applicants for a mathematics chair in Lund, in 1871. A friend of Lie was supposed to hand in the application, but was negligent in his duties and the application arrived one day late. The act stayed in the office of the registrar, however, and Bäcklund was allowed to study Lie's papers, among which he found a description of Lie's contact transformations. The *Bäcklund transformation*, a generalization of Lie's concept, ensued. Bäcklund later used his considerable powers of analysis on problems in mechanics and electricity. None of Bäcklund's students continued with mathematics. We will now return to Wiman's students, Beurling and Fritz Carlson.

**Arne Beurling** (1905-1986), was a student of Holmgren as well as of Wiman, but took up Wiman's interest in analytic functions. His thesis, in which he started the development of the concept of extremal length, was delayed for a year because Beurling's father wanted him to join him for an alligator safari in Panama. An additional reason was that some of the results in the thesis had been anticipated by the



Arne Beurling

Finnish mathematician Lars Ahlfors, requiring Beurling to extend his investigations. Beurling had a brilliant career, first as professor in Uppsala, then as a member of the Institute for Advanced Study. He is probably unique in Swedish mathematics in that he might have influenced the course of history through his ingenious cryptanalytic efforts during World War II. The most dramatic event was the breaking of the German "Geheimschreiber", T-52, which made it possible for the Swedish military command to read top level German communications for two or three years during the most critical part of the war (the story is described in detail in Beckman, 2003). There were other Swedish mathematicians involved as well: **Gunnar Blom** and **C-O Segerdahl**, both students of Cramér, and Beurling's student Bo Kjellberg.

In his own country, Beurling is considered to be one of the handful of really brilliant mathematicians that Sweden has produced, but unfortunately, except for a small group of *cognoscenti*, he is little known internationally. Part of the reason might be that his most productive time coincided with a trend away from concrete, down-to-earth analysis. Also, he seldom published, and had a tendency to keep his ideas to himself, guarding them with suspicion and jealousy. He had a lasting influence on Swedish mathematics through his many students, one of which was to join him on the Swedish mathematical Olympus.

The second of Wiman's students, **Fritz Carlson** (1888-1952, professor in Stockholm), also continued in the path of his teacher's interest in analytic functions, and can indeed be seen as continuing Mittag-Leffler's program in the area. Carlson had two students, with an international reputation in different fields, **Hans Rådström** (1918-1969, professor in Linköping) and **Tord Ganelius** (1925-, professor in Göteborg). Ganelius is best known for his Tauberian theorems. He

had several students, the best known of which was **Björn Dahlberg** (1950-1998, professor in Göteborg), an imaginative analyst who produced a result on the mutual singularity of harmonic and boundary surface measure in Lipschitz domains.

Anders Wiman had a colleague in Uppsala, **Erik Holmgren** (1873-1943, professor in Uppsala), whose name would live on because of an oft-cited theorem. His interest was partial differential equations, and he proved a uniqueness theorem which plays an important part in the modern theory of the subject.

**Torsten Carleman** (1892-1949, professor in Stockholm), was easily Holmgren's



Torsten Carleman

most famous student. He wrote about integral equations, partial differential equations, analytic functions, quasi-analytic functions, harmonic analysis, and kinetic gas theory, and made fundamental contributions in all of these areas. Like Beurling however, he did not enjoy the international reputation that we the Swedes think he deserves. Again, part of the reason was the concurrent development of more abstract theories, which attracted more attention. Other causes may have been that he was actually ahead of his time, as for example with his generalization of Holmgren's uniqueness theorem, which only drew attention through later developments in the area. Carleman's personality was complicated. To quote Gårding: he was neither at ease with the world nor with himself. Carleman was director of the Mittag-Leffler Institute for 20 years, but there was hardly any activity, due a great deal to the lack of funds.

Carleman did not have many students. One of them however, **Åke Pleijel** (1913-1989, professor at KTH, in Lund and in Uppsala), continued and extended Carleman's work in spectral theory. Pleijel is perhaps best known for his

results on the asymptotic distribution of eigenvalues for the Laplacian on Riemannian manifolds and for ordinary differential equations.

To finish the line of Holmgren's descendants, we will mention **Jaak Peetre** (1935-, professor in Lund and in Stockholm). Peetre started out in PDEs as a student of Pleijel, but then branched out into interpolation theory and other fields, broadening his areas of interest substantially.

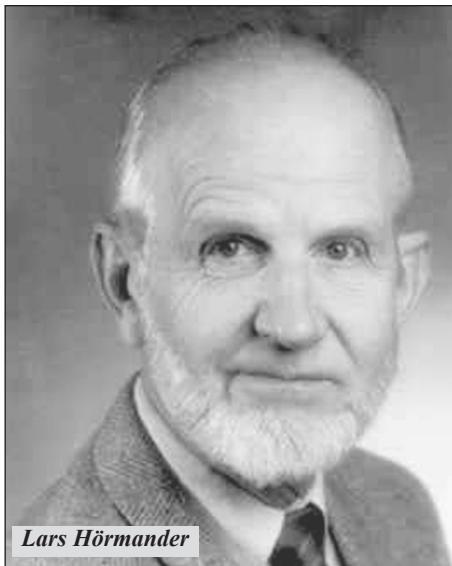
### The Riesz Legacy

The first Riesz student to reach fame was **Einar Hille** (1894-1980, professor at Yale). A couple of years after his Ph.D. he left for a postdoc position at Harvard and was lost to Sweden for ever, although the author took a graduate course in algebra from him in Uppsala in the early 1960's. Hille published widely, but is of course best known for his book on semigroup theory.

**Otto Frostman** (1907-1977, professor in Stockholm) wrote a beautiful thesis on potential theory. Using modern measure theory he could complete and considerably extend Gauss's ideas from the 1820's to kernels of a very general type. Several good papers followed the thesis, but after his time as docent in Lund, Frostman had to support himself as a high school teacher until a chair in Stockholm became available. For a 15-year period, he was also director of the Institute Mittag-Leffler.

**Lars Gårding** (1918-, professor in Lund) soon developed into one of the pillars of Swedish mathematics in the last half of the century. He worked initially in group representation theory - albeit with strong connections to PDEs - but soon made a name for himself in PDEs, where he published widely on elliptic and hyperbolic equations and systems of equations. Presumably, his name will live on forever in connection with the *Gårding inequality*. Later work on hyperbolic equations includes studies on lacunas, partly together with Atiyah and Bott. Gårding also used his considerable literary skills to advantage; one example is the already cited book on Swedish mathematics before 1950. Less known abroad are probably his plays and essays, with mathematical and philosophical themes. His colourful and outspoken personality, combined with a solid interest in music and culture, has never failed to make a lasting impression on anybody who has come his way.

Sweden's only Fields medallist so far, **Lars Hörmander** (1931-, professor in Stockholm and Lund), received his PhD in Lund in 1955, and was a full professor in Stockholm in 1957, at the age of 25. His thesis on linear partial differential equations was a tour-de-force, and he has continued in the same vein for decades, pub-



**Lars Hörmander**

lishing widely on PDEs, several complex variables, Fourier integral operators, and convexity. His early monograph on linear PDEs grew with time into a formidable four-volume *oeuvre*. After some political skirmishes, Hörmander left Stockholm for the Institute of Advanced Study, causing the Swedish parliament to institute a new law, *Lex Hörmander*, aiming at curtailing the brain-drain. After a few years however, Hörmander returned to his *Alma Mater*. Through his dozen or so Swedish PhD students, he has over 80 descendants (MGP does not list all of them). It would probably not be too much of an exaggeration to say that Hörmander is the Swedish mathematician who has had the largest impact ever.

Two of his early students, Germund Dahlquist and Vidar Thomée, moved away from their master's voice to continue their careers in different parts of numerical mathematics, more about which will be explained later. Other students include **Jan Boman** (1931-, professor in Stockholm) and **Christer Kiselman** (1939-, professor in Uppsala). Kiselman has carried the torch of several - indeed, sometimes infinitely many - complex variables, and has in turn had over a dozen students.

### The Beurling Legacy

Beurling's students were given problems in very different areas, attesting to their master's wide interests.

**Carl-Gustav Esseen** (1918-2001, professor at KTH and in Uppsala), wrote a thesis on the best remainder in the law of large numbers, and continued in mathematical statistics.

**Göran Borg** (1913-1997, professor at KTH), was among the first to successfully study the inverse spectral problem for ODEs. Due to the isolation caused by the war, his results did perhaps not have an immediate following, receiving their recognition much later.

**Bo Kjellberg** (1920-1999, professor at

KTH), took up Beurling's interest in Nevanlinna theory, studying the relations between maximum and minimum moduli of analytic functions in the plane.

The most famous of Beurling's students, and the only one to reach - and surpass - his level, is **Lennart Carleson** (1928-, professor in Stockholm and Uppsala). Carleson started out doing bone-hard analysis, studying exceptional sets of vari-



**Lennart Carleson**

ous classes of harmonic and analytic functions. Interpolation problems for holomorphic functions led to the solution of the *corona problem*. For the general mathematical public, Carleson is best known for proving the i.e. convergence of the Fourier series of L2 functions, a problem which had eluded analysts for more than half a century. Outside harmonic analysis he has, in particular in his later years, been very active in dynamical systems. Carleson seems to be the only one to have profited by *Lex Hörmander*, having been awarded a personal professorship by the Government in the 1960s. As a thesis adviser he has been prolific, with over two dozen students, and his descendants, close to 100 in number, have populated the mathematics departments of Swedish universities and university colleges. He was also in practice the lone editor of the *Acta Mathematica* for close to a 30-year period. In addition to traditional mathematical activities, Carleson contributed greatly by infusing life into the slumbering **Institut Mittag-Leffler**. By finding funds to refurbish the Mittag-Leffler villa and building apartments for guests, and by conceiving the format of the institute's scientific programs, he managed to create a centre for mathematical research of a unique character.

Carleson's most productive student has been **Hans Wallin** (1936-, professor in Umeå), both when it comes to number of papers and number of students (MGP lists 32 descendants, of which 24 were students). The rest of us, including **Gunnar Aronsson** (1939-, professor in

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Linköping), **Lars Inge Hedberg** (1935-, professor in Linköping), **Ingemar Wik** (1934-, Umeå), and the author, have been more modest in these respects.

The last one of Beurling's students, **Yngve Domar** (1928-, professor in Uppsala), wrote a beautiful thesis that did not receive the attention it probably deserved. Domar had a productive career in Uppsala, doing harmonic analysis and synthesis. He had 12 PhD students, one of whom, **Matts Essén** (1932-2003, professor in Uppsala), came under the influence of Bo Kjellberg and continued his career in majorization problems for harmonic and analytic functions.

### Numerical Mathematics

Numerical analysis took off as an independent subject in the 1960s, and three more or less independent schools were created. My impression is that Sweden has had a very good reputation in this area. A result of these activities was the creation of **BIT**, a Nordic journal for numerical mathematics with **Carl-Erik Fröberg** (1918-, professor in Lund) as one of the driving forces.

Although a student of Hörmander, **Germund Dahlquist** (1925-, professor at KTH) worked in numerical methods of ODEs and associated numerical algebra problems from the beginning. Among his students, in excess of 20, was **Åke Björck** (1934-, professor in Linköping).

**Heinz-Otto Kreiss** (1930-, professor in Uppsala and at KTH), came from Germany but took a PhD at the KTH and went on to found a thriving school of numerical analysis of PDEs in Uppsala. He has also had long stints abroad. Among his students are **Bertil Gustafsson** (1939-, professor in Uppsala) and **Olof Widlund** (1939- ), who left Sweden early on for a career at the Courant Institute.

Another of Hörmander's students was **Vidar Thomée** (1933-, professor in Göteborg), who started doing more traditional Hörmander mathematics, but then branched out and founded a very successful school of PDE numerical analysis in Göteborg, with an international reputation and a number of good students to his credit.

### New Areas

In the author's youth, Swedish mathematics was dominated by analysis. True, there was **Trygve Nagell** (1895-198) in Uppsala doing number theory, and **Olof Hanner** (1924?- ) in Göteborg doing general topology, but both represented rather modest-scale activities.

This started to change when **Jan-Erik Roos** (1935-, professor in Stockholm) went away from Lund to study in Paris, and found out that there was something called algebra and algebraic geometry. He

came back and founded a school in the subject in Stockholm, which is very active to this day.

**Bernt Lindström** (1933-, professor at KTH) was formally a student of Otto Frostman, but wrote a thesis on combinatorial methods in number theory. He had several students, thus creating a Swedish school of combinatorics, represented today by at least three professors.

### Foreign Influx

With the fall of the iron curtain, a number of Russian mathematicians immigrated to Sweden, changing the face of mathematics here inalterably. Also, professors have been recruited from Germany, Holland, Italy, Norway, Poland, Romania, the UK, and the United States (I have probably forgotten one or two). Due to my self-imposed age limit, the names will not be mentioned here, with the exception of **Vladimir Maz'ya** (1937- ), who was drafted to Linköping by Lars Inge Hedberg. Maz'ya, working mostly in PDEs, and enormously productive by Swedish standards (several books by his hand have appeared in the last few years), has given analysis in Sweden and in Linköping new impetus and new directions.

### Women Mathematicians

As mentioned above, Mittag-Leffler managed to convince the board of the then private Stockholm University (known as Stockholm's högskola in those days) to appoint **Sonya Kovalevsky** (1850-1892)



Sonya Kovalevsky

to a chair of mathematics. Unfortunately, she died at a relatively young age, before really making an imprint here.

With Kovalevsky as a model, one would have expected locally groomed women mathematicians to appear soon after.

These however, failed to materialize, and the female gender has been sadly under-represented in the professorial ranks. A few have received PhDs though, and some have become lecturers. One can mention **Sonja Lyttkens** (student of Carleson), **Anna-Lisa Wold-Arrhenius** (Domar), and **Kersti Haliste** (Carleson). (There are some unmentionables also.)

When in the 1990s, there were finally one or two women appointed to mathematics and applied mathematics chairs, they were imports from Germany and Finland.

### Swedish mathematics of late

For several decades after 1950, the number of professors of mathematics in Sweden was very small, on the order of 10-15, and most of them worked in analysis. Life was dominated, and the standard set, by the three giants, Carleson, Gårding, and Hörmander. This meant that you seldom published, and preferably only your best and polished results. In retrospect, this time seems idyllic, although I do not think we, the lesser souls, thought so at the time, the worries of finding a bread-winning position occupied our minds too much. Since then, Swedish mathematics has taken on a completely different, international flavour, for good and for bad (on balance, surely for good). The number of professors is more than 75, although due to a strange law, some of these are teaching positions, and professorships in name only. The breadth of areas of mathematics being studied has increased enormously, while some of the traditional subjects have receded almost into oblivion. The tradition of publishing has altered, the pressure to publish has increased, and so has the competition for positions. It is not uncommon that a lectureship draws 50 applicants. In short, we have joined the world.

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Kjell-Ove Widman [widman@ml.kva.se] received his PhD under Lennart Carleson in Uppsala in 1966. Professorships in Uppsala and Linköping were followed by a longish stint abroad. He then returned to Sweden to become Director of Institut Mittag-Leffler in 1995. His scientific interests include partial differential equations, calculus of variations, and cryptography.

## SOCIETIES CORNER

The Societies Corner is a column concerning mathematical societies in European countries. Over the last five years, this column has presented numerous articles that described the history of a particular society or the society's present day work. Many European Mathematical Societies have been portrayed, and we wait for the contributions from those societies that have yet to present.

However, until now some important aspects of the societies' lives have not appeared in the column.

The life of a society does not only consist of its historical development and its current active work. A number of events happen in the every day life of a society.

Do you remember the general assembly of your society three years ago and a fascinating discussion that took place there? How about a farewell party and an exciting toast by the President, or a controversial decision about something undertaken by the council last year? Several anecdotes concerning famous mathematicians or former presidents of the society, connected with their work in the society? And what about your society competition for the paper written by school pupils, and some papers presented there? What about another activity? A host of funny or important stories, so many interesting events happened...

Don't keep them to yourself! Please, write about them and submit to the Society column. Let others enjoy your stories! Let others know about the everyday life of the society! If it is not written down now, after some years it may be forgotten.

Of course, we will not stop publishing articles of the same style as before, we just want to have something more.

If you feel that anything like that would interest others, please do not hesitate and



Krzysztof Ciesielski

contact the column editor, Krzysztof Ciesielski (e-mail: [ciesiels@im.uj.edu.pl](mailto:ciesiels@im.uj.edu.pl)) in the first instance.

## France: "Sauvons la recherche"

*Colette Moeglin (Paris, Jussieu)  
and  
Marie-Francoise Roy (Rennes, France)*



Colette Moeglin



Marie-Francoise Roy

During the first semester of 2004, a very significant initiative entitled 'Sauvons la Recherche' ('Let's save scientific research') has taken place in France, inducing an unexpected and powerful social movement of popular support for scientists' requests. Several dozen thousand scientists, as well as several hundred thousand non-scientists, have signed a petition, with opinion polls indicating that more than 80% of the population are in support of the scientists' requests. These requests were extremely simple. The first one was that the government cancels its recent funding restrictions and gives the funding that was initially promised back to the research institutions. The second one was that the 550 research positions proposed by the government as non permanent positions are turned into civil servants positions as was the case in preceding years. Finally, the third was that a national discussion is organized under the name 'Etats généraux de la recherche'. In the French political and social tradition, Etats généraux is thought of as a large bottom-up discussion converging to national proposals for further actions and changes, following the model of the famous initial Etats Généraux that was prelude to the French revolution. When the influence of the initiative grew bigger, the opening requests, initiated in research institutions such as INRA and CNRS, were rapidly added to by a request for the creation of 1000 new positions for hir-

ing young scientists in Universities.

On March 9, given the lack of satisfactory response from the government, over 2000 scientists in charge of research responsibilities (heads of research laboratories or research programs) collectively resigned from their responsibilities. This was considered as a major event by the media. A few weeks later, the government lost the regional elections in a spectacular way. A change of policy concerning research was one of the three main changes recommended by President Chirac to take into account the electoral regional failure (the two others being the funding of young artists, and the support of unemployed people, two other topics for which important demonstrations and actions had taken place in the preceding months). The ministry in charge of research was changed and the requests of the scientists accepted within a few days.

The current situation, beyond all initial hopes, is that all the requests motivating the 'Let's save scientific research' initiative have been accepted.

The Etats Généraux de la Recherche are currently being prepared in local research groups, universities and research institutions and should define propositions for the fall. A 'programmation law' making scientific research a national priority should be voted before the end of the year and various changes to the organization of scientific research in France should follow.



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The French mathematical community is naturally active in this context. The French mathematical societies SMF (Société Mathématique de France) and SMAI (Société de Mathématiques Appliquées & Industrielles) have supported the 'Let's save scientific research' initiative from its beginning and encouraged their members to be active in the various demonstrations, contacts with the population and preparation of the 'Etats généraux'.

The organization of research and of scientific curricula has some very specific features in France. Firstly in many areas of science, like physics, biology and chemistry, a majority of scientists are not university professors but rather full time researchers, and moreover, civil servants paid by the state. The main research institution is CNRS, which covers all aspects of scientific research from archaeology and linguistics to biology, computer science and mathematics. However smaller and more specialized research institutes like INRA for biology and medicine, and INRIA for computer science, are also present, and the relative roles of these institutes and CNRS is not clear. Important research programs are defined and directed by the Ministry of Research, independently of research institutes. The role of Universities, as well as regional institutions, in the orientation of research has to be redefined, and the duality between full time permanent research positions and university positions is being questioned.

Another specific French feature is the existence of 'classes préparatoires'. Rather than going to universities, the best scientific students prepare competitive exams in specific classes, taught by very competent full time professors in the secondary school administrative system, having no contact with research. The engineering schools they enter after this preparation are mostly independent of the universities. The very best of these students, if they enter into major 'grandes écoles' such as Ecoles Normales Supérieures, Ecole Polytechnique and a few others, will be in contact with scientific research in the remainder of their studies. However, an important proportion of these good students will stay apart from the university and scientific research system during their whole curriculum. On the other hand, students deciding to choose universities are often poorly motivated and not adequately prepared and do not meet the expectations of the university professors who are active scientists.

## Status and Perspectives

The present context is moreover characterized by two important facts: the number of scientists retiring from research or teaching positions is growing, and the number of students

choosing scientific studies in universities is dropping.

In this general context, French mathematical data can quickly be summarized. The community of professional mathematicians (university teachers and full time researchers) has about 3500 members, only 10% of them being full time CNRS researchers, and about half of these are concentrated in Paris or the Ile de France region. In spite of the small number of full time researchers in mathematics, CNRS plays a very important role in the organization of French mathematical research: about 85% of the mathematicians belong to one of about 60 joint CNRS-Universities research laboratories, and the important equipments of the French mathematical community are supported by CNRS (CIRM, IHP, the network of math libraries, the project Numdam for numeration of old mathematical publication, the network of computer engineers for mathematics). 25% of INRIA (around 100 people) are mathematicians. The loss of students, although less spectacular than in physics for example, is high in mathematics, reaching about 25% in the last four years. The number of university positions in mathematics started to decrease in 2003 and about 100 positions have disappeared. The mechanism is the following: mathematicians retire or are promoted and their position is given to another department where student numbers are greater (sport and theatre are currently popular among students).

A vicious circle is thus currently at work: less students, less positions for mathematicians, less motivation to study mathematics.... and in a few years the French mathematical atmosphere, recently active and full of promise, could switch to long-term depression.

This situation is paradoxical since the importance of mathematics in a growing num-

ber of scientific and technological research areas is widely acknowledged. Industry and services need more and more people with a good mathematical background and an ability to adapt to the rapid changes in science and technology. French mathematicians have to open their eyes and ears and redefine new curricula for students and new scientific collaborations with the outer world.

A forum discussion 'Let's save scientific research' has been proposed by SMF and approved by SMAI, cf. <http://smf.emath.fr/Forum/>. Concrete suggestions following this discussion will be proposed before the summer. On the European scale, a petition 'For a European Research' in support of a European Research Council (ERC) for basic research has been launched at <http://fer.apinc.org/>. European mathematical societies and individual European mathematicians are very welcome to participate in our French discussions and bring us some fresh air and new ideas! Please visit and send your contribution (by email to [smf-debats@smf.ihp.jussieu.fr](mailto:smf-debats@smf.ihp.jussieu.fr)).

*Colette Moeglin [moeglin@math.jussieu.fr] graduated at Univ. Paris 6 under Jacques Dixmier with a thesis on primitive ideals of enveloping algebras. Her current research is concerned with automorphic forms and representations of  $p$ -adic groups. She is employed by CNRS as Directrice de Recherche at the Institute of Mathematics at Paris-Jussieu, where she also acts as Directrice Adjointe and as editor-in-chief of JMJ, the Journal of the institute.*

*Marie-Françoise Roy [marie.francoise.roy@univ-rennes1.fr] works on algorithmics within real algebraic geometry. She is a professor of mathematics at the University of Rennes 1 and a member of the council of the SMF.*



Monsieur François Fillon  
Ministre de l'Education Nationale  
de l'Enseignement Supérieur et de la Recherche  
110 rue de Grenelle  
75357 Paris 07 SP

Paris, le 13 avril 2004

Monsieur le Ministre,

L'organisation future de la recherche scientifique en France va faire l'objet de discussions et de décisions dans les semaines qui viennent. Nous nous félicitons de ces nouvelles perspectives.

Par la présente, nous avons l'honneur de solliciter un entretien avec vous pour vous présenter quelques particularités de l'organisation actuelle des sciences mathématiques qu'il nous semble important de voir prises en considération dans le cadre des adaptations nécessaires. Il nous revient aussi de vous

# PORTUGAL IN THE FIRST YEARS OF THE INTERNATIONAL MATHEMATICAL UNION

(*Enrique Hernández-Manfredini and Anabela Ramos, Universidade de Aveiro, Portugal*)

In [5], Graciano de Oliveira describes the essentials of the life, aims and activities of the Portuguese Mathematical Society, together with the adverse historical context that seriously hindered its normal development and obstructed Portuguese mathematics during the dictatorship years. De Oliveira pays special attention to the history of mathematics in Portugal in the 1940's. Digging a little further back in time, and following Costa & Malonek [2], and Ramos & Malonek [6], [7], these notes aim to attract more attention to the efforts made by Portugal to establish contacts with the international scientific community in the post-Great War period; an aspect of Portuguese scientific history which has often not received due attention, if not been completely forgotten.

We also intend these notes to be a contribution to recovering the memory of Francisco Miranda da Costa Lobo, (1864-1945), whose dedication and efforts to make these contacts possible and to stimulate mathematical life in Portugal in the 1920's seem to have faded from memory as well.

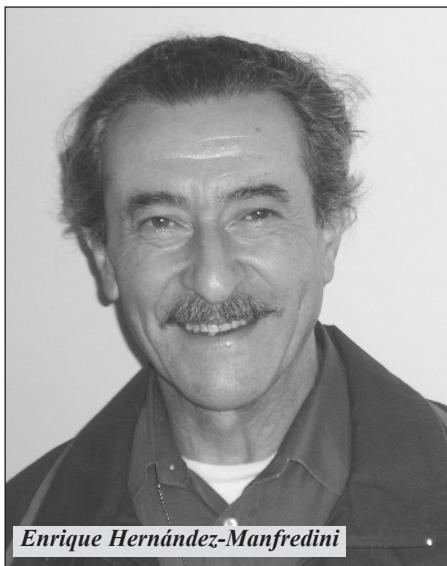


Francisco Miranda  
da Costa Lobo.  
(1864-1945)

Costa Lobo was not only a first-class scientist, but also an important ambassador for Portugal in the international scientific field, without whom an important chapter in the history of Portuguese academic life would, no doubt, be missing. Costa Lobo, an astronomer, was Director of the *Observatório Astronómico de Coimbra*, and was also president of the *Instituto de Coimbra* from 1915. The *Instituto*, recognised by the International Association of Scientific Academies as an academy of sciences, had an interest in international intellectual life. Under the influence of its director, the *Instituto* focused its interests in the area of mathematics.

At the time, the *Instituto* made a considerable contribution to Portuguese mathematical life, and opened the country to the international mathematical world: In its 1920 General Assembly, the *Instituto* named Emile Picard, Charles de la Vallé Poussin, and Gabriel Koenigs, as Honorary foreign members.

Later, after the 1924 International



Enrique Hernández-Manfredini

Congress of Mathematicians (ICM) in Toronto, the *Instituto*'s Assembly named as Corresponding Members the following renowned mathematicians: Élie Joseph Cartan, from France; John Charles Fields, from Canada; Karl Rudolf Fueter, from Switzerland; Tullio Levi-Civita, from Italy; Salvatore Pincherle, also from Italy; John Lighton Synge, from Ireland; Grace Chisolm Young, from England; William Henry Young, from England, and Clément Servais, from Belgium.

Within the next couple of months, the Assembly would also add to the list, the names of Nicolau Kryloff, from Kiev University, who gave a two-month course on Advanced Analysis at the University of Coimbra, and Jacques Hadamard, from France. Finally, in 1925, the file was completed with the name of the mathematical physicist Sir John Larmor, from England.

Some of the above were to become Presidents of different ICMs: Pincherle in 1928 in Bologna; Fueter in 1932, in Zurich; and Hadamard, Honorary President of the 1950 Congress in Cambridge, USA. The presence of these personalities in Portugal allowed a much needed interchange of information and widened Portugal's international scientific horizon. In particular, one can see works by de la Vallé, G. C. Young, Kryloff, and Servais, published by the *Instituto*'s magazine. This impressive list of collaborators is a token of Costa Lobo's dedication and commitment to developing mathematics in Portugal, as well as to forging relations between his country and the mathematical world.

It was Costa Lobo's connection with the



Anabela Ramos

International Mathematical Union (IMU) and his involvement in several Congresses of the International Research Council (IRC), which provided him with the opportunity to become acquainted with several foreign mathematicians, with whom he developed strong ties of friendship. This is especially true of his participation in the Toronto Congress in 1924. The long trip by sea provided an environment that was propitious for such developments.

Costa Lobo's activities and contacts enabled him to make the state of mathematical research abroad known in his country, as well as to report about the way Mathematical Education was conducted in foreign universities. He was quite impressed with several aspects of the University of Toronto, (its organization, academic life, interaction with other universities, the subjects offered in the different courses, in particular those in mathematics), and describes them in detail in Lobo [4]. This can also be seen in Costa & Malonek [2].

Mathematical activities in Portugal in the 1920's, and especially, Portugal's early association with the IMU are rather forgotten matters. Yet, it is to be noted that Portugal, was involved in the very creation of the IMU. First, taking part in the Constitutive Assembly of the International Research Council in Brussels, in 1919, where preliminary moves towards the creation of the IMU were made, and later, in 1920, as a participant in the ICM in Strasbourg, where the IMU was definitively founded. On this occasion, the Portuguese representative was Costa Lobo, who presented a communication *Sur la courbe décrite par le pôle sur la surface*

*de la terre*. Later, in 1924, Portugal attended the Congress that took place in Toronto, represented by Costa Lobo and Fernando de Vasconcelos. The former presented a communication on *Nouvelles théories physiques: application à l'astronomie*; the latter, one entitled *Sur quelques points de l'histoire des mathématiques des Egipcien et aussi sur le Sidhantas des Indiens*.

It will be remembered that political issues and sensitivities proper of the post-war period pervaded the life of the IMU and the IRC's meetings. They were haunted by animosities towards the countries that had fought against the allied powers. As a consequence, these countries were banned from membership of the IMU and from participation in the 1924 Toronto Congress. Regarding these political issues within the context of the IMU and the IRC, it may be of interest to note that Costa Lobo did not directly address or comment on them. However, his views about war related topics were the ones that could be expected from a citizen of one of the allied countries. (cf. Lobo [3]).

In the year that followed the Toronto Congress, it has to be mentioned the rôle played by Costa Lobo in the realization in Coimbra, in 1925, of the Second Joint Congress of the *Associação Portuguesa para o Avanço das Ciências*<sup>1</sup>, together with its Spanish counterpart, the *Asociación Española para el Avance de las Ciencias*<sup>2</sup>.

The congress took place from 14th to 19th of July and embraced a wide spectrum of topics: mathematics, astronomy and physics of the Earth, physico-chemistry, natural sciences, philological, philosophical and historical sciences, and medical sciences and applications. Apart from the peninsular representatives, the congress was attended, amongst others, by representatives of The French Institute, The Royal Academy of Belgium and the French and English Associations for the Advancement of Science.

On this occasion, Costa Lobo shared responsibilities with an important and internationally known Portuguese mathematician: Francisco Gomes Teixeira. The latter took part as President of the *Associação Portuguesa*, while Costa Lobo acted as President of the Congress' Executive Committee.

Although we lose track of Portuguese participation in the 1928, 1932 and 1936 ICMs, we note the presence of Portuguese mathematicians at the 1950 Cambridge-USA Congress. Communications were submitted by Hugo Ribeiro, A. Gião, A. Monteiro, M. Peixito, and R. L. Gomes. However, these scientists took part in the Congress as individuals and not as representatives of Portugal. Moreover, only Hugo Ribeiro was personally present, as he



University of Coimbra

was living in the USA. The others, exiled in Brazil, limited themselves to sending their contributions. This was an indication that political pressure exerted on the Portuguese universities by the Salazar regime had begun to show its effects. It is well known that during the Salazar regime, intellectual life and scientific research activities in Portugal were severely hampered. For more details of how this situation arose and subsequently evolved, see de Oliveira [5], Bebiano [1], and Ramos & Malonek [6].

In these notes we have focussed our attention on a specific part of the history of mathematics in Portugal in the 1920's. Details of ensuing events, especially the foundation and life of the Portuguese Mathematical Society can, again, be found in de Oliveira [5].

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*Enrique Hernández Manfredini lectures in Mathematical Logic and Foundations of Mathematics at the University of Aveiro-Portugal. He graduated in mathematics education at Chile University and obtained his Ph. D. in mathematics from (the former) Bedford College, London University. He has worked at the Chile University, the (former) Technical University of Santiago-Chile, the Cuban Academy of Sciences, the University of Havana, British Telecom, and at a variety of secondary and further education colleges. His main interest lies in the realm of the foundations of mathematics.*

*Anabela Ramos is an assistant at the Mathematics Department of Aveiro University, where she tutors and lectures in mathematical analysis. She completed her first degree in mathematics, and later her master's degree, at the University of Coimbra. Her MSc thesis dealt with one of Pedro Nunes' works: *O De Erratis Orontii Finaei*. She has previously worked as a secondary school mathematics teacher. Her main interest focuses on Portuguese mathematical history.*

<sup>1</sup> Portuguese Association for the Advancement of Science.

<sup>2</sup> Spanish Association for the Advancement of Science

# An interview with Ari Laptev

Interviewer:  
Marta Sanz-Solé  
(Universitat de Barcelona)

Ari Laptev (Stockholm) is the Chairman of the 4ECM Organizing Committee.

## Role of the European Congress of Mathematics (ECM)

**You are chairing the Organizing Committee of the 4ECM, one of the major mathematical events in Europe in 2004. What is in your opinion the role and the necessity of big, interdisciplinary meetings like ECMS?**

I used to be quite sceptical about large-scale meetings in general and EMS Congresses in particular. However, my involvement in organizing the 4ECM has radically changed my attitude. I see now how useful such meetings can be. Compared with specialized scientific conferences, where one meets close colleagues, big meetings have quite a different function. They give an overview of new tendencies in Mathematics and play an important role in developing its different areas. There is an opinion that Mathematics has become too large and that it is difficult for the majority of participants to follow most of the talks at these meetings. During the last century many new fields have been created, and established branches of Mathematics have split into sub fields. However, development during the last twenty years has to a large extent been the opposite. There is a clear tendency towards unification of different branches of mathematical research. Many new break-



Ari Laptev



Marta Sanz-Solé

throughs were achieved by combining techniques from different areas of Mathematics. Therefore I think that IMU and EMS Congresses will play an even more important role than before, allowing mathematicians to focus on problems of general interest.

**Since 1897, mathematicians have met periodically at International Congresses (ICMs), with programmes including all disciplines. Mathematics is the only scientific field keeping this tradition. The much younger ECMS might be seen as ICMs on a smaller scale. What makes ECMS so special?**

At the time when it became clear that Stockholm might be the next host for the European Congress of Mathematics, the opinion of some of my colleagues was that EMS Congresses are not needed, precisely for this reason. They claimed that such meetings imitate IMU Congresses on a smaller scale, are not of the highest quality and therefore unnecessary. I disagree. Looking back at the three EMS Congresses organized in Paris, Budapest and Barcelona, it is noticeable that they were all different from the IMU Congresses in character as well. When organizing the 4ECM we also hoped to introduce new features. For example, already at the very early organizational stage we

decided to involve the European Networks by giving members an opportunity to speak at the Congress.

In order to receive a grant from Brussels or ESF, one has to gather a very representative bunch of teams from different European countries working in the same area of Mathematics. The competition is extremely high but once the project is awarded, the Network obtains quite substantial funding for developing a specific area of research. However, they have very little interaction with mathematicians who do not belong to their Network. Therefore, European Congresses can play a coordinating role for these programmes.

By inviting the speakers from different European Networks to the 4ECM, we want to give European Networks the possibility of communicating with each other. We also hoped that by inviting such speakers we could make the 4ECM more attractive to the members of the corresponding research groups. Indeed, as we see now, many members of these groups will be participating in the 4ECM and will have their own satellite meetings before or after the Congress.

This time we only have half a day for the Network presentations. I personally think that it would have been even better if we

could have devoted a whole day to the European Network activities. For example, half a day for general Invited Lectures and another half for networks mini-symposia, where the members of the corresponding Network could choose the speakers themselves. One of the advantages of such mini-symposia compared with "usual" Network meetings could be that such a meeting would gather a broader audience, allowing people to both invite speakers from parallel groups and participate themselves in other Network mini-symposia.

We also kept in mind that including European Network presentations might be important for future EMS relations with the authorities from Brussels and with the ESF in Strasbourg. We are expecting some representatives from Brussels to attend the 4ECM Opening Ceremony.

Another new feature of the 4ECM is the involvement of scientists from areas outside Mathematics. Nobel Institutes in Physics and Chemistry of the Swedish Royal Academy of Sciences are supporting the Congress. This enables us to invite prominent specialists in Physics, Chemistry and Biology, who will share their views on the significance of Mathematics in their subject in a lecture of their own choice.

Finally, I would also like to mention that there are 10 prizes awarded to young mathematicians at EMS Congresses. They are different in character compared with Fields Prizes and allow us to draw attention to a group of young European mathematicians who are at the beginning of their scientific career and have already made substantial contributions in their areas of research.

I am sure that the organizers of the 5ECM in Amsterdam will also have new ideas, which will contribute to the future style of the EMS Congresses.

## Mathematics in Science and Technology

*The theme of the 4ECM - Mathematics in Science and Technology - might be a statement addressed to those who think that Mathematics can develop without any cross connections and interactions with scientists and engineers. At the same time, it is carrying a clear message about the new opportunities and developments for Mathematics in the 21st century. Could you comment on this?*

In his article written for the *Barcelona Intelligencer*, Jean-Pierre Bourguignon pointed out the importance of interactions between different areas within Mathematics and also the interaction of Mathematics with other subjects. He writes: "Mathematicians are already and as I see it, will be more and more, confronted with the need to broaden the conception they developed of their discipline and how they practice it."

With the title "Mathematics in Science and Technology" we wanted to emphasize the importance of developing Mathematics for other Sciences and Engineering, and try to identify the place of Mathematics within the modern scientific community. The attitude that Mathematics can be good only for mathematicians should be changed and we all have to try to be more open minded towards a large number of new areas in Science and Engineering developed during the last 20 years. There is a danger that mathematicians will lose respect from other scientists if they are not prepared to collaborate.

There is also a danger that Mathematics as a subject could be put aside and become insignificant in the main stream of the development of Science. We should not take the importance of Mathematics for granted; we have to continue to prove it.

As I already mentioned, when

organizing the 4ECM we had the idea of using the fact that Sweden is the country of Nobel Prizes and invited representatives from other Sciences to give Plenary talks. We believe that a partnership with specialists from other scientific areas could be extremely fruitful.

We have made an effort to establish better contact with Applied Mathematics by inviting Plenary and Invited Speakers covering a variety of applications.

## Visibility of Mathematics

*The celebration of a large scientific meeting like the 4ECM is an excellent opportunity to increase the visibility of Mathematics. What aspects of the scientific programme help the most to show why Mathematics is useful and why a knowledge-based society needs mathematically well-educated people?*

It is a very good opportunity indeed. Looking at the list of registered participants and at some of the applications to Poster sessions, I can see many scientists who cannot be classified as Pure Mathematicians. Obviously, this means that people from areas of research related to Mathematics are interested in the 4ECM and in developing contacts with mathematicians. This is a promising tendency and once more shows the importance of big general meetings of Mathematics where people from different areas can meet each other.



When trying to predict the future, we sometimes have discussions in our Department about how much one will need Mathematics in perhaps 50 years time. Will future engineers need to know the multiplication table? For example, this skill is hardly a

requirement for a supermarket cashier nowadays, but it definitely was very important in the past. Will the teaching of Mathematics survive or will it die out in a similar way to the teaching of Latin? It is difficult to say. However, it is clear that the training of logical thinking will always be necessary.

Moreover, progress of Science and Engineering relies more and more on complicated computations, which always have to be evaluated. It is difficult to imagine that this can be done without deep knowledge of Mathematics.

### Young mathematicians

***One cannot think of the future of science without the incoming flow of the younger generations. Why do young people feel attracted to do research in Mathematics? What is the importance of Congresses showing both the unity and the diversity of Mathematics and its most recent breakthroughs?***

It is a mystery how society continuously produces a number of people who are gifted in Mathematics and who have the necessary passion to pursue it. At the beginning of May we had a meeting at our Department at KTH evaluating applications for PhD positions. There were 30 applications. Although our financial possibilities allowed us to accept only four applications, we were very happy to see that Mathematics is an attractive subject for young people in Sweden. However, such an attraction is based on many factors, which are often insignificant when taking separately, but become very encouraging when viewed together. I believe that the visibility of the EMS Congresses with their awards to young researchers plays an important part in making Mathematics attractive to young people.

Needless to say that more than 200 grants, which were distributed to mostly young participants from Central and Eastern

European countries, will support the popularity of Mathematics in countries where in general, conditions for mathematicians are still very poor.

***You live in a country with intensive activity and influence in the development of Science and Technology. Do education policy makers put enough effort into ensuring good mathematical education at Swedish schools? Are scholars aware of the attractive, exciting side of Mathematics?***

It is clear that we have a big problem with mathematical education in Swedish schools and it looks like it is a common phenomenon for many other European countries too. One of the main reasons behind such development is that nowadays the status of teachers is rather low. As a result, this profession is not very attractive to talented people. Our government tries to improve this situation but it is not an easy problem, especially if one is not prepared to invest a substantial amount of money in it. A relatively "cheap" solution - investing into research in mathematical education - has not had much of an effect.

Being a good teacher in any subject, in particular Mathematics, is a talent that is based on a solid knowledge of the subject. The resource of talented teachers is limited and it is vital for any society not to lose such people and to make the teaching profession more attractive. At University level, we try to do our best to put future Engineers on the "right track", but it becomes more and more difficult because students have not been sufficiently prepared in Mathematics at school.

***Stockholm is known as a "City of Science". Do you plan to present the 4ECM to journalists, to let people know that it will be the "City of Mathematics" around the dates of the congress?***

We shall try our best and hope

that we shall be able to involve the media. The possibility of getting extra publicity is also an added advantage of inviting Nobel Prize winners to give lectures at the Congress. The Swedish Academy of Science has already shown its interest in publicizing the 4ECM. However, it is difficult to compete with the European Football Cup in Portugal, the last week of which coincides with the 4ECM. You can guess which event will be the most attractive to journalists.

### And the outcome?

***You and your colleagues on the Organizing Committee are devoting enormous efforts into the preparation of this important event, and many of the country's Institutions are sponsoring the Congress. What kind of tangible benefits could you expect from all this? (For instance, increasing student's interest for Mathematics, gaining government support to improve mathematical education and research, strengthening scientific cooperation between different departments across the country, etc.)***

I do not think that one should try to measure how profitable the 4ECM will be for Mathematics in Europe, Sweden or for our Department at KTH. However, I believe that a meeting of this dimension, as well as everything important which is happening in Mathematics, is definitely helpful for its visibility. I believe that ultimately it will play an important role in the future funding of Mathematics at every level.

We expect that the 4ECM will be very good for Swedish PhD students. Many of them are participating in the Congress, and meeting prominent mathematicians personally will definitely inspire their commitment to Mathematics.

***Thank you very much for this interview. I am sure that the 4ECM will be a success.***

# ERCOM: European Research Centres on Mathematics

A lot of mathematical activity is hosted at the 24 European research centres - quite different in size and scope - represented in the EMS committee ERCOM. The Newsletter wishes to give these institutions the opportunity to present themselves in the coming issues and years. This will hopefully give the readers an idea of how they might use the possibilities offered by these centres: as organiser or participant of a workshop or a seminar or as a visitor. Quick information about the ERCOM institutions and about the ERCOM committee of their directors is found on the web site <http://www.crm.es/ERCOM>.



## EURANDOM, a European research institute for the stochastic sciences “The feeling of sparkling Champagne”

### History and people

The stochastic sciences have been developing so fast in the past decades that an increasing need was felt for international cooperation and bundling of research resources. This led to the establishment of EURANDOM, in July 1997. The institute opened its doors in September 1998. The first scientific director was Willem van Zwet. Since October 2000, Frank den Hollander has been the scientific director, with Henry Wynn (London, United Kingdom) as scientific co-director.



Frank den Hollander, scientific director

Peter Bickel (Berkeley, USA) chairs the scientific council, which consists of renowned European scientists in the areas of probability, statistics and stochastic operations research. In July 2004, Don Dawson (Ottawa, Canada) will take over this position.

### Financial resources

EURANDOM has two main financiers: the Netherlands Organisation for Scientific Research (NWO) and Eindhoven

University of Technology (TU/e). Together



**TU/e** technische universiteit eindhoven

they provide 60% of the budget. Both parties have a representative on the Board of EURANDOM and they jointly appoint a third external member. Other funding sources are: European science organisations, industrial contracts, EU-funds, ESF-networks, etc.

### Mission

The mission of EURANDOM is to foster research in the stochastic sciences and their applications. It achieves this mission by helping talented young researchers find their way to tenured positions in academia and industry. It does this by carrying out and facilitating research through post-doctoral and graduate appointments, visitor exchange and workshops, and by taking initiatives for collaborative research at the European level.

### Focus

From the summer of 2004 onwards, the research at EURANDOM will be realigned into three parallel programmes, each with three themes:

#### *Random Spatial Structures (RSS):*

- Critical Phenomena

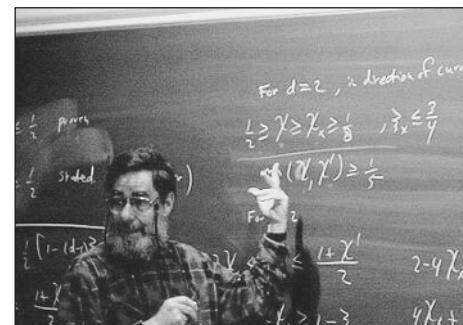
- Disordered Systems
- Combinatorial Probability

#### *Queuing and Performance Analysis (QPA):*

- Performance Analysis of Production Systems
- Performance Analysis of Communication Systems
- Queuing Theory

#### *Statistical Information and Modelling (SIM):*

- Statistical Learning
- Biomedical and Bio-molecular Statistics
- Industrial Statistics.



Harry Kesten (Cornell University, USA), EURANDOM Chair in 2002

**RSS** focuses on systems consisting of a large number of interacting random components, possibly with disorder. It aims to capture their large space-time behaviour through a combination of probabilistic, combinatorial and ergodic techniques, with special emphasis on critical phenomena and universality.

**QPA** is concerned with the performance analysis of stochastic networks, in particular, production and communication systems. Queuing theory and stochastic analysis provide the means to deal with control and optimization issues.

**SIM** combines analytic and algebraic methods to address a variety of statistical questions, including statistical classification, bootstrap, micro-arrays, gene expression, design of experiments, predictive maintenance and quantum statistics.

Each programme hosts six to eight post-doctorates and graduate students, is supervised by senior scientific advisors, and is guided by an international steering committee. Through the choice of themes, special attention is given to the interfaces with physics, biology, telecommunication and industry.

In addition to the above three research programmes, EURANDOM runs two research projects, one on reinsurance and one on battery management.



#### Staff and research

EURANDOM offers an attractive environment for carrying out research on random phenomena. Since its start, some 60 post-doctorates and graduate students have been working at EURANDOM. About 80% of the junior staff leaves EURANDOM for tenured positions in academia or industry (11 so far in The Netherlands). At any time, about 25 junior researchers are working at EURANDOM.

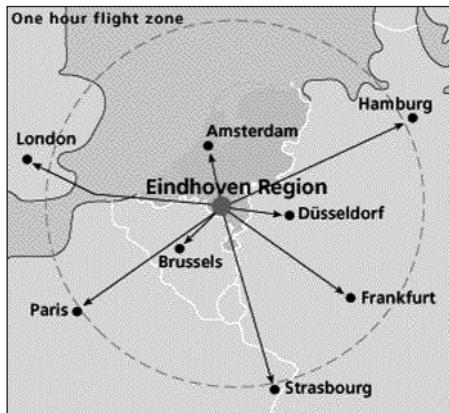
Vacancies at EURANDOM may occur at any time during the year. The opening of positions is not restricted by deadlines, and depends on the number of junior staff per programme at a given moment of time. Candidates who wish to apply for an external research grant are welcome. The institute offers assistance in the application procedure. Post-doctoral appointments are typically for two years, though shorter periods are possible as well. Graduate students are appointed for three to four years.

The EURANDOM visitor programme is another important element that makes the institute into a lively place. Senior and junior researchers come from all over the world, either on invitation or upon request, and participate in the various activities. Workshop topics are chosen by the scientific advisors and by the steering committees of the research programmes. Ideas for topics are collected from post-doctorates

and senior visitors, and through various European networks and programmes in which EURANDOM participates.

#### Links to research in The Netherlands

EURANDOM is located on the TU/e campus in Eindhoven. EURANDOM is an independent foundation. The location on the university campus allows for special ties with the university, in particular with the Department of Mathematics and Computer Science. Through the appointment of scientific advisors, the institute maintains intensive relations with other mathematics departments in The Netherlands.



#### Links to research in Europe

European cooperation runs via the scientific council and the steering committees of the research programmes, via the appointment of post-doctorates, graduate students, and through the workshop and visitors programme. Each year EURANDOM organises some 10 workshops on a range of topics, with on average 40-45 participants from around the world. Each year some 40 junior and senior researchers from outside The Netherlands visit EURANDOM for periods of one week up to several months (on average 60-70 visitor weeks per year). EURANDOM thus provides a stimulating environment to enhance collaboration within Europe.

EURANDOM participates in an EC Thematic Network (Pro-Enbis), two EC Networks of Excellence (PASCAL and EURO-NGI), a Dutch-German Bilateral Research Group (BRG), and chairs an ESF Network (RDSES). EURANDOM has



close ties with MaPhySto (Aarhus, Denmark), the Stochastic Centre (Göteborg, Sweden), and the Norwegian Computing Centre (Oslo), which share a strong focus on the stochastic sciences.

Several national research organizations (e.g. DFG, FWO, NSF) have supported EURANDOM through grants for post-doctorates and visitors to carry out their research at EURANDOM.

#### Location and facilities

EURANDOM is situated in a spacious building, full of light, glass and colours, on the TU/e campus in Eindhoven.



Post-doctorates get together on a daily basis, run a bi-weekly post-doctorate seminar, and participate in the busy schedule of seminars and mini-courses the institute offers. Social events are organised at regular intervals, ranging from hiking on bike (with or without umbrella), to admiring tulip exhibitions, ice-skating, canoeing, etc.



Junior staff and long-term visitors have an office with computer facilities and are assisted at various levels, such as finding accommodation and arranging a visa and work permit. There is a small in-house library, carrying most international stochastic science journals. Extensive library facilities are available on campus, much of it on-line.

The TU/e acts formally as employer of EURANDOM personnel; therefore all staff can use the facilities on campus (library, sporting, restaurant, etc.).

EURANDOM looks forward to welcoming you at the institute. Come and share with us the "feeling of sparkling Champagne". For more detailed information on the institute and its current activities, visit our website: <http://www.eurandom.tue.nl>.

# Abel Prize 2004 to Atiyah and Singer

The Norwegian Academy of Science and Letters has decided to award the Abel Prize for 2004 jointly to

Sir Michael Francis Atiyah, University of Edinburgh  
and

Isadore M. Singer, Massachusetts Institute of Technology

*"for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics."*



I.Singer, M. Atiyah and King Harald  
during the Prize Ceremony  
(Photo: Knut Falch/Scanspix)

The second Abel Prize has been awarded jointly to Michael Francis Atiyah and Isadore M. Singer. The Atiyah-Singer index theorem is one of the great landmarks of twentieth century mathematics, profoundly influencing many of the most important later developments in topology, differential geometry and quantum field theory. Its authors, both jointly and individually, have been instrumental in repairing a rift between the worlds of pure mathematics and theoretical particle physics, initiating a cross-fertilization that has been one of the most exciting developments of the last decades.

We describe the world by measuring quantities and forces that vary over time and space. The rules of nature are often expressed by formulas involving their rates of change, so-called differential equations. Such formulas may have an "index", the number of solutions of the formulas minus the number of restrictions that they impose on the values of the quantities being computed. The index theorem calculated this number in terms of the geometry

of the surrounding space.

A simple case is illustrated by a famous paradoxical etching of M.C. Escher, "Ascending and Descending", where the people, going uphill all the time, still manage to circle the castle courtyard. The index theorem would have told them this was impossible!

The Atiyah-Singer index theorem was the culmination and crowning achievement of a development of ideas more than 100-years old, from Stokes's theorem, which students learn in calculus classes, to sophisticated modern theories like Hodge's theory of harmonic integrals and Hirzebruch's signature theorem.

The problem solved by the Atiyah-Singer theorem is truly ubiquitous. In the 40 years since its discovery, the

theorem has had innumerable applications, first within mathematics and then, beginning in the late 70's, in theoretical physics: gauge theory, instantons, monopoles, string theory, the theory of anomalies, etc.

At first, the applications in physics came as a complete surprise to both the mathematics and physics communities. Now the index theorem has become an integral part of their cultures. Atiyah and Singer, together and individually, have been tireless in their attempts to explain the insights of physicists to mathematicians. At the same time, they brought modern differential geometry and analysis as it applies to quantum field theory to the attention of physicists and suggested new directions within physics itself. This cross-fertilization continues to be fruitful for both sciences.

Michael Francis Atiyah and Isadore M. Singer are among the most influential mathematicians of the last century, and are still working. Over a period of 20 years they worked together on the index theorem and its ramifications, and with their theorem they changed



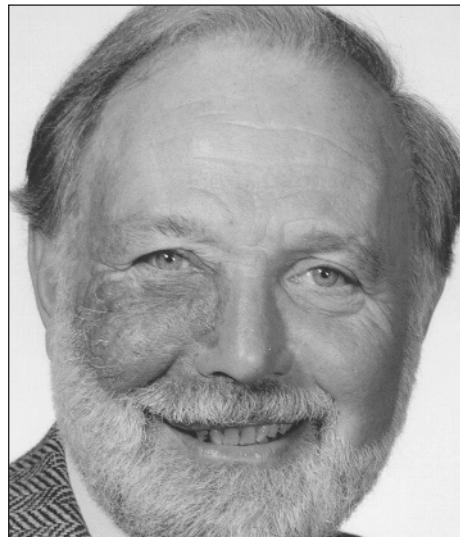
Wreath ceremony at the  
Abel monument, Oslo

the landscape of mathematics.

Atiyah and Singer came originally from different fields of mathematics - Atiyah from algebraic geometry and topology, Singer from analysis. Their main contributions in their respective areas are also highly recognized. Atiyah's early work on meromorphic forms on algebraic varieties, and his important 1961 paper on Thom complexes, are such examples. Atiyah's pioneering work together with Friedrich Hirzebruch on the development of the topological analogue of Grothendieck's K-theory, had numerous applications to classical problems of topology and turned out later to be deeply connected with the index theorem.

Singer, jointly with Richard V. Kadison, initiated the subject of triangular operator algebras. Singer's name is also associated with the Ambrose-Singer holonomy theorem and the Ray-Singer torsion invariant. Singer, together with Henry P. McKean, pointed out the deep geometrical information hidden in heat kernels, a discovery that had great impact.

**Isadore M. Singer** was born in Detroit in 1924, and received his undergraduate degree from the University of



Michigan in 1944. After obtaining his Ph.D. from the University of Chicago in 1950, he joined the faculty at the Massachusetts Institute of Technology (MIT). Singer has spent most of his professional life at MIT, where he is currently an Institute Professor.

Singer is a member of the American EMS June 2004

Academy of Art and Sciences, the American Philosophical Society and the National Academy of Sciences (NAS). He served on the Council of NAS, the Governing Board of the National Research Council, and the White House Science Council. Singer was vice president of the American Mathematical Society from 1970 to 1972.

In 1992 Singer received the American Mathematical Society's Award for Distinguished Public Service. The citation recognized his "outstanding contribution to his profession, to science more broadly and to the public good."

Among the other awards he has received are the Bôcher Prize (1969) and the Steele Prize for Lifetime Achievement (2000), both from the American Mathematical Society, the Eugene Wigner Medal (1988), and the National Medal of Science (1983). When Singer was awarded the Steele Prize, his response, published in the Notices of the AMS, was: "For me the classroom is an important counterpart to research. I enjoy teaching undergraduates at all levels, and I have a host of graduate students, many of whom have ended up teaching me more than I have taught them." Singer has also written influential textbooks that have inspired generations of mathematicians.

**Michael Francis Atiyah** was born in London in 1929. Atiyah got his B.A. and his doctorate from Trinity College,



Cambridge. He spent the greatest part of his academic career in Cambridge and Oxford. He has held many promi-

nent positions, among them the highly prestigious Savilian Chair of Geometry at Oxford and that of Master of Trinity College, Cambridge. Atiyah has also been professor of mathematics at the Institute for Advanced Study in Princeton.

Atiyah rejuvenated British mathematics during his years at Oxford and Cambridge. He was also the driving force behind the creation of the Isaac Newton Institute for Mathematical Sciences in Cambridge and became its first director. Atiyah is now retired and an honorary professor at the University of Edinburgh.

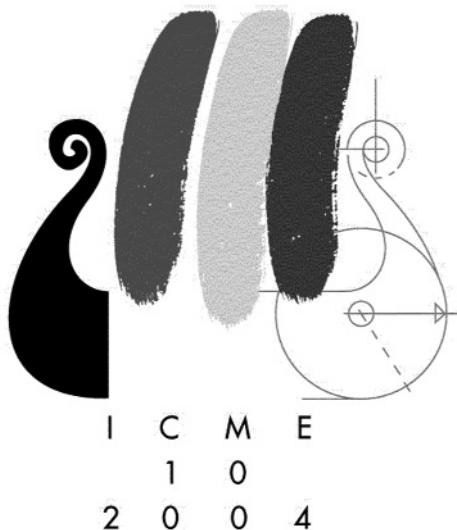
Michael Francis Atiyah has received many honours during his career, including the Fields Medal (1966). He was elected a Fellow of the Royal Society in 1962, at the age of 32. He was awarded the Royal Medal of the Society in 1968 and its Copley Medal in 1988. Atiyah was president of the Royal Society from 1990 to 1995. Atiyah has also served as president of the London Mathematical Society (1974 - 1976) and has played an important role in the shaping of today's European Mathematical Society (EMS).

Atiyah was responsible for the founding of the Inter-Academy Panel that brought together many of the world's academies of science. The Inter-Academy Panel has now been permanently established and will play a major role in the integration of scientific policy throughout the world. Atiyah also instigated the formation of the Association of European Academies (ALLEA). Atiyah has been president of Pugwash Conferences on Science and World Affairs.

Among the prizes he has received are the Feltrinelli Prize from the Accademia Nazionale dei Lincei (1981) and the King Faisal International Prize for Science (1987). Michael Francis Atiyah was knighted in 1983 and made a member of the Order of Merit in 1992.

*The Newsletter plans to publish an interview with the laureates in its next issue.*

# ICME-10 and ICMI awards



The 10th International Congress on Mathematics Education, ICME-10, in Denmark this summer, has got the following visions:

- A well organised congress celebrating the ICME tradition.
- A congress with a strong scientific programme representing the state-of-the-art in both mathematics education research and teaching practices.
- A strong Nordic representation amongst participants.
- A large representation of mathematics teachers from all educational levels.
- A congress that makes a difference to the practice of teaching mathematics.
- A gender balanced congress.
- A congress that gives space and time for informal discussions among the participants.
- A financially balanced congress.

The last vision might need some further development. Two months before the congress the number of registrations is a little below the

budget, but not that bad: 484 from the Nordic Countries, 590 from the rest of Europe and 1032 from the rest of the world including 355 from the USA. If you want the exact number of participants from your country check at [www.icme-10.dk](http://www.icme-10.dk).

The programme will be as planned in the second announcement.

## The ICMI Felix Klein and Hans Freudenthal medals for 2003

The International Commission on Mathematical Instruction (ICMI), founded in Rome in 1908, has, for the first time in its history, established prizes recognizing outstanding achievement in mathematics education research. The Felix Klein Medal, named for the first president of ICMI (1908-1920), honours a lifetime achievement. The Hans Freudenthal Medal, named for the eighth president of ICMI (1967-1970), recognizes a major cumulative program of research. These awards are to be made in each odd numbered year, with presentation of the medals, and invited addresses by the medallists at the following International Congress on Mathematical Education (ICME).

These awards, which pay tribute to outstanding scholarship in mathematics education, serve not only to encourage the efforts of others, but also to contribute to the development, through the public recognition of exemplars, of high standards for the field. The awards represent the judgment of an (anonymous) jury of distinguished scholars of international stature, chaired by Prof. Michèle

Artigue of the University Paris 7.

ICMI is proud to announce the first awardees of the Klein and Freudenthal Medals.

The Felix Klein Medal for 2003 is awarded to **Guy Brousseau**, Professor Emeritus of the University Institute for Teacher Education of Aquitaine in Bordeaux, for his lifetime development of the theory of *didactic situations*, and its applications to the teaching and learning of mathematics.

The Hans Freudenthal Medal for 2003 is awarded to **Celia Hoyles**, Professor at the Institute of Education of the University of London, for her seminal research on instructional uses of technology in mathematics education.

Citations of the work of these medalists can be found below. Presentation of the medals, and invited addresses of the medalists, will occur at ICME-10 in Copenhagen, July 4-11, 2004.

## Citation for the 2003 ICMI Felix Klein Medal to Guy Brousseau

The first Felix Klein Award of the International Commission on Mathematical Instruction (ICMI) is awarded to Professor Guy Brousseau. This distinction recognises the essential contribution Guy Brousseau has given to the development of mathematics education as a scientific field of research, through his theoretical and experimental work over four decades, and to the sustained effort he has made throughout his professional life to apply the fruits of his research to the mathematics education of both students and

teachers.

Born in 1933, Guy Brousseau began his career as an elementary teacher in 1953. In the late sixties, after graduating in mathematics, he entered the University of Bordeaux. In 1986 he earned a 'doctorat d'état,' and in 1991 became a full professor at the newly created University Institute for Teacher Education (IUFM) in Bordeaux, where he worked until 1998. He is now Professor Emeritus at the IUFM of Aquitaine. He is also Doctor Honoris Causa of the University of Montréal.



**Guy Brousseau**

From the early seventies, Guy Brousseau emerged as one of the leading and most original researchers in the new field of mathematics education, convinced on the one hand that this field must be developed as a genuine field of research, with both fundamental and applied dimensions, and on the other hand that it must remain close to the discipline of mathematics. His notable theoretical achievement was the elaboration of the theory of *didactic situations*, a theory he initiated in the early seventies, and which he has continued to develop with unfailing energy and creativity. At a time

when the dominant vision was cognitive, strongly influenced by the Piagetian epistemology, he stressed that what the field needed for its development was not a purely cognitive theory but one allowing us also to understand the social interactions between students, teachers and knowledge that take place in the classroom and condition what is learned by students and how it can be learned. This is the aim of the theory of didactic situations, which has progressively matured, becoming the impressive and complex theory that it is today. To be sure, this was a collective work, but each time there were substantial advances, the critical source was Guy Brousseau.

This theory, visionary in its integration of epistemological, cognitive and social dimensions, has been a constant source of inspiration for many researchers throughout the world. Its main constructs, such as the concepts of *adidactic* and *didactic situations*, of *didactic contract*, of *devolution* and *institutionalization* have been made widely accessible through the translation of Guy Brousseau's principal texts into many different languages and, more recently, the publication by Kluwer in 1997 of the book, *'Theory of didactical situations in mathematics - 1970-1990'*.

Although the research Guy Brousseau has inspired currently embraces the entire range of mathematics education from elementary to post-secondary, his major contributions deal with the elementary level, where they cover all mathematical domains from numbers and geometry to probability. Their production owes much to a specific structure – the COREM (Center for Observation and Research in Mathematics Education) – that he created in 1972 and directed until 1997.

COREM provided an original organisation of the relationships between theoretical and experimental work.

Guy Brousseau is not only an exceptional and inspired researcher in the field, he is also a scholar who has dedicated his life to mathematics education, tirelessly supporting the development of the field, not only in France but in many countries, supporting new doctoral programs, helping and supervising young international researchers (he supervised more than 50 doctoral theses), contributing in a vital way to the development of mathematical and didactic knowledge of students and teachers. He has been until the nineties intensely involved in the activities of the CIEAEM (Commission Internationale pour l'Etude et l'Amélioration de l'Enseignement des Mathématiques) and he was its secretary from 1981 to 1984. At a national level, he was deeply involved in the experience of the IREM (Research Institutes in Mathematics Education), from their foundation in the late sixties. He had a decisive influence on the activities and resources these institutes have developed for promoting high quality mathematics training of elementary teachers for more than 30 years.

### **Citation for the 2003 ICMI Freudenthal Medal to Celia Hoyles**

The first Hans Freudenthal Award of the International Commission on Mathematical Instruction (ICMI) is awarded to Professor Celia Hoyles. This distinction recognises the outstanding contribution that Celia Hoyles has made to research in the domain of technology and mathematics education, both in terms of theoretical advances and through the development and piloting of national and international projects in this field,

aimed at improving through technology the mathematics education of the general population, from young children to adults in the workplace.



**Celia Hoyles**

Celia Hoyles studied mathematics at the University of Manchester, winning the Dalton prize for the best first-class degree in Mathematics. She began her career as a secondary teacher, and then became a lecturer at the Polytechnic of North London. She entered the field of mathematics education research, earning a Masters and Doctorate, and became Professor of Mathematics Education at the Institute of Education, University of London in 1984.

Her early research in the area of technology and mathematics education, like that of many researchers, began by exploring the potential offered by Logo, and she soon became an international leader in this area. Two books published in 1986 and 1992 (edited) attested to the productivity of her research with Logo. This was followed, in 1996, by the publication of *Windows on Mathematical Meanings: Learning Cultures and Computers*, co-authored with Richard Noss, which inspired

major theoretical advances in the field, such as the notions of *webbing* and *situated abstraction*, ideas that are well known to researchers irrespective of the specific technologies they are studying.

From the mid nineties, her research on technology integrated the new possibilities offered by information and communication technologies as well as the new relationships children develop with technology. She has recently co-directed successively two projects funded by the European Union: the Playground project in which children from different countries designed, built and shared their own video games, and the current WebLabs project, which aims at designing and evaluating virtual laboratories where children in different countries build and explore mathematical and scientific ideas collaboratively at a distance. As an international leader in the area of technology and mathematics education, she was recently appointed by the ICMI Executive Committee as co-chair of a new ICMI Study on this theme.

However, Celia Hoyles' contribution to research in mathematics education is considerably broader than this focus on technology. Since the mid nineties, she has been involved in two further major areas of research. The first, a series of studies on children's understanding of proof, has pioneered some novel methodological strategies linking quantitative and qualitative approaches that include longitudinal analyses of development. The second area has involved researching the mathematics used at work and she now co-directs a new project, *Techno-Mathematical Literacies in the Workplace*, which aims to develop this research by implementing and evaluating some theoretically

designed workplace training using a range of new media.

In recent years Celia Hoyles has become increasingly involved in working alongside mathematicians and teachers in policy-making. She was elected Chair of the *Joint Mathematical Council of the U.K.* in October 1999 and she is a member of the *Advisory Committee on Mathematics Education* (ACME) that speaks for the whole of the mathematics community to the Government on policy matters related to mathematics, from primary to higher education. In 2002, she played a major role in ACME's first report to the Government on the Continuing Professional Development of Teachers of Mathematics, and contributed to the comprehensive review of 14-19 mathematics in the UK. In recognition of her contributions, Celia has recently been awarded the *Order of the British Empire* for "Services to Mathematics Education".

Celia Hoyles belongs to that special breed of mathematics educators who, even while engaging with theoretical questions, do not lose sight of practice; and reciprocally, while engaged in advancing practice, do not forget the lessons they have learned from theory and from empirical research. Celia Hoyles' commitment to the improvement of mathematics education, in her country and beyond, can be felt in every detail of her multi-faceted, diverse professional activity. Her enthusiasm and vision are universally admired by those who have been in direct contact with her. It is thanks to people like Celia Hoyles, with a clear sense of mission and the ability to build bridges between research and practice while contributing to both, that the community of mathematics education has acquired, over the years, a better-defined identity.

# Personal column

*Please send information on mathematical awards and deaths to the editor.*

## AWARDS

In November 2003, the African Mathematics Union (AMU) and the International Conference of Mathematical Sciences presented medals to a group of promising young African Mathematicians. The winners are:

**Olatunde Akiulade** (Nigeria; mathematical physics)  
**Bamidele Awojoyogbe** (Nigeria; mathematical physics)  
**Abba Gumel** (Nigeria, now Canada; applied mathematics)  
**Oluwole D. Makinde** (Nigeria, now South Africa; applied mathematics)  
**Guy M. Kniel** (Gabon; statistics)  
**James A. Oguntuase** (Nigeria, now Italy; pure mathematics)

The French Academy of Sciences has awarded several prizes in mathematics for 2003: **Claire Voisin** (Paris) has been awarded the Prix Sophie Germain; **Louis Boutet de Monvel** (Paris) received the Prix Fondée par l'État; The Prix Jacques Hadamard was awarded to **Wendelin Werner** (Paris-Sud); **Claude Bardos** was awarded the Prix Marcel Dassault; **Gilles Lebeau** (Nice) received the Prix Ampère de l'Électricité de France; The Prix Gabrielle Sand et M. Guido Triossi was awarded to **Damien Gaboriau** (CNRS) and **Jean-Marc Delort** (Paris-Nord) received the Prix Langevin.

The 2003 Fermat Prize for Mathematical Research has been awarded to **Luigi Ambrosio** (Scuola Normale Superiore) for his contributions to the calculus of variations and geometric measure theory and their relations with partial differential equations.

Two scientists were awarded the Max Planck Research Award 2003 in Mathematics from the Max Planck Society:  
**Stephan Luckhaus** (Leipzig, Germany) for his trail blazing model showing clear cut solutions for the transport of water and other substances in soil. These solutions can also describe the growth of tumour cells in healthy tissue, which, up until now, was only described theoretically; and  
**Wolfgang Lück** (Münster, Germany) for his landmark research on algebraic topology.

In 2003 the first John von Neumann Awards were presented to **Marina von Neumann Whitman**, **Charles Simony** and to **George Dyson**.

**Mikhail L. Gromov** (IHES, Bures-sur-Yvette, France) and **Jay Gould** (Courant Institute, NYU, NY, USA) have been named recipients of the 2003-2004 Frederic Esser Nemmers Prize in Mathematics.

The Institut de Recherche Mathématique Avancée (IRMA) has created an annual prize in memory of the mathematician Paul André Meyer (1934-2003). The prize honours an outstanding young probabilist, working in the field of stochastic processes. The first IRMA prize in Memory of Paul André Meyer was awarded in February 2004 to **Thomas Duquesne** (Paris-Sud).

The 2004 Emil Artin Junior Prize in Mathematics has been awarded to **Gurgin R. Asatryan** (Yerevan, Armenia).

The Norwegian Academy of Science and Letters has awarded the second Abel Prize (2004) jointly to **Sir Michael Atiyah**

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(Edinburgh, UK) and to **Isadore M. Singer** (MIT, USA) for their discovery and proof of the index theorem. More information can be found in this issue of the Newsletter.

The 2004 Gödel Prize for outstanding papers in the area of theoretical computer science, sponsored jointly by the European Association for Theoretical Computer Science (EATCS) and the Special Interest Group on Algorithms and Computing Theory of the Association of Computing Machinery (ACM-SIGACT), has been awarded to **Maurice Herlihy** (Brown Univ., RI, USA), **Nir Shavit** (Sun Labs, USA), **Michael Saks** (Rutgers, NJ, USA) and **Fotios Zaharoglou** (Univ. CA, San Diego, USA) for important papers on the topological structure of asynchronous computability.

Former Newsletter Editor **Robin Wilson** (Open University, UK) has been appointed Gresham Professor of Geometry for the years 2004-07. Congratulations!

### Erratum:

The editor wishes to apologize for an incorrect statement in issue 50 about awards given in Poland in 2003. The corrected announcements are:

**Michał Szurek** (Warsaw) has received the Dickstein Great Prize of the Polish Mathematical Society for his achievements in popularizing mathematics.

**Paweł Wilczyński** (Kraków) has been awarded the first Marcinkiewicz Prize of the Polish Mathematical Society for students' research papers.

## DEATHS

We regret to announce the deaths of:

Martine Babillot (July 2003)

Andrey Andreevich Bolibruch (11 November 2003)

Armand Borel (11 August 2003)

David Fowler (13 April 2004)

Miguel de Guzman (14 April 2004)

John Hammersley (2 May 2004)

Hans Hermes (10 November 2003)

Eckehart Hotzel (28 September 2003)

Martin Kneser (16 February 2004)

John Lewis (21 January 2004)

Hans-Joachim Nastold (26 January 2004)

Peter Owens (30 December 2003)

Paul Wauters (26 October 2003)

Heiner Zieschang (5 April 2004)

# Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses [vberinde@ubm.ro](mailto:vberinde@ubm.ro) or [vasile\\_berinde@yahoo.com](mailto:vasile_berinde@yahoo.com). Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.

July 2004

**June 26 - July 1: 7th International Conference of The Mathematics Education into the 21st Century Project, Ciechocinek, Poland.**

**Information:**

e-mail: [arogerson@vsg.edu.au](mailto:arogerson@vsg.edu.au)

Web site (for previous conferences only): <http://math.unipa.it/~grim/21project.htm>  
[For details, see EMS Newsletter 50]

**27 June - 2 July: 4th European Congress of Mathematics, Stockholm, Sweden.**

**Information:**

Web site: <http://www.math.kth.se/4ecm>  
[For details, see also EMS Newsletter 50, 51]

**June 30-July 2: Convexity and Discrete Geometry. A conference on the occasion of the sixtieth birthday of Tudor Zamfirescu**

**Theme:** This conference will reflect Tudor Zamfirescu's lifelong passion for geometry, his broad range of mathematical interests, and his commitment to students and young mathematicians

**Location:** Eötvös Loránd University, Faculty of Science, Pázmány Péter sétány 1/a H-1117 Budapest, Hungary

**Organizers:** K. Bezdek (Univ. of Calgary & Eötvös Univ.); K. Böröczky (Eötvös Univ.); A. Heppes (Eötvös Univ.); Éva Vásárhelyi (Eötvös Univ.)

**Confirmed Speakers:** K. Bezdek (Univ. of Calgary, Canada); P. Gritzmann (Tech. Univ. of Munich, Germany); P.M. Gruber (Tech. Univ. of Vienna, Austria); A. Heppes (Eötvös Univ., Hungary);

J.-I. Itoh (Kumamoto Univ., Japan); S. Marcus (Univ. of Bucuresti, Romania); J. Myjak (Univ. dell'Aquila, Italy)

**Deadline:** for preliminary registration (express of interest) is April 10, 2004.

**Information:** Eva Vásárhelyi, e-mail: [vasar@ludens.elte.hu](mailto:vasar@ludens.elte.hu)

**1-15: Moonshine - the First Quarter Century and Beyond. A Workshop on the Moonshine Conjectures and Vertex Algebras, International Centre for Mathematical Sciences, Edinburgh UK**

**Information:**

e-mail: [icms@maths.ed.ac.uk](mailto:icms@maths.ed.ac.uk)

Web site: <http://www.ma.hw.ac.uk/icms/meetings/2004/moonshine>

[For details, see EMS Newsletter 51]

**3-10: Conference on Symplectic Topology ECM Satellite Conference Stare Jabłonki, Poland**

**Information:**

e-mail: [symp@univ.szczecin.pl](mailto:symp@univ.szczecin.pl)

Web site: <http://symp.univ.szczecin.pl>

[For details, see EMS Newsletter 51]

**15-23: EMS Conference at Będlewo (Poland). Analysis on metric measure spaces.**

**Information:**

Web site: <http://impan.gov.pl/Bedlewo/>

**26-31: 6th World Congress of the Bernoulli Society and the 67th Annual Meeting of the Institute of Mathematical Statistics, Barcelona (Spain)**

**Information:**

e-mail: [wc2004@imub.ub.es](mailto:wc2004@imub.ub.es)

Web site: <http://www.imub.ub.es/events/wc2004/>

[For details, see EMS Newsletter 49]

August 2004

**16-20 Workshop "Automorphisms of Curves", Lorentz Center, Leiden, The Netherlands.**

**Organizers:** Gunther Cornelissen and Frans Oort (Universiteit Utrecht)

**Aim:** We will discuss the latest developments in the theory of group actions on algebraic curves, a field that has been lifted to a new level of sophistication in recent years

**Topics:** deformation theory, lifting to characteristic zero, relation to moduli of curves, patching method, techniques from

fundamental groups, equivariant families, non-archimedean uniformization, and group theoretic aspects

**Keynote participants** include: A. Abbes; P. Bradley; T. Chinburg; R. Guralnick; D. Harbater; F. Kato; S. Maugeais; R. Pries; M. Raynaud; N. Stalder; S. Wewers; M. Zieve

**Information:** e-mail: [cornelis@math.uu.nl](mailto:cornelis@math.uu.nl) or [oort@math.uu.nl](mailto:oort@math.uu.nl)

Web site: <http://pascal.lc.leidenuniv.nl/>

**16-21: Bunyakovsky International Conference, Kyiv, Ukraine**

**Theme:** Honouring the bicentenary of B.Ya.Bunyakovsky (1804-1889), eminent mathematician of Tsarist Russia, born in Ukraine. The scope of the conference will include the range of the problems Bunyakovsky was interested in and made his own contribution

**Topics:** (1) mathematical analysis; (2) number theory; (3) theory of probability and its applications, actuarial mathematics; (4) mathematics education and mathematical terminology

**Languages:** English, Ukrainian, Russian

**Organizers:** Institute of Mathematics of the National Academy of Sciences of Ukraine, Ministry of Science and Education of Ukraine, National Technical University of Ukraine "KPI", Vinnytsya National Technical University, T. Shevchenko Kyiv National University, Ukrainian Mathematical Society

**Programme committee:** Y. Berezansky; O. Boholyubov; V. Vyshensky; O. Hannyushkin; M. Gorbachuk; M. Zgurovsky; V. Klochko; V. Korolyuk; V. Melnyk; B. Mokin; M. Portenko; A. Yurachkivsky; M. Yadrenko (all from Ukraine); L. Brylevskaya (St-Petersburg, Russia); E. Seneta (Sydney, Australia); R. Andrushkiw (New York, USA)

**Organising committee:** A. Samoilenco (Chairman); M. Gorbachuk and M. Swirzhevsky (Vice-Chairmen); H. Syta (Secretary); V. Boyko; Y. Vynnyshyn;

I. Yegorchenko; T. Karataeva; O. Magda; V. Ostrovskyi; N. Ryabova; S. Spichak - Institute of Mathematics of the NASU; N. Pankratova - National Technical University of Ukraine "KPI"; V. Grabko - Vinnytsya National Technical University

**Location:** Kyiv, Institute of Mathematics of National Academy of Sciences of Ukraine Tereshchenkivska Str., 3 01601, Kyiv-4, Ukraine

**Note:** From August 20 to 21, conference participants have the opportunity to take part in a two days' after-conference excursion to Bar, the native place of Viktor Bunyakovsky, and to Vinnytsya, staying the night in the City Hotel in Vinnytsya

**Deadline:** for abstracts May 15, 2004

**Information:**

Web site: <http://www.imath.kiev.ua/~syta/bunyak>

**18-21: The Thirteenth International**

**Workshop on Matrices and Statistics, in Celebration of Ingram Olkin's 80th Birthday, Będlewo, near Poznań, Poland**  
**Information:**

e-mail: matrix04@main.amu.edu.pl  
Web site: <http://matrix04.amu.edu.pl/>  
[For details, see EMS Newsletter 50]

**23-September 2: International Conference-School on Geometry and Analysis dedicated to the 75th anniversary of Academician Yu. G. Reshetnyak, Novosibirsk, Russia**

**Information:** contact Sergei Vodopyanov (Chairman of the Organizing Committee), e-mail: [angeom@math.nsc.ru](mailto:angeom@math.nsc.ru)  
Web site: <http://www.math.nsc.ru/conference/ag/2004/indengl.htm>  
[For details, see EMS Newsletter 51]

**24-27: International Conference on Nonlinear Operators, Differential Equations and Applications (ICN-ODEA-2004), Cluj-Napoca, Romania**

**Information:** e-mail: [nodeacj@math.ubbcluj.ro](mailto:nodeacj@math.ubbcluj.ro)  
Web site: <http://www.math.ubbcluj.ro/~mserban/confan.htm>  
[For details, see EMS Newsletter 49]

**30 - September 3: 7th French-Romanian Colloquium in Applied Mathematics, Craiova (Romania)**

The previous editions of this Colloquium were organized in Iasi (1992), ENS-Paris (1994), Cluj (1996), Metz (1998), Constantza (2000), and Perpignan (2002). The present edition is organized by the Department of Mathematics of the Faculty of Mathematics and Informatics, University of Craiova (Romania), under the patronage of the Institute of Statistics of the Romanian Academy, Université Pierre et Marie Curie (Paris 6), and the Société de Mathématiques Appliquées et Industrielles (SMAI).

**Scientific Committee:** D. Cioranescu (chairman, Paris 6), M. Iosifescu (chairman, Romanian Academy), V. Radulescu (chairman, Craiova), V. Barbu (Iasi), S. Basarab (Bucharest), F. Bethuel (Paris 6), H. Brezis (Paris 6), P. Deheuvels (Paris 6), J.-M. Deshouillers (Bordeaux), C. Duhamel (Paris), D. Gaspar (Timisoara), U. Herkenrath (Duisburg), I. Ionescu (Chambéry), Y. Maday (Paris 6), C. Niculescu (Craiova), A. Rascanu (Iasi), I.A. Rus (Cluj), G. Salagean (Cluj)

**Invited Speakers:** V. Bally (Paris), C. Bandle (Basel), J.-Y. Chemin (Paris), Ph. G. Ciarlet (Paris and Hong Kong), A. Damlamian (Paris), G. Dinca (Bucharest), C. Faciu (Bucharest), O. Goubet (Amiens), G. Haiman (Lille), D. Iftimie (Lyon), P. Jebelean (Timisoara), C. Le Bris (Paris), B. Miara (Noisy-le-Grand), S. Micu (Craiova), G. Nenciu (Bucharest), D. Polisevschi (Bucharest), R. Precup (Cluj), T. Ratiu (Lausanne), M. Sofonea (Perpignan), M. Théra

(Limoges)

**Local Organizing Committee:** V. Radulescu (chairman), D. Busneag, N. Constantinescu, M. Cosulschi, D. Ebanca, M. Gabroveanu, M. Ghergu, A. Matei, M. Preda, N. Tandareanu, C. Vladimirescu, I. Vladimirescu.

**Invited Sessions:**

- 1) Contrôle des systèmes gouvernés par des équations aux dérivées partielles, organized by Marius Tucsnak (Nancy I)
- 2) Homogénéisation et applications aux sciences des matériaux, organized by Horia Ene (Bucharest)
- 3) Mathématiques financières, organized by Radu Tunaru (London)
- 4) Biomécanique, organized by Marc Thiriet (Paris)
- 5) Inégalités et applications, organized by Constantin Niculescu (Craiova)

**Information:** To participate in the Colloquium it is necessary to fill out the Registration Form and send it to the Local Organizing Committee.

Abstracts of conference lectures and communications will have been published by the start of the Colloquium. The Organizing Committee plans to publish the Proceedings of the Colloquium in the local mathematical journal, Annals of the University of Craiova, Series Mathematics and Informatics. Additional information about the Colloquium may be obtained at: <http://www.inf.ucv.ro/colloque2004>

**Contacts:** M. Basarab, Laboratoire Jacques-Louis Lions, Université Paris 6, 175 rue du Chevaleret, 75013 Paris, France.

e-mail: [matei@ann.jussieu.fr](mailto:matei@ann.jussieu.fr)  
V. Radulescu, Department of Mathematics, University of Craiova, 200 585 Craiova, Romania, e-mail: [colloque@inf.ucv.ro](mailto:colloque@inf.ucv.ro)

**September 2004**

**3-5: EMS-Czech Union of mathematicians and Physicists (Mathematics Research Section). Mathematical Weekend, Prague (Czech Republic).**

**Information:** Web site: <http://mvs.jcmf.cz/emsweekend/>

**8-11: Dixièmes journées montoises d'informatique théorique, à Liège (Tenth Mons theoretical computer science days, in Liège)**

**Information:** e-mail: [M.Rigo@ulg.ac.be](mailto:M.Rigo@ulg.ac.be); Web site: [www.jm2004.ulg.ac.be](http://www.jm2004.ulg.ac.be)  
[For details, see EMS Newsletter 50]

**13 - 18: 3º CIME Course - Stochastic Geometry, Martina Franca, Taranto, Italy**

**Information:** e-mail: [cime@math.unifi.it](mailto:cime@math.unifi.it)  
Web site: <http://www.math.unifi.it/CIME>  
Address: Fondazione C.I.M.E. c/o Dipart.di Matematica "U. Dini"  
Viale Morgagni, 67/A - 50134 FIRENZE (ITALY)

Tel. +39-55-434975 / +39-55-4237111

FAX +39-55-434975 / +39-55-4222695

[For details, see EMS Newsletter 51]

**20-24: 12th French-German-Spanish Conference on Optimization Avignon, France**

**Information:**

e-mail: [alberto.seeger@univ-avignon.fr](mailto:alberto.seeger@univ-avignon.fr)  
Web site: <http://www.fgs2004.univ-avignon.fr>

[For details, see EMS Newsletter 50]

**23-26: 4th International Conference on Applied Mathematics (ICAM-4), Baia Mare, Romania (previous editions in 1998, 2000 and 2002)**

**Information:** e-mail: [mariettag@ubm.ro](mailto:mariettag@ubm.ro); [icam4@ubm.ro](mailto:icam4@ubm.ro)

Web site: <http://www.ubm.ro/site-ro/facultati/departament/manifestari/icam4/index.html>

[For details, see EMS Newsletter 49]

**26-October 1: Potential theory and related topics, Hejnice, Czech Republic**

**Aim:** to introduce progress, prospects and perspectives of the field

**Scope:** various aspects of potential theory and related fields

**Main speakers:** L. Beznea (Romania); A. O'Farrell (Northern Ireland); S. Gardiner (Northern Ireland); W. Hansen (Germany); O. Martio (Finland); M. Rockner (Germany)

**Note:** During the conference there will be a chance to congratulate our colleague Ivan Netuka on the occasion of his birthday. His works will briefly be described by Jurgen Bliedtner (Germany)

**Format:** invited and contributed talks

**Sessions:** only plenary section

**Organizers:** Technical University Liberec and other institutions

**Organizing committee:** M. Brzezina (Technical Univ. Liberec);

M. Dont (Czech Technical Univ., Prague); K. Janssen (Heinrich-Heine-Univ., Dusseldorf); J. Lukes (Charles Univ., Praha); D. Medkova (Czech Academy of Sciences, Prague); Karl-Theodor Sturm (Rheinische Friedrich-Wilhelms-Univ., Bonn); Jiri Vesely (Charles Univ., Praha)

**Proceedings:** will not be published

**Location:** Hejnice, Czech Republic

**Information:**

e-mail: [jvesely@karlin.mff.cuni.cz](mailto:jvesely@karlin.mff.cuni.cz)  
Web site: <http://www.karlin.mff.cuni.cz/PTRT04>

**29-October 2: XII Annual Congress of the Portuguese Statistical Society, Evora, Portugal**

**Theme:** all subjects in Probability and Statistics and their applications

**Aim:** to promote the exchange of ideas and the spread of recent scientific developments

**Main speakers:** L. Centeno (Portugal); D. Cox (UK); B. Efron (USA); J. Mexia

(Portugal); D. Peña (Spain); J. Branco (Portugal; tutorial on Cluster Analysis)

**Format:** invited conferences, contributed talks, tutorial

**Sessions:** plenary conferences, 3 or 4 parallel sessions, poster sessions, tutorial

**Languages:** Portuguese and English

**Call for papers:** if you wish to present a contributed presentation, please send an abstract before 15 May 2004 to the e-mail: spe2004@uevora.pt, indicating keywords and the preferred form of presentation (talk or poster). Maximum one A4 page in Word format with a 12 font and 1.5 line spacing

**Organizers:** Portuguese Statistical Society (SPE), University of Evora

**Executive and Scientific Committee:** F. Rosado (Lisboa, Congress President); C. Braumann (Evora); M. E. Graça Martins (Lisboa); P. Oliveira (Coimbra); C. D. Paulino (Lisboa)

**Local Organising Committee:** C. Braumann; R. Alpizar-Jara; F. Mendes; M. Oliveira

**Proceedings:** refereed, to be published after the Congress (assuming, as it happened in all the previous 11 Congresses, that adequate financial support will be available)

**Location:** Evora Hotel (Evora), special accommodation fares at the Hotel for participants and accompanying persons

**Social program:** dinner, excursion, other events

**Reduced registration fees:** for students and members of the Society

**Notes:** registration and accommodation forms can be downloaded from the web site

**Deadlines:** for abstracts, 15 May 2004; registration with reduced fee if sent before 15 May 2004

**Information:** e-mail: spe2004@uevora.pt  
Web site: [www.eventos.uevora.pt/spe2004](http://www.eventos.uevora.pt/spe2004)  
Address: Comissão Organizadora XII Congresso SPE, Departamento de Matemática, Universidade de Evora, Rua Romão Ramalho 59, 7000-671 EVORA, PORTUGAL Fax: +351-266745393; Phone: +351-266745370

October 2004

#### **6-9: HYKE Conference on Complex Flows, Centre de Recerca Matemática, Bellaterra, Spain**

**Aim:** The main objective of the conference is to highlight new developments of either numerical or analytical nature in kinetic and hydrodynamic equations. We would like to foster the interaction with applications, making an emphasis on two applications: granular media and astrophysical flows, with special sessions devoted to them.

**Co-ordinators:** J.A. Carrillo (ICREA-UAB); A. Marquina (Univ. de Valencia)

**Programme Committee:** A.V. Boleyev (Karlstadt Univ.); F. Bouchut (École

Normale Supérieure, Paris); D. Kroener (Univ. Freiburg); S. Noelle (RWTH Aachen); L. Pareschi (Univ. di Ferrara); M. Pulvirenti (Univ. di Roma); J. Soler (Univ. de Granada); C. Villani (École Normale Supérieure, Lyon)

**Speakers:** E. Caglioti (Univ. di Roma I, Italy); B. Despres (Univ. Paris VI, France); L. Desvilletes (ENS Cachan, France); F. Filbet (Univ. d'Orléans, France); J.A. Font (Univ. de Valencia, Spain); A. Goldshtain (Technion Haifa, Israel); L. Gosse (IAC Bari, Italy); T. Goudon (Univ. des Sciences et Technologies Lille 1, France); C. Helzel (IAM Bonn, Germany); J.M. Ibáñez (Univ. de Valencia, Spain); P.E. Jabin (ENS Paris, France); K.H. Karlsen (Univ. of Bergen, Norway); D. Levermore (Univ. of Maryland, USA); A. Mangeney (Institut de Physique du Globe de Paris, France); J.M. Martí (Univ. de Valencia, Spain); C. Mouhot (ENS Lyons, France); S. Osher (UCLA, USA); T. Poeschel (Humboldt-Universität - Charité, Germany); S. Rjasanow (Univ. Saarland, Germany); G. Russo (Univ. di

Catania, Italy); O. Sánchez (Univ. de Granada, Spain); A. Santos (Universidad de Extremadura, Spain); H.J. Schroll (Lund Univ., Sweden); S. Serna (Univ. de Valencia, Spain); B. Sjogreen (KTH Stockholm, Sweden); M. Torrillhon (ETHZ, Switzerland); G. Toscani (Univ. di Pavia, Italy); J.J.L. Velázquez (Univ. Complutense de Madrid, Spain)

**Oral Presentations and Posters:** There are 10 free time slots of 25' for oral presentations. Also, there will be a poster session throughout the whole conference. We would appreciate your registration as soon as possible and that a title of your presentation is sent no later than June 6th. We will send notification of acceptance and the type of your contribution, before June 15th.

**Deadlines:** For application for financial support, June 5, 2004. For registration and payment, June 30, 2004

**Information:**

e-mail: ComplexFlows@crm.es

Web site: <http://www.crm.es/ComplexFlows>

# Recent books

edited by Ivan Netuka and Vladimír Souček (Prague)

Books submitted for review should be sent to the following address:

Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic

**Y. A. Abramovich, C. D. Aliprantis:** *An Invitation to Operator Theory*, Graduate Studies in Mathematics, vol. 50, American Mathematical Society, Providence, 2002, 530 pp., US\$69, ISBN 0-8218-2146-6

The book is an immense graduate text devoted to the basic notions of linear operator theory in Banach spaces and Banach lattices. Basic concepts of the theory (Banach spaces, operators and functionals, Banach lattices, bases, fundamentals of measure theory and basic operator theory) are contained in preliminary chapters. The chapters that follow these are on operators on *AL*- and *AM*-spaces and special classes of operators (finite-rank operators, multiplication operators, the Fredholm and strictly singular operators) and on integral operators. The next part of the book is devoted to spectral theory (including holomorphic functional calculus and the Riesz-Schauder theory), positive matrices and irreducible operators. The last two chapters also contain some recent results, published for the first time in the book. The first of these chapters deals with the famous invariant subspace problem, while the last one presents a comprehensive study of operators satisfying the Daugavet equation. Every chapter ends with a large number of exercises and problems of various levels of difficulty. There are more than 600 exercises offering useful practise to the reader, supplementing examples and counterexamples, or extending the theory presented in the text. Complete solutions of all exercises are given in a separate companion volume "Problems in operator theory" (cf. the next review). The book is written in an understandable way and it requires only a basic knowledge of functional analysis, measure theory and topology, together with a suitable background in real analysis. It can be warmly recommended to graduate students and everybody who wishes to become acquainted with basic elements and deeper properties of linear operators on Banach spaces and Banach lattices. (jl)

**Y. A. Abramovich, C. D. Aliprantis:** *Problems in Operator Theory*, Graduate Studies in Mathematics, vol. 51, American Mathematical Society, Providence, 2002, 386 pp., US\$49, ISBN 0-8218-2147-4

This book contains complete solutions to more than 600 exercises, given in the companion volume "An invitation to operator theory" (cf. previous review) by the same authors. It can also be viewed as a supplementary text to courses in operator theory and their relatives. (jl)

**I. Agricola, T. Friedrich:** *Global Analysis: Differential Forms in Analysis, Geometry*  
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**and Physics**, Graduate Studies in Mathematics, vol. 52, American Mathematical Society, Providence, 2002, 343 pp., US\$59, ISBN 0-8218-2951-3

The book is based on courses taught at the Humboldt University for students of mathematics and physics. Topics treated in the book cover analysis, differential geometry, Lie groups and Lie algebras and main applications in mathematical physics. The main tools used throughout are differential forms and their calculus. They are introduced first (together with the Stokes theorem) on open subsets in  $R^n$ , then on submanifolds in  $R^n$ . The next essential chapter addresses Pfaffian systems. Fundamental theorems on curves and surfaces in  $R^3$  are discussed carefully using all the advantages offered by the language of differential forms. To allow the reader to deal with symmetry questions, a brief introduction to Lie groups and homogeneous spaces is given. The last three chapters use material prepared earlier in the book for a discussion of basic applications in mathematical physics – symplectic geometry and its relations to mechanics, statistical mechanics and thermodynamics, and finally electrodynamics. The book is nice to read. It is very clearly written, understandable and contains a lot of pictures. Each chapter ends with many carefully chosen exercises. The book can be useful for students as well as for teachers preparing courses on the topics covered. (vs)

**M. Aguilar, S. Gitler, C. Prieto:** *Algebraic Topology from a Homotopical Viewpoint*, Universitext, Springer, New York, 2002, 478 pp., €79,95, ISBN 0-387-95450-3

This is a basic course on algebraic topology, and I would like to stress at the beginning that it is excellently written and composed and can be strongly recommended to anybody wishing to learn the field. It is already clear from the title that the authors adopted the homotopical approach to algebraic topology. This means that the book is more topological and less algebraic. Important technical tools here are infinite symmetric products and Moore spaces. For example, homology groups are defined as homotopy groups of infinite symmetric product of the space under consideration. Eilenberg-Mac Lane spaces are infinite symmetric products of Moore spaces, and via Eilenberg-Mac Lane spaces, cohomology groups are defined. Taking the approach chosen in the book into account, the authors were well aware of the fact that they must pay special attention to computations. The reader can find many examples, calculations, and also a number of exercises. (There is a whole chapter devoted to cellular homology.) However, we do not want to create an impression that the book covers only homology and cohomology. The book deals also with K-theory and its many applications, and later on, generalized cohomology theories are treated. The authors

have also included the theory of characteristic classes of vector bundles. The book has two appendices (A: Proof of the Dold-Thom theorem, B: Proof of the Bott periodicity theorem), references consisting of 83 items and a long list of symbols. There are many remarks and comments making the orientation of the reader in the field of algebraic topology easier, and suggesting directions for further studies. The reading requires rather modest prerequisites; some basic point set topology and some basic information about groups. Let us mention also that the book had a predecessor, namely a preliminary version in Spanish. (jiva)

**J.P. Allouche, J. Shallit:** *Automatic Sequences: Theory, Applications, Generalizations*, Cambridge University Press, Cambridge, 2003, 571 pp., £37,50 ISBN 0-521-82332-3

The book can serve both as an introduction into the study of automatic sequences, and as a systematic survey of known results and applications. The topic mentioned in the title is far from being the only interesting theme of the volume. Related topics range from combinatorics on words and formal languages through number theory and formal power series to physics. The basic concept is defined in the fifth chapter using finite automata formalism. In the following chapter an equivalent characterization in terms of fixed points of morphisms is given. Properties of automatic sequences bring together in a natural way results from number theory and theoretical computer science, for example, questions of transcendence of numbers and power series. Relations between these areas are often neglected due to different conventions in language and notation. The book presents a lot of results from both fields for the first time in a unified framework. Material is presented in a clear and attractive way with motivation, exercises, open questions, and a very rich bibliography. The presentation is self-contained in a remarkable degree. The book contains a review of various areas needed such as periodicity in words, subword complexity, algebraic and transcendental numbers, numeration systems, finite automata, Turing machines or continued fractions. Famous sequences like Thue-Morse, Fibonacci or Rudin-Shapiro are also studied. The book will be of use for beginners as well as for advanced students and researchers. (sh)

**F. J. Almgren, Jr.:** *Plateau's Problem: An Invitation to Varifold Geometry*, revised edition, Student Mathematical Library, vol. 13, American Mathematical Society, Providence, 2001, 77 pp., US\$19, ISBN 0-8218-2747-2

This is a small booklet describing germs of geometric measure theory and their applications to the solution of the Plateau problem in calculus of variation. It means that we are looking for a "surface" of smallest area having a given closed curve as its boundary. The book starts with many examples of soap films illustrating an unexpected behaviour of solutions. The author then introduces differential forms and their integration over rectifiable sets. It leads in a natural way to a notion of a current (i.e., an element of the dual to the space of  $k$ -forms in a suitable topology) and it

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is an inspiration for the definition of a varifold. The last part describes a solution of the Plateau problem in the context of varifolds. The first version of the book appeared in the 60's. The new revised edition contains an expanded bibliography, reflecting the evolution of the topic. After all these years, the book is still a beautiful introduction to the Plateau problem and it is a pleasure to read. (vs)

**K.T. Arasu, A. Seress, Eds.: *Codes and Designs*, Ohio State University Mathematical Research Institute Publications 10, Walter de Gruyter, Berlin, New York, 2002, 322 pp., €128, ISBN 3-11-017396-4**

This volume contains the proceedings of a conference honouring Dijen K. Ray-Chaudhuri on the occasion of his 65th birthday held at Ohio State University in 2002. Dijen Ray-Chaudhuri is a well-known combinatorist who is most renowned for BCH-codes, which appear in every textbook on coding theory. He is also an influential teacher. The volume contains 22 contributions, written mostly by his former students, and relates to various aspects of algebraic combinatorics, coding theory, tilings, block designs and graph theory. Highlights of Dijen Ray-Chaudhuri's research are reviewed in the first contribution by A. Seress, which also includes a list of his papers. It is a nice volume honouring a very fine mathematician. (jneš)

**S. Ariki: *Representations of Quantum Algebras and Combinatorics of Young Tableaux*, University Lecture Series, vol. 26, American Mathematical Society, Providence, 2002, 158 pp., US\$31, ISBN 0-8218-3232-8**

The main part of the book is devoted to a study of representations of quantum algebras. The author uses the quantum deformation of the Kac-Moody Lie algebra  $A_k^{(1)}$  as the main working example. The first three chapters contain a brief review of concepts needed for a description of  $A_k^{(1)}$  by generators and relations. The next three chapters contain a discussion of crystal bases for integrable representations of  $A_k^{(1)}$  and a description of behavior of crystal bases under tensor products. The existence of crystal bases is proved in chapters 7, 8 and 9. The proof is based on relations between crystal bases and the Lusztig canonical bases. The next two chapters contain a discussion of the third possible approach to crystal bases, which uses a combinatorial construction based on (multi)-Young diagrams, due to Misra and Miwa. The final part of the book contains the proof of a conjecture (due to Leclerc, Lascoux and Thibon) concerning the representation theory of cyclotomic Hecke algebras of classical type. The book offers a nice introduction to combinatorics related representation theories and their applications. (vs)

**I. Bauer, F. Catanese, Y. Kawamata, T. Peternell, Y.-T. Siu, (Eds.): *Complex Geometry*, Springer, Berlin, 2002, 340 pp., €79,95, ISBN 3-540-43259-0**

The book contains a collection of papers dedicated to Hans Grauert on the occasion of his 70th birthday. The reader can also find here a

list of his publications, a long list of his doctoral students (40 altogether), together with the program of the conference organized in Göttingen on this occasion in April 2000. There are 17 research contributions on many topics from complex geometry (often related to work of H. Grauert) by W. Barth; T. Bauer and his coworkers; I. Bauer, F. Catanese and R. Pignatelli; A. Bonifant and J. Fornaess; C. Ciliberto and K. Hulek; J.P. Demailly; H. Flenner and M. Lübke; A. Huckleberry and J. Wolf; Y. Kawamata; S. Kebekus; T. Peternell and A. Sommese; K. Oguiso and De-Qi Zhang; T. Oshawa, S. Schröer and B. Siebert; Y.-T. Siu; E. Viehweg and K. Zuo, and by J. Wisniewski. (vs)

**M. C. Beltrametti, F. Catanese, C. Ciliberto, A. Lanteri, C. Pedrini (Eds.): *Algebraic Geometry*, Walter de Gruyter, Berlin, 2002, 355 pp., €148, ISBN 3-11-017180-5**

The book is based on talks given at the conference in memory of Paolo Francia, held in Genova in September 2001, and it contains invited papers dedicated to Paolo Francia. The contents cover a wide spectrum of algebraic geometry. The life and work of Paolo Francia is briefly described in the introduction, together with the list of his publications. The book contains 18 contributions, by L. Badescu and M. Schneider (Formal functions, connectivity and homogeneous spaces), L. Barbieri-Viale (On algebraic 1-motives related to Hodge cycles), A. Beauville (The Szapiro inequality for higher genus fibrations), G. Borelli (On regular surfaces of general type with  $p_g = 2$  and non-birational bicanonical map), F. Catanese and F.O. Schreyer (Canonical projections on irregular algebraic surfaces), C. Ciliberto and M. M. Lopes (On surfaces with  $p_g = 2$ ,  $q=1$  and non-birational bicanonical map), A. Conte, M. Marchisio and J. Murre (On unirationality of double covers of fixed degree and large dimension), A. Corti and M. Reid (Weighted Grasmanians), T. de Fernex, L. Ein (Resolution of indeterminacy of pairs), V. Guletskii and C. Pedrini (The Chow motive and the Gedeaux surface), Y. Kawamata (Francia's flip and derived categories), K. Konno (On the quadric hull of a canonical surface), A. Langer (A note on Bogomolov's instability and Higgs sheaves), A. Lanteri and R. Mallavibarrena (Jets of antimulticanonical bundle on Del Pezzo surfaces of degree  $< 2$ ), M. M. Lopes and R. Pardini (A survey on the bicanonical map of surfaces with  $p_g = 0$  and  $K^2 > 2$ ), F. Russo (The antibirational involutions of the plane and the classifications of real Del Pezzo surfaces), V. V. Shokurov (Letters of a bi-rationalist: IV. Geometry of log flips), A. J. Sommese, J. Verschelde and Ch. Wampler (A method for tracking singular paths with application to the numerical irreducible decomposition). In addition the book contains information on the conference consisting of a list of lectures and a list of participants. (jbu)

**N. Bourbaki: *Elements of Mathematics. Lie Groups and Lie Algebras*, Chapters 4-6, Springer, Berlin, 2002, 300 pp., €99,95, ISBN 3-540-42650-7**

The book is one of the Bourbaki volumes, translated into English. The first three chap-

ters of the book contain part of a theory needed in a study of Lie groups and Lie algebras. Chapter 4 is devoted to Coxeter groups and Tits systems. Chapter 5 contains a discussion of groups generated by reflections. Finally, Chapter 6 treats root systems, ending with their classification. (vs)

**O. Bratteli, P. Jorgensen: *Wavelets Through a Looking Glass, Applied and Numerical Harmonic Analysis*, Birkhäuser, Boston, 2002, 398 pp., €85,98, ISBN 0-8176-4280-3**

The book gives a general presentation of some recent developments in wavelet theory, with an emphasis on techniques that are both fundamental and relatively timeless, having a geometric and spectral-theoretic flavour. The exposition is clearly motivated and unfolds systematically, aided by numerous graphics. Excellent graphics show how wavelets depend on the spectra of the transfer operators. Some new results are presented for the first time (e.g., results on homotopy of multiresolutions, on approximation theory, and the spectrum of associated transfer and subdivision operators). The book is divided into six chapters. Each chapter, and some sections within chapters, open with tutorials or primer of varying length. The tutorials are written in a style that is much more informal. They are in fact meant as friendly invitations to the topics to follow, with the emphasis on friendliness. Key topics of wavelet theory are examined: connected components in the variety of wavelets, the geometry of winding numbers, the Galerkin projection method, classical functions of Weierstrass and Hurwitz and their role in describing the eigenvalue-spectrum of the transfer operator, Perron-Frobenius theory, and quadrature mirror filters. Concise background material for each chapter, open problems, exercises, bibliography, and comprehensive index, make this work a fine pedagogical and reference source. The book also describes important applications to signal processing, communications engineering, computer graphics algorithms, qubit algorithms and chaos theory, and is aimed at a broad readership of graduate students, practitioners, and researchers in applied mathematics and engineering. The book is also useful for other mathematicians with an interest in the interface between mathematics and communication theory. (knaj)

**E. Bujalance, F.-J. Cirre, J.-M. Gamboa, G. Gromadzki: *Symmetry Types of Hyperelliptic Riemann Surfaces*, Mémoires de la Société Mathématique de France, Vol 91, Société Mathématique de France, Paris, 2001, 122 pp., FRF 150, ISBN 2-85629-112-0**

The aim of the publication is to present classification of all real structures of hyper-elliptic Riemann surfaces, together with their full group of analytic and antianalytic automorphisms, their topological invariants, their description in terms of polynomials equations and explicit formulae for the corresponding real structures. A real structure  $\tau$  on a Riemann surface  $X$  is an antianalytic involution on  $X$ . A real form on  $X$  is a class of equivalence of  $\tau$  modulo conjugation by elements of the full group of analytic and antianalytic automorphisms. For example, if the curve  $X$

is defined by a set of polynomials with real coefficients, then the complex conjugation gives real structure on  $X$ . Most complex curves have no real forms but some have more than one. A number of connected components of the fixed point set of the involution  $\tau$  and connectivity of its complement in  $X$  together form an invariant, which characterizes the conjugacy class of  $\tau$ . The symmetry type of  $X$  is the (finite) set of these invariants for all real forms of  $X$ . In the first chapter, the authors introduce combinatorial methods used in the classification. The second chapter is devoted to a description of the full group of automorphisms and computation of the number of real forms. The third chapter contains a computation of symmetry types of hyper-elliptic Riemann surfaces, divided into ten different subcases. (vs)

**C.H. Clemens: *A Scrapbook of Complex Curve Theory***, Graduate Studies in Mathematics, vol. 55, American Mathematical Society, Providence, 2003, 188 pp., US\$39, ISBN 0-8218-3307-3

This is very nice book by C.H. Clemens and would make an indispensable tool for anyone willing to enter the realm of algebraic geometry. Although not written in the tight mathematical form Definition-Theorem-Proof, it covers and explains motivation for the introduction of algebraic language to problems in geometry using many suitable examples. The first chapter recalls all notions of classical geometry related to conic curves - projective space, linear system of conics, cross ratio, hyperbolic space and rational points on quadrics. The second chapter describes cubic (i.e., elliptic) curves. A unifying picture emerges by comparison of elliptic curves over complex numbers (i.e., the variation of Hodge structure) and elliptic curves over finite fields (i.e., generating function for the number of rational points over successive extensions of finite fields). The third and fourth chapters offer an introduction to elliptic, modular and theta function theory, moduli spaces and basic facts of cohomology theory - Jacobians, duality in sheaf cohomology, Abel-Jacobi map, etc. The last two chapters contain topics from geometry of higher genus Riemannian surfaces, e.g. Prym varieties, Schottky groups and hyperelliptic curves of higher genus. (ps)

**J. Cnops: *An Introduction to Dirac Operators on Manifolds***, Progress in Mathematical Physics, vol. 24, Birkhäuser, Boston, 2002, 211 pp., €72,90, ISBN 0-8176-4298-6

A Dirac operator on a Riemannian manifold  $M$  acts on sections of a bundle  $S$  of left modules over the canonical Clifford bundle  $Cl(M)$ . This construction allows for many special cases, a choice of the Dirac operator depends on a choice of the bundle  $S$ . The book is devoted to such cases, where the manifold  $M$  inherits its Riemannian structure from an embedding to a (pseudo)-Euclidean space  $R^{p,q}$ . The special case of embedded manifolds makes it possible to study various special constructions of the bundle  $S$ . The embedding of  $M$  into  $R^{p,q}$  also allows the use of the Clifford algebra  $Cl_{p,q}$  of  $R^{p,q}$  for various computations related to the normal bundle of  $M$ . The first chapter of

the book treats Clifford algebras of a given vector space with a non-degenerate scalar product. Embedded manifolds, corresponding covariant derivatives and spinor fields are introduced in the second chapter. Various versions of Dirac operators are introduced and their properties are discussed in the third chapter. A description of conformal mappings using Clifford algebra  $2 \times 2$  matrices and their relation to Dirac operators are given in Chapter 4. The next two chapters contain a discussion of the unique continuation property of solutions of a Dirac equation and their boundary values. (vs)

**P. M. Cohn: *Basic Algebra. Groups, Rings and Fields***, Springer, London, 2003, 465 pp., €64,95, ISBN 1-85233-587-4

This book, together with its sequel, "Further Algebra and Applications" (to appear), is a new and revised version of the author's famous text "Algebra (Vols. 2 and 3)", which is now out of print. Besides basics on groups, rings, modules and fields, the book contains classical Galois theory of equations, an introduction to quadratic forms and ordered fields, valuation theory, and classical commutative ring theory (including primary decomposition and Hilbert Nullstellensatz). The final chapter deals with infinite field extensions. The book contains numerous exercises, and moreover, motivation and many illuminating comments on the subject. There is no doubt that the book will take the position of its predecessor in being one of the most outstanding introductory algebra textbooks. (jtr)

**J. S. Cohen: *Computer Algebra and Symbolic Computation: Mathematical Methods***, A. K. Peters, Natick, 2003, 448 pp., US\$59, ISBN 1-56881-159-4

The book is a readable introduction to computer algebra. It presents a theoretical basis of recently published *Computer Algebra and Symbolic Computation: Elementary Algorithms*, of the same author. The book explores applications of algorithms to automatic simplification, greatest common divisor calculation, resultant computation, polynomial decomposition, and factorisation. After a review of basic background concepts, algorithms for manipulation and evaluation of numerical objects are given. Automatic algebraic and trigonometric simplification is worked out thoroughly and it gives the reader rather detailed information about one of the essential algorithms implemented in computer algebra software. Another of the important numerical objects - single variable polynomials - is discussed in two chapters. Attention is paid in particular to the algorithm for polynomial division, which is the basis to the algorithm for polynomial expansion, Euclidean algorithm, algorithm for partial fraction expansion, and algorithm for performing arithmetic operations for expressions in simple algebraic number fields. This is followed by a generalization of these concepts and algorithms to multivariate polynomials. Two important concepts of computer algebra, the resultant and Gröbner basis, are introduced. The book culminates in the last chapter, devoted to polynomial factorisation and a modern algorithm for it. The book will be use-

ful for undergraduate students of mathematics and computer science. The text is also accessible to a more general audience interested in computer algebra and its applications. (mer)

**E. von Collani (Ed.): *Defining the Science of Stochastics***, Sigma Series in Stochastics, Vol. 1, Heldermann Verlag, Lemgo, 2004, 260 pp., €30, ISBN 3-88538-301-2

Although the words "stochastic" and "stochastics" are frequently used, their exact meaning may not be generally known. The Greek noun *stochos* means target, aim, or guess. *Stochastikos* is a person who is skilful in predicting. From this point of view, the title 'Ars conjectandi' of the famous book by Jacob Bernoulli has the Greek equivalent 'στοχαστική τέχνη' (stochastic techniques). Formerly, the word "stochastic" was used mainly in the phrase "stochastic process". Today one feels that "stochastic" means more than only "random". The book contains contributions devoted to the problem of how to define the science of stochastics. The articles are revised and updated versions of lectures delivered at the symposium "Defining the Science of Stochastics", which took place in Germany in October 2000. The publication is divided into three parts, each one having three papers. The first part describes the present state of statistics and probability theory in comparison to the science of stochastics. The second part deals with randomness, contingency, and white noise. The third part is devoted to stochastics in public education. The authors of individual contributions are S. Kotz, U. Herkenrath, V. Kalashnikov, J. Kohlas, C. S. Calude, T. Hida, M. Dumitrescu, and E. von Collani (2x). The authors emphasize history and nature of statistical science and propose to establish "Stochastics" as an independent, mathematically based exact science, which investigates the inherent variability exhibited by any real phenomenon. A great variety of topics in the published papers, reflects the fast growing diversification of statistical sciences. It might be that the reader finds some of them to be too far from his/her specialization, but everybody will find some contributions interesting and stimulating. In Biographical Notes, on p. 215, one reads that Thomas Bayes lived 1702-1761. According to the inscription on his tomb in Bunhill Field cemetery in London, he died on April 7, 1761, at age of 59. Some authors introduce that he was probably born in 1701. (ja)

**J.-M. Delort: *Global Solutions for Small Nonlinear Long Range Perturbations of Two Dimensional Schrödinger Equations***, Mémoires de la Société Mathématique de France 91, Société Mathématique de France, Paris, 2002, 94 pp., €23, ISBN 2-85629-125-2

In the book, a global existence of solutions to the two dimensional Schrödinger equations is proved. The equation allows for a quadratic nonlinearity containing the space derivatives. The initial data are small. The setting is 'critical' in the sense that the nonlinearity becomes non-integrable when applied to the fundamental solution of the Schrödinger operator. To overcome this difficulty, a suitably chosen function is subtracted from the solution to cancel out the 'worst' terms. The main technique

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of the book is classical harmonic analysis. Symbolic calculus, dyadic decomposition, carefully developed linear theory, together with some nonlinear estimates (e.g., for products) are the keystones of the final success, and the solution is found in a suitable weighted space. The book is rather technical but is mostly self-contained. (dpr)

**Y. Eliashberg, N. Mishachev: *Introduction to the h-Principle*, Graduate Studies in Mathematics, vol. 48, American Mathematical Society, Providence, 2002, 206 pp., US\$30, ISBN 0-8218-3227-1**

The Gromov “h-principle” was introduced in the 80’s, in Gromov’s book on partial differential relations. This is really a principle, not a theorem, and it should be adapted to every new application. It describes the behaviour of a system of partial differential equations or inequalities having a big space of solutions (e.g., dense in the space of functions or fields considered). It was inspired by work on the  $C^1$ -isometric embedding theory (Nash and Kuiper) and the immersion theory in differential topology (Smale and Hirsch). The Gromov book is not easy to understand. The book under review offers the reader a nice and understandable treatment of two methods based on the “h-principle” – the holonomic approximation methods and convex integration theory (which was treated in a more general situation in the book by D. Spring, published in 1998). These two methods cover many interesting applications. The book contains a discussion of applications of the holonomic approximation method in symplectic and contact geometry, including a review of basic notions needed for the applications. The book is very interesting, readable and should be of interest to more than just geometers. (vs)

**F. Fauvet, C. Mitschi, Eds.: *From Combinatorics to Dynamical Systems, IRMA Lectures in Mathematics and Theoretical Physics 3*, Walter de Gruyter, Berlin, 2003, 241 pp., €36,95, ISBN 3-11-017875-3**

This is the proceedings of the conference “Journées de calcul formel en l’honneur de Jean Thomann”, which took place at IRMA (Institut de Recherche Mathématique Avancée) in Strasbourg in March 2003. The meeting was devoted to dynamical systems, computer algebra, and theoretical physics. The first paper, by M. Espie, J.-C. Novelli and G. Racinet, contains computations of certain graded Lie algebras related to multiple zeta values. The article by E. Corel gives an algebraic and algorithmic treatment of formal exponents at an irregular singularity. The article by J. L. Martins studies irregular meromorphic connections in higher dimensions. The paper by M. A. Barkatou, F. Chyzak and M. Loday-Richaud describes various algorithms for the rank reduction of differential systems. The article by M. Canalis-Durand presents a “pattern recognition algorithm” for Gevrey power series. The article by D. Boucher and J.-A. Weil gives a new proof of the non-integrability of the planar three-body problem. The paper by L. Breinig uses the Quasi Polynomial formalism to obtain effective integrability for some dynamical systems. The article by R. Conte, M. Musete and T.-L.

Yee, presents an example of an analytical trajectory in a chaotic system. The last article, by J. Della Dora and M. Mirica-Ruse, presents formalization for the concept of a hybrid system. (pkur)

**T. Forster: *Logic, Induction and Sets*, London Mathematical Society Student Texts 56, Cambridge University Press, Cambridge, 2003, 234 pp., £18,95, ISBN 0-521-53361-9, ISBN 0-521-82621-7**

The book is an introduction to mathematical logic and set theory. It contains everything that is expected from such a book - language, syntax, semantics, model, deducibility, completeness theorem, normal form theorem, recursion and axioms of ZFC. The style of the book differs from similar textbooks. The central notion in the book is a recursive data type, which means a set defined by a recursion. This notion is introduced early, hence some knowledge of set theory is tacitly assumed at the very beginning (and it also induces some circular arguments at the end). The whole presentation is not as rigorous as could be expected in textbooks of this type. On the other hand, the book contains a lot of philosophical explanations pointing to aspects usually ignored. Another interesting point is that the set of exercises contains some, which are rare to meet, for example: “Dress up the traditional proof that  $\sqrt{2}$  is irrational into a proof by well-founded induction on  $N \times N$ .” Once you are familiar with a classical textbook on the topics, reading this one as a second choice is fun. (psi)

**R. Garnier, J. Taylor: *Discrete Mathematics for New Technology*, second edition, Institute of Physics Publishing, Bristol, 2002, 749 pp., £25, ISBN 0-7503-0652-1**

The book is designed for students of computer science. It contains main mathematical topics needed in their undergraduate study. The main core of mathematics contained in the book belongs to standard material needed for any student of mathematics. The choice of topics was inspired by a first-year mathematics course for undergraduate students of computer science at the University of Nottingham. The reader can find here sections on logic, mathematical proofs, sets, relations, relational databases, functions and functional dependence, matrix algebra and systems of linear equations, basic algebraic structures, Boolean algebra and graph theory and its applications. The book also contains applications (relational databases, normal form of databases). In the second edition, the authors added a lot of new exercises and examples, illustrating discussed concepts. The book contains a lot of well-ordered and nicely illustrated material. (vs)

**J. C. Gower, G. B. Dijksterhuis: *Procrustes Problems*, Oxford Statistical Science Series 30, Oxford University Press, Oxford, 2004, 233 pp., £55, ISBN 0-19-851058-6**

Let  $\mathbf{X}_1$  and  $\mathbf{X}_2$  be given matrices. The simplest Procrustes problem is to find a matrix  $\mathbf{T}$  that minimizes  $\|\mathbf{X}_1\mathbf{T} - \mathbf{X}_2\|$ . In analogy with the story of Procrustes, son of Poseidon, we can imagine that  $\mathbf{X}_1$  is the unfortunate traveller,  $\mathbf{X}_2$  is the Procrustean bed, and  $\mathbf{T}$  is the treatment

(racking, hammering, or amputation). Under classical conditions, minimization of  $\|\mathbf{X}_1\mathbf{T} - \mathbf{X}_2\|$  is the multivariate multiple regression problem with the solution  $\mathbf{T} = (\mathbf{X}_1'\mathbf{X}_1)^{-1}\mathbf{X}_1'\mathbf{X}_2$ . The two-sided form of the Procrustes problem is to minimize  $\|\mathbf{X}_1\mathbf{T}_1 - \mathbf{X}_2\mathbf{T}_2\|$  under some conditions. A further variant consists of the double Procrustes problem concerning minimization of  $\|\mathbf{T}_2\mathbf{X}_1\mathbf{T}_1 - \mathbf{X}_2\|$ . The solutions of the mentioned problems depend on the set of matrices  $\mathbf{T}$ , which are allowed to be used. In some cases  $\mathbf{T}$  is constrained to be an orthogonal matrix, which represents a generalized rotation, in other problems  $\mathbf{T}$  must be a permutation matrix and so on. Generally we can say that Procrustean methods are used to transform one set of data to represent another set of data as closely as possible. The book also contains oblique Procrustes problems, weighting, scaling, and missing values. Further topics are accuracy and stability, and some applications. The authors inform that the process of writing the book was similar to the machinations of Procrustes, the material seemed to be continuously stretching and so it was necessary to chop things. However, this is probably the fate of many mathematical papers and books. (ja)

**F. von Haeseler: *Automatic Sequences*, de Gruyter Expositions in Mathematics 36, Walter de Gruyter, Berlin, 2003, 191 pp., €78,50, ISBN 3-11-015629-6**

Automatic sequences can be equivalently defined in terms of finite automata (they are produced by them), substitutions (they are their fixed points), or by their kernel (which is finite). The book studies a generalized concept of automatic sequences, where the sequence is indexed by elements of a finitely generated group, instead of natural numbers. Accordingly, ordinary substitutions are replaced by mappings based on expanding group endomorphisms, and a corresponding notion analogous to finite automata is developed. These concepts are introduced and discussed in the first three chapters. Chapter 4 studies automatic subsets and automatic maps. The final chapter combines automatic sequences with some additional algebraic structures, and deals especially with Mahler equations over the ring of formal power series. The book is written quite succinctly, and is not very easy to read. A certain degree of mathematical maturity is required just to grasp the basic definition of the automatic sequence. The volume will be useful mainly for advanced researchers in the field, interested in broadening their view of the subject. (sh)

**I. Chajda, G. Eigenthaler, H. Länger: *Congruence Classes in Universal Algebra*, Research and Exposition in Mathematics, vol. 26, Heldermann, Lemgo, 2003, 217 pp., €28, ISBN 3-88538-226-1**

The book explores a number of different features of congruence on universal algebras. It covers the subject from the early works of Maltsev to recent results. One can find a good amount of material concerning properties of blocks (in general and in varieties), relationship to quotients and subalgebras or local properties of congruence. The first two chapters explain basic notions and give quite a few

examples of algebras. The study of congruence starts with the classical notions of permutability, distributivity, modularity and direct decomposability. Further chapters deal with the role of subalgebras, single blocks, constant terms or ideals. Special chapters are devoted to extension properties of congruence, regular and coherent algebras, local conditions and one-block congruence. (eb)

**P. Imkeller, J.-S. von Storch, Eds.: *Stochastic Climate Models, Progress in Probability*, vol. 149, Birkhäuser, Basel, 2001, 398 pp., DM 196, ISBN 3-7643-6520-X**  
 The proceedings of the workshop on Stochastic Climate Models, held in Chorin in 1999, present an exiting and stimulating summary of probabilistic developments in climate physics. Stochastic processes, stochastic and partial differential equations, random dynamical systems, local and large deviations asymptotic procedures, have comprised the most active mathematical areas in the field over the past 30 years, since Klaus Hasselmann suggested a climate stochastic model where weather fluctuations randomly force the climate in the same way that fluid molecules force Brownian pollen particles. Hence the Langevin stochastic equation and stochastic calculus, enter the model in a very natural way. The structure of the book reflects its aim to review and explain recent mathematical additions to the list of tools for climate modelling, to help the climate physicists to understand what these tools are about. Chapter 1 by D. Olbers, K. Fraedrich, J. S. von Storch and R. Temam exhibits the most important climate models, from those simple ones to more complex ones. Chapter 2 by L. Arnolds, M. Denker, M. Kesseböhmer, Y. Kiefer, Ch. Rödenbeck, Ch. Beck and H. Kantz, searches for possible sources of stochasticity in the models from the mathematical as well as physical point of view. Chapter 3 by P. Imkeller, J. Duan, P. E. Kloeden, B. Schmalfuss, J. Zabzyk and P. Müller, is a very informative compendium of probabilistic tools mentioned above that are already at work in the climate dynamics. Chapter 4 by J. Egger, J.A. Freund, A. Neiman, L. Schimanski-Geier, A.H. Monahan, L. Pandolfo, P. Imkeller, P. Sardeshmukh, P. Penland, M. Newman and W.A. Woyczyński, completes the presentation of climate mathematics with a discussion of some of the more special models for the fluctuating eastward flows or planetary waves. The book is strongly recommended as an excellent source of information and inspiration, both to mathematicians and physicists interested in the field. (jste)

**P.-Y. Jeanne: *Optique géométrique pour des systèmes semi-linéaires avec invariance de jauge*, Mémoires de la Société Mathématique de France, Société Mathématique de France, Paris, 2002, 160 pp., €23, ISBN 2-85629-123-6**

The main topic of the book is a study of the properties of solutions of field equations for a scalar field or a spinor field coupled to a Yang-Mills field, and construction of approximate solutions of a special shape. First, in the 60's, solutions that rapidly oscillated in high frequencies were found by Y. Choquet-

Bruhat, using a modification of WKB methods for solutions of linear partial differential equations. Meanwhile, justification for ideas coming from geometric optics was developed for solutions of certain semi-linear or quasi-linear equations. The first part of the book contains an exposition of a structure of the equations. The second part describes families of approximate solutions (to any order), in the shape of single-phase expansions, rapidly oscillating at high frequency. The third part contains a description of exact solutions, asymptotic to previous expansions. (vs)

**J.-M. Kantor (Ed.): *Ou en sont les mathématiques?*, Société Mathématique de France, Paris, 2002, 440 pp., ISBN 2-7117-8994-2**

The Société Mathématique de France has published the journal *Gazette des mathématiciens* since the 70's. Its aim is a diffusion of mathematical knowledge. The book under review brings a sample of papers from the journal since its foundation. Papers are complemented by comments written by a contemporary specialist, describing recent evolution in the field. As a whole, it is a fascinating read. Papers (written in French) describe eighteen different topics of enormous interest, reviewed in a readable and understandable way, and designed for a general audience. The topics include classical themes (the third and the fifteenth Hilbert problems, theory of invariants, arithmetic algebraic geometry), important mathematical topics (stochastic calculus, deformation of various mathematical structures, symplectic geometry and topology, the work of M. Gromov, convex polytopes, singularities of maps, error correcting codes), very modern themes (mathematical ideas coming from quantum field theory, quantum groups and invariants of nodes, Wiles' proof of the Taniyama-Weil conjecture), as well as some applications (image recognition, mathematics in meteorology, topology in molecular biology). It is a very nice collection, which can contribute in an essential way to the general mathematical education of any reader. (vs)

**A. Katok: *Combinatorial Constructions in Ergodic Theory and Dynamics*, University Lecture Series, vol. 30, American Mathematical Society, Providence, 2003, 121 pp., US\$29, ISBN 0-8218-3496-7**

The book is an updated version of the unpublished notes "Constructions in ergodic theory", written by the author in collaboration with E.A. Robinson Jr., in 1982-83. The first part entitled "Approximations and genericness in ergodic theory", deals with the approximations of measure-preserving transformations by periodic processes, which are permutations of partitions of the space. Several types of such periodic processes are considered and the speed of approximation is evaluated. The main theorem says that the set of measure-preserving transformations of a Lebesgue space, which admit a periodic approximation of a given type and speed, is residual. The second part entitled "Cocycles, cohomology and combinatorial constructions", deals with algebraic constructions in ergodic theory. The concepts of rigidity, stability and effectiveness are investigated. Main applications include Diophantine translations of the torus, Anosov

diffeomorphisms, horocycle flows and interval exchange transformations. (pkur)

**B. Lapeyre, É. Pardoux, R. Sentis: *Introduction to Monte-Carlo Methods for Transport and Diffusion Equations*, Oxford Texts in Applied and Engineering Mathematics, Oxford University Press, Oxford, 2003, 163 pp., £60, ISBN 0-19-852592-3, ISBN 0-19-852593-1**

The book is devoted to the applications of Monte Carlo methods (i.e., simulation techniques based on random numbers), for solutions of partial differential equations. A probabilistic representation of solutions of these equations is given in the following way: the transport or diffusion type equations are interpreted as the Fokker-Planck equations associated with Markov processes. Transport equations in particle physics, the nonlinear Boltzmann transport equation, and the links between second order partial differential diffusion equations and Brownian motion, are all investigated. For each type of problem, limits of methods are discussed and specific techniques used in practice are described. An essential point is that even if Monte-Carlo methods need not converge, it is always possible to control the reliability of the result using an inexpensive additional calculation. The reader should have a good background in mathematical analysis. (jste)

**W. K. Li W.: *Diagnostic Checks in Time Series*, Monographs on Statistics and Applied Probability 102, Chapman & Hall/CRC, Boca Raton, London, 2004, 216 pp., US\$69,95, ISBN 1-58488-337-5**

There are many books on time series analysis but this is the first monograph specialized to diagnostic checking. Construction of a model for time series data usually consists of three steps. At the beginning, a preliminary model is chosen. Then the parameters are estimated. The third stage is called model diagnostic checking. It involves techniques like residual plots and procedures for testing if the residuals are approximately uncorrelated. If it is found that the model is not adequate, the process starts with the first step again, this time with some new information from the previous analysis. The book describes a rich variety of diagnostic checks. It starts with the univariate and multivariate linear models. Although attention is paid mainly to ARMA models, the author also writes about periodic autoregression and Granger causality tests. A chapter is devoted to robust modelling and diagnostic checking, from a robust portmanteau test to the trimmed portmanteau statistic. An important part of the book describes results on nonlinear models. It contains goodness-of-fit tests, tests for general nonlinear structure, tests for linear vs. specific nonlinear time series, and methods for choosing two different families of nonlinear models. Then the models for conditional heteroscedasticity are presented. Finally, fractionally differenced processes are described and a few special topics are mentioned like non-Gaussian time series and power transformations. Unit root and co-integration tests are not included. The author is a known specialist in time series modelling. His approach is a practical one and

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each topic is presented from a model builder's point of view. A long list of references is also appreciated as an important part of the publication. The monograph is very useful for statisticians working in time series analysis. (ja)

**S. Markvorsen, M. Min-Oo:** *Global Riemannian Geometry: Curvature and Topology*, Advanced Courses in Mathematics CRM Barcelona, Birkhäuser, Basel, 2003, 87 pp., €24, ISBN 3-7643-2170-9

The small booklet contains lecture notes from two advanced courses. The first one was written by S. Markvorsen (Distance geometric analysis on manifolds). It contains a comparison theory for distance functions on immersed submanifolds in Riemannian manifolds and its relations to seemingly unrelated topics. These include isoperimetric inequalities, diffusion processes (mean exit time comparison), transience, warped products, and all that related to the Laplacian. The second lecture notes are written by M. Min-Oo (The Dirac operator in geometry and physics). The first part describes traditional results on the Dirac operator, its index formula and the Lichnerowicz formula. The second part treats the Gromov notion of K-area and the corresponding fundamental K-area inequality, while the third part discusses various aspects of the famous positive mass theorem. Both reviews are very useful for the orientation of the reader in the field, details should be found in the corresponding literature. (vs)

**P. Monk:** *Finite Element Methods for Maxwell's Equations*, Numerical Mathematics and Scientific Computation, Clarendon Press, Oxford, 2003, 450 pp., £59,95, ISBN 0-19-850888-3

The aim of this book is to provide an up-to-date and sound theoretical foundation for finite element methods in computational electromagnetism. The emphasis is on finite element methods for scattering problems that involve the solution of Maxwell's equations on infinite domains. Suitable variational formulations are developed and justified mathematically. The main focus of the book is an error analysis of edge finite element methods that are particularly well suited to Maxwell's equations. The methods are justified for Lipschitz polyhedral domains that can cause strong singularities in the solution. The book is divided into 14 chapters. After the preparatory work in Chapters 2 and 3 (functional analysis and abstract error estimates, Sobolev spaces and vector function spaces), Chapter 4 contains a discussion of a simple model problem for Maxwell's equations. Chapters 5 and 6 form a central part of the book. They present a detailed description of the original Nédélec finite element spaces. In Chapter 7, the author gives the finite element discretization of the cavity problem and presents two convergence proofs for this method. Chapter 8 presents basic topics concerning finite elements and gives a brief introduction to  $hp$  finite element methods for Maxwell's equations. A central task of computational electromagnetism is the approximation of scattering problems. The presentation of these problems starts in Chapter 9, with classical scattering by a sphere, where the author derives the famous

integral representation of the solution to Maxwell's equations called the Stratton-Chu formula. In addition, the author derives classical series representations of the solution of Maxwell's equations. These are used in Chapter 10 to derive a semi-discrete method for the scattering problem utilizing the electromagnetic equivalent of the Dirichlet to Neumann map. A fully discrete domain-decomposed version of this algorithm is proposed and analysed in Chapter 11. In Chapter 12, the author studies a coupled integral equation and finite element method due to Hazard, Lenoir and Cutzach. Chapter 13 is devoted to a discussion of some issues related to practical aspects of solving Maxwell's equations. The final Chapter gives a short introduction to inverse problems in electromagnetism. The book will be of interest to mathematicians, physicists and researchers concerned with Maxwell's equations. The three chapters, 4, 5 and 7, could be useful in a graduate course on finite element methods. (knaj)

**S. Montgomery, H.-J. Schneider:** *New Directions in Hopf Algebras*, Mathematical Sciences Research Institute Publications 43, Cambridge University Press, Cambridge, 2002, 485 pp., £55, ISBN 0-521-81512-6

Hopf algebras are abstractions of algebras of functions on a group. As such, they were introduced to topology in the first half of the last century. Roughly speaking, Hopf algebra is an associative algebra that also carries a "comultiplication", and these two structures are compatible in an appropriate sense. The second advent of Hopf algebras was marked by a seminal talk on quantum groups, presented by V. Drinfel'd in Berkeley in 1986. Since then, Hopf algebras have found a great number of applications, not only in the theory of quantum groups (which are particular types of Hopf algebras) but also in low-dimensional topology, algebra and geometry. The book under review is a collection of expository articles that attempt to summarize the recent developments and offer a new understanding of classical topics. The contributions were written by N. Andruskiewitsch, S. Gelaki, G. Letzler, A. Masuoka, D. Nikshych, F. van Oystaeyen, D. Radford, P. Schauenburg, H.-J. Schneider, M. Takeuchi, L. Vainerman and Y. Zhang. Most of the authors were participants of the Hopf Algebras Workshop held at MSRI as a part of the 1999-2000 Year on Noncommutative Algebra. (mm)

**P.T. Nagy, K. Strambach:** *Loops in Group Theory and Lie Theory*, de Gruyter Expositions in Mathematics 35, Walter de Gruyter, Berlin, 2002, 361 pp., US\$148, ISBN 3-11-017010-8

The book is a comprehensive treatment on various topological and geometrical aspects of the theory of loops (i.e., generally non-associative binary structures with neutral elements and unique division). The loops are represented as stable transversals (alias sections) in their one-sided multiplication groups, and these are equipped with additional structure. Thus topological, differentiable, algebraic loops are treated via Lie groups, algebraic groups, symmetric spaces, etc. The book is divided into two parts. The first one presents

basic material, with special attention paid to Bol loops and Bruck loops. The second part is devoted to smooth loops on low dimensional manifolds. (tk)

**V. Paulsen:** *Completely Bounded Maps and Operator Algebras*, Cambridge Studies in Advanced Mathematics 78, Cambridge University Press, Cambridge, 2003, 300 pp., £47,50, ISBN 0-521-81669-6

This book is intended as an (advanced) introductory text on the main results and ideas in some of the major topics of modern operator theory. The first half of the book is devoted to the study of completely positive and completely bounded maps between  $C^*$ -algebras and their connections with the dilation theory. The adverb completely is related to a collection of matrix norms on the algebra. The second part explains connections with various types of similarity problems (e.g. the Kadison conjecture and Pisier's theory), and properties of operator systems (e.g., characterization of existence of isometric representations of operator algebras - Blecher, Ruan, Sinclair theory, which is applied to new proofs of classical results on interpolation of analytic functions). Another application is Pisier's theory of the universal operator algebra of an operator space and his results on similarity and factorization degree. The book is carefully written, proofs are often accompanied with notes helping to explain the situation. Several theorems are proved by various methods, e.g., there are five different proofs of the von Neumann inequality. The text can be used either for a graduate course in operator theory or for an independent study, since it contains 205 exercises. The author recommends that the reader be acquainted with R. G. Douglas' book "Banach Algebra Techniques in Operator Theory". (jmil)

**R. Plato:** *Concise Numerical Mathematics*, Graduate Studies in Mathematics, vol. 57, American Mathematical Society, Providence, 2003, 453 pp., US\$85, ISBN 0-8218-2953-X

This excellent textbook is a concise introduction to the fundamental concepts and methods of numerical mathematics. The author covers many important topics using well-chosen examples and exercises. Many of the presented methods are illustrated by figures. For numerous algorithms, practical and relevant work estimates are given and pseudo codes are provided that can be used for implementations. The author presents approximately 120 exercises with different levels of difficulty. Topics covered include interpolation, the fast Fourier transform, iterative methods for solving systems of linear and nonlinear equations, numerical methods for solving ODE's, numerical methods for matrix eigenvalue problems, approximation theory, and computer arithmetic. The book is suitable as a text for a first course in numerical methods, for mathematics students or students in neighbouring fields, such as engineering, physics, and computer science. In general, the author assumes knowledge of calculus and linear algebra only. (knaj)

**I. Reiner:** *Maximal Orders*, London Mathematical Society Monographs 28,

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Clarendon Press, Oxford, 2003, 395 pp., £70, ISBN 0-19-852675-3

Let  $R$  be a noetherian integral domain with quotient field  $K$ , and let  $A$  be a finite dimensional  $K$ -algebra. Recall that an  $R$ -order in the  $K$ -algebra  $A$ , is a sub-ring  $\Lambda$  of  $A$ , having the same unit element as  $A$ , and such that  $K \cdot \Lambda = \{ \sum \alpha_i m_i \mid \text{finite sum}, \alpha_i \in K, m_i \in \Lambda \} = A$ . A maximal  $R$ -order in  $A$  is defined in the usual way as such an  $R$ -order that is not properly contained in any other  $R$ -order in  $A$ . After algebraic preliminaries, basic properties of orders are investigated in Chapter 2. Maximal orders in skew-fields, over discrete valuation rings and over Dedekind domains, are studied in Chapters 3, 5 and 6, respectively. Chapter 3 is devoted to Morita equivalence and Chapter 7 deals with crossed-product algebras. The last two parts investigate simple algebras over global fields and a local and global theory of hereditary orders. (lbi)

**A. Shen, N.K. Vereshchagin:** *Computable Functions*, Student Mathematics Library, vol. 19, American Mathematical Society, Providence, 2002, 166 pp., US\$29, ISBN 0-8218-2732-4

The book is based on lectures for undergraduate students at the Moscow State University Mathematics Department and the content covers a majority of basic notions of general theory of computation, excluding computational complexity. It begins with a definition of computable functions, based on an informal definition of algorithms. Then, the classic notions of topics such as decidability, universal functions, fixed point theorem, enumerable sets, oracle computations and arithmetical hierarchy, are presented, clarified and developed. A precise definition of an algorithm is given in the ninth chapter (Turing machines); the word problem is treated here as well. Chapter 10 (Arithmeticity of computable function), develops the subject on the mentioned precise base; Tarski's theorem and Gödels' undecidability theorem are presented. The last chapter is devoted to recursive functions. The book can appropriately provide knowledge of the theme to programmers and to students of mathematics and computer science. (jml?)

**W. Sieg, R. Sommer, C. Talcott, Eds.:** *Reflections on the Foundations of Mathematics. Essays in Honour of Solomon Feferman*, Lecture Notes in Logic 15, A.K. Peters, Natick, 2002, 444 pp., US\$45, ISBN 1-56881-170-5

The book consists of contributions submitted mostly by participants of the symposium on the foundations of mathematics, organized to honour Solomon Feferman, which was held at Stanford University, in December 1998. The volume is organized into four parts: Proof-theoretic analysis, Logic and computation, Applicative and self-applicative theories, and Philosophy of modern mathematical and logical thought. For example, in the first part we can find contributions concerning ordinal analysis (such as Avigad's and Buchholz's contributions), Simpson's paper that treats predicative analysis, where Feferman's system IR is compared with Friedman's ATR<sub>0</sub>, and Friedman's article on combinatorial finite trees. To demonstrate the variety of themes,

let us mention the Fenstad contribution "Computability theory: structure and algorithms", Rathjen's "Explicit mathematics with monotone inductive definitions. A survey", and Mancosu's "On the constructivity of proofs. A debate among Behmann, Bernays, Gödel, and Kaufmann." (jmle)

**Y. Talpaert:** *Differential Geometry with Applications to Mechanics and Physics*, Monographs and Textbooks in Pure and Applied Mathematics, vol. 237, Marcel Dekker, Inc., New York, 2001, 454 pp., US\$185, ISBN 0-8247-0385-5

The book contains a careful exposition of basic facts concerning manifolds, differential forms and their integration, Stokes theorem, exterior differential systems, Lie derivatives and basic facts on Lie groups and Lie algebras, Riemannian geometry and their use in physics. Comparing the new English version with the original French edition, the last three chapters were substantially reworked and extended (these are chapters on Riemannian geometry, Lagrange and Hamilton mechanics, and symplectic geometry). The book is written in a very understandable and systematic way, with a lot of figures. A very good feature of the book is a collection of more than 130 exercises and problems, which are completely solved, resp. answered. The book can be recommended for a wide range of students as a first book to read on the subject. It can be also useful for the preparation of courses on the topic. (vs)

**L. Vainerman, Ed.:** *Locally Compact Quantum Groups and Groupoids*, IRMA Lectures in Mathematics and Theoretical Physics 2, Walter de Gruyter, Berlin, 2003, 247 pp., €34,53, ISBN 3-11-017690-4

These are the proceedings of the workshop "Quantum Groups, Hopf Algebras and their Applications", which was held in Strasbourg in February 2002. They contain an "Introduction of the editor" and 7 original articles. All of them represent substantial contributions to the theory of quantum groups and groupoids, and every specialist in the field should be familiar with them. The introduction describes the development of the theory of quantum groups and groupoids in a very concise and deep way, so that even a mathematician who is not a specialist on this topic can adequately understand why the theory has proceeded in this way or that. For the non-specialist, let us also mention that the paper by S. Vaes and L. Vainerman (On low dimensional locally compact quantum groups) contains Preliminaries, where we can find basic information about these groups. This paper represents the continuation of the research of both authors on extensions of locally compact quantum groups. J. Kustermans and E. Koelink (Quantum SUq(1,1) and its Pontryagin dual), present an overview of the quantum group SUq(1,1) and study its Pontryagin dual. A. Van Daele, in his paper (Multiplier Hopf\*-algebras with positive integrals: A laboratory for locally compact quantum groups), gives a survey of the theory of algebraic quantum groups and their relations with locally compact quantum groups. The remaining four papers deal with quantum

groupoids (M. Enock: Quantum groupoids and pseudo-multiplicative unitaries; P. Schauenberg: Morita base change in quantum groupoids; K. Szlachányi: Galois actions by finite quantum groupoids; J.-M. Vallin: Multiplicative partial isometries and finite quantum groupoids). (jiva)

**S. H. Weintraub:** *Representation Theory of Finite Groups: Algebra and Arithmetic*, Graduate Studies in Mathematics, vol. 59, American Mathematical Society, Providence, 2003, 212 pp., US\$45, ISBN 0-8218-3222-0

The book is an introduction to the group representation theory with emphasis on finite groups. Since group representations are just modules over group algebras, the classical results on semisimple group representations are naturally obtained in Chapter 3, as particular cases of more general results concerning the structure of modules over semisimple rings. The latter results are presented in Chapter 2. Chapter 4 deals with induced representations and culminates in the proof of Brauer's theorem (concerning irreducible complex representations of  $G$  of exponent  $n$  being defined over  $\mathbb{Q}$  ( $\sqrt[n]{1}$ )). The remaining three chapters provide an introduction to modular representation theory, again by applying the powerful module theoretic approach. The book is well written, and contains many examples worked out in detail (for example, determining all irreducible complex and modular representations of  $A_5$ ). It is a suitable text for a year long graduate course on the subject. (jtrl)

**V. I. Zhukovskiy:** *Lyapunov Functions in Differential Games, Stability and Control: Theory, Methods and Applications*, vol. 19, Taylor & Francis, London, 2003, 281 pp., £70, ISBN 0-415-27341-2

The book is devoted to the linear theory of differential games under uncertainty. It is supposed that nothing is known on their nature (i.e., the stochastic approach is ruled out). The presence of uncertainties moves the problem of the choice of strategy to the field of "multi-criteria" dynamical problems. The results are mainly given for quadratic payoff functions, two players and non-cooperative games. The book is divided into two parts. The first one describes the foundations of differential games under uncertainties. The central role is played by the notion of a vector guarantee. Two approaches, based on the analogue of the vector saddle point and the vector maximin, are used for solving the multicriteria dynamical problems. The second part is devoted to the concept of equilibrium of objections and counter-objections, as well on the active equilibrium. The reader should have a basic knowledge of ordinary differential equations and optimization. For example, the dynamic programming approach (i.e., the Belman equation) and the method of Lyapunov functions are frequently used. Both parts end with results, comments on the history of ideas, and references (131 items). These are well oriented in works of the Russian and Ukrainian schools. Each part is also accompanied by exercises, with their solutions given at the end of the book. (jmil)