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NOTICE FOR MATHEMATICAL SOCIETIES

Labels for the next issue will be prepared during the second half of August 2003. Please send your updated lists before then to Ms Tuulikki Mäkeläinen, Department of Mathematics, P.O. Box 4, FIN-00014 University of Helsinki, Finland; e-mail: makelain@cc.helsinki.fi

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EMS Agenda

2003

30 June-4 July
EMS Summer School at Porto (Portugal)
Dynamical Systems
Organiser: Maria Pires de Carvalho, e-mail: mpcarval@fc.up.pt
webpage: http://www.fc.up.pt/cmup/sds

6-13 July
CIME-EMS Summer School at Bressanone/Brixen (Italy)
Stochastic Methods in Finance
Organisers: Marco Frittelli and Wolfgang J. Runggaldier
e-mail: runggal@math.eunipd.it
webpage: www.math.unifi.it/~cime

15 August
Deadline for submission of material for the September issue of the EMS *Newsletter*
Contact: Robin Wilson, e-mail: r.j.wilson@open.ac.uk

12-14 September
SPM-EMS Weekend Meeting at the Calouste Gulbenkian Foundation, Lisbon (Portugal)
Organiser: Rui Loja Fernandes, e-mail: rfern@math.ist.utl.pt

14-15 September
EMS Executive Committee meeting at Lisbon (Portugal)
Contact: Helge Holden, e-mail: holden@math.ntnu.no

15 November
Deadline for submission of material for the December issue of the EMS *Newsletter*
Contact: Robin Wilson, e-mail: r.j.wilson@open.ac.uk

2004

31 January
Closing date for nominations for delegates to the EMS Council to represent the individual membership.
Contact: Tuulikki Mäkeläinen, e-mail: makelain@kantti.helsinki.fi

25-27 June
EMS Council Meeting, Stockholm (Sweden)

27 June-2 July
4th European Congress of Mathematics, Stockholm
webpage: http://www.math.kth.se/4ecm

2-6 September
EMS Summer School at Universidad de Cantabria, Santander (Spain)
Empirical processes: theory and statistical applications

Editorial: A European Sense of Identity?

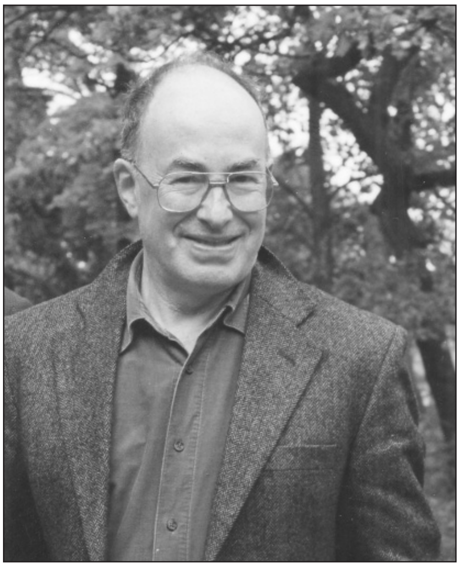
David Salinger (Leeds, UK)

The statutes of the EMS state:

The purpose of the Society is to promote the development of all aspects of mathematics in the countries of Europe, with particular emphasis on those which are best handled on an international level.

The Society will concentrate on those activities which transcend national frontiers and it will in no way seek to interfere with the national activities of the member societies.

In particular, the Society will, in the European context, aim to promote mathematical research (pure and applied), assist and advise on problems of mathematical education, concern itself with the broader relations of mathematics to society, foster the interaction between mathematicians of different countries, ***establish a sense of identity amongst European mathematicians****, and represent the mathematical community in supra-national institutions.*



I have emphasised one aim, because it seems to me that it is different from the others, in that it is overtly political. Why should we foster a sense of identity among European mathematicians? If we accept this as a purpose, how can we tell whether what we do achieves this purpose?

Mathematics, it can be argued, is international and we should regard highly good mathematics, whatever its provenance. So any attempt at creating a 'European identity' is, in principle, divisive and should be avoided.

I think this is a valid argument and that there is a danger of such division: but the Society is reaching out to non-European Societies (in particular, the AMS and SIAM) to facilitate European mathematicians' membership of those societies (and *vice versa*). As a fledgling society, some members fear that we may be overwhelmed by more established and richer partners, but, as we grow, this will become less of a problem (if it is one at all).

But there are positive reasons for fostering a European sense of identity. It is the acceptance that, if one European country has problems, then it is incumbent on all of us to help. A small example is the response to the flood damage of Prague's mathematical library. A more routine example is the way we devote funds regularly to give conference grants to mathematicians from the poorer European countries – we also do this indirectly through applications to UNESCO and the European Framework programmes.

The European prizes are another way in which we try to foster a European identity. The Executive committee has had lively debates in trying to frame an exact definition of 'European' in this context. The twin demands of nationality and place of work are difficult to reconcile in a sensible way. There is a further problem with the word 'European': which countries does it include? The Society has interpreted this term very broadly when it comes to membership: we do not limit ourselves to members and potential members of the European Union. Again, we are seeking to unite, not divide, the mathematical community.

The European congresses of mathematics are occasions when mathematicians from around the world, but particularly from Europe, meet. There we discuss not only mathematics, but common problems of all kinds, informally in the corridors, formally at round-table discussions. The Stockholm congress in 2004 will bring the Framework 5 networks into the proceedings.

The mathematical community cannot avoid politics, if it wishes to prosper, in that it cannot afford to be absent from the European arena, either for funding, or in matters of mathematical education. We have to remind officials from time to time that mathematics is basic to all science, and that it has its own special needs which the Framework programmes should accommodate. Often, the needs of mathematics are overlooked (as in the thematic part of Framework 6) and would be even more disregarded if it were not for the tireless efforts of our colleagues. This is an area in which the Society can claim some success.

In mathematical education, the Bologna programme will impinge on all of us, but, so far, the Society has not been able to arrive at a position representative of the mathematical community, partly because opinion is divided in our community, and partly because key individuals on the Executive Committee have been overloaded with crucial applications to the Framework 6 programme on behalf of the Society. But in many countries, mathematicians are already having to come to terms with degree programmes curtailed in the name of a common European educational area.

On a small scale, by its efforts as a team, the Executive Committee finds a common purpose. In working together, it sees what common problems need solving and is aware of the diverse strengths coming from different traditions which can be brought to bear upon on those problems, whether totally solvable or not. In short, the EMS Executive has a European identity. By participating in EMS events, members can share that sense of identity.

European Mathematical Society Prizes, 2004

David Salinger (EMS Publicity Officer)

A call for nominations for the 2004 EMS prizes has been issued and is on the European Mathematical Society's webpages (*www.emis.de*) and in the March 2003 issue of the EMS *Newsletter*.

Ten prizes (covering all areas of mathematics) were awarded by the city of Paris when the inaugural European Congress of Mathematics was held there in 1992, and the same number of EMS prizes have been awarded at each European Congress since then. These prizes are for young mathematicians, but, in response to comments at the last Congress, the age limit has now been increased to 35 and a further allowance of up to three years can be made for career breaks. The prizes are awarded to European mathematicians, meaning those whose nationality is European or whose normal place of work is in Europe. (When discussing the rules, some members of the Executive Committee felt unhappy about restricting the prizes to Europe – even if that was taken in the widest possible sense – but the Committee as a whole felt that the time was not yet ripe for removing this restriction.)

Several of the EMS prizewinners have become Fields medallists. The names of all prizewinners can be found in the information about past congresses on the EMIS website.

Nominations have to be made (to the 4ecm Organising Committee, Prof. Ari Laptev, Department of Mathematics, Royal Institute of Technology, SE-100 44 Stockholm, Sweden) by 1 February 2004, and the nominees will be judged on the strength of work published by December 2003.

Stochastic Methods in Finance

SUMMER SCHOOL of the
Centro Internazionale Matematico Estivo
and the European Mathematical Society
Supported by UNESCO-Roste

Cusanus Akademie Bressanone (Bolzano)
6th-13th July 2003

The aim of the SUMMER SCHOOL is to provide a broad and accurate knowledge of some of the most up to date and relevant topics in Mathematical Finance. Particular attention will be devoted to the investigation of innovative methods from stochastic analysis that play a fundamental role in the mathematical modelling of finance or insurance.

The school will provide an occasion to facilitate and stimulate discussion between all participants, in particular, young researchers and lecturers, for whom limited financial support will be available.

Courses

- Partial and asymmetric information. (Kerry Back, University of St.Louis, USA)
- Stochastic Methods in Credit Risk Modeling, Valuation and Hedging (Tomasz Bielecki, Northeastern Illinois University, USA)
- Finance and Insurance (Christian Hipp, University of Karlsruhe, Germany)
- Nonlinear expectations and risk measures (Shige Peng, Shandong University, China)
- Utility Maximization in Incomplete Markets (Walter Schachermayer, Technical University of Vienna, Austria)

Course directors

Prof. Marco Frittelli (Università di Firenze, Italy) marco.frittelli@dmd.unifi.it
Prof. Wolfgang Runggaldier (Università di Padova, Italy) runggal@math.unipd.it

Webpage: http://www.math.unifi.it/cime

European Mathematical Society

Introducing the Committee

(part 2)

Victor Buchstaber graduated from the Moscow State University (MSU) in 1969 and went on to postgraduate study there, with advisors Sergei P. Novikov and Dmitri B. Fukhs. He received his PhD in 1970 and a DSc in 1984.

He has been Research Leader of the topology division of the Steklov Mathematical Institute of the Russian Academy of Science, Professor in Higher Geometry and Topology at the MSU, and Head of the Mathematical Modelling Division at the National Scientific and Research Institute for Physical, Technical and Radio-technical Measurements.

He has been on the Council of the Moscow Mathematical Society, Deputy Editor-in-Chief of *Uspekhi Mat. Nauk*, and Head of the Expert Committee in Mathematics in the Russian Foundation for Basic Research.

Doina Cioranescu has been Directeur du Recherche at the Centre National de la Recherche Scientifique (CNRS), Laboratoire d'Analyse Numérique Paris VI, since 1981. She graduated from the University of Bucharest in partial differential equations in 1966, and became associate researcher at the CNRS in 1971, receiving her Doctorat d'État in 1978 at Université Paris VI.

Her research interests are mainly concentrated in the modelling of non-Newtonian fluids, homogenisation theory (in particular, applied to perforated domains and reticulated structures), and control problems in heterogeneous media. She has been involved in several international scientific projects, including being scientific coordinator of the CE Eurohomogenisation Science programme. Since 1997 she has been a member of the management committee of the French Society of Applied Mathematics. In January she was joint organiser of the EMS-SMAI-SMF meeting *Mathématiques Appliquées – Applications des Mathématiques* in Nice.

Pavel Exner graduated from Charles University, Prague, in 1969, and has worked in several places, including a dozen years at the Laboratory of Theoretical Physics in Dubna. He currently heads a mathematical-physics research group at the Czech Academy of Sciences.

His scientific interests are primarily concerned with mathematical problems and methods of quantum theory – in particular, unstable systems, scattering theory, functional integration and quantum mechanics on graphs, surfaces, quantum waveguides, and has written about 130 research papers and three books on these subjects. He is a member of IAMP and secretary of the IUPAP commission for mathematical physics.

Marta Sanz-Solé is Professor of Mathematics at the University of Barcelona. She received her PhD there in 1978. Her research interests lie in probability theory, in particular in stochastic analysis – stochastic partial differential equations, analysis on theWiener space, and Malliavin calculus, and she has been active in the creation of a research group in stochastic analysis in Barcelona. She has been a visiting professor at the University of North Carolina at Chapel Hill, the Mathematical Sciences Research Institute in Berkeley, Université Pierre et Marie Curie in Paris, Université d'Angers, Clermont-Ferrand II, Messina, among others.

From 1993 to 1996 she was Dean of the Faculty of Mathematics, and from 2000 to 2003 Vice-President of the Division of Sciences, at the University of Barcelona. She has been a member of the EMS Executive Committee since 1997, and was organisational secretary of *3ecm*, the Third European Congress of Mathematics, in Barcelona in 2000.

Mina Teicher is Professor of Mathematics and Computer Science at Bar-Ilan University, Israel, specialising in algebraic geometry and topology, with applications to theoretical physics (shape of the universe), computer vision, and mathematical models in brain research. She is a world leader in braid techniques and fundamental groups.

She obtained her PhD from Tel-Aviv University and did postdoctoral studies in Princeton. She has received many awards and grants, including the prestigious Batsheva de Rothschild award, Excellency Center of the Israel National Academy of Sciences, and a European Commission funded network which was rated first (out of 430) in all fields of science and the humanities.

She has been Chair of the Department of Mathematics and Computer Science at Bar-Ilan University, Director of the Emmy Noether Research Institute for Mathematics, and a member of the Israeli Council for Higher Education, the National Forum for Brain Research, the Israeli National Forum 'Academia-Industry' and the council of the Wolf Foundation, and Advisor for the German Science Foundation, the European Union and the Italian Science Foundation. She is Vice-president for Research at Bar-Ilan University and has created bilateral programmes between Bar-Ilan and leading centres in Germany, Italy, Poland and India.

Fourth European Congress of Mathematics

David Salinger (EMS Publicity Officer)

In November 2000 in De Morgan House (London), the Executive Committee of the European Mathematical Society accepted on behalf of Council the offer of holding 4ecm in Stockholm. It is now just over a year to the Congress – 27 June to 2 July 2004 – and much of the framework is already in place. There are a poster and a website: *http://www.math.kth.se/4ecm/*

We are honoured that Lennart Carleson has agreed to be President of the Scientific Committee. The President of the Organising Committee is Ari Laptev, whose good-humoured reports have reassured the Executive Committee that the Congress is in very capable hands.

The Congress has a subtitle: *Mathematics in Science and Technology*. Although it will try to cover all of mathematics as usual (a daunting task), there will be a special place this time for applications of mathematics; this is in keeping with the Society's recent efforts to put applied mathematics high on its agenda. Since Stockholm is the home of the Nobel Prizes, a number of Nobel prizewinners have been asked to speak on the role of mathematics in their discipline. Creating room for this has meant that there will be no round-table discussions at 4ecm, but this is a decision affecting 4ecm alone: it will be open to the organisers of 5ecm to reinstate the round tables.

Another feature of 4ecm is that the current European networks have been invited to hold their annual meetings in association with the Congress: some of them may present their work there.

There will be satellite conferences: so far there have been six expressions of interest.

4ecm will be the occasion for the presentation of the European Mathematics Prizes. Nina Uralt'seva (St. Petersburg) has been appointed Chair of the Prize Committee and a call for nominations for prizes is on the website. The Felix Klein Prize will also be presented at the Congress, and the committee for that is now complete.

Everything is shaping up for a really stimulating Congress. All it needs is for the mathematical community to turn up in droves, as it did in Barcelona.

2003 Abel Prize awarded to Jean-Pierre Serre

The Niels Henrik Abel Memorial Fund was established on 1 January 2002, to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize amount is 6 million Norwegian kroner (about 750,000 Euro) and the name of the first Abel laureate was announced at the Norwegian Academy of Science and Letters on 3 April 2003.

The Prize committee, appointed for a period of two years, consisted of: *Erling Stoermer* (Chair), University of Oslo (Norway), *John Macleod Ball*, University of Oxford (UK), *Friedrich Ernst Peter Hirzebruch*, Max Planck Institute for Mathematics (Germany), *David Mumford*, Brown University (USA), and *Jacob Palis*, Instituto Nacional de Matemática Pura e Aplicada, IMPA (Brazil).

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2003 to ***Jean-Pierre Serre****, Collège de France, Paris (France), 'for playing a key role in shaping the modern form of many parts of mathematics, including topology, algebraic geometry and number theory'. The Prize was presented by HM King Harald in a special ceremony at the University of Oslo on 3 June 2003.*

Jean-Pierre Serre was born in 1926 in Bages, France. He studied at the École Normale Supérieure and received his DSc in 1951 from the Sorbonne in Paris. After holding a position at the Centre National de la Recherche Scientifique, he became an associate professor at the Université de Nancy. In 1956 he was appointed a professor at the Collège de France. He was made a Commander Légion d'Honneur and High Officer Ordre National du Mérite. He has been elected to many national academies, in particular, the academies of France, Sweden, the United States and the Netherlands. He was awarded the Fields Medal in 1954 (the youngest recipient ever), the Prix Gaston Julia in 1970, the Balzan Prize in 1985, the Steele Prize in 1995, and the Wolf Prize in 2000. He has been awarded honorary degrees from many universities, most recently from the University of Oslo in 2002 in connection with the Abel Bicentennial.

Serre developed revolutionary algebraic methods for studying topology, and in particular studied the transformations between spheres of higher dimensions. He is responsible for a spectacular clarification of the work of the Italian algebraic geometers, by introducing and developing the right algebraic machinery for determining when their geometric construction works. Serre's powerful technique, with its new language and viewpoint, ushered in a golden age for algebraic geometry.

For the past four decades Serre's magnificent work and vision of number theory have been instrumental in bringing that subject to its current glory. This work connects and extends in many ways the mathematical ideas introduced by Abel, in particular his proof of the impossibility of solving the fifth-degree equation by radicals, and his analytic techniques for the study of polynomial equations in two variables. Serre's research has been vital in setting the stage for many of the most celebrated recent breakthroughs, including the proof by Wiles of Fermat's last theorem.

Although Serre's effort has been directed to more conceptual mathematics, his contributions have connections with important applications. The practical issues of finding efficient error-correcting codes and of public-key cryptography make use of solutions of polynomial equations (specifically over finite fields) and Serre's work has substantially deepened our understanding of this topic.

Many events took place in 'Abel week 2003', including:

- an Abel lecture by Jean-Pierre Serre entitled 'Prime numbers, equations and modular forms'
- a wreath-laying by Serre at Gustav Vigeland's Abel monument in the Palace grounds
- a one-day Abel symposium at the University of Oslo
- a special banquet at Akershus Castle, attended by the King and Queen
- a Maths Carnival on Universitetsplassen in Central Oslo, involving children of all ages and the awarding of prizes to Year 9 and upper secondary school pupils for the mathematical contests 'KappAbel' and 'The Abel Competition'

More details of Serre's work and of the events of Abel week 2003 can be found on the website *http://www.abelprisen.no*

Popularising mathematics: From eight to infinity

V. L. Hansen

It is rare to succeed in getting mathematics into ordinary conversation without meeting all kinds of reservations. In order to raise public awareness of mathematics effectively, it is necessary to modify such attitudes. Here we point to some possible topics for general mathematical conversation.

Introduction

"One side will make you grow taller, and the other side will make you grow shorter", says the caterpillar to Alice in the fantasy *Alice's Adventures in Wonderland* as it gets down and crawls away from a marvellous mushroom upon which it has been sitting. In an earlier episode, Alice has been reduced to a height of three inches and she would like to regain her height. Therefore she breaks off a piece of each of the two sides of the mushroom, incidentally something she has difficulties identifying, since the mushroom is entirely round. First Alice takes a bit of one of the pieces and gets a shock when her chin slams down on her foot. In a hurry she eats a bit of the other piece and shoots up to become taller than the trees. Alice now discovers that she can get exactly the height she desires by carefully eating soon from one piece of the mushroom and soon from the other, alternating between getting taller and shorter, and finally regaining her normal height.

Few people link this to mathematics, but it reflects a result obtained in 1837 by the German mathematician Dirichlet, on what is nowadays known as *conditionally convergent infinite series* – namely, that one can assign an arbitrary value to infinite sums with alternating signs of magnitudes tending to zero, by changing the order in which the magnitudes are added. Yes, mathematics can indeed be fanciful, and as a matter of fact, *Alice's Adventures in Wonderland* was written, under the pseudonym Lewis Carroll, by an English mathematician at the University of Oxford in 1865. The fantasy about Alice is not the only place in literature where you can find mathematics. And recently, mathematics has even entered into stage plays [18] and movies [19].

On the special position of mathematics

When the opportunity arises, it can be fruitful to incorporate extracts from literature or examples from the arts in the teaching and dissemination of mathematics. Only in this way can mathematics eventually find a place in relaxed conversations among laymen without immediately being rejected as incomprehensible and relegated to strictly mathematical social contexts.

Mathematics occupies a special position among the sciences and in the educational system. This position is determined by the fact that mathematics is an *a priori* science building on ideal elements abstracted from sensory experiences, and at the same time mathematics is intimately connected to the experimental sciences, traditionally not least the natural sciences and the engineering sciences. Mathematics can be decisive when formulating theories giving insight into observed phenomena, and often forms the basis for further conquests in these sciences because of its power for deduction and calculation. The revolution in the natural sciences in the 1600s and the subsequent technological conquests were to an overwhelming degree based on mathematics. The unsurpassed strength of mathematics in the description of phenomena from the outside world lies in the fascinating interplay between the concrete and the abstract. In the teaching of mathematics, and when explaining the essence of mathematics to the public, it is important to get the abstract structures in mathematics linked to concrete manifestations of mathematical relations in the outside world. Maybe the impression can then be avoided that abstraction in mathematics is falsely identified with pure mathematics, and concretisation in mathematics just as falsely with applied mathematics. The booklet [3] is a contribution in that spirit.

On the element of surprise in mathematics

Without a doubt, mathematics makes the longest-lasting impression when it is used to explain a counter-intuitive phenomenon. The element of surprise in mathematics therefore deserves particular attention.

As an example, most people – even teachers of mathematics – find it close to unbelievable that a rope around the Earth along the equator has to be only 2ð metres (i.e., a little more than 6 metres) longer, in order to hang in the air 1 metre over the surface all the way around the globe.

Another easy example of a surprising fact in mathematics can be found in connection with figures of constant width. At first one probably thinks that this property is restricted to the circle. But if you round off an equilateral triangle by substituting each of its sides with the circular arc centred at the opposite corner, one obtains a figure with constant width. This rounded triangle is called a *Reuleaux triangle* after its inventor, the German engineer Reuleaux. It has had several technological applications and was among others exploited by the German engineer Wankel in his construction of an internal combustion engine in 1957. Corresponding figures with constant width can be constructed from regular polygons with an arbitrary odd number of edges. In the United Kingdom, the regular heptagon has been the starting point for rounded heptagonal coins (50p and 20p).

Figures of constant width were also used by the physicist Richard Feynmann to illustrate the dangers in adapting figures to given measurements without knowledge of the shape of the figure. Feynmann was a member of the commission appointed to investigate the possible causes that the space shuttle Challenger on its tenth mission, on 28 January 1986, exploded shortly after take off. A problematic adaptation of figures might have caused a leaky assembly in one of the lifting rockets.

Mathematics in symbols

Mathematics in symbols is a topic offering good possibilities for conversations with a mathematical touch. Through symbols, mathematics may serve as a common language by which you can convey a message in a world with many different ordinary languages. But only rarely do you find any mention of mathematics in art catalogues or in other contexts where the symbols appear. The octagon, as a symbol for eternity, is only one of many examples of this.

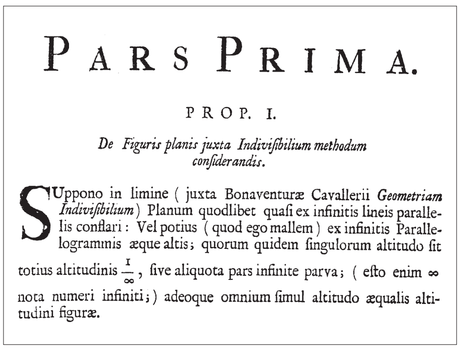
The eternal octagon

Once you notice it, you will find the octagon in very many places – in domes, cupolas and spires, often in religious sanctuaries. This representation is strong in Castel del Monte, built in the Bari province in Italy in the first half of the 1200s for the holy Roman emperor Friedrich II. The castle has the shape of an octagon, and at each of the eight corners there is an octagonal tower [9]. The octagon is also found in the beautiful Al-Aqsa Mosque in Jerusalem from around the year 700, considered to be one of the three most important sites in Islam.

In Christian art and architecture, the octagon is a symbol for the *eighth day* (in Latin, *octave dies*), which gives the day when the risen Christ appeared to his disciples for the second time after his resurrection on Easter Sunday. In the Jewish counting of the week, in use at the time of Christ, Sunday is the first day of the week, and hence the eighth day is the Sunday after Easter Sunday. In a much favoured interpretation by the Catholic Fathers of the Church, the Christian Sunday is both the first day of the week and the eighth day of the week, and every Sunday is a celebration of the resurrection of Christ, which is combined with the hope of eternal life. Also in Islam, the number eight is a symbol of eternal life.

Such interpretations are rooted in old traditions associated with the number eight in oriental and antique beliefs. According to Babylonian beliefs, the soul wanders after physical death through the seven heavens, which corresponds to the material heavenly bodies in the Babylonian picture of the universe, eventually to reach the eighth and highest heaven. In Christian adaptation of these traditions, the number eight (octave) becomes the symbol for the eternal salvation and fuses with eternity, or infinity. This is reflected in the mathematical symbol for infinity, a figure eight lying down, used for the first time in 1655 by the English mathematician John Wallis.

The International Congress of Mathematics, Beijing, 2002



PARS PRIMA.

PROP. I.

De Figuris planis juxta Indivisibilium methodum considerandis.

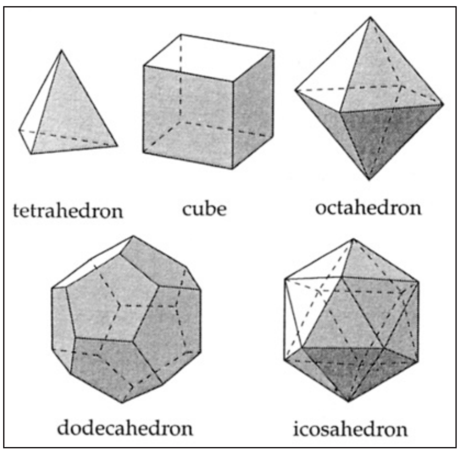
SUppono in limine (juxta Bonaventuræ Cavallerii *Geometriam Indivisibilium*) Planum quodlibet quasi ex infinitis lineis parallelis conflari: Vel potius (quod ego mallem) ex infinitis Parallelogrammis æque altis; quorum quidem singulorum altitudo sit totius altitudinis $\frac{1}{\infty}$, sive aliquota pars infinite parva; (esto enim ∞ nota numeri infiniti;) adeoque omnium simul altitudo æqualis altitudini figuræ.

The first appearance of ∞

The regular polyhedra

Polyhedra fascinate many people. A delightful and comprehensive study of polyhedra has been made by Cromwell [4]. Already Pythagoras in the 500s BC knew that there can exist only five types of regular polyhedra, with respectively 4 (tetra), 6 (hexa), 8 (octa), 12 (dodeca) and 20 (icosa) polygonal lateral faces. In the dialogue *Timaeus*, Plato associated the tetrahedron, the octahedron, the hexahedron [cube] and the icosahedron with the four elements in Greek philosophy, fire, air, earth, and water, respectively, while in the dodecahedron, he saw an image of the universe itself. This has inspired some globe makers to represent the universe in the shape of a dodecahedron.

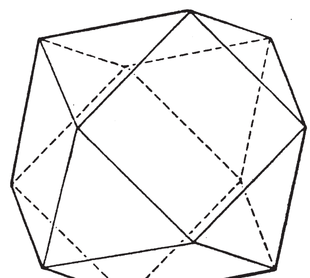
It is not difficult to speculate as to whether there exist more than the five regular polyhedra. Neither is it difficult to convince people that no kinds of experiments are sufficient to get a final answer to this question. It can only be settled by a mathematical proof. A proof can be based on the theorem that the alternating sum of the number of vertices, edges and polygonal faces in the surface of a convex polyhedron equals 2, stated by Euler in 1750; see e.g. [10].



The five regular polyhedra

The cuboctahedron

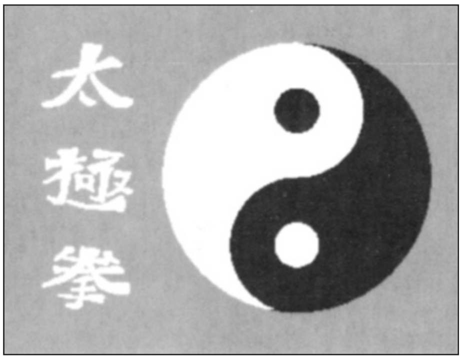
There are several other polyhedra that are ascribed a symbolic meaning in various cultures. A single example has to suffice.



Cuboctahedron

If the eight vertices are cut off a cube by planes through the midpoints in the twelve edges in the cube one gets a polyhedron with eight equilateral triangles and six squares as lateral faces. This polyhedron, which is easy to construct, is known as the *cuboctahedron*, since dually it can also be constructed from an octahedron by cutting off the six vertices in the octahedron in a similar manner. The cuboctahedron is one of the thirteen semiregular polyhedra that have been known since Archimedes. In Japan cuboctahedra have been widely used as decorations in furniture and buildings. Lamps in the shape of cuboctahedra were used in Japan already in the 1200s, and they are still used today in certain religious ceremonies in memory of the dead [16].

The yin-yang symbol
Yin and yang are old principles in Chinese cosmology and philosophy that represent the dark and the light, night and day, female and male. Originally yin indicated a northern hill side, where the sun does not shine, while yang is the south side of the hill. Everything in the world is viewed as an interaction between yin and yang, as found among others in the famous oracular book *I Ching* (Book of Changes), central in the teaching of Confucius (551-479 BC). In this ancient Chinese system of divination, an oracle can be cast by flipping three coins and the oracle is one of sixty-four different hexagrams each composed of two trigrams. The three lines in a trigram are either straight (yang) or broken (yin), thus giving eight different trigrams and sixty-four hexagrams. In Taoism, the eight trigrams are linked to immortality. Yin and Yang as philosophical notions date back to the 400s BC.

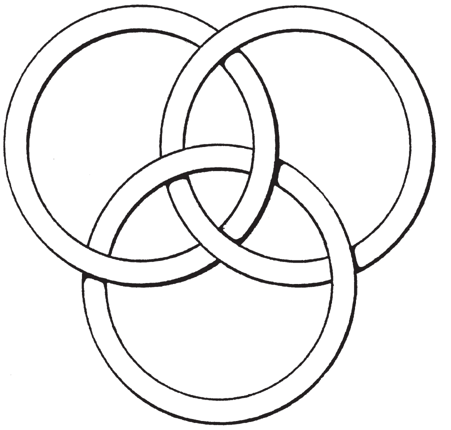


The mathematical symbol for yin-yang is a circle divided into two equal parts by a curve made up of two smaller semicircles with their centres on a diameter of the larger circle. The mathematics in this beautiful and very well known symbol is simple, but nevertheless it contains some basic geometrical forms, and thereby offers possibilities for mathematical conversations.

The Borromean link
Braids, knots and links form a good topic for raising public awareness of mathematics, as demonstrated in the CD-Rom [2].

Here we shall only mention the Borromean link. This fascinating link consists of a system of three interlocking rings, in which no pair of rings interlocks. In other words, the three rings in a Borromean link completely fall apart if any one of the rings is removed from the system.

The Borromean link is named after the Italian noble family Borromeo, who gained a fortune by trade and banking in Milano in the beginning of the 1400s. In the link found in the coat of arms of the Borromeo family, the three rings apparently interlock in the way described, but a careful study reveals that the link often has been changed so that it does not completely fall apart if an arbitrary one of the rings is removed from the system. The



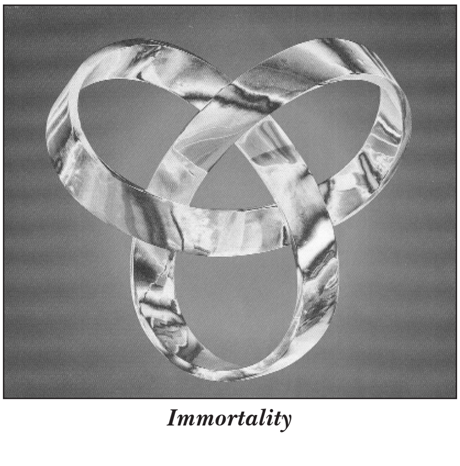
Borromean rings

link in the coat of arms of the Borromeo family is a symbol of collaboration.

Mathematics in art and design
Artists, architects and designers give life to abstract ideas in concrete works of art, buildings, furniture, jewellery, tools for daily life, or, by presenting human activities and phenomena from the real world, dynamically in films and television. The visual expressions are realised in the form of paintings, images, sculptures, etc. From a mathematical point of view, all these expressions represent geometrical figures.

For a mathematician, the emphasis is on finding the abstract forms behind the concrete figures. In contrast, the emphasis of an artist or a designer is to realise the abstract forms in concrete figures. At the philosophical level many interesting conversations and discussions about mathematics and its manifestations in the visual arts can take place following this line of thought.

More down to earth, the mathematics of perspective invites many explanations of concrete works of art [8], and the beautiful patterns in Islamic art inspire discussions on geometry and symmetry [1]. In relation to the works of the Dutch artist Escher, it is possible to enter into conversations about mathematics at a relatively advanced level, such as the Poincaré disc model of the hyperbolic plane [5], [7]. The sculptures of the Australian-British artist John Robinson – such as his wonderful sculpture *Immortality* that exhibits a Möbius band shaped as a clover-leaf knot – offer good possibilities for conversations on knots [2].



Immortality

In his fascinating book [17], Wilson tells many stories of mathematics in connection with mathematical motifs on postage stamps.

Mathematics in nature
The old Greek thinkers – in particular Plato (427-347 BC) – were convinced that nature follows mathematical laws. This belief has dominated thinking about natural phenomena ever since, as witnessed so strongly in major works by physicists, such as Galileo Galilei in the early 1600s, Isaac Newton in his work on gravitation in *Principia Mathematica* (1687), and Maxwell in his major work on electromagnetism in 1865; cf. [10]. In his remarkable book *On Growth and Form* (1917), the zoologist D'Arcy Wentworth Thompson concluded that wherever we cast our glance we find beautiful geometrical forms in nature. With this point of departure, many conversations on mathematics at different levels are possible. I shall discuss a few phenomena that can be described in broad terms without assuming special mathematical knowledge, but nevertheless point to fairly advanced mathematics.

Spiral curves and the spider's web
Two basic motions of a point in the Euclidean plane are motion in a line, and periodic motion (rotation) around a central point. Combining these motions, one gets spiral motions along *spiral curves* around a *spiral point*. Constant velocity in both the linear motion and the rotation gives an *Archimedean spiral* around the spiral point. Linear motion with exponentially growing velocity and rotation with constant velocity gives a *logarithmic spiral*.

A logarithmic spiral is also known as an *equiangular spiral*, since it has the following characteristic property: *the angle between the tangent to the spiral and the line to the spiral point is constant*.

Approximate Archimedean and logarithmic spirals enter in the spider's construction of its web. First, the spider constructs a Y-shaped figure of threads fastened to fixed positions in the surroundings, and meeting at the centre of the web. Next, it constructs a frame around the centre of the web and then a system of radial threads to the frame of the web, temporarily held together by a non-sticky logarithmic spiral, which it constructs by working its way out from the centre to the frame of the web. Finally, the spider works its way back to the centre along a sticky Archimedean spiral, while eating the logarithmic spiral used during the construction.

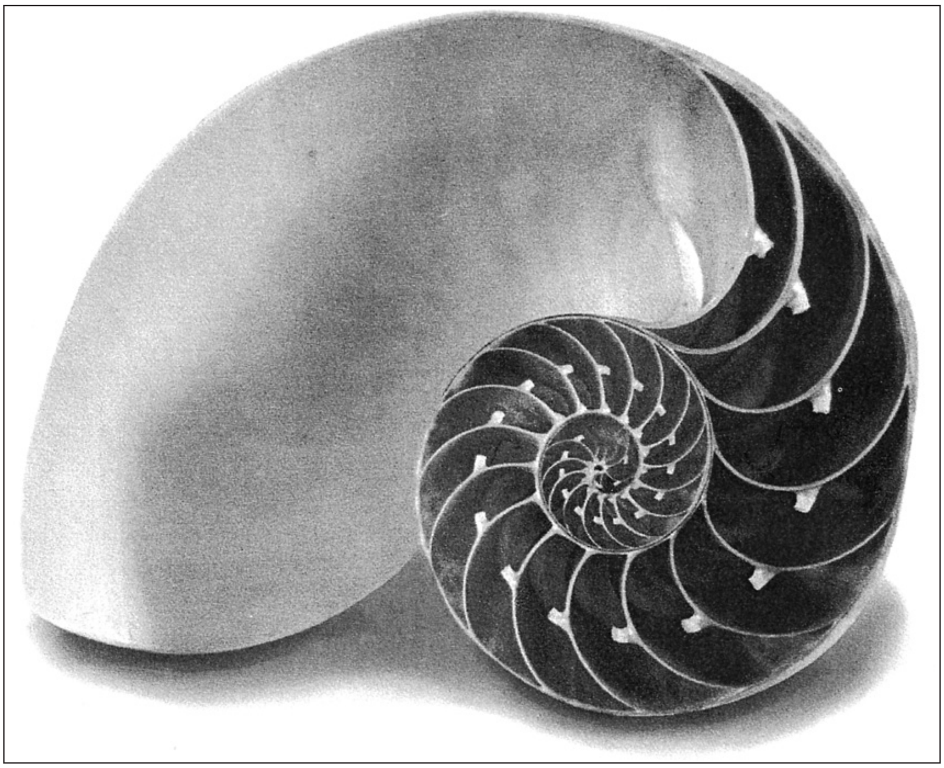
Helices, twining plants and an optical illusion
A space curve on a cylinder, for which all the tangent lines intersects the generators of the cylinder in a constant angle, is a *helix*. This is the curve followed by twining plants such as bindweed (right-handed helix) and hops (left-handed helix).

As we look up a long circular cylinder

in the wall of a circular tower enclosing a spiral staircase, all the generators of the cylinder appear to form a system of lines in the ceiling of the tower radiating from the central point, and the helix in the banister of the spiral staircase appears as a logarithmic (equiangular) spiral. A magnificent instance of this is found in the spiral staircase in Museo do Popo Galergo, Santiago, Spain, where you have three spiral staircases in the same tower.

Curvature and growth phenomena in nature
Many growth phenomena in nature exhibit curvature. As an example, we discuss the shape of the shell of the primitive cuttlefish *nautilus*.

The fastest way to introduce the notion of curvature at a particular point of a plane curve is by approximating the curve as closely as possible in a neighbourhood of the point with a circle, called the *circle of curvature*, and then measuring the curvature as the reciprocal of the radius in the approximating circle. If the curve is flat in a neighbourhood of the point, or if the point is an inflection point, the approximating circle degenerates to a straight line, the *tangent* of the curve; in such situations, the curvature is set to 0. Finally, the curvature of a curve has a sign: positive if the curve turns to the left (anticlockwise) in a neighbourhood of the point in question, and negative if it turns to the right (clockwise).



Nautilus shell

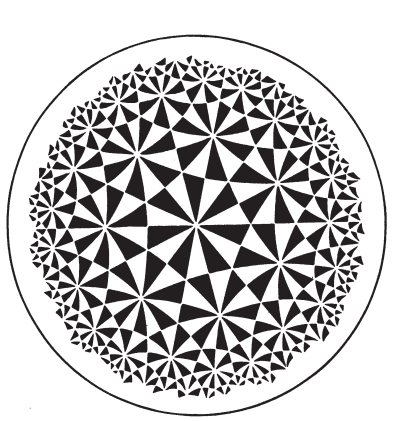
Now to the mathematics of a nautilus shell. We start out with an equiangular spiral. At each point of the spiral, take the circle of curvature and replace it with the similar circle centred at the spiral and orthogonal to both the spiral and the plane of the spiral. For exactly one angle in the equiangular spiral, the resulting surface winds up into a solid with the outside of one layer exactly fitting against the inside of the next. The surface looks like the shell of a nautilus.

In the Spring of 2001, I discovered that this unique angle is very close to half the golden angle (the smaller of the two angles obtained by dividing the circumference of the unit circle in golden ratio), so close in fact that it just allows for thickness of the shell. More details can be found on my home page [14].

Geometry of space
Questions about geometries alternative to Euclidean geometry easily arise in philosophical discussions on the nature of space and in popularisations of physics. It is difficult material to disseminate, but still, one can come far by immediately incorporating a concrete model of a non-Euclidean geometry into the considerations. In this respect, I find the disc model of the *hyperbolic plane* suggested by Poincaré (1887) particularly useful. In addition to its mathematical uses, it was the inspiration for Escher in his four woodcuts Circle Limit I-IV [7, p. 180].

In [13, Ch. 4], I gave a short account of the Poincaré disc model, which has successfully been introduced in Denmark at the upper high-school level, and where you encounter some of the surprising relations in hyperbolic geometry. In particular, the striking difference between tilings with congruent regular n-gons in the Euclidean plane, where you can tile with such n-gons only for $n = 3, 4, 6$, and in the hyperbolic plane, where you can tile with such n-gons for each integer $n \geq 3$.

At a more advanced level, one can introduce a hyperbolic structure on surfaces topologically equivalent to the surface of a sphere with p handles, for each



The Poincaré disc model

genus $p = 2$, by pairwise identification of the edges in a regular hyperbolic $4p$-gon; cf. [11].

The appearance of non-Euclidean geometries raised the question of which geometry provides the best model of the physical world. This question was illustrated in an inspired poster designed by Nadja Kutz; cf. [6].

Eternity and infinity
Eternity and, more generally, infinity are notions both expressing the absence of limits. Starting from the classical paradoxes of Zeno, which were designed to show that the sensory world is an illusion, there are good possibilities for conversations about mathematical notions associated with infinity. And one can get far even with fairly difficult topics such as infinite sums; cf. [15].

The harmonic series
A conversation about infinite sums (series) inevitably gets on to the *harmonic series*:

$$1 + 1/2 + 1/3 + 1/4 + \ldots + 1/n + \ldots .$$

As you know, this series is *divergent*, i.e., the Nth *partial sum*

$$S_N = 1 + 1/2 + 1/3 + 1/4 + \ldots + 1/N$$

increases beyond any bound, for increasing N.

The following proof of this fact made a very deep impression on me when I first met it. First notice, that for each number k, the part of the series from $1/(k+1)$ to $1/2k$ contains k terms, each greater than or equal to $1/2k$ so that

$$1/(k+1) + 1/(k+2) + \ldots + 1/2k >$$
$$1/2k + 1/2k + \ldots + 1/2k = 1/2.$$

With this fact at your disposal, it is not difficult to divide the harmonic series into infinitely many parts, each greater than or equal to ½, proving that the partial sums in the series grow beyond any bound when more and more terms are added.

The alternating harmonic series
The *alternating* harmonic series

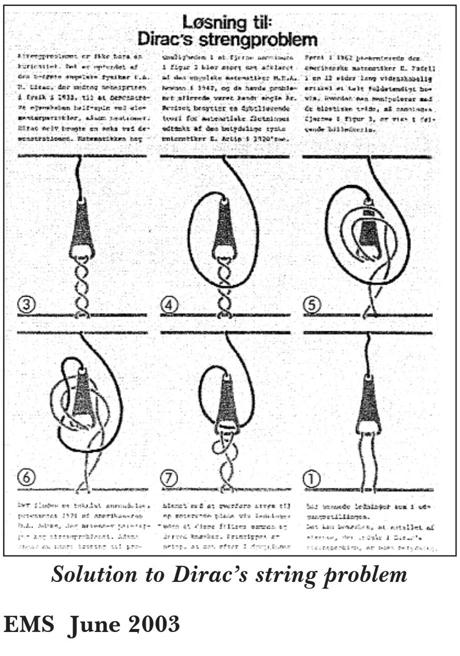
$$1 - 1/2 + 1/3 - 1/4 + \ldots - (-1)^n\, 1/n + \ldots$$

offers great surprises. As shown in 1837 by the German mathematician Dirichlet, this series can be *rearranged* – that is, the order of the terms can be changed, so that the rearrangement is a convergent series with a sum arbitrarily prescribed in advance. The proof goes by observing that the *positive series* (the series of terms with positive sign) increases beyond any upper bound, and that the *negative series* (the series of terms with negative sign) decreases beyond any lower bound, when more and more terms are added to the partial sums in the two series. Now, say that we want to obtain the sum S in a rearrangement of the alternating harmonic series. Then we first take as many terms from the positive series as are needed just to exceed S. Then take as many of the terms from the negative series as needed just to come below S. Continue this way by taking as many terms as needed from the positive series, from where we stopped earlier, just again to exceed S. Then take as many terms as needed from the negative series, from where we stopped earlier, just again to come below S, and so on. Since $\{^1/_n\}$ tends to 0 as n increases, the series just described is convergent with sum S.

The alternating harmonic series is the mathematical idea behind the episode from *Alice's Adventures in Wonderland*, mentioned at the beginning of this paper.

Mathematics as a sixth sense

In 1982, *Dirac's string problem* was presented on a shopping bag from a Danish supermarket chain [12]. This problem illustrates the property of half-spin of certain elementary particles, mathematically predicted by the physicist P. A. M. Dirac in the 1920s for elementary particles such as the electron and the neutron. To convince colleagues sceptical of his theory, Dirac conceived a model to illustrate a corresponding phenomenon in the macroscopical world. This model consists of a solid object (Dirac used a pair of scissors) attached to two posts by loose (or elastic) strings, say with one string from one end of the object and two strings from the other end to the two posts. Dirac then demonstrated that a double twisting of the strings could be removed by passing the strings over and round the object, while he was not able to remove a single twisting of the strings in this way. Rather deep mathematics from topology is needed to explain the phenomenon, and in this case mathematics comes in as a 'sixth sense', by which we 'sense' (understand) counter-intuitive phenomena. An application to the problem of transferring electrical current to a rotating plate, without the wires getting tangled and breaking, was patented in 1971.



Solution to Dirac's string problem

Making mathematics visible

In many countries there are increasing demands to make the role and importance of the subjects taught in the school system visible to the general public. If the curriculum and the methods of teaching are not to stagnate, it is important that an informed debate takes place in society about the individual subjects in school.

For subjects strongly depending on mathematics, this is a great challenge. Where mathematicians (and scientists in general) put emphasis on explanations (proofs), the public (including politicians) are mostly interested in results and consequences. It is doubtless necessary to go part of the way to avoid technical language in presenting mathematics for a wider audience.

On the initiative of the International Mathematical Union, the year 2000 was declared *World Mathematical Year*. During the year many efforts were made to reach the general public through poster campaigns in metros, public lectures, articles in general magazines, etc. and valuable experience was gained. The mathematical year clearly demonstrated the value of an international exchange of ideas in such matters, and the issue of raising public awareness of mathematics is now on the agenda all over the world from eight to infinity.

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Thought for the Day

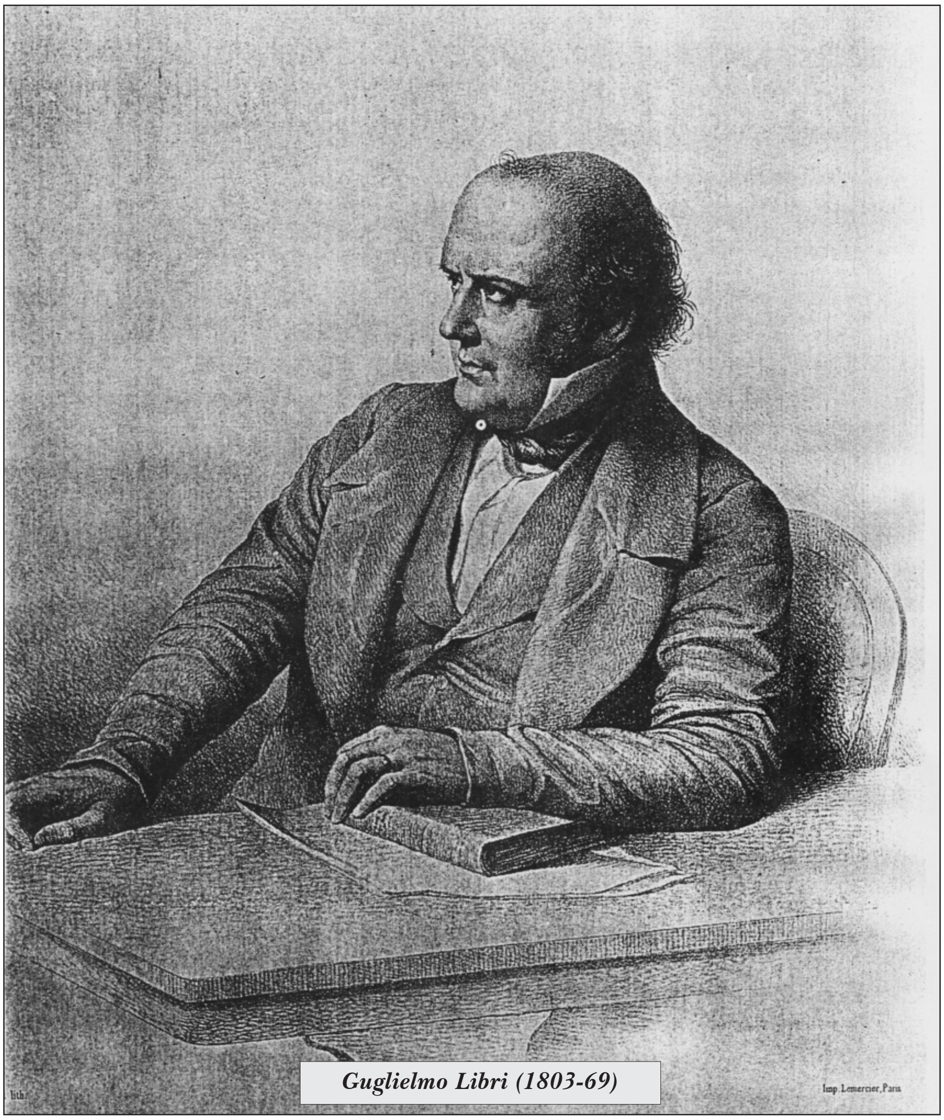
The following remarks appeared in the Editor's column *Internet: The Editor's Choice* in a recent issue of the *Mitteilungen der Deutschen Mathematiker-Vereinigung*, in connection with the ICM in Beijing.

It is all too familiar. You go to a colloquium talk, supposedly meant for a "general mathematical audience" and "graduate students", and you don't understand a word. OK, being clueless for one hour per week is not the end of the world, but what if you go to a National Meeting?, or worse, THE BIG HAPPENING of mathematics, ICM, and try to go to ALL the plenary talks?

I recently came back from ICM 2002, in Beijing, and once again was struck by the fact that the Tower of Babel hit us mathematicians particularly hard. In addition to the intrinsic compartimization, overspecialization and splintering of math, most mathematicians (even, or perhaps especially, the most prominent) have no idea how to give a general talk.

Brought to book: the curious story of Guglielmo Libri (1803-69)

Adrian Rice



Guglielmo Libri (1803-69)

This year marks the 200th birthday of an intriguing and somewhat enigmatic mathematician. For despite being one of the most colourful figures in the history of mathematics, he is also one of the most obscure. An Italian aristocrat of noble descent, he spent most of his life in exile, first in France, and later in England. His career was scarred by turbulence and controversy, yet in his lifetime he managed to achieve fame as a mathematician, a historian of mathematics and, most notoriously, a mathematical bibliophile. In this article, we look at the work and fascinating life of the Italian mathematician Guglielmo Libri.

Guglielmo Bruto Icilio Timoleone Libri Carrucci Dalla Sommaia was born into a noble Tuscan family on 2 January 1803. After an excellent and wide-ranging elementary education, he entered the University of Pisa in 1816, initially studying law, but then concentrating on mathematics and the natural sciences. Academically precocious, his graduation coincided with the publication of his first paper, *Memoria … sopra la teoria dei numeri* in 1820. This paper attracted much praise from scholars both at home and abroad, including Charles Babbage in England, Augustin-Louis Cauchy in France, and Carl Friedrich Gauss in Germany.

In 1823, aged only 20, he was appointed professor of mathematical physics at the University of Pisa. However, he was not a good teacher and, not relishing the position, managed to obtain permanent leave after just one academic year. Yet, despite the brevity of his tenure, he retained the title (and salary) of the professorship for the rest of his life.

Travelling to Paris in 1824-25, he became personally acquainted with many of the greatest scientists of the day, including Pierre-Simon Laplace, Siméon Denis Poisson, André-Marie Ampère, Joseph Fourier and François Arago. These friendships proved valuable to him when, in 1830, he was forced into exile from Tuscany after being involved in a plot to impose a new liberal constitution on the Grand Duke of Tuscany. Fleeing to France, Libri took refuge amongst his scientific friends in Paris, of whom Arago was particularly generous, providing the penniless refugee with funds for a time.

Meanwhile, his mathematical reputation continued to grow. In March 1833, shortly after being granted French citizenship, Libri was elected a full member of the *Académie des Sciences* to replace the recently deceased Adrien-Marie Legendre. He threw himself wholeheartedly into the *Académie*'s proceedings, but his ambitious and arrogant nature, together with the fact that he was not a native Frenchman, soon led to resentment amongst many of his colleagues.

Nevertheless, with Arago's help, he obtained a teaching position at the prestigious Collège de France in 1833, and was also elected as *professeur suppléant* (assistant professor) in the calculus of probability at the Sorbonne in 1834. But, for reasons unknown, his friendship with Arago did not last, and by 1835 they were bitter enemies. Unfortunately for Libri, Arago was an extremely powerful figure within the *Académie*, and opposing him meant opposing those who were under his influence or patronage. One of these was the mathematician Joseph Liouville, who set about undermining Libri's mathematical reputation within French scientific society. Indeed, the feuding between Liouville and Libri was to become a characteristic feature of the French academic world throughout the 1830s and 1840s.

François Arago (1786-1853)

Joseph Liouville (1809-82)

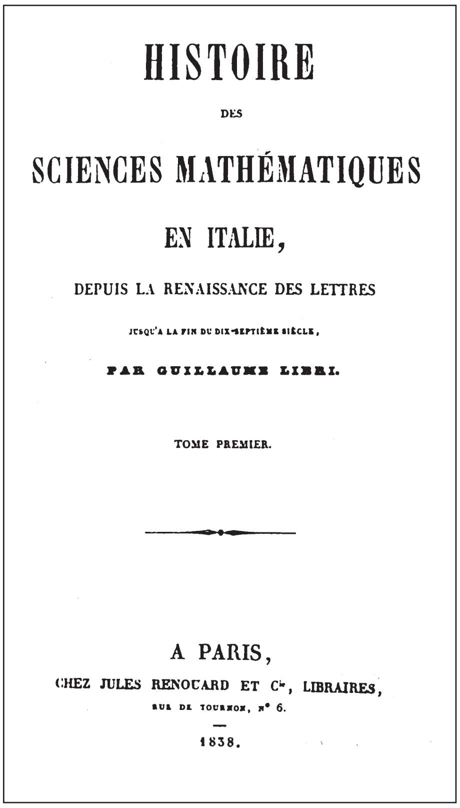
Libri's mathematical research covered several areas, but chiefly included number theory and mathematical physics. He was also extremely interested in the theory of equations although, in the words of Jesper Lützen,

it is now generally agreed that there is at most one good idea in Libri's mathematics: the attempts to transfer theorems from the solution of algebraic equations (Galois theory) to the field of differential equations, but Libri never got far with his idea. [1]

Indeed, it was found that, although his work was non-trivial, its originality was often questionable and his methods of proof were not always the most efficient or elegant. Liouville, in particular, who produced important and innovative work in several of the same areas, was able to prove many of Libri's results in a far less convoluted fashion.

The two mathematicians clashed repeatedly during stormy sessions of the *Académie*, bombarding each other with abstruse mathematical arguments. A little-known consequence of these disputes is that Liouville made his famous announcement of Evariste Galois's important work on theory of equations in response to an attack by Libri in 1843. [2] Somewhat unsettled by the prolonged assaults, not just from Liouville, but also from other mathematical scientists including Arago, Charles-François Sturm and Peter Lejeune Dirichlet, Libri was gradually to abandon mathematics and to look for new research areas.

He had always been particularly interested in the history of mathematics, and it was here that he produced the work for which he is probably best remembered. His four-volume *Histoire des Sciences Mathématiques en Italie* was published between 1838 and 1841. A wide-ranging and provocative study of the mathematical produce of the Italian peninsula from antiquity until the seventeenth century, the work was never completely finished. With a strong emphasis on original sources, Libri leads us from the Etruscans to Galileo via the invention of spectacles, the travels of Marco Polo and the genius of Leonardo da Vinci (whom he wrongly credits with the invention of the + and – signs and the discovery of blood circulation). Along the way, he revealed for the first time the originality of Fibonacci's geometry, and that the Italians Giovanni Porta and Branca had envisaged steam power before James Watt invented the steam engine.



HISTOIRE

DES

SCIENCES MATHÉMATIQUES

EN ITALIE,

DEPUIS LA RENAISSANCE DES LETTRES

JUSQU'A LA FIN DU DIX-SEPTIÈME SIÈCLE,

PAR GUILLAUME LIBRI.

TOME PREMIER.

A PARIS,

CHEZ JULES RENOUARD ET C^ie^, LIBRAIRES,

RUE DE TOURNON, N° 6.

1838.

The title page of Libri's Histoire

However, in his aim to raise foreign awareness of the richness of Italian science and the extent to which European *savans* were indebted to their Italian forebears, Libri was only moderately successful, and because of his tendency to praise Italian achievements at the expense of others, the *Histoire* must today be read with caution. As Benedetto Croce wrote in 1947, the *Histoire* was

excellent as a rich collection of uncommon knowledge and as a work of versatile learning and vivid mind, [but] not, however, as an example of what a history of science should be.

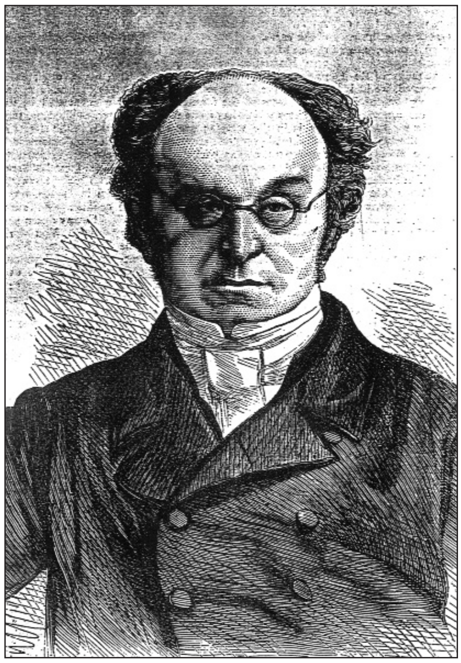
As Libri's fascination with the history of mathematics grew, so too did his interest and expertise in rare books and mathematical manuscripts. Throughout the 1830s and 1840s he rapidly acquired a substantial collection, including some valuable finds, such as previously lost papers of Fermat, Descartes, Euler, D'Alembert and Arbogast, found in a bookshop in Metz. Further manuscripts by Galileo, Leibniz, Mersenne, Gassendi, and even Napoleon were acquired, until by 1841 he estimated his library to contain around 1800 manuscripts, ranging from a 9th-century copy of Boethius to papers of Lagrange and Laplace. By 1847, his collection of printed books and incunabula amounted to around 40,000 volumes. As he said in 1846,

I have books everywhere, even behind the bed.

Thanks to the influence of his good friend François Guizot, a minister in the French government, Libri was appointed Inspector of French Public Libraries in 1841. But it was this post that was to lead to his downfall. When rumours began to circulate that he had used the position to steal large quantities of valuable books and manuscripts for his own collection, his enemies began to compile evidence against him, and a police investigation was begun.

Libri had nothing to fear as long as his friend was in power, but in February 1848 came the revolution that toppled the regime of Louis-Philippe. Fearing imminent arrest, Libri fled to London within twenty-four hours, arranging for around 30,000 of his most precious books and manuscripts to be packed and sent after him.

Once again, he was a refugee in a foreign land. However, on his arrival in London, he was immediately welcomed by fellow Italian ex-patriot Antonio Panizzi, who occupied the position of Director of the Library of the British Museum. Panizzi was well connected in London's literary circles, having briefly held the Chair of Italian at University College. It was through this connection that Libri was introduced to the British mathematician and logician Augustus De Morgan, who became his most vigorous supporter.



Augustus De Morgan (1806-71)

British public opinion in general, although divided, was largely favourable towards Libri at first. Although *The Times* was opposed, articles in Libri's defence appeared in the *Daily News*, *The Guardian* and *The Examiner*, as well as a host of articles by De Morgan in *The Athenæum*, where Libri was portrayed as a patriot whose belief in Italian precedence had aroused the animosity of French dogmatists such as Arago. His real crime, De Morgan claimed, was that

in science he would not be a Frenchman, but remained an Italian. One of his great objects was to place Italian discovery, which the French historians had not treated fairly, in its proper rank.

He continued,

We suspect that political animosity generated this slander, and a real belief in the minds of bad men that collectors always steal, and that the charge was therefore sure to be true.

Perhaps ironically, in order to support himself financially while in England, Libri was forced to sell a large proportion of his collection at a series of auctions. Many of these items found their way into De Morgan's library! Not surprisingly, these sales attracted further attention and criticism from the French government, and for the next few years, Libri and De Morgan published a barrage of articles and pamphlets attacking the French authorities and protesting Libri's innocence of the charges made against him. As De Morgan dryly put it, the accusation had created a new form of syllogism:

Jack lost a dog; Tom sold a cat; therefore Tom stole Jack's dog.

If the French prosecutors ever read the stream of publications issued by Libri and De Morgan on the affair, it made no difference. On 22 June 1850, a French court found Libri guilty *in absentia*, and sentenced him to ten years imprisonment. With De Morgan's unwavering support, Libri continued to argue his case from across the English Channel, but he never succeeded in clearing his name. He remained in England for the next eighteen years, finally returning to his native Tuscany (by now part of a unified Italy) in 1868. In failing health, he moved to a villa in Fiesole, where on 28 September 1869, his eventful life finally came to an end.

But the question remained: was Libri really a thief? Up to this point, all the evidence that had been compiled against him had been purely circumstantial. But his death initiated a resurgence of interest in the affair. As De Morgan observed when reporting Libri's death to the astronomer John Herschel,

of course, he being removed, the accusations which he put down [now] begin to revive.

De Morgan did not live to see the eventual outcome of this revival, as he himself died in 1871. But in a detailed investigation that stretched through the 1870s and 1880s, the French archivist Léopold Delisle finally managed to piece together the missing strands of evidence, the absence of which had previously prevented any adequate confirmation of the allegations, to prove conclusively the truth of the charges. Complex negotiations between the British, French and Italian governments eventually resulted in the return of the stolen manuscripts to France in 1888, finally bringing an end to the regrettable saga of *L'affaire Libri*.

How then can one sum up the life and work of Guglielmo Libri? So wide-ranging were his interests and abilities, yet so bizarre and inexplicable were his actions, it is difficult to draw any definitive conclusions about him. Over the years there have been several who have attempted biographical appraisals, but perhaps the most succinct was one of the earliest, made shortly after Libri's death by the French critic Philarète Chasles, who offered the following assessment:

Admirable in the salons and incomparably friendly, flexible, with gentle epigrams of sweet humour, elegant flattery, a good writer in both French and Italian, a profound mathematician, geometer, physicist, knowing history through and through, a very analytical and comparative mind…; more expert than an auctioneer or a bookseller in the science of books, this man had only one misfortune: he was essentially a thief.

Further Reading

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NOTES

[1] In 1839, Libri claimed to have shown that all properties of algebraic equations could be transferred to linear ordinary differential equations. The only result he mentioned explicitly was: if a homogeneous linear differential equation of order $m + n$ is given, and if we know another homogeneous linear differential equation of order m all of whose solutions are also solutions of the first equation, then it is possible to find a third homogeneous linear differential equation of order n such that the solutions of the two latter span the solutions of the first.

[2] See J. Liouville, Réplique à M. Libri, *Comptes rendus* **17** (1843), 445-449.

Compositio Mathematica

Gerard van der Geer (Managing editor)

From January 2004, the journal *Compositio Mathematica* will be published by the London Mathematical Society on behalf of its owners, the Foundation Compositio Mathematica. The Foundation is based in the Netherlands and its Board members are all mathematicians. The journal goes back to 1934 when L. E. J. Brouwer, expelled from the editorial board of *Mathematische Annalen* by Hilbert, founded his own journal.

The aim of the journal is to publish first-class mathematical research papers, and during the last 25 years it has earned an excellent name. By tradition, the journal focuses on papers in the mainstream of pure mathematics and includes the fields of algebra, number theory, topology, algebraic and analytic geometry, and geometric analysis.

Through the years the journal has been published by Dutch publishers, and in recent years Kluwer Academic Publishers took care of this although the ownership of the journal remained with the Foundation. Alarmed by rising prices, the Foundation has changed policy and now the London Mathematical Society will be managing the journal on their behalf, but without taking on ownership of the journal. There will be no change to the editorial policy or to the extent of the mathematics published in the journal, but there will be a change of format to a small A4 size with about 256 pp per issue. It will be published in one volume of six issues per year.

Prices for subscriptions to the 2004 volume are to be a third less than those for the 2003 volumes. The prices are 1200 euros, US$1200 or £750 for the full subscription to print plus electronic versions. The intention is that this price reduction is not just a one-off introductory offer but a permanent change to a lower pricing policy. The journal will be distributed via Cambridge University Press.

With this notice, we hope to persuade you to pass on the information to your library committees and to support this move to lower prices and learned-society publishing by taking up new subscriptions. We have agreed to buy the electronic archive of papers dating back to 1997 from Kluwer and these will be made available to subscribers from January 2005. This will provide continuity for libraries that have cancelled their subscriptions in recent years.

Mathematicians are encouraged to submit good papers to the journal. Information about the aims and scope of the journal, the Editorial Board and the procedure for submission of papers can all be found via the Foundation's website at *http://www.compositio.nl*

Interview with D. V. Anosov (part 2)

conducted by R. I. Grigorchuk

Dmitry Anosov

This is the second part of R. I. Grigorchuk's interview with Professor Anosov; the first part appeared in issue 47.

Stephen Smale played an important role in your scientific career. How did this happen?

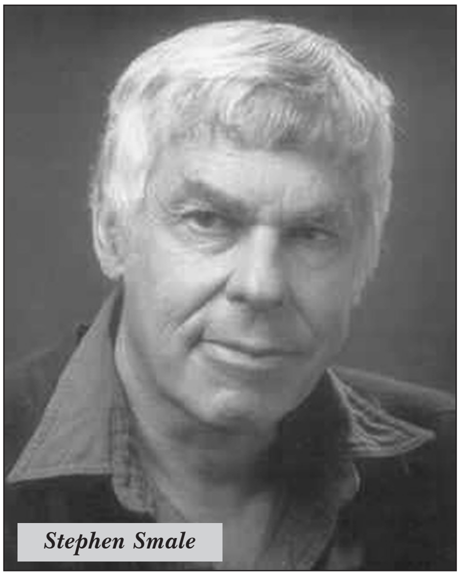
First, I need to explain what preceded our acquaintance.

Having found out from the 'Theory of oscillations' structurally stable flows in a planar region, and having some understanding of closed smooth manifolds, I naturally started thinking of structurally stable systems on n-dimensional smooth closed manifolds. These thoughts were naive. I myself came to the two conjectures that S. Smale dared to publish, which were undoubtedly known to many, that structurally stable systems are dense in the space of all $\mathbf{C}^n$ dynamical systems, and that structurally stable systems are Morse-Smale systems (using a later term). These conjectures were immediate generalisations of the Andronov-Pontryagin theorem on structurally stable systems in a bounded planar region, and of the analogous theorem by Peixoto on structurally stable flows on surfaces which appeared around 1960. Note that Smale proved the Morse-Smale inequalities, which were quite meaningful and required a non-trivial argument for Morse-Smale systems. When publishing them, Smale also published his conjectures. I (and probably many others) did not prove anything about these systems, which saved us from publishing wrong conjectures.

While attempting to show the genericity of Morse-Smale systems, I proved what is now called the Kupka-Smale theorem: or maybe I thought I proved it. The proof is based on the method of 'bringing to general position by small perturbations', which I had learned by that time, and I found the proof not difficult. Later, in 1964, when talking to Peixoto I mentioned it but he did not support me, which was not surprising — he had just published a simplified proof of it and was probably aware of some concealed underwater rocks. So he was partly right. I'm not sure now that my sketches contained a complete proof, but I was also partly right – if, under threat of death (or at least expulsion from graduate school) I had been ordered to write a complete proof of the Kupka-Smale theorem, I would have done it, and would have found and dealt with all those problems. Later I used my earlier considerations in order to prove Abraham's theorem on bumpy metrics, which hung in the air for fifteen years. This was not much of an achievement, but even so, during those years an incorrect argument was published by two well-known authors.

I must confess one of my misunderstandings. In 1959 I constructed the following example of a map f of the interval [0,1] to itself which is not invertible and not smooth: $f(x) = 3x$ ($0 \le x \le 1/3$); $2 - 3x$ ($1/3 \le x \le 2/3$); $3x - 2$ ($2/3 \le x \le 1$). I realised that the cascade $\{f_n\}$ (the dynamical system with discrete time n) is topologically transitive, has countably many periodic trajectories, and that it did not look as though it could be approximated by something like a Morse-Smale cascade – 'something like', because the map is not bijective; also, the graph has 'corners', so we cannot speak of a C^1 approximation. But I attributed this to f being not smooth and not invertible. Why didn't I (who thought in manifolds) think of the expanding map of the circle $\mathbf{R}/\mathbf{Z}$ into itself, sending x to $3x$ (mod $\mathbf{Z}$)? The issue of non-differentiability would disappear, and the role of orbit instability would probably become obvious. Then one could address the non-invertibility, and in higher dimensions – in dimension 1, there is no invertible example. In fact, why didn't I become Smale for a couple of hours? That is too much, but it seems that I could get to circle expandings, and to their connections to the one-sided Bernoulli shift and to the map $x \to \{3x\}$ (where {.} denotes the fractional part), so loved by number theorists. However, I did not think of it.

This was all about reflections, which stayed 'reflections for the soul'. In parallel, I worked on the theory of differential equations – without much spark, but with some success. The latter is due to the fact that Pontryagin and Mishchenko fortunately gave me several problems to work on, gradually increasing their difficulty and leaving me more room for initiative. Besides, by 1961 I had become rather knowledgeable about a number of questions in the classical theory of differential equations. Some things I used in my papers, but usually I learned not only what 'worked' for me but also many related things. In particular, I knew the works of A. M. Lyapunov and O. Perron on stability and conditional stability, and not only those parts related to equilibria and periodic trajectories. I also knew N. N. Bogolyubov's papers on averaging and integral manifolds; the former were related to my PhD thesis, while the latter were not in my immediate area but also influenced me.



Stephen Smale

For some reason, as well as the areas of topology and differential equations, I was attracted to Riemannian geometry, and I read (at least to a first approximation) the famous book of E. Cartan, whereas most people read other books.

So this is approximately what my status was by September 1961 when the International Symposium on Non-linear Oscillations took place in Kiev. At first I hesitated whether to go to it. I had reported on my last work (on averaging in multi-frequency systems) in the spring of 1961 at the fourth (and last) All-union Mathematical Congress in Leningrad, and that time I defended my PhD thesis on that topic. I did not want to repeat myself by speaking on the same subject. However, Smale, who was already famous in topology, but not yet in dynamical systems, wrote to Novikov that he was coming to Kiev and wanted to stop in Moscow and meet him, and maybe other young Moscow mathematicians. But not myself – he did not know about me, and there was little to know – I'd published some papers, but not in topics of interest to Smale and not of a high enough level to attract his attention. I went to Kiev mainly to assure him that Novikov and other advanced young people would be in Moscow.

The organisers had published the abstracts as booklets by the beginning of the conference (it required quite an effort then), and the abstracts of the foreigners were translated into Russian. While standing in the registration line and looking over someone's shoulders ahead, I read the title of one of them: 'S. Smale. A structurally stable, differentiable homeomorphism with infinitely many periodic points'. At this moment the world turned over for me and a new life started.

In addition to his formal lecture, Smale was kind enough to explain his discovery ('Smale's horseshoe') in more detail to a group of interested participants. There were several people from Gorky; from Moscow I remember only myself and M. M. Postnikov. Then he went to Moscow, met Novikov, Arnold, Sinai and myself at the Steklov Mathematical Institute and gave us still more details. He also remarked that a hyperbolic automorphism of the 2-torus disproves his naive conjecture of the density of Morse-Smale systems and stated two conjectures on the structural stability of this automorphism and of the geodesic flow on a closed surface (or on a closed n-dimensional manifold?) of negative (constant or variable?) curvature. We know now that Smale's horseshoe, and the torus (n-dimensional) with a hyperbolic automorphism, and the phase space of the geodesic flow are hyperbolic sets, but Smale did not know that – such a general notion did not exist then. (Even later, in 1966, after Smale had already introduced this general notion, he was doubtful – or rather, careful – about the existence of invariant stable and unstable manifolds for all trajectories from a hyperbolic set, not only for periodic ones.) But I am sure that intuitively he felt that these three examples had something in common.

I must tell you about the path that led Smale to the horseshoe. As he explained later, after he had stated his unfortunate conjectures, several people told him that he was wrong. He named only two: R. Thom and N. Levinson, who had different explanations.

Thom suggested that Smale should consider a hyperbolic toral automorphism f. It is trivial that the Lefschetz number $|L(f^n)| \to \infty$ as $n \to \infty$; therefore it holds for any g close to f. This allows one to expect that the number of periodic points of g of period $\leq n$ also tends to ∞ as $n \leq \infty$. We must exclude the possibility that there are finitely many periodic points, but the local (Kronecker-Poincaré) indices of f^n at these points are unbounded. This is impossible for Morse-Smale diffeomorphisms; it is easy to show that this is impossible for g which is C^1 close to f; this argument was obvious to Smale. We immediately deduce that f is not approximated by Morse-Smale diffeomorphisms.

On the other hand, Levinson's letter made Smale think hard. Levinson informed Smale of papers by M. Cartwright and J. E. Littlewood, and also by himself, which proved the existence of infinitely many periodic solutions for certain second-order differential equations with periodic perturbing force (which leads to a flow in 3-space), and, Levinson emphasised, this property and all major properties of the 'phase portrait' are preserved under small perturbations. According to him, this more-or-less followed from Cartwright and Littlewood's argument, but Levinson even proved it directly (for a slightly different equation). What would a mathematician do after receiving Levinson's message? He would probably start reading the corresponding papers. This is not easy, especially with Cartwright and Littlewood's papers. If one is interested only in the existence of a certain example, it is enough to read the less formidable paper by Levinson, but even that paper is hardly a gift to the reader.

Smale chose a different road. He felt that he was a god who had to create a world with certain phenomena: how could he do it? He extracted from the papers of the three authors (or maybe only from Levinson's letter?) only that strong friction and a large perturbing force are present in the corresponding examples, and at the same time, the trajectories stay in a bounded region in the phase space. The first can be modelled geometrically by assuming that trajectories approach fast in one direction and diverge fast in another. We can consider not a continuous motion, but rather iterations of a map (in this case a certain natural map – time shift by the period – is present), so that we can imagine a square becoming a long and narrow rectangle. But since the trajectories stay in a bounded region in the phase space, we should bend the rectangle so that it does not leave the region. Smale might have known the 'baker's transformation' of abstract ergodic theory: it is the following map f from the unit square $S = [0, 1]^2$ to itself: first the square is stretched horizontally and contracted vertically into the rectangle $R = [0, 2] \times [0, \frac{1}{2}]$; then R is cut into two rectangles, $R_1 = [0, 1] \times [0, \frac{1}{2}]$ and $R_2 = [1, 2] \times [0, \frac{1}{2}]$, and without moving R_1, R_2 is mapped onto $S \setminus R_1$ by $(x, y) \to (x - 1, y + \frac{1}{2})$.

The iterates f^n of this map perfectly imitate the random process consisting of an infinite sequence of coin tossings, so that its trajectories have strikingly apparent 'quasi-regular' behaviour. In abstract ergodic theory, the discontinuities of f are not troublesome. Contraction and expansion suited Smale, while the cutting of R did not – he needed a smooth map. But in ergodic theory, they want preservation of area (the corresponding measure). Smale did not need that (I think he knew that the Lebesgue measure is decreased under the time shift by the period in the Littlewood-Cartwright-Levinson case). He could therefore contract the rectangle vertically even more, bend the obtained strip, and try to put back it inside S. Experimentation with pictures – a friend of mine said that Smale 'is great at drawing pictures' – quickly led to the horseshoe.

Although Smale did not intend to go deeply into the theory of ODE, he somehow managed to learn about homoclinic points. It was not trivial then. I remember that in 1956, while speaking at the III All-union Mathematical congress, E. A. Leontovich-Andronova (the widow and co-worker of A. A. Andronov and a sister of another famous physicist, M. A. Leontovich) said that difficulties in the investigation of multi-dimensional systems are related to homoclinic points, and L. S. Pontryagin asked publicly: 'And what is that?' Strangely I do not remember the answer – maybe E. A. L.-A. refrained from answering. Of course, L. S. P. was a newcomer in the theory of ODE, but he had still been studying it for several years. And one more vivid demonstration – there are no homoclinic points in the book of Nemytsky-Stepanov!

L. S. Pontryagin

Smale realised that a horseshoe appears near a transverse homoclinic point. He might have mentioned it during our discussions in 1961. At that time it was only a plausible picture; later he wrote a paper on it. But the paper imposed certain conditions on the corresponding periodic point that actually were not necessary. L. P. Shilnikov (of Nizhni Novgorod) found a treatment of this question that made the conditions unnecessary: it can be easily achieved using the general theory of hyperbolic sets (in particular, the theorem on families of ε-orbits). Shilnikov later studied later non-transverse homoclinic points, where this does not work. He was the first to give them special attention, and although the most famous discoveries in this area belong to Newhouse, Shilnikov's contributions around that time were also significant.

By the way, I have always had good contact with mathematicians in Gorky, and some of my interests were close to theirs. Although Smale saw me only a few times, he understood that, and even counted me as a member of the 'Gorky school' (founded by Andronov), even though he knew that I lived in Moscow.

Finally, Smale described the coding of the horseshoe trajectories by a topological Bernoulli shift. (He might have known the coding for the baker's transformation with a similar shift with the standard invariant measure.) Later, a similar coding was widely used in more complicated situations arising in the hyperbolic theory, starting with the works of R. Adler-B. Weiss and Sinai. There are other studies in this direction that deal with 'genuine' hyperbolic sets, as well as with sets with weaker hyperbolic properties. In the general case the symbolic description is somewhat worse, since the phase space of a Bernoulli shift is 0-dimensional, whereas a hyperbolic set may have positive dimension. Certain 'glueing' must be performed when passing from the symbolic system to the original one. The usefulness of this depends on whether it is possible to get a good description of the glueing, but even without going into details, it is possible to have the glueing affect a very small ('negligible') portion of symbolic sequences.

As far as I know, Smale did not describe the details of his considerations. I think, though, that in this case they can be reconstructed as above. But how he guessed the structural stability of the other two systems (the hyperbolic toral automorphism and the geodesic flow on a surface of negative curvature) remains a mystery to me.

Here is how things developed further. Arnold and Sinai came up with a proof of the structural stability of a hyperbolic automorphism of the 2-torus – unfortunately, wrong. Before discovering their mistake, and taking their theorem at face value (the theorem is actually correct), I could not help but notice a certain similarity with the recent D. M. Grobman-Ph. Hartman theorem on the local structural stability of a multi-dimensional saddle. (Bogolyubov's integral manifolds were somewhere at the edge of peripheral vision.) Finally my thoughts went in the following direction: let a diffeomorphism g of the torus be C^1-close to a hyperbolic automorphism f, and assume that the latter is structurally stable; then for each point x, there must be an orbit $\{g^n y\}$ somewhere near the orbit $\{f^n x\}$. How can one be certain of its existence without assuming that f is structurally stable? How many such orbits $\{g^n y\}$ are there? As soon as I thought about it, everything became clear.

Clear to me, but I might have been the only one for whom it was obvious then. I heard that English conservatives claim that good laws and institutions are not so important, the right person in the right place is important (the English version of 'the cadre decides everything'). Of course, conservatives meant politics and economics, and in my case this formula has worked in the world of science. Through my background I turned out to be the best person for this question – I knew manifolds (within 'no-man's land') and classical stuff about differential equations. Finally, the hyperbolicity of geodesics in a space of constant negative curvature (at least in the two-dimensional case) was already known to every educated mathematician, but I learned from Appendix III to Cartan's book that hyperbolicity is also present in variable curvature.

I was not allowed to go to the International Congress of Mathematicians in Stockholm in 1962, although the powers-that-be were generally not as suspicious of young people then as later, and several of my contemporaries went. Probably the question of my trip arose too late, and I could not get an invitation. However, Arnold and Sinai informed the community of my work.

On the other hand, I was able to go to the USA in the autumn of 1964. I accompanied Pontryagin and his wife, but they spent part of the time with Gamkrelidze and Mishchenko who went for a much longer time, so I could fly to California and spend some time there. At Berkeley I, of course, spent time with Smale, who was kind enough to give my name to those dynamical systems for which the whole phase space is a hyperbolic set, and by doing so moved me to one of the top slots among mathematicians by the number of 'Math. Reviews' citations. (Later W. Thurston contributed to that by his 'pseudo-Anosov maps'. By doing this, Smale and Thurston created a discrepancy with the 'scientifically based' citation index which, as far as I know, is not very high for me. So, am I famous in the mathematical world or not?) I also met Ch. Pugh, who was then finishing the proof of the closing lemma, but later he turned seriously to hyperbolicity and co-discovered 'stable ergodicity'. Anosov systems are stably ergodic, but Pugh and Shub discovered that there are other such systems.

I repeat that, as I recollect, Smale for some time could not express in rigorous terms what he felt intuitively. My main mathematical achievement was thus that I explained to Smale what his achievement was.

Next year Smale formulated the general notion of a hyperbolic set, which became one of the central notions in the theory of dynamical systems. Together with the previously known hyperbolic equilibria and closed orbits, locally maximal hyperbolic sets acquired the role of principal structural elements that must be studied first when describing the phase portrait. Smale hoped initially that, with this expansion of the list of structural elements, the corresponding modifications of his conjectures would become correct. But as soon as he and his co-workers started analysing how these elements are connected to each other by trajectories travelling from one element to another, the conjectures were again disproved, as well as new hasty attempts to modify them.

Currently, few people, except J. Palis, risk making conjectures about the structure of 'typical' dynamical systems. (Palis's conjectures are continuously changing.) Hyperbolicity still plays an important role in the above-mentioned examples, but it is weakened hyperbolicity by comparison with the version of the 1960s. So the grand hopes of the 1960s did not come true. (Only the conjecture on the 'phase portrait' of structurally stable systems was proved: see the references below.) However, the horizon widened tremendously, although not all systems (by far) fall inside the wider field of vision. Smale's famous survey, which to a large extent is devoted to the hyperbolic theory and which for a time became the main source of inspiration for many in dynamical systems, is a bright monument to that era.

That is more or less it about my contacts with Smale and the emergence of the general concept of hyperbolicity as it happened in pure mathematics. I have not described the long prehistory that starts with the discovery of homoclinic points in Poincaré's memoir 'On the three body problem and equations of dynamics', and with Hadamard's paper on surfaces of negative curvature: enough is said about them in the literature. However, the literature connected with applications tells the history differently. The discovery of homoclinic points by Poincaré is still mentioned, but there is not a word about geodesic flows, and the main development starts with E. Lorenz's paper 'Deterministic non-periodic flow', published in 1963 in the *J. Atmospheric Sciences*, a periodical that is not very popular among pure mathematicians. As a result of numerical experiments with a specific 3rd-order system motivated by hydrodynamic considerations, Lorenz discovered several interesting phenomena. But his discovery became of wide interest only about ten years later (after hyperbolicity and related effects had become known to theorists), and applied and theoretical mathematicians became interested in it for different reasons. It proved to the applied people the existence of strange (or chaotic) attractors: hyperbolic strange attractors discovered by mathematicians were unknown to them, because such examples did not appear in the problems of natural sciences. (Not then and not now: the impression is that the Lord prefers to accept a weakened hyperbolicity rather than labour with the limitations on the topology of the genuine uniformly hyperbolic attractor of the 1960s.) Mathematicians, on the other hand, became interested in the Lorenz attractor – not as one more manifestation of known possible behaviours of trajectories, but quite the opposite, as the object which although close to hyperbolic attractors in many ways (those that impressed the applied people), is different from them in other ways: mathematicians are interested in such nuances.

Tell us more about your first works before the hyperbolic period.

We have departed from our chronological exposition. But for a person connected to mixing, etc., this comes naturally.

I have already mentioned one of my early student papers. There were others, roughly of the same level; I will not talk about them. I will describe my Candidate thesis. (There were two scientific degrees in the USSR – a Candidate and a Doctor: the latter is roughly equivalent to an American full professor, a Candidate is lower.) Usually, a topic for a Candidate thesis is offered by the scientific supervisor. This happened to me as well, and was the last time that L.S.P. offered me a topic.

M. M. Postnikov wrote that L.S.P. gave students questions for which he already knew the answers; this guaranteed the timely completion of the thesis. Postnikov himself, having realised that L.S.P. already knew everything, rebelled and wrote a thesis on a different topic – close to L.S.P.'s interests, of course, but the results of which L. S. P. did not know in advance.

I also became an exception to this usual practice. In graduate school it was proposed that I study multi-frequency averaging with slow motions influencing the fast ones. These again were singular perturbations, but in a completely different situation. The task was general and not well defined – my teachers obviously had no idea what the answer could be (convergence in initial conditions by measure). As to the time of my defence, it took place several months before the official end of the graduate school, after which to my regret I had to be expelled. Graduate student life at MIAN (during the second and third years when there is no philosophy or foreign language) is so good! – no requirements at all – but now I had to go to work, and in the morning (!) – and I am an 'owl' who works well at night. For a while I came alone in the morning and took a nap on a sofa. Fortunately, the MIAN leadership was never strict on issues of attendance and within reason muted the memoranda from above. So, in the winter of 1961-62 I could work in a more suitable regime – in the evening and at home (where, by Soviet standards, I had good conditions – a separate room).

My first paper on hyperbolicity and structural stability was written at home and at night. Later and previously, useful ideas came to me at a different time of the day and not at home, but nothing ever came to me at my work place in MIAN. On the other hand, I could do other things there, useful and even necessary. (The main one is to discuss scientific questions with colleagues. I could also edit texts, write reports, have a preliminary look at the literature from the library and decide whether to take it home for careful study, and now there is also a computer.)

I thought through many issues for my thesis while attending with many other graduate students (from MIAN and from other institutes of the AS) the mandatory lectures on dialectic and historical materialism that were given in a large hall at the Institute of Philosophy on Volhonka street, and also on the way home from those lectures. This once again shows the role of the only true Marxist-Leninist philosophy in the development of science.

Later, A. I. Neishtadt brilliantly extended my work, and in 2001 we were awarded the A. M. Lyapunov prize of the Russian Academy of Sciences for our investigations. It is curious that I received that prize for my Candidate thesis (and partly for Neishtadt as well) and that 40 years (25 for Neishtadt) had passed since its publication!

What can you say about your students and colleagues?

Formally I worked at the MSU in 1968-73 and from 1996. My main achievement is the joint seminar with A. B. Katok on dynamical systems, which was a study seminar at first and later became a scientific seminar. After Katok emigrated, A. M. Stepin became the leader of the seminar, and for a while R. I. Grigorchuk. The seminar continued to function when I did not formally work at the MSU. It has existed for 30 years, although this is not a record for the MSU.

About the students: I have never supervised anybody that started from their early undergraduate student years. M. I. Monastyrsky was my student during the first period of my pedagogical career. I was not quite fortunate with him – I always had a strong analytical component, and he was not inclined that way. Later he became a student of N. N. Meiman, a pure analyst. Nevertheless, Monastyrsky 'survived' and managed to find his 'ecological niche', actually in physical questions. Here he got lucky – some issues in physics were discovered (with his participation) that required more-or-less pure topological considerations. He is also widely known to mathematicians and theoretical physicists as the author of historical books, not on ancient history but on the Fields medals and Riemann (his biography as well as his heritage).

Then I had a couple of talented students who wrote one or two papers under my supervision, but later quit science for one reason or another. But around the same time several people appeared who were initially students of others but later, without quitting their advisors, became some way or another connected with me – sometimes formally, sometimes not. Some of these not only stayed in science, but also earned considerable recognition. The first was Katok, who cannot be considered my student – we collaborated as equals, but I was older and this showed at first. Then came M. I. Brin, Ya. B. Pesin, E. A. Satayev and A. V. Kochergin, who initially were Katok's students. A bit later (it seems that I already did not work at the MSU) Grigorchuk came to our seminar, who had initially been Stepin's student. The second period of my pedagogical career has been less successful in terms of immediate students.

I already mentioned my colleagues. At that time I communicated a lot with the late V. M. Alekseyev. From the end of the 1980s I had close ties with A. A. Bolibruch.

What interfered, and what helped, with your work?

Let me add two things.

When visiting MIAN, the famous Lefschetz

S. Lefschetz

said that the institute was good because one could do nothing there. Imagine how awful this sounds to the bureaucrats! Lefschetz was not only an outstanding scientist but also an outstanding science administrator. Princeton has not only the Institute for Advanced Study but also Fine Hall, a period of whose history is closely connected with Lefschetz. So, he understood these issues better than bureaucrats. This was what helped – now about things that have significantly interfered lately. The situation of the first post-war years (as I learned from older people) has been repeated during peace time. There are older scientists: some of them left, but many didn't, and most of those who did don't break their ties and try to compensate for their absence by spending part of the year in Russia. There are also young people (fewer than before, and definitely fewer than we wish), but there is a definite lack of people in the middle – both by age and rank – and this affects everything. This started even before 'democracy', from the end of the 1970s when few were hired by the MSU or MIAN, and those very people would carry the main load now. It is pointless to discuss how everything has deteriorated during the past decade.

Tell us about the department that you head.
In his best years Pontryagin undoubtedly dominated all of his subordinates, and the ODE department was thus a 'theatre of one actor'. He had no objection to somebody playing his role, and this is what I did after graduate school. Usually, though, the rest of the department worked on topics close to him and stayed in his shadow. This system remained in place even when he grew old and could not be the leader as before. And when, in a theatre of one actor, his performance lost former brilliance, the quality of the shows went down. They even started talking of the department's degeneration.

Curiously, an American mathematician who spent several months at the institute, wrote that everyone in the ODE department was a KGB agent except Gamkrelidze and me! I do not know where he obtained this information about the KGB. If it were not simply silly I would add that only a man of the KGB, and of high rank, could know such things. But it is true that our department did not look good then, and that Gamkrelidze and I were the exceptions, at least with respect to our research interests. (Unlike me, Gamkrelidze worked in the theory of optimal processes, but his interests in this wide field lay off the mainstream of the main group of the department.) Gamkrelidze became head of the department after L.S.P.'s death. In essence we have a different organisation now — everyone plays his own role. Also, we hired new people. The main changes took place during Gamkrelidzee's tenure, but I helped him as much as I could. (For some time before I became the Chief, the department had a laboratory for dynamical systems which I headed. Thus, even formally, I had some administrative freedom and could do some things myself.


Moscow State University

How and why was the Chair of dynamical systems created at the Moscow State University?
This was Bolibruch's, Mishchenko's and my initiative. We reckoned that the theory of dynamical systems, which historically appeared in ODE, had actually become a major independent scientific field. This was formally expressed by the fact that there was an 'ODE and dynamical systems' section at several recent congresses of mathematicians – as you see, dynamical systems play the role of an equal partner and not a part of ODE. Indeed, the theory of dynamical systems has many connections with functional analysis, probability theory, algebra, complex analysis, geometry and topology. The seminar that I founded with Katok at the MSU about 30 years ago has mainly been (and is now) affiliated with the Chair of the theory of functions and functional analysis (TFFA).

The leadership of the RAS (Yu. S. Osipov and A. A. Gonchar) supported our initiative. The decision had to be made by the MSU rector V. A. Sadovnichy, who also understood us. At its start it was understood that the Chair would be quite small (it is probably the smallest one in the department of mechanics and mathematics), but would grow later. We hope that this will happen. We cooperate with the old Chairs of differential equations and TFFA, and also with several university researchers who deal with analytical mechanics and probability. We have only begun working, so it is too early to talk about the results.

Many are interested in your work at the school reform commission.
There is no such commission. There is a commission of the Division of Mathematical Sciences of the RAS on school mathematics, of which I am the Chair. And, to believe official statements, there is no reform, there is modernisation. However, this modernisation touches on all sides of school (and not only school) education, so it is a reform, and a very deep one. Its realisation would destroy whatever was best in our education. Some even think that financial support from abroad for the reformers is connected with the hopes for precisely this result, after which Russia will quickly lose the remnants of past greatness and can be practically discounted. I do not share this point of view, since a similar process started earlier and has already gone pretty far in several western countries and first of all in the USA – well, it could be in the interests of Soviet or Chinese communists, but did they have a hand in this?

Rather, a malignant tumour has grown in education. It did not appear here, but could spread here metastatically, which would be even more dangerous to us than the original cancer to the place where it appeared.

Somebody may object: has the present state of American education led to any loss of American greatness? Answer: the drawbacks of the American educational system are partly compensated by other factors that Russia lacks; one of them is the import of specialists from abroad. Recall, however, what happened after the launch of the first Soviet sputnik. And quite recently, the Glenn commission issued a report whose title speaks for itself: 'Before it's too late'. Right now, the main US problems are connected with something else – terrorism – but it will not remain like that.

Our proposed reform-modernisation has aspects that one cannot disagree with. First is the branching of the final grade's education into several directions or, as they say now, into 'profiles'. This was so under the Czar, but in the Soviet period the idea of profiling somehow turned out to be ideologically bad. Further, only a proportion of children (in my time, at most a quarter) received high-school education earlier, those who aimed at colleges, primarily technically oriented; now it is different, and it clearly requires a change. The main mistake of the reformers of the 1960s was that they wanted to increase the content of the school material while the student population grew, and a reasonable simplification and reduction of that material was in order. Much was said, and new programmes and textbooks were prepared without sufficient thought. But even if it had been done well, the reform was unfortunate because of the principal mistake I mentioned above – they meant some ideal educational model realisable with nonexistent populations of students and teachers. And finally, when preparing people for a life in society, the school must familiarise them with the organisation of the society.

Professionally our commission deals only with the contents of school mathematical programme: the curriculum, schedule, textbooks. But we also have to discuss other sides of the reform. Speaking of the educational curriculum, unlike previous commissions, this one has no right to mandate anything. We can only express our opinion on issues, and this is what we do. Our statements have so far had little effect, but I have witnessed many epochal reforms 'for ages' (not only in education) that were followed by other reforms. I believe the same thing will happen here.

What is the role of the Moscow Mathematical Society (MMS), of whose governing board you are a member?

Presently, the leading roles in the mathematical life in Moscow are played by MIAN, the MSU and MMS; the Independent Moscow University (see below) has become more visible recently. They all differ in their objectives, administration, and type of activities: this combination is some sort of pluralism. Rephrasing a quote from a famous academician A. Yu. Ishlinsky (his area is mechanics), who spoke of the role of the scientist in industry, we can say that good consequences of pluralism are not apparent if it exists, but become apparent when pluralism does not exist.

What about the Independent University?

The Independent Moscow University (IMU) is a part of the conglomerate called the 'Moscow centre for continuing mathematical education' (MCCME). The 'continuity' in the name reflects the fact that the centre works with schoolchildren, as well as with college students. This work has many sides to it: I will give only one example.

In 2001 and 2002 a summer school was held near Dubna for high-school juniors and seniors and for college freshmen and sophomores. In between swimming and sports they had a number of lectures, accompanied by seminar-type classes. These lectures were quite different in topic and style; for example, I was into education (vector fields and related notions, mainly topological, and complex analysis), V. I. Arnold talked about new things (for example, in 2001 it was asteroidal geometry 'and all that'). Notes were taken, and a considerable portion has already been published as booklets. In general, I praise the publishing aspects of MCCME (which is wide with respect to the range and level of books and booklets – from quite popular to the newest).

Now the IMU. For a number of reasons it is not an ordinary university, but supplements courses for MSU students and for students from other colleges (including some high-school students). I gave several courses there. No matter what one calls it, these courses are useful and important. One can find out only from there about a number of new developments in mathematics (and about some older things as well). Several of our scientists, and foreign visitors, have given lectures and minicourses there, and in this respect the IMU is definitely the leader. (The MSU leads in long-term topics courses – so to say, sprinters and long distance runners.)

The IMU consists of two colleges: mathematics and mathematical physics; the former is larger, while the second uses MIAN as a base. When it was created, it was hoped to cover other fields of natural sciences, but this has not yet happened.

Bibliography

My book [1] contains my doctoral thesis. Smale's famous survey is [2]. We both mention works on geodesic flows on manifolds of negative curvature, beginning with Hadamard, and give other references, for example on structural stability; later I published a historical paper on the latter subject [3]. More new information on this subject and hyperbolic theory can be found in [4], which also contains some information on the Lorenz attractor (including references for my early publications). My Candidate thesis is [5]. M. M. Postnikov's article is [6].

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Mathematical Society of Japan Prizes, 2003

The Spring Prize and the Algebra Prize of the Mathematical Society of Japan (MSJ) were awarded at its Annual Meeting in Tokyo in March 2003.

The Spring Prize is awarded each year to a mathematician who is not older than forty years old and who has made an outstanding contribution to mathematics. The 2003 Spring Prize was awarded to **Tomotada Ohtsuki** of Tokyo University for his distinguished contributions to the study of quantum invariants for 3-manifolds.

After the discovery of the Jones polynomial, much work has been devoted to topological invariants of knots and 3-manifolds. Tomotada Ohtsuki entered this field in the beginning of the 1990s with his notion of finite type invariants for 3-manifolds. He defined an arithmetic perturbative expansion of 3-manifold invariants due to Witten and Reshstikhin-Turaev for homology 3-spheres, and showed that each coefficient of this expansion is of finite type in his sense. Later, in collaboration with Le Tu Quoc Thang and Jun Murakami, Ohtsuki established universal finite type invariants for 3-manifolds with values in the algebra of Jacobi diagrams, based on the Vassiliev-Kontsevich invariants for framed links. Ohtsuki's achievements clarified greatly combinatorial aspects of quantum invariants for 3-manifolds and shed new light on our understanding of the class of homology 3-spheres.

The Algebra Prize is awarded every year to one or two algebraists in recognition of outstanding contributions in algebra. This prize was established in 1998 with funds donated to the MSJ, and awarded to Takeshi Saito and Hiroshi Umemura in 1998, to Kazuhiro Fujiwara and Masahiko Miyamoto in 1999, to Koichiro Harada in 2000, to Tamotsu Ikeda and Toshiaki Shoji in 2001, and to Masato Kurihara in 2002. The 2003 Algebra prize was awarded to **Kei-ichi Watanabe** of Nihon University for his outstanding contributions to commutative ring theory and its applications to singularity theory.

The Latvian Mathematical Society after 10 years

Alexander Šostak

In January 2003 the Latvian Mathematical Society (LMS) marked its 10th anniversary. Compared with such bodies as the London Mathematical Society (established in 1865) or our neighbour the Estonian Mathematical Society (which celebrated its 75th anniversary in 2001), we are still in our infancy, being one of the youngest mathematical societies in Europe. However, mathematics and mathematicians existed in Latvia long before the LMS was founded, and therefore I would like to say some words about the prehistory of the LMS, mentioning some facts about mathematics and mathematicians related to Latvia before 1993 when the LMS was founded.

Piers Bohl

Probably, the first outstanding mathematician whose life and work was closely related to Latvia was Piers Bohl. He was born in 1865 in Valka (a town on the border between Latvia and Estonia) in the family of a poor German merchant. In 1884, after graduating from a German school in Viljandi (Estonia) he entered the faculty of physics and mathematics at the University of Tartu [1]. Two years later, in 1886, Piers presented his first research work 'Invariants of linear differential equations and their applications', for which he was awarded the 'Golden Medal'. Bohl graduated from Tartu University in 1887, earning a Master's degree at the same university in 1893. In his Master's thesis, *Über die Darstellung von Functionen einer Variabeln durch trigonometrische Reichen mit mehreren eines Variabeln proportionalen Argumenten*, he introduced a class of functions which, ten years later, were rediscovered by the French scientist E. Esklangon under the name *quasiperiodic functions*.

In the period 1887-95 Piers Bohl worked as a teacher of mathematics in countryside schools, but in 1895 he was invited to be the Head of the Department of Mathematics at the Riga Polytechnic Institute – at that time the only institute of higher education in the territory of Latvia. In 1900 he was awarded the degree of Doctor in Applied Mathematics from Tartu University for his research work in the applications of differential equations to mechanics. He continued intense work in different areas of mathematics – in particular, in the field of differential equations. In his paper 'Über die Bewegung eines mechanischen Systems in die Nähe einer Gleichgewichtslage' (*J. reine, angew. Math.* **127** (1904), 179-276), he studied the existence and smoothness problems of stable and unstable manifolds for a quasilinear system of differential equations. In the course of studying these problems, as auxiliary results he established that *a sphere is not a retract of a ball* and proved that *a continuous mapping of a ball into itself has a fixed point*, a famous result which is often called the 'Brouwer fixed-point theorem', although Brouwer published the result seven years later (see L.E.J. Brower, 'Über die Abbildung von Mannigfaltigkeiten', *Math. Ann.* **71** (1911), 97-115. In his paper, 'Über ein Dreikörperproblem', *Z. Math. Phys.* **54** (1906), 381-418, Bohl proved a famous theorem about quasiperiodic functions, as well as some important results about differential equations with quasiperiodic coefficients. Of course, here we have been able to mention only a few results of this insightful mathematician.

It is worth mentioning that Piers Bohl was known also as a very strong chess player. One of the chess openings discovered by him is known as the 'Riga version of the Spanish game'. During the First World War, Riga Polytechnic Institute was evacuated to Moscow, and Bohl spent three years in the capital city of Russia. He returned to (now already independent) Latvia in 1919 and was elected Professor of the Engineering faculty of the recently founded University of Latvia. Unfortunately, two years later in December 1921, Piers Bohl died from a cerebral haemorrhage.

[1] We use here modern geographical names. At the time when Bohl studied in Tartu this city was called by its German name Dorpat, but later, by the time he defended his Doctoral Thesis, it had been renamed by the Russians as Yurjev.

The first Society of Latvian Physicists and Mathematicians.

Of all the mathematicians who were active during the time known as the first republic of Latvia (1918-40) we shall mention Edgars Lejnieks, Alfreds Meders, Eižens Leimanis, N. Brazma (Brower), Kārlis Zalts and J. Cizarevičs. None of them is known as a world-famous mathematician or as the founder of a 'mathematical school', but each played an enormous role in the development of the mathematical culture and education in a newly founded state – the Republic of Latvia. The first time that Latvia gained its independence was after the World War I in 1918, and this was also the first time that Latvian came into use as the official language and that students could acquire education in their native language.

Edgars Lejnieks was born in Riga in 1889. After graduating from Moscow University in 1907, he worked for a time in Göttingen University. In 1919 he returned to Riga and was elected the first dean of the faculty of mathematics and natural sciences at the University of Latvia. In 1923 he organised the first congress of Latvian mathematicians. Lejnieks is also recognised as the founder of the library of the University of Latvia, which now contains the largest collection of books and journals in mathematics and physics in Latvia (more than 180,000 items).

On 10 March 1939 the Latvian Society of Physicists and Mathematicians (LSPM) was founded. The first Chair of the Society was the physicist F. Gulbis. The mathematician members of this society included E. Leimanis, A. Lūsis, A. Meders, P. Putniņš, N. Brazma and E. Grinbergs: on the whole, the physicists and astronomers were better represented and more active in that society than the mathematicians. The Society held about ten scientific sessions. In December 1940 the activity of the Society's activities were stopped by the Soviet authorities, who aimed to exclude the appearance of national thought in all its possible forms. The Society had a brief recovery in January 1943, but in May 1944 it was again closed. The total time that the Society existed was 26 months. The conclusion of these not very cheerful glimpses into the history was that after Latvia regained its independence in 1991, the physicists decided to establish the Latvian Physical Society as the renewed LSPM, while the mathematicians founded the Latvian Mathematical Society as a new body.

Mathematics in Latvia during the Soviet period (1945-91)

In the years 1940-45 some mathematicians emigrated to the West, while others found themselves in Stalin's camps. Thus the face of mathematics in Latvia started to produce other persons. Some of them originated from Latvia, but after graduating from the University of Latvia they often went to develop their theses at other universities – usually Moscow (Āriņš, Klokov, Reiziņš *et. al*) and Leningrad (Detlovs, *et al.*), where the level of mathematical education and researches was very high. At the same time, a considerable number of mathematicians, some outstanding ones among them, came to Latvia from other parts of the USSR. This process was especially intense during the first fifteen years after the incorporation of Latvia into the USSR.

There were two main stimuli for this process. On the one hand, the living standards in Latvia (and in the Baltic on the whole) were relatively higher than in the 'old part' of the Soviet Union, and hence there was a natural interest by scientists (and others!) to come to live and to work in the Baltic. On the other hand, the politics of the Soviet government (with regard to the incorporated states) was to assimilate the local population, and it therefore encouraged such a movement of people. In any case, this process was beneficial for mathematics in Latvia, since the Latvian mathematical community was replenished with such serious mathematicians as A. Myshkis, B. Plotkin, I. Rubinštein, M. Goldman, S. Krachkovskij, L. Ladyzhenskij and others. Some of them created strong schools here in different areas of mathematics: we should especially mention A. Myshkis, B. Plotkin, I. Rubinštein and M. Goldman, whose strong influence on mathematical life in Latvia is still felt nowadays. Unfortunately, their professional activity was also often hampered by the authorities.

We now mention briefly some of the mathematicians who 'defined the mathematical face of Latvia' in this period and afterwards.

Eižens Āriņš (1911-84), a Latvian, was born in Krasnojarsk, in Siberia, where his father was in exile. In 1920 the family returned to Riga. After having graduated from the University of Latvia, Āriņš continued his education at postgraduate level at the Moscow State University. He developed his Candidate (PhD) thesis under the supervision of L. V. Keldysh, a famous Russian topologist. Āriņš was a mathematician with a diverse spectrum of interests. He wrote papers in the descriptive theory of functions (his thesis was devoted to partially continuous functions on products of topological spaces), theoretical computer science, and cybernetics. Besides, Ariņš had a great organisational talent: in particular, he is remembered as the founder and the first director of the computer centre at the University of Latvia, one of the first such institutions in the Soviet Union.

Emanuels Grinbergs (1911-82) was born in St Petersburg into a family of diplomats and artists. In 1930 he entered the University of Latvia, and graduated from it in 1934. Later he studied at the *Ecole Normale Superieure* in Paris. Grinbergs was elected as docent at the University of Latvia in 1940. In 1943 he defended his doctoral thesis, 'On oscillations, superoscillations and characteristic points', but unfortunately was then drafted into the German Army. In 1944 the Soviets again entered Latvia, and his presence in the German Army (over which he had no choice) was regarded as an unforgivable sin. He was sent to Stalin's camp in Kutaisi in the Caucasus.

Although permitted to return to Latvia in 1947, he was only allowed menial work at a factory. Only after Stalin's death in 1954 was Grinbergs permitted to work at the University. In 1956 he defended his Candidate thesis, 'Problems of analysis and synthesis in simple linear circuits' (his first thesis was not acknowledged by the Soviet authorities, since it had been defended during the period of German occupation), and was then appointed as Head of the section in the computer centre of the University of Latvia. The main contributions of Grinbergs and his collaborators were in the theory of electrical circuits, in research on non-linear circuit theory, in magnetic hydrodynamics, in telephony, and in the development of numerical methods in the analysis of Markov processes. A significant achievement of Grinbergs' group was the development of analytic methods for calculations of planar contours of 3-dimensional components of the hulls of ocean-going ships. These methods were highly acclaimed throughout the Soviet Union and were used by many shipbuilding companies. However, Grinbergs' main interest was in graph theory, where he had several publications and an archive of about 20,000 (!) pages of unpublished papers.

The first mathematician who graduated from the University of Latvia and spent all his working life lecturing at that university was **Arvids Lūsis** (1900-69), whose brother Jânis became a well-known geneticist. Born to a family of peasants, he received his first education at a village school, and then entered the University of Latvia from which he graduated in 1924. During the following 45 years he lectured in different courses of theoretical mechanics and applied mathematics. His main scientific interests were in differential and integral equations, but he is mostly remembered as a the teacher of many Latvian mathematicians who started their research work in the second half of the 20th century.

Anatoly Myshkis, at that time one of the leading USSR specialists in the field of differential equations, came to Riga and started to work at the University of Latvia in 1948. He was soon promoted to the post of Head of the Department of Higher Mathematics. His outstanding skills of lecturing and competence drew to him many good young mathematicians and students. As a result, a very strong group of researchers in the field of differential equations was created, and can still be felt, more than 50 years later. It is unfortunate that he soon had to relinquish the post of head of the department, due to political insinuation, and then in 1956 was forced to leave the University of Latvia and Latvia forever. Nevertheless, his influence on the development of mathematics there was (and still is) extremely strong. In particular, most of the people researching into differential equations (H. Kalis, L. Reiziņš, A. Reinfelds, A. Lepins, *et al.*) can consider themselves as the mathematical offspring of Myshkis.

One of the outstanding representatives of this school was **Linards Reiziņš** (1924-91). Linards' studies at school were interrupted by the Second World War. To escape being drafted into the German Army, he went to the North of Latvia and then to Estonia. Near the small town of Paide he was arrested by German soldiers and taken as a prisoner to Riga. He was released in 1942, and soon after, he passed his high-school final exams. After the war he entered the University of Latvia, and after graduating in 1948 he continued his studies at postgraduate level under the guidance of Lūsis, specialising in the field of differential equations. In his Candidate thesis he studied the qualitative behaviour of homogeneous differential equations and obtained results that were highly regarded by specialists.

A large part of Reiziņš' research energy was directed to the investigation of conditions under which two systems of differential equations are dynamically (topologically) equivalent. Among other results he found a formula giving the relationship between the solutions of full and truncated systems of differential equations. During his later years he worked on Pfaff's equations.

Reiziņš was the author of two monographs. He was the scientific advisor of several students who later became successful researchers and administrators – among them, Andrejs Reinfelds (the director of the Institute of Mathematics), Ojārs Judrups (from 1991-2002 the dean of the faculty of physics and mathematics at the University of Latvia) and Kārlis Dobelis (a former rector of the Liepāja Pedagogical University).

Ernsts Fogels worked in number theory, and his main interest was the ζ-function. Trying to solve the famous problem on the location of the zeros of the ζ-function, Fogels obtained a number of important results as by-products. In particular, he discovered new effective proofs of the Gauss-Dirichlet formula on the number of classes of positively definite quadratic forms, as well as the de la Vallée-Poussin formula for the asymptotic location of prime numbers in an arithmetic progression.

In the 1960s and 1970s, much interest among Latvian mathematicians was focused on functional analysis. Especially, noteworthy was the work of **Sergej Krachkovski** and **Michael Goldman**, whose scientific interests included Fredholm-type functional equations and the spectral theory of linear operators. Further work in functional analysis was done by Imants Kārkliņš, a student of D. Raikov, and Andris Liepiņš.

In 1960 a very talented algebraist, **Boris Plotkin**, came to Riga. His scientific competence, energy and enthusiasm attracted many young and not-so-young mathematicians. Under his supervision several dozen of Candidate (= PhD) and doctoral theses were worked out. (Recall that in the USSR there were two levels of scientific degree – Candidate of Sciences and a Doctor of Science; the demands for the second one were usually very high.)

In 1967 a large algebraic conference was organised in Riga by Plotkin and his collaborators. One of his important accomplishments was the founding and guiding of the 'Riga Algebraic Seminar' – well known among specialists throughout the whole Soviet Union and in other countries. Among active members of this seminar were many of Plotkin's former students: I. Strazdiņš, A. Tokarenko, L. Simonyan, Ja. Livchak, A. Kushkuley, L. Gringlass, J. Cīrulis, A. Pekelis, A. Bērziņš, R. Lipyanskij, Je. Plotkin, V. Shteinbuk, L. Krop and I. Ripss. The last of these merits special attention.

The most talented of Plotkin's students, Ripss, was very sensitive and keenly felt the injustice reigning around him. On 13 April 1969 he attempted self-immolation in the centre of Riga, near the Monument of Freedom, protesting the Soviet invasion of Czechoslovakia in August 1968. The militia beat him severely and he was later committed to a mental hospital; then, after some days, he was imprisoned. The conditions in the prison were terrible, but nevertheless Ripss found the strength and energy to continue his work in mathematics and to obtain some brilliant results. In particular, while in prison, the 20-year-old mathematician solved the famous long-standing Magnus problem on dimensional subgroups, constructing a counter-example for the case $n = 4$. This was a major event in 20th-century mathematics. However, for political reasons, this work could not be published in the Soviet Union and only later was it published in Israel, creating a great deal of interest among mathematicians.

One of the consequences of Ripss' action was that Plotkin, the author of five monographs and an outstanding scientist and lecturer, lost his job at the University of Latvia: the 'crime' of a student was also the 'sin' of the teacher, arising from 'bad education' – this was an unwritten rule of the Soviet system. Fortunately, Plotkin had influential friends who helped him to find a job in another institution, the Riga Institution of Aviation. As a result, the 'Algebraic centre' in Latvia moved from the University of Latvia – its appropriate home – to the Institute of Aviation.

Many of the members of the Riga Algebraic Seminar have now emigrated from Latvia, mainly to the USA and Israel. Since 1996 Plotkin has been living and working in Jerusalem. However, he still keeps close contact with colleagues and former students in Latvia and regularly comes to Riga and gives talks at the sessions and seminars of the LMS.

Mathematics in independent Latvia

In the late 1980s in Latvia, as well as in other Baltic republics, much of the country's intellectual and artistic energies were focused on the independence movement. This movement in Latvia was called *Atmoda* (Awakening) and showed itself in different forms. In particular, organising national scientific and professional Societies was among the most vivid and popular ideas. (However, as the experience of Estonian mathematicians shows, the idea of organising such societies during the Soviet Era was usually put to an end by the state authorities, who were afraid of any national movement.)

One of the first bodies of this kind was the Union of Scientists of Latvia, created in 1988. The Latvian Mathematical Society was founded on 15 January 1993 by the decision of the Constituent Assembly of mathematicians of Latvia. Before its foundation much preliminary work was done by a group of enthusiasts, including Uldis Raitums (the first Chair of the Society, from 1993-97), Andrejs Reinfelds (the present Chair), Jānis Cīrulis, and the author of these notes (the Chair from 1997-2000). The Society is governed by a Board of seven members, one of which is the Chair of the Society. As its main aims, the society considers the following activities:

Participants of the 4th conference of the LMS in Ventspils. The castle is mid-14th century of the Livonian Order.

- Consolidating the mathematical community of Latvia, to make Latvian mathematicians feel as a body and not as isolated individuals. In particular, the Society aims to stimulate exchange of ideas between mathematicians representing different areas and fields of science. This is especially important in a country of the size of Latvia, where the number of mathematicians is small. Besides being interested in various fields of mathematics, they usually keep closer scientific contacts with foreign colleagues and not with their compatriot colleagues.
- Stimulating the development of Latvian mathematical terminology. This is especially important since the overwhelming majority of mathematical publications are written in English, while our lectures for students are usually given in Latvian.
- Representing Latvian mathematicians and mathematics developed in Latvia in international circles such as the European Mathematical Society, and establishing to keeping contacts with mathematical societies in other countries.
- Supplying libraries with current mathematical publications. This was especially urgent during the first years of independence, when the financial situation of our universities and our Academy was extremely poor and they could afford no new publications. During these years the Latvian Mathematical Society received books, journals and other kinds of information, through some agreements in the form of exchange, as donations, or by greatly reduced prices. In particular, we have got such important materials for every working mathematician as *Mathematical Reviews* and *Zentralblatt-MATH*. (Here especially should be noted the activities of the AMS by the FSU program, as well as support by the EMS and personally by Bernd Wegner, the Editor-in-chief of *Zentralblatt-MATH*.

As one of its main goals, the Society views the organisation of seminars and conferences. During the first ten years of its existence, the LMS organised more than one hundred seminars, at which both local mathematicians and their foreign colleagues have given talks. In particular, the following speakers have given talks at our seminar: Wiesław Želazko and Tomasz Kubiak (Poland), Wesley Kotzè (SA), Andrzhej Szymanski (ASV), Mati Abel, Arne Kokk and Heino Turnpu (Estonia), Ulrich Höhle, Hans Porst, Horst Herrlich and Helmut Neunzert (Germany), Boriss Plotkin (Latvia-Israel), Juris Steprāns (Canada), and many others. The topics presented at these seminars covered practically all fields of mathematics.

The LMS organises biannual conferences. The guiding principle of these conferences is that they are not specialised: everyone working in mathematics in Latvia is welcome to give a talk. The working language of the conference is Latvian – this has both positive effects (stimulating the development of mathematical terminology in Latvian) and negative ones: there are usually no foreign guests at these conferences – the only exception was the third conference held in 2000, where about 20 foreign guests were invited by Andžans to commemorate the 50th anniversary of the movement of Mathematical Olympiads in Latvia. The first two conferences (in October 1995 and 1997) were held in Riga, the capital of Latvia, where about 80% of Latvian mathematicians live and work, while later we decided to go to other regions, in order to propagate mathematics more widely while gaining a better conception about the problems of mathematicians in the rest of the country. For example, our third conference in April 2000 was in Jelgava, a town known for its University of Agriculture. The fourth conference in April 2002 was in Ventspils, a town in the west of the country, in the region called Kurzeme, an important harbour on the Baltic Sea. The main organisers of these conferences were Andrejs Reinfelds and local organisers: Aivars Aboltiņš from the University of Agriculture in Jelgava, and

Jānis Vucāns, the rector of Ventspils University. The next conference will be held in 2004 in Daugavpils, a city in the Southeast of Latvia in the region called Latgale.

So far about the activities of the society as a whole. However, any professional society gains its strength through its members and their scientific and social activities. So, here are the main fields in which Latvian Mathematicians are working.

We first mention *ordinary differential equations*, interest in which was essentially influenced by P. Bohl and later by A. Myshkis and L. Reiziņš (see above). The leading people in this area are now Andrejs Reinfelds, Yurij Klokov, Arnold Lepin and Felix Sadyrbajev. Reinfelds' main scientific interests are the qualitative theory of differential equations, dynamical systems and impulse differential equations; Klokov, Lepin and Sadyrbajev work in the qualitative theory of ordinary differential equations, studying boundary-value problems (Klokov, Sadyrbajev) and the topological properties of solutions of differential equations (Lepin).

Problems of *optimal control for partial differential equations and calculus of variations* are investigated by Uldis Raitums, who also works on the problem of *G*-convergence of elliptic operators.

In recent decades one of the most rapidly developing areas has been *mathematical simulation*, represented by Andris Buiķis, Jānis Cepītis, Andrejs Reinfelds, Aivars Zemītis and their students. One of the principal aims of this group is to develop mathematics for the needs of industry, specifically for the wood industry that is among the most important branches in Latvia. Evident confirmation of the success of the activities in this direction were provided by the seminar 'Baltic days: Mathematics for industry' in June 1995 with the participation of ECMI experts, and the 14th ECMI conference in September 2002. Both of these events took place in Jurmala, near Riga, and were organised under close collaboration with the LMS. For more information about these and related activities of Latvian mathematicians, see ECMI Newsletter **29** (March 2001), 20-23.

Research in *partial differential equations* received a strong impulse from Isaak Rubinshtein, who came to Riga in 1961 and left for Israel in 1974. The main representatives of this field are now Andris Buiķis, Harijs Kalis, Maksimilian Antimirov, Vladimir Gudkov and Andrej Kolyshkin.

Important results in the *theory of functions* were obtained by Eduards Riekstiņš (1919-92). In particular, he studied problems of asymptotic expansion of functions: he was the author of five monographs related to this subject. The fate of Riekstiņš during the war and the first years after it has much in common with Reiziņš' fate which we discussed above. Interesting results in the field of the theory of functions were obtained by Georgs Engelis (special functions, operator theory), Teodors Cīrulis and Svetlana Asmuss (approximation and expansion) and their collaborators.

Jelgava Palace. At present the Latvian University of Agriculture is in this palace. The 3rd coference of the LMS took place here in April 2000.

The 3rd LMS conference in Jelgava, April 2000

Researches in *mathematical logic*, and in particular in the theory of recursive functions, started in the early 1950s by Vilnis Detlovs who, after graduating from the University of Latvia, completed his Candidate thesis under the supervision of A. A. Markov (Junior) at the University of Leningrad. One of Detlovs' first students, Jānis Bārzdiņš, developed his Candidate thesis in Novosibirsk (Russia) under the supervision of a well-known Russian logician B. A. Trahtenbrot: the thesis was devoted to the problem of universality in automata theory. Bārzdiņš continued his professional education at the Moscow State University where he wrote his Dr Sc thesis, collaborating with the famous Andrej Kolmogoroff. The most active in the field of mathematical logic is currently Jānis Cīrulis, who has important results in algebraic and many-valued logics as well as research papers in different areas of *abstract algebra*.

The field of *algebraic geometry* is represented by the talented mathematician Aivars Bērziņš, who, in particular, has found conditions under which two extensions of a given field have the same geometry.

Set-theoretic topology, the direction started in Latvia by Michael Goldman in the 1960s is now developed mainly by the author of these notes and his students. Our group is currently working in classical set-theoretic and in categorical topology, as well as in *many-valued* (*or fuzzy*) *topology*.

The field of *numerical analysis* is represented by Harijs Kalis, Andris Buiķis, Teodors Cīrulis and Ojārs Lietuvietis.

Stochastic aspects of Probability theory is the field of research by Jevgenij Tsarkov and his group including Kārlis Šadurskis, at present the Minister of Education and Science in Latvia.

Mathematical statistics is represented by Jevgenij Tsarkov, Aivars Lorencs, Jānis Lapiņš and their students and collaborators.

It should also be noted that a large amount of work in the field of *theoretical computer science* is carried out by a group of

Latvian scientists, headed by Jānis Bārzdiņš and Rusiņš Mārtiņš Freivalds. In particular, the results obtained by Freivalds and his collaborators in the theory of probabilistic automata and of quantum computers are well known and highly regarded by colleagues all over the world. Problems of structural synthesis of probabilistic automata are studied also by Aivars Lorencs.

As a separate direction in mathematics in Latvia, we should mention *modern elementary mathematics*, whose founder and undoubted leader is Agnis Andžāns. Every year Andžāns and his group organise two olympiads for school pupils interested in mathematics. In one of these olympiads all pupils can participate on their own initiative. To participate in the other one, a pupil has to get good results in a series of local olympiads. Many advanced textbooks and problem books in elementary mathematics have been published. As evidence of the immense work done by Andžāns and his group (including Aivars Bērziņš, Andris Cībulis and Līga Ramana) are the relatively high places which the Latvian team usually earns at international mathematical competitions. In 1998 the World Federation of National Mathematical Competitions selected Andþâns as a recipient of a Paul Erdös Award to honour his 'significant contribution to the enrichment of mathematical learning in Latvia'. Andžāns was the main organiser of the 2nd International Conference on 'Creativity in Mathematical Education and Education of Gifted Students', which took place in Riga last July.

Last, but not least, we wish to emphasise the important role played by an outstanding teacher of mathematics, Jānis Mencis (Senior) in the development of *mathematical education* in Latvia. His son, Jānis Mencis (Junior), has followed in his father's footsteps and is also the author of several mathematical textbooks for schools. For their enormous contributions to the rise in level of the mathematical culture of pupils in primary and secondary schools in Latvia, Jäanis Mencis (Senior) and Agnis Andžāns were awarded the 'Three Star Order', the highest national award from the Latvian Government.

Acknowledgements
When preparing this article we used materials written by L. Reiziņš, J. Dambītis, I. Rabinovičs, I. Heniņa, D. Taimiņa. In particular, see the electronic version of 'Mathematics in Latvia through the centuries' by D. Taimiņa and I. Heniņa in *www.math.cornell.edu/~dtaimina/mathin.lv. html* Information about the first Society of Latvian Physicists and Mathematicians was obtained from a physicist Jānis Jansons. The author acknowledges discussions with Jānis Cepītis, Jānis Cīrulis, Boriss Plotkin, Uldis Raitums and Andrejs Reinfelds, which helped to shape these notes. We express special gratitude to Juris Steprāns, a Canadian mathematician of Latvian descent, who helped to improve the language style of these notes. In conclusion we emphasise that, although the author has tried to be unbiased and has discussed the contents of these notes with colleagues, the flavour of subjectivity is inevitable in such a paper.

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Slovenian Mathematical Society: some further comments

Peter Legisa

In my report on our Society, in the previous issue, I concluded with an unfortunate personal critical opinion of connections with our northern and western neighbours. It stirred dust and I should like to do some explaining.

It is a fact of life that undergraduates who study abroad tend to stay abroad. For obvious reasons, many undergraduates study at universities directly across the border. The concern of losing too much talent is one of the reasons why Slovenia is now establishing the third university near our western border. (Language is the main obstruction to the arrival of foreign undergraduates in our country.) Graduate students, who are more likely to return, disperse far and wide, and in countries to the north and west cooperation seems to be better with the more distant centres. With hindsight I connected these two facts, plus some political concerns, in a way that strongly resembled the proverbial elephant in a china shop. No wonder that mathematicians who invested a lot of effort in our cooperation have been upset.

I received a letter from Prof. Dr. Heinz W. Engl, President of the Austrian Mathematical Society. He presented counter-examples to several of my conjectures. He also helped to dispel many of my concerns and offered more contacts, a gesture that I appreciate very much. My colleagues also informed me that there was much more cooperation than I was aware of, especially with Austria. They even warned me against including a list of names, since contacts were so numerous I would almost surely omit somebody. They also told me that personal relations with mathematicians across the border are excellent and that even in political discussions agreement was quite common.

To correct the imbalance in my previous writing, I shall state some facts.

- Prof. Josip Globevnik from Ljubljana gave an invited plenary lecture at the International Congress of the Austrian Mathematical Society in Linz in 1993. We can consider this as a formal contact between the two societies.
- The extremely fruitful cooperation with Montanuniversität in Leoben resulted in a joint book, several PhD degrees and exchanges of graduate students. Prof. Wilfried Imrich (Leoben) recently held a colloquium talk in Ljubljana to a large audience. The LL [Leoben-Ljubljana] Seminar in Discrete Mathematics is an annual (or semi-annual) meeting with 30-40 participants and a history going back two decades.
- There is a long established cooperation between universities in Graz and Maribor (as well as Ljubljana), producing at least one PhD degree. We work with other Austrian universities as well.
- There were two joint regional conferences: one, concerning technology-supported mathematics teaching, was organised in 2000 in Portorož on the Adriatic coast (by the universities in Maribor and Klagenfurt/Celovec).
- If we look towards Italy: Prof. Tomaž Pisanski from Ljubljana lectured at the University of Udine for three years. Exchanges in graduate students produced at least one PhD degree in Italy and a MSc degree in Slovenia.
- Several of our colleagues have been guests in the Abdus Salam International Centre for Theoretical Physics in Trieste. There have been other joint ventures in numerical analysis, discrete mathematics, geometric topology, etc.
- There is good research co-operation with Hungary, another bordering country.

The title 'Links in my report' was inaccurate, since I limited myself to regional contacts. I never mentioned the prevailing connections of the Slovenian mathematical community with other countries. (A high point was reached this year when Prof. Dana S. Scott of Carnegie-Mellon University (US) was awarded an Honorary PhD degree by the University of Ljubljana.)

Finally, I turn to the future. Slovenia voted overwhelmingly for membership in the EU and that means better opportunities for exchanges of graduate students and researchers in all directions. I am also glad to quote Prof. Engl of the Austrian Mathematical Society:

Our next international congress in Klagenfurt in 2005 will be specifically devoted to contacts to our neighbours in the south and in the east, just as we do this year with our Italian colleagues in Bolzano.

Forthcoming conferences

compiled by Vasile Berinde (Baia Mare, Romania)

*Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to one of the following addresses: **vberinde@ubm.ro** or **vasile_berinde @yahoo.com**. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared.*

July 2003

1-5: EuroConference VBAC 2003, Porto, Portugal
Information: ttp://www.math.ist.utl.pt/~cfloren/VBAC2003.html
[For details, see EMS Newsletter 47]

1-10: PI-rings: structure and combinatorial aspects (summer course)
Bellaterra (Barcelona), Catalonia
Information: e-mail: PI-rings@crm.es
website: http://www.crm.es/PI-rings
[For details, see EMS Newsletter 47]

3-8: Wavelets and Splines, St Petersburg, Russia
Information: e-mail: ws@imi.ras.ru
website: http://www.pdmi.ras.ru/EIMI/2003/ws
[For details, see EMS Newsletter 47]

6-12: Journées Arithmétiques XXIII, Graz, Austria
Information: e-mail: ja03@tugraz.at
website: http://ja03.math.tugraz.at/
[For details, see EMS Newsletter 47]

9-12: XV Italian Meeting on Game Theory and Applications, Urbino, Italy
Information: e-mail: XVimgta@uniurb.it
website: www.econ.uniurb.it/imgta
[For details, see EMS Newsletter 46]

13-28: VI Diffiety School in the geometry of Partial Differential Equations, Santo Stefano del Sole (Avellino), Italy
Information: e-mail: school@diffiety.org;
website: http://www.diffiety.ac.ru or http://www.diffiety.org.
[For details, see EMS Newsletter 47]

14-18: International Conference on Algebras, Modules and Rings, Lisboa, Portugal [in memory of António Almeida Costa, on the centenary of his birth]
Information: e-mail: : lisboa03@cii.fc.ul.pt;
website: http://caul.cii.fc.ul.pt/lisboa2003/
Subscribe to the conference mailing list at http://caul.cii.fc.ul.pt/alg.announce.html
[For details, see EMS Newsletter 46]

20-27: Hodge Theory in a New Century - A Euro Conference celebrating the Centenary of Sir William Hodge (1903-75), Edinburgh, UK
Information: e-mail: icms@maths.ed.ac.uk
http://www.ma.hw.ac.uk/icms/meetings/2003/HODGE/index.html
[For details, see EMS Newsletter 47]

21-25: The Władysław Orlicz Centenary Conference and Function Spaces VII, Poznań, Poland
Topics: Banach space, geometry and topology of Banach spaces, operators and interpolations in Banach spaces, Orlicz spaces and other function spaces, decomposition of functions, approximation and related topics
Main speakers:
1. Conference in Honour of Orlicz, G. Bennet (USA), J. Diestel (USA), P. Domanski (Poland), F. Hernandez (Spain), N. Kalton (USA), B. Kashin (Russia), H. König (Germany), S. Konyagin (Russia), J. Kurzweil (Czech Republic), A. Pinkus (Israel), K. Urbanik (Poland), P. Wojtaszczyk (Poland);
2. Function Spaces VII, J. Appell (Germany), A. Kaminska (USA), D. Pallaschke (Germany), L. Persson (Sweden), A. Rozhdestvensky (Russia), B. Sims (Australia), F. Sukochev (Australia), H. Triebel (Germany)
Programme Committee:
1. Conference in Honour of Orlicz, Z. Ciesielski (Poland), L. Drewnowski (Poland), W. Johnson (USA), F. Hernandez (Spain), J. Lindenstrauss (Israel), N. Kalton (USA), J. Musielak (Poland), G. Pissier (France), A. Pelczynski (Poland), P. Ulyanov (Russia);
2. Function Spaces VII (all from Poland), J. Musielak, H. Hudzik, L. Skrzypczak
Organising Committee (all from Poland)**:** Z. Palka (Chair), B. Bojarski (Vice-Chair), S. Janeczko (Vice-Chair), L. Skrzypczak (Secretary), P. Domanski, H. Hudzik, J. Kaczmarek, J. Kakol, I. Kubiaczyk, M. Mastylo, P. Pych-Tyberska, S. Szufla, R. Urbanski, J. Werbowski, A. Waszak, W. Wnuk
Proceedings: to be published
Location: main building of the Faculty of Mathematics and Computer Science, Adam Mickiewicz University, **Poznań,** Poland
Deadlines: for abstracts and registration, passed
Information: e-mail: orlicz@amu.edu.pl
website: http://orlicz.amu.edu.pl

August 2003

3-9: The Third International Conference 'Creativity in Mathematics Education and the Education of Gifted Students', Rousse, Bulgaria.
Information: e-mail: conf_orgcom@ami.ru.acad.bg or emily@ami.ru.acad.bg
website: www.cmeegs3.rousse.bg, www.ami.ru.acad.bg/conference2003/ www.nk-conference.ru.acad.bg
[For details, see EMS Newsletter 47]

5-10: Workshops Loops '03, Prague, Czech Republic
Information:
e-mail: loops03@karlin.mff.cuni.cz
website: http://www.karlin.mff.cuni.cz/~loops/workshops.html
[For details, see EMS Newsletter 47]

10-17: Loops '03, Prague, Czech Republic
Information:
e-mail: loops03@karlin.mff.cuni.cz
website: http://www.karlin.mff.cuni.cz/~loops
[For details, see EMS Newsletter 47]

11-15: Workshop on Finsler Geometry and Its Applications, Debrecen, Hungary
Information: e-mail: kozma@math.klte.hu
http://www.math.klte.hu/finsler/
[For details, see EMS Newsletter 47]

18-22: ENUMATH 2003, European Conference on Numerical Mathematics and Advanced Applications, Prague, Czech Republic
Information:
e-mail: enumath@karlin.mff.cuni.cz
http://www.karlin.mff.cuni.cz/~enumath/
[For details, see EMS Newsletter 47]

18-22: 7th International Symposium on Orthogonal polynomials, Special Functions and Applications, Copenhagen, Denmark
Information: e-mail: opsfa@math.ku.dk,
website:
http://www.math.ku.dk/conf/opsfa2003
[For details, see EMS Newsletter 46]

September 2003

2-5: Symposium on 'Cartesian Set Theory', Paris
Information:
e-mail: editions.europeenne@wanadoo.fr
[For details, see EMS Newsletter 46]

2-6: Barcelona Conference on Asymptotic Statistics, Bellaterra (Barcelona), Catalonia
Information: e-mail: bas2003@crm.es
web site: http://www.crm.es/bas2003
[For details, see EMS Newsletter 46]

12-16: International Conference of Computational Methods in Sciences and Engineering (iccmse 2003), Kastoria, Greece
Information: iccmse@uop.gr
website: http://www.uop.gr/~iccmse/ or http://kastoria.teikoz.gr/~iccmse/
[For details, see EMS Newsletter 47]

15-21: First MASSEE Congress, Borovetz, Bulgaria
Information:
http://www.math.bas.bg/massee2003/
[For details, see EMS Newsletter 47]

16-20: Barcelona Conference on Set Theory, Bellaterra (Barcelona), Catalonia
Information: e-mail: : set-theory@crm.es
website: http://www.crm.es/set-theory
[For details, see EMS Newsletter 46]

17-19: Conference on Computational Modelling in Medicine, Edinburgh, UK
Information: e-mail: icms@maths.ed.ac.uk
http://www.ma.hw.ac.uk/icms/current/index.html
[For details, see EMS Newsletter 47]

Recent books

edited by Ivan Netuka and Vladimír Souček

Books submitted for review should be sent to the following address:
Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic.

The review of the book by J.-P. Pier (see below) appeared in EMS Newsletter 47 with an incorrect title. We apologise for this unfortunate mistake.

T. Andreescu and Z. Feng (eds.), ***Mathematical Olympiads 1998-1999: Problems and Solutions From Around the World, MAA Problem Books, Mathematical Association of America, Washington, 2000, 290 pp., £19.95, ISBN 0-88385-803-7***
This is the authors' second volume in the series, and is a sequel to *Mathematical Contests 1997-1998: Olympiad problems and solutions from around the world*. In the first part we find solutions to challenging problems from 1998 featured in the previous book. In the second part, problems from 1999 are presented without answers. As the authors mention, different nations have different mathematical cultures, and it is interesting for students to compare their knowledge of mathematics with that of students from other countries. As a trainer of mathematics competition teams, I often need to find problems related to some particular topic. It is hard to find relevant ones in books with unsorted questions, so I was impressed by the appendix containing a classification of problems. The glossary at the end of the book will be valuable for students.

The book will be particularly appreciated by students preparing for competitions, as well as by high-school teachers involved with the preparation of students. If the series continues, the material will be a valuable source for a study of the development of mathematical culture in various countries. From this viewpoint, it will also be of interest to advanced mathematicians, and a welcome addition to their bookshelf. (mči)

T. Andreescu and Z. Feng (eds.), ***Mathematical Olympiads 1999-2000: Problems and Solutions from Around the World, Mathematical Association of America, Washington, 2002, 323 pp., £21.95, ISBN 0-88358-805-3***
The editors of this book have collected more than 500 problems from Mathematical Olympiads organised in various countries in 1999-2000. The problems from 1999 appear with solutions. The book is a sequel to the above publication containing problems from 1998-1999. The problems are arranged according to individual countries and not according to topics. The first editor T. Andreescu is the coauthor of the successful book *Mathematical Olympiad Challenges* (see below). (lbo)

T. Andreescu and R. Gelca, ***Mathematical Olympiad Challenges, Birkhäuser, Boston, 2002, 260 pp., 45 euro, ISBN 0-8176-4155-6 and 3-7643-4155-6***
The book is the third of a successful collection of mathematical problems. A review of the first edition (*Math. Bohem.* **127**, p.504) reads: 'The authors of the book are two Romanian mathematicians living in the USA since 1991. Both of them had taken part successfully in a number of mathematical competitions and after completing their University studies have engaged themselves in organisation of mathematical competitions for high school students, first in Romania and now in USA. They participate intensively in the preparation of American students talented for Mathematics for their participation in International Mathematical Olympiads'.

In this book they have collected about 400 problems from various fields of mathematics, divided into three chapters: Geometry and trigonometry, Algebra and analysis, and Number theory and combinatorics. Each chapter is divided into sections, (cyclic quadrilaterals, power of a point, periodicity, mean value theorem, Pell equations, etc.). Each section presents mathematical assertions used subsequently, several solved problems, and further unsolved problems: solutions of the latter can be found in the other part of the book. The problems come from mathematical journals and competitions in various countries, as well as from International Mathematical Olympiads.

The book can be heartily recommended to students who wish to succeed in mathematical competitions, and also to teachers preparing their students for demanding mathematical competitions at both national and international levels. (lbo)

V. I. Arnold and S. P. Novikov (eds.), ***Dynamical Systems IV, 2nd edition, Encyclopaedia of Mathematical Sciences 4, Springer, Berlin, 2001, 335 pp., DM 159, ISBN 3-540-62635-2***
This is the second, expanded and revised edition of the book. It consists of three contributions: *Symplectic geometry* by V. I. Arnold and A. B. Givental', Geometric quantisation by A. A. Kirillov, and Integrable systems I by B. A. Dubrovin, I. M. Krichever and S. P. Novikov.

In the first part, a survey of fundamental concepts of symplectic geometry is given. Starting with linear symplectic geometry in the first chapter, the theory of symplectic manifolds is developed in Chapter 2 and applied to mechanics in Chapter 3. An overview of contact geometry is presented in Chapter 4. The last two chapters are devoted to the study of Lagrangian and Legendre singularities and Lagrangian and Legendre cobordisms. The whole contribution is very nicely written, with many figures appealing to geometric intuition. The second part offers an overview of the theory of geometric quantisation by one of its founders. There is a nice introductory part containing mathematical models of classical and quantum mechanics, the statement of quantisation problems and explaining a connection with orbits in representation theory. Then prequantizations, polarisations and quantisation are introduced. This part ends with definition and properties of quantisation operators, and there is a supplement to this edition containing some topics investigated and exploited in recent years. The last part surveys the modern theory of integrable systems, based on the inverse scattering method. In the first chapter, hamiltonian systems and classical methods of integration are presented, and Chapter 2 describes modern ideas on integrability of evolution systems. There are four appendices devoted to special problems and to historical remarks on methods from algebraic geometry, Hamiltonian systems and spectral theory. As is standard in volumes of the *Encyclopaedia*, all parts bring an excellent description of the chosen topics, with many examples, mostly without proofs. They can be found in the books and papers cited in a comprehensive bibliography. (jbu)

B. C. Berndt and R. A. Rankin (eds.), ***Ramanujan: Essays and Surveys, History of Mathematics 22, American Mathematical Society, Providence, 2001, 347 pp., US$79, ISBN 0-8218-2624-7***
During his short life Srinivasa Ramanujan (1887-1920) published 37 papers (not counting solutions of problems for the *Journal of the Indian Mathematical Society*). Nevertheless, he was a mathematical genius who heavily influenced many fields of mathematics and stimulated their development for many years ahead. A description of his work can be found in Hardy's *Collected Papers of S. Ramanujan* (1927) and many further studies. We recall, for example, the outstanding volumes published by B. C. Berndt (*Ramanujan's Notebooks I-V*, Springer, 1985-98), containing a detailed discussion of Ramanujan's preserved notes.

This book contains 28 papers covering Ramanujan's exceptional life and magnificent mathematical work from many angles, and including several documentary photos. Let us mention explicitly the papers by B. C. Berndt (The books studied by Ramanujan in India), S. Janaki Ammal (Mrs. Ramanujan), The notebooks of S. Ramanujan), R. A. Rankin (The Ramanujans' family record, Ramanujan as a patient, Ramanujan's manuscripts and notebooks I-II), A. Selberg (Reflections around Ramanujan's centenary), R. Askey (Ramanujan and hypergeometric and basic hypergeometric series) and G. N. Watson (An account of the mock theta functions).

The book can be heartily recommended to mathematicians working in number theory, special functions and mathematical analysis, as well as to those interested in history of mathematics. (bn)

A. Borel, ***Essays in the History of Lie Groups and Algebraic Groups, History of Mathematics 21, American Mathematical Society, Providence, 2001, 184 pp., US$39, ISBN 0-8218-0288-7***
This book consists of essays on topics belonging mainly to the first century of the history of Lie groups and algebraic groups. Some of them have been published before. In the first chapter, the author describes the historical development of the theory of Lie groups and Lie algebras, starting from the local theory of Sophus Lie. Chapter 2 is devoted to problems of full reducibility and invariants of the group $SL_2(\mathbf{C})$. In Chapter 3, the author explains the principal role played by Hermann Weyl in the development of theory of Lie groups and their representations; an important impact of his papers on the work of Élie Cartan is also mentioned. Chapter 4 concentrates on Élie Cartan's work on symmetric spaces and Lie groups, semisimple groups and global theory, as well as the role of the theory of Lie groups in the Cartan calculus of exterior forms; the theory of bounded symmetric domains is also presented here. Chapter 5 contains a review of the theory of linear algebraic groups in the 19th century, and Chapter 6 describes the main lines of a spectacular development of the theory in the 20th century. Chevalley's work

in the theory of Lie groups and algebraic groups is presented in Chapter 7. The last chapter is based on E. R. Kolchin's work on algebraic groups and Galois theory.

The book is an interesting collection of papers describing ideas that were essential to the development of the theory. (jbu)

B. Buffoni and J. Toland, *Introduction à la Théorie Globale des Bifurcations, Cahiers Mathématiques de l'EPFL, Presses Polytechniques et Univesitaires Romandes, Lausanne, 2002, 130 pp., 33,88 euros, ISBN 2-88074-494-6*

This small book is based on a course at École Polytechnique Fédéral de Lausanne during the winter semester 1999-2000. It is a simply written introduction to bifurcation theory with applications. The style is telegraphic and elegant, and the book is therefore strongly recommended to students and other interested readers as a first reading in this field. It may also help with the design of a course for less advanced (undergraduate) students. The reader is supposed to have a basic knowledge in linear functional analysis, while other tools, such as differential calculus in Banach spaces and the local theory of analytic sets, are adequately contained in the book (20 and 38 pages from 122). Readers will learn the famous theorem of Crandall-Rabinowitz in local bifurcation theory and how to apply it to the Euler-Bernoulli beam model, how to model surface waves on an infinite ocean (Stokes' waves) as a bifurcation problem, and they will see demonstrations of results of global bifurcation theory to Stokes' waves. A few results in the last section are presented without proof. (ef)

D. Burago, Yu. Burago and S. Ivanov, *A Course in Metric Geometry, Graduate Studies in Mathematics 33, American Mathematical Society, Providence, 2001, 415 pp., US$44, ISBN 0-8218-2129-6*

This book contains ten chapters. The first is devoted to the standard advanced introduction to metric spaces. Hausdorff measure and dimension are explained, and the Banach-Hausdorff-Tarski paradox is also mentioned informally. Chapter 2 deals with the very general 'length structure' defined on a topological space and later studied in metric spaces: examples range widely from Euclidean geometry to the Finsler geometry. The famous theorem of Hopf and Rinow is generalised from the Riemannian case to a length space. Chapter 3 considers the glueings of length spaces, the quotient metrics, polyhedral spaces, metric graphs (including so-called Cayley graphs), direct products and warped products on length spaces, concluding with the abstract study of angles. Chapter 4 is devoted to spaces of bounded curvature (Alexandrov spaces): here the curvature is not defined explicitly – instead, spaces of non-positive curvature and non-negative curvature are characterised by the geometric behaviour of their distance functions, when compared with the Euclidean case. The first variation formula (known from differential geometry) is derived for Alexandrov spaces. Chapter 5 is on Riemannian metrics, Finsler metrics, sub-Riemannian metrics and a detailed study of hyperbolic plane; the Besicovitch inequality is proved at the end. Chapter 6 deals with the curvature of Riemannian metrics and the related comparison theorems. Chapter 7 investigates the 'space of metric spaces' (Lipschitz distance, Gromov-Hausdorff distance and convergence). Chapter 8, 'Large-scale geometry', deals mainly with Gromov hyperbolic spaces and with periodic metrics. Chapter 9 is concerned with the spaces of curvature bounded from above, mainly Hadamard spaces (again in the most general setting); as an 'example', the theory of semi-dispersing billiards is treated. The last section treats spaces of curvature bounded from below: the main results are general versions of the Toponogov theorem and the Gromov-Bishop inequality.

The book is written in an informal precise style, with many exercises. This is a really excellent book which should be read by every expert in geometry, and which can be warmly recommended even to undergraduate students. A tribute is paid to the fundamental contributions to many parts of geometry and topology by mathematicians of Russian origin. (ok)

C. Chatfield, *Time-Series Forecasting, Chapman & Hall/CRC, Boca Raton, 2000, 267 pp., US$69.95, ISBN 1-58488-063-5*

This book deals with forecasting in time series and is aimed at readers with a statistical background and basic knowledge of time series. It starts with a short introduction to time-series analysis, giving simple descriptive techniques and basic facts. It is followed by a survey of univariate time-series models. Then follow ARMA and ARIMA models, including long-memory models, state-space models, non-linear models, GARCH models, and other models with changing variance, as well as neural networks and chaos. The next chapters describe univariate and multivariate forecasting methods and explore relationships between different methods, providing many up-to-date references. Criteria for comparing and evaluating forecasting methods are introduced, and there is a discussion of a problem on how to choose the 'best' method for a particular situation. Prediction intervals are also considered, and the effect of model uncertainty on the accuracy of the forecast is treated; the results are summarised at the end of the final chapter.

This book can serve as a textbook on forecast model building, and is an outstanding reference source for both researchers and practitioners, especially in economics, operations research and industry. (zp)

S. S. Cheng, *Partial Difference Equations, Advances in Discrete Mathematics and Applications 3, Taylor & Francis, London, 2003, 267 pp., £50, ISBN 0-415-29884-9*

The main topic of this book is a class of functional relations with recursive structures, called partial difference equations. The author describes mathematical methods that help in dealing with recurrence relations governing the behaviour of variables such as population size or stock price.

The book begins with introductory examples of partial difference equations: most of these stem from familiar concepts of heat diffusion, heat control, temperature distribution, population growth and gambling. Basic definitions and a classification of partial difference equations are given in Chapter 2, while the following chapter introduces operators and studies their properties. Chapter 4 is concerned with univariate sequences or bivariate sequences that satisfy monotone or convex functional relations; maximum and discrete Wirtinger's inequalities are derived. In Chapter 5, the author employs methods of operators, and a number of other methods, for constructing explicit solutions for a number of basic partial difference equations. Certain methods useful for deriving stability criteria appear in Chapter 6. Chapter 7 presents several criteria for the existence of solutions, mainly those that are positive, bounded, or for travelling waves. In the final chapter, the author derives a number of conditions for the non-existence of solutions of certain types. Every chapter ends with notes and remarks.

The book offers a concise introduction to tools and techniques that were used to obtain results in the theory of partial difference equations. It will be useful for anyone who has mastered the usual sophomore mathematical concepts. (kn)

E. N. Chukwu, *Optimal Control of the Growth of Wealth of Nations, Stability and Control: Theory, Methods and Applications 17, Taylor & Francis, London, 2003, 384 pp., £75, ISBN 0-415-26966-0*

The aim of this research monograph is to identify realistic dynamic models for the growth of wealth of nations. For this purpose, the text derives the dynamics of the key economic variables (gross national product, employment, interest rates, capital stock, inflation, balance of payment), using rational expectations and market principles. The models are validated using economic time series of several countries, enabling one to deal with the growth of wealth of nations as examples of mathematical optimal control problems. The corresponding government control variables include taxation, government outlay, money supply and tariffs. The control variables of private firms include autonomous consumption, investment, net export, money holding, wages and productivity; linear models using MATLAB are included for this purpose. The controllability and other properties of the systems are discussed as a simple explanation of an economic situation. The text compares the extent of government intervention with the activity of private firms in the framework of controllabilty of the economy.

The monograph is suitable for graduate students and researchers in applied mathematics and economics dealing with problems of the global economy. (tc)

L. Conlon, *Differentiable Manifolds, 2nd edition, Birkhäuser Advanced Texts, Birkhäuser, Boston, 2001, 418 pp., DM 130, ISBN 0-8176-4134-3 and 3-7643-4134-3*

This is a well-organised and nicely written text containing the basic topics needed for further work in differential geometry and global analysis. This extended second edition starts with topological manifolds and their properties, including covering spaces and the fundamental group. Smooth manifolds and smooth maps are introduced, after a review of the local theory of smooth functions. The local theory of local flows is then used to treat foliations and flows on manifolds. The next chapter, on Lie groups and Lie algebras, introduces an important tool for future use, and includes key examples of Lie groups and their homogeneous spaces. The following three chapters introduce differential forms and their integration on manifolds, including de Rham cohomology, Mayer-Vietoris sequences, Poincaré duality, the de Rham theorem and degree theory; the Frobenius theorem is then reformulated in terms of differential forms. The last two chapters present basic facts from Riemannian geometry and introduce principal fibre bundles. Appendices contain basic facts on universal coverings, inverse function theorem, ODEs, and proofs of the de Rham theorem for Èech and singular cohomology. This book is very suitable for students wishing to learn the subject, and interested teachers can find well chosen and nicely presented material for their courses. (vs)

N. J. Cutland, ***Loeb Measures in Practice: Recent Advances, Lecture Notes in Mathematics 1751, Springer, Berlin, 2000, 111 pp., DM 44, ISBN 3-540-41384-7***
This book presents an expanded version of the European Mathematical Society lectures given by the author at Helsinki in 1997. It describes important and interesting applications of the Robinson non-standard analysis in measure theory, (stochastic) PDEs, Malliavin calculus and financial mathematics.

It starts with a condensed review of non-standard analysis, the Loeb construction of a Loeb measure from a non-standard measure (including simple constructions of the Lebesgue, Haar and Wiener measures from the Loeb measure), Loeb integration and simple applications. The three following chapters contain applications of the principles in Chapter 1 to three different fields: fluid mechanics (deterministic and stochastic Navier-Stokes equations), stochastic calculus of variations (infinite-dimensional Ornstein-Uhlenbeck process, Malliavin calculus) and financial mathematics (Cox-Ross-Rubinstein and Black-Scholes models, convergence of market models). These applications show clearly that methods of non-standard analysis can be used very efficiently in many fields of 'ordinary' mathematics. The basic principles of possible applications can be well understood from the interesting material presented here, and the book will inspire mathematicians and mathematical physicists. (vs)

G. Da Prato and J. Zabczyk, ***Second Order Partial Differential Equations in Hilbert Spaces, London Mathematical Society Lecture Notes Series 293, Cambridge University Press, Cambridge, 2002, 379 pp., £29.95, ISBN 0-521-77729-1***
This book is devoted to parabolic and elliptic equations for functions defined on infinite-dimensional Hilbert spaces. The first attempt to generalise the theory of partial differential equations to functions defined on infinite-dimensional spaces was made by R. Gateaux and P. Lévy around 1920. A different approach, developed by L. Gross and Yu. Daleckij about 30 years ago, is used here; its main tools are probability measures in Banach spaces, semigroups of linear operators, stochastic evolution equations, and interpolation spaces.

The book is divided into three parts. Part 1 presents the existence, uniqueness and regularity of solutions in the space of bounded and uniformly continuous functions. The theory of Sobolev spaces with respect to a Gaussian measure is developed in Part 2, enabling a study of equations with very irregular coefficients arising in different applications, such as reaction-diffusion systems and stochastic quantisation. The final part contains some applications to control theory.

This book presents the state of the art of the theory. To make the presentation as self-contained as possible, it includes essential background material. The reader will find numerous comments and references to more specialised results not included here. It can be warmly recommended to anyone interested in the field. (rl)

A. Del Centina, ***The Abel's Parisian Manuscript, Cultura e Memoria 25, Leo S. Olschki Publisher, Firenze, 2002, 185 pp., ISBN 88-222-5090-7***
This bilingual (Italian-English) book contains a reproduction of the famous 1826 Paris manuscript by the Norwegian mathematician Niels Henrik Abel (1802-29). The manuscript *Memoire sur une propriété d'une classe très étendue de fonctions transcendantes* is deposited in the Moreniana Library in Florence, and has been republished for the bicentenary of Abel's birth. In its time, the paper had a great impact on mathematical research, opening new research directions in the field.

The book contains the following chapters: The enhancement of the documents of the Moreniana library, Niels Henrik Abel, a biographical portrait, Genius and orderliness, Abel in Berlin and Paris, The mathematical significance of Abel's Paris memoir (A very superficial tale), Abel's manuscripts in the Libri collection: their history and their fate, The papers of Guglielmo Libri in the Moreniana library of Florence. They contain new contributions concerning Abel's life and work, written by mathematicians and historians from Norway and Italy. A short history of the Moreniana Library is also included. The final part of the book contains a reprint of Abel's manuscript. (mnem)

J. F. van Diejen and L. Vinet (eds.), ***Calogero-Moser-Sutherland Models, CRM Series in Mathematical Physics, Springer, New York, 2000, 561 pp., DM 189, ISBN 0-387-98968-4***
Thirty years ago, Calogero and Sutherland (on a quantum level) and Moser (on a classical level) showed the integrability of certain N-particle models. Perelomov and Olshanetsky then constructed similar integrable models connected with root systems of simple Lie groups. Soon after, a relativistic deformation (Ruijsenaars and Schneider) and a long-range spin model (Haldane and Shastry) were discovered. These origins were followed by a broad and intensive activity in the field in recent years, leading to some very interesting connections with other parts of mathematical and theoretical physics, as well as of pure mathematics.

The meeting organised in 1997 at CRM (Montréal) brought together people working in many different areas connected with the original models. Its proceedings contains 39 contributions, covering a broad range of topics. There are papers by Calogero and Sutherland, as well as longer papers on solutions of the quantised KZ difference equation by multidimensional q-hypergeometric integrals (E. Mukhin, A. Varchenko), relativistic Lamé functions (S. Ruijsenaars), combinatorial theory of the Jack and Macdonald symmetric polynomials (L. Lapointe, L. Vinet), generalisations of the elliptic Calogero-Moser and Ruijsenaars-Schneider systems based on a special inverse problem (I. Krichever), a connection between a Painlevé VI equation and the elliptic Calogero system (A. M. Levin, M. A. Olshanetsky), generalisations of R-matrices for Calogero-Moser systems (J. Avan) and many other interesting papers. The book offers a good overview of this modern and quickly developing area between mathematics and physics, and will be of great interest to a wide range of mathematicians and theoretical physicists. (vs)

G. Ellingsrud, W. Fulton and A. Vistoli (eds), ***Recent Progress in Intersection Theory, Trends in Mathematics, Birkhäuser, Boston-Basel-Berlin, 2000, Hardback, CHF 148.00, ISBN 0-8176-4122-X,***
This collection of articles originated in the International Conference in Intersection Theory, organised in Bologna in December 1997. The main aim of the conference was to bring postgraduate students to the borders of contemporary research in intersection theory. Four preparatory courses were prepared, by M. Brion (Equivariant Chow groups and applications), H. Flenner (Joins and intersections), E. M. Friedlander (Intersection products for spaces of algebraic cycles), and R. Laterveer (Bigraded Chow (co)homology). With exception of the first one, the texts of these courses (in slightly modified form) appear here. In addition, the collection contains eight more research articles from intersection theory.

Anyone reading this collection will need a good knowledge of algebraic geometry and commutative algebra. On the other hand, the authors were well chosen: excellent specialists who are also very good writers. Because of the high mathematical quality of the articles, this collection should be attractive to specialists in algebraic geometry and related fields. (jiva)

G. Fischer and J. Piontkowski, ***Ruled Varieties. An Introduction to Algebraic Differential Geometry, Friedrich Vieweg & Sohn, Wiesbaden, 2001,141 pp., DM 68, ISBN 3-528-03138-7***
Ruled varieties form a natural generalisation of ruled surfaces. A ruled variety is a variety of arbitrary dimension which is swept out by moving linear subspaces of ambient space; the ruling is an extrinsic property of a variety related to a chosen embedding into ambient affine or projective space.

In this book, complex projective algebraic varieties are mainly considered: the Hermitian Fubini-Study metric and relative curvature are not necessary here. It is sufficient to consider the second fundamental form, which is the differential of the Gauss map. In comparison with the book by P. Griffiths and J. Harris, *Principles of algebraic geometry*, this book offers a more detailed and elementary explanation of related results, and also brings a report on recent progress in the area. An introductory chapter contains a review of facts needed from classical differential and projective geometry. Chapter 1 is devoted to Grassmanianns, and an elementary theory of ruled surfaces can be found in Chapter 2. In the final chapter, tangent and secant varieties are studied, and the third and higher fundamental forms are used to describe their properties. The book is very well written, and I can recommend it to anyone interested in these topics. (jbu)

L. C. Grove, ***Classical Groups and Geometric Algebra, Graduate Studies in Mathematics 39, American Mathematical Society, Providence, 2001, 169 pp., US$35, ISBN 0-8218-2019-2***
This book contains a detailed study of classical linear groups over arbitrary fields, both finite and infinite. The first part describes the properties of classical groups over fields with characteristic different from 2. The discussion starts with symplectic groups, Clifford algebras are treated together with orthogonal groups, and the discussion ends with unitary groups. The final chapters are then devoted to orthogonal groups and Clifford algebras in characteristic 2. The author concentrates on individual classes of classical groups, and general tools of Lie theory are not used here. Relationships with Chevalley groups are indi-

cated.

This text is written for a course at graduate level, and contains many exercises. The reader will need a good knowledge of linear algebra and some parts of general algebra. (vs)

M. Günter, ***Customer-based IP Service Monitoring with Mobile Software Agents, Whitestein Series in Software Agent Technologies, Birkhäuser, Basel, 2002, 160 pp., EUR 27,10, ISBN 3-7643-6917-5***

Software agents are software artefacts with specific properties, which usually include: autonomy (the agent is a software application able to perform autonomous activities), mobility (the agents are mobile agents (MA) that can autonomously move through a computer network) and cloning (the agent can produce its new instances). Further properties can be associated with the agent intelligence in the sense of artificial intelligence, and a basic precondition for this is the ability of agents to build autonomously their communication, but such properties are not the topics of this book. The author describes in detail the implementation of MA able to monitor the behaviour and effectiveness of services of nodes of the network, related to requirements of a particular user of the network. The implementation of MA and the supporting infrastructure, including the security issues, are also discussed in detail. The implemented mobile agents can monitor aspects of the network behaviour that interest a user.

The book contains results of experiments measuring various network properties (such as one-way delay, delay variation, round-trip times and bottleneck bandwidth) and the overhead connected with the agent activities. In the final chapter, the author mentions several similar related projects, such as Internet2 Initiative, Qbone, and related IP measurement methodologies, and proposes ways of applying his MA here. He also discusses the applications of his systems in network management and formulates several open problems. The book does not indicate whether the developed system has been used in real-life long-term mass service practice. The bibliography contains more than 150 items.

The book is an excerpt from the author's Ph.D. thesis *Management of Multi-Provider Internet Services with Software Agents*, published in July 2001. It is available, with the developed source code, at http://www.iam.unibe.ch/~mguenter/phd.html. (jk)

S. Helgason, ***Differential Geometry, Lie Groups, and Symmetric Spaces, Graduate Studies in Mathematics 34, American Mathematical Society, Providence, 2001, 641 pp., US$69, ISBN 0-8218-2848-7***

This is a new edition of a volume published by Academic Press under the same title in 1978. The author has corrected some inaccuracies and has added a number of footnotes that appear in the final section. It is useful to recall that the author's second book *Groups and geometric analysis* (Academic Press, 1984) was reprinted by the AMS in 2000, as Volume 83 in the series *Mathematical Surveys and Monographs*. An essentially different version of this book appeared in 1994 as Volume 39 of the same series, under the title *Geometric analysis on symmetric spaces*. (ok)

N. Higson and J. Roe, ***Analytic K-Homology, Oxford Mathematical Monographs, Oxford University Press, 2000, 405 pp., £65, ISBN 0-19-851176-0***

K-homology can be formally described as the homology theory dual to Atiyah-Hirzebruch's *K*-theory (even though this fact does not seem to be extremely useful). It turns out, especially in recent years, that *K*-homology finds many very interesting applications, both in topology and geometry. Using *K*-homology techniques it has been possible to attack problems that had been considered rather hopeless, and to present shorter and more conceptual proofs of earlier theorems. (We mention that *K*-homology enables us to write a very short proof of the Atiyah-Singer index theorem.)

This book combines two interpretations of *K*-homology, both closely related to functional analysis: the well-known Brown-Douglas-Fillmore theorem, and Kasparov's *K*-homology. Here we underline that the Kasparov product plays an especially important role. The most interesting applications are concentrated around the index theory, and we find here index theory for hypersurfaces, the index theorem for Spinc-manifolds, Toeplitz index theorems, and index theory on strongly pseudoconvex domains. The last chapter is devoted to the higher index theory, with applications in Riemannian geometry. Because the whole book deals with complex *K*-homology, the authors include an appendix suggesting a possible passage to real *K*-homology. However, they have decided not to describe systematically Kasparov's bivariant *KK*-theory.

Concerning prerequisites, the reader needs to be familiar with basic functional analysis and algebraic topology (or homological algebra), but this is not sufficient if the reader is interested in applications: here more specialised knowledge is required. But the book is carefully written, and the authors include many notes and commentaries that enable the reader to understand the main lines of the exposition. At the end of each chapter are exercises that enhance the main text and are often rather difficult. In summary, this is a very nice book that will interest specialists from several branches of mathematics and be attractive for postgraduate students. (jiva)

A. Holme, ***Geometry. Our Cultural Heritage, Springer, Berlin, 2002, 378 pp., 34,95 euros, ISBN 3-540-41949-7***

This book discusses central themes from the history of classical and modern geometry, and is divided into two parts.

Part 1 (*A Cultural Heritage*) contains six chapters on the roots of geometric ideas, their development and their historical context. Many interesting themes are included: geometry in prehistory, Egypt and Babylonia, Greek geometry (the Pythagoreans, Platonic solids, classical problems and their solutions), the Hellenic Era (Euclid, Archimedes, Apollonius of Perga, Heron of Alexandria, etc.), and European geometry from the Middle Ages to modern times. We find here some problems that ancient mathematicians thought about and how they solved them. We also find legends, stories and tales (about Plato, Pythagoras, Euclid, Archimedes, etc.).

Part 2 (*Introduction to Geometry*) contains twelve chapters on the development of some aspects of modern geometry (for example, the creation of axiomatic geometry, projective geometry, models and properties of non-Euclidean geometry, the main properties of algebraic curves, solutions of classical problems, the beginning of geometrical algebra and fractal geometry, and an introduction to catastrophe theory). This part is a very interesting introduction to modern geometry and its historical aspects.

The book can be recommended as an excellent textbook for several possible courses (such as Historical topics in geometry, or Introduction to modern geometry). It can be recommended to historians of mathematics, and to professional mathematicians, teachers and students wanting to understand the origins of modern geometry. (mnem)

R. Iorio and V. Iorio, ***Fourier Analysis and Partial Differential Equations, Cambridge Studies in Advanced Mathematics 70, Cambridge University Press, 2001, 411 pp., £45, ISBN 0-521-62116-X***

This book arises from several courses and seminar talks over the years, and is a greatly modified version of the authors' previous work (*Equacoes diferenciais parciais*, 1978). Its aim is to introduce students to the basic concepts of Fourier analysis and illustrate this theory in the study of linear and non-linear evolution equations.

After a preliminary chapter, the basic theory of Fourier series is presented. Chapter 3 develops the theory of periodic distributions. Since it is done in the context of (continuous) functions and series, the reader is not assumed to know the Lebesgue integral; a separate section presents material from topology, necessary for characterising the topology of distributions.

The second part of the book is devoted to applications of the previous chapters. Chapter 4 provides applications to linear evolution equations with periodic boundary conditions in Sobolev spaces, and includes a section on (unbounded) operator theory and semigroup theory. The following two chapters deal with non-linear equations. In Sobolev spaces as phase spaces, the local and global existence and uniqueness of solutions are proved by means of fixed-point theorems and *a priori* estimates. In particular, the Schrödinger and Korteweg-de Vries equations are treated and their Hamiltonian structure is explained.

The third part deals with partial differential equations on $\mathbf{R}^n$. Chapter 7 presents the theory of distributions, Fourier transforms and Sobolev spaces, simplified with references to some analogous periodic cases developed in Chapter 3. As applications, linear heat and Schrödinger equations, and the non-linear Korteweg-de Vries and Benjamin-Ono equations are studied in greater detail.

The reader should have a basic knowledge of functional analysis (Banach and Hilbert spaces, bounded operators), ordinary differential equations, and, for the third part of the book, the Lebesgue integral. Developing the periodic distribution theory based on sequences first is an interesting feature of the book. The exercises in each section form an integral part of the text. It would have been helpful to have a list of symbols and a longer index. Students and those beginning their academic career should find this text interesting and stimulating. (ef)

H. Iwaniec, ***Spectral Methods of Automorphic Forms, 2nd edition, Graduate Studies in Mathematics 53, American Mathematical Society, Providence, 2002, 220 pp., US$49, ISBN 0-8218-3160-7***

This is a very readable introduction to the spectral theory of automorphic forms on $SL_2(\mathbf{R})$, based on the author's higher graduate courses. The book begins with harmonic analysis on the upper half-plane H, Fuchsian groups and Eisenstein series, and proceeds to develop spectral theory on $\tilde{A}\backslash H$, beginning with cusp forms and continuing with analytic theory of Eisenstein series and their residues. Establishing the Kuznetsov formula and the Selberg trace formula, the author estimates Fourier coefficients of Maass forms, eigenvalues of the hyperbolic Laplacian, and the cusp forms themselves. An appendix covers the necessary analytic background. This is a remarkable book, written in a clear style by one of the world leaders in number-theoretical applications of the spectral theory of automorphic forms. (jnek)

M. Jarnicki and P. Pflug, ***Extension of Holomorphic Functions, de Gruyter Expositions in Mathematics 34, Walter de Gruyter, Berlin, 2000, pp., DM 248, ISBN 3-11-015363-7***
A domain of holomorphy for holomorphic functions of several complex variables is a domain G in $\mathbf{C}^n$ on which there exists a holomorphic function that cannot be continued across any boundary point of G. Every domain is a domain of holomorphy for $n = 1$, but this is not true in several complex variables. Moreover, studying the continuation of holomorphic functions leads directly to multivalued holomorphic functions and envelopes of holomorphy.

A classical method for treating them in one variable is to study holomorphic functions on Riemann surfaces. This book contains a study of extensions of holomorphic functions of several complex variables in the framework of Riemann domains over $\mathbf{C}^n$. The first chapter introduces the domain of holomorphy for a family of holomorphic functions, and its characterisation in terms of holomorphic convexity. Chapter 2 discusses pseudoconvex domains and a solution of the Levi problem for Riemann domains over $\mathbf{C}^n$, while Chapter 3 studies envelopes of holomorphy for special types of domains. The final chapter describes the structure of envelopes of holomorphy.

The book contains many open problems, and can be recommended to those interested in the theory of several complex variables. (vs)

S. Kantorovitz, ***Introduction to Modern Analysis, Oxford Graduate Texts in Mathematics 8, Oxford University Press, 2003, 434 pp., £45, ISBN 0-19-852656-3***
This book in the *Oxford Graduate Texts in Mathematics* series is based on lectures in different related subjects given since 1964 at Yale, the University of Illinois and Bar Ilan University. The series is aimed at presenting texts of high mathematical quality, especially in areas that are not sufficiently covered in the current literature.

The reader will find here a carefully written exposition of classical topics (Lebesgue integration, Hilbert spaces, and elements of functional analysis), and results less frequently covered such as Haar measure, the Gelfand-Naimark-Segal representation theorem and the von Neumann double commutant theorem. Many results appear in exercises (almost 130 in total). One third of the book is devoted to two long applications: probability and distributions (Appl. II). The book can thus be viewed as a multi-purpose text for advanced undergraduate and graduate students, and can be recommended for mathematical libraries. (jive)

W. Kierat and U. Sztaba, ***Distributions, Integral Transforms and Applications, Analytical Methods and Special Functions 7, Taylor & Francis, London, 2003, 148 pp., £45, ISBN 0-415-26958-X***
This book is an approachable introduction to the theory of distributions and integral transforms. The principal intention is to emphasise the remarkable connections between distribution theory, classical analysis and the theory of differential equations. The theory is developed from its beginnings to the point where fundamental and deep results are proved, such as the Schwartz kernel theorem and the Malgrange-Ehrenpreis theorem. Answers are given to natural questions arising from the topics presented in the text. In cases where classical analysis is insufficient, practical hints are given on using the theory of distributions.

The book is divided into seven chapters. In Chapter 1 the authors define distributions and introduce some fundamental notions. Chapters 2 and 3 are devoted to local properties of distributions, tensor products and convolution products of distributions. In Chapter 4 the authors give some applications to differential equations. The next two chapters deal with integral transforms, such as the Cauchy, Fourier and Hilbert transforms. In the final chapter the author present some aspects of the theory of orthogonal expansions of some classes of distributions. An appendix contains basic facts from functional analysis.

The book will be understandable to a student familiar with advanced calculus, and will be of interest to mathematicians, physicists and researchers concerned with distribution theory and its many applications to the applied sciences. (kn)

W. Kühnel, ***Differential Geometry. Curves–Surfaces–Manifolds, Student Mathematical Library 16, American Mathematical Society, Providence, 2002, 358 pp., US$49, ISBN 0-8218-2656-5***
This book (a translation of *Differential-geometrie*, Vieweg & Sohn Verlag, 1999) is a nice introduction to classical differential geometry of curves and surfaces, containing also the theory of curves and surfaces in Minkowski space and basic results in the theory of Riemannian manifolds, the Riemannian connections, examples of tensors, spaces of constant Riemannian curvature and Einstein spaces. The text is illustrated with many examples, and several unsolved exercises appear in each chapter. The whole exposition is clear and the book can be recommended to all students in the latter part of their undergraduate curriculum. (lbo)

V. Lakshmikantham and S. Köksal, ***Monotone Flows and Rapid Convergence for Nonlinear Partial Differential Equations, Series in Mathematical Analysis and Applications 7, Taylor & Francis, London, 2003, 318 pp., £55, ISBN 0-415-30528-4***
This book deals in a unified way with monotone iterative techniques and generalised quasilinearisations. Starting from a sub- (or super-) solution, this may create a sequence of approximate solutions converging pointwise monotonically to the exact solution. Modification of this process to the non-monotone case is also possible, leading to a more sophisticated iterative scheme. Sometimes, quadratic convergence can be achieved, which has possible numerical importance.

The first part of this book addresses these issues in the classical framework, while the second part uses variational techniques relying on the weak solution. In each case, second-order elliptic, parabolic and hyperbolic equations or systems are scrutinised (possibly on unbounded domains), and impulsive parabolic equations are addressed. The first part allows semilinear equations or systems in non-divergence form, while the second part treats equations or systems (possibly quasilinear) in the divergence form. Reading this book requires a certain basic knowledge of partial differential equations and a knowledge of the standard notation in this subject is assumed, as there is no list of symbols. A fresh compressed style of explanation arises from reducing the number of definitions and from rather sketched assertions whose more detailed content can be read from the proofs. This book will be a useful reference for applied scientists and mathematically oriented engineers. (trou)

S. Lang, ***Introduction to Differentiable Manifolds, 2nd edition, Universitext, Springer, New York, 2002, 250 pp., 59,95 euros, ISBN 0-387-95477-5***
This book has ten chapters with the following titles: Differential calculus, Manifolds, Vector bundles, Vector fields and differential equations, Operations on vector fields and differential forms, The theorem of Frobenius, Metrics, Integration of differential forms, Stokes' theorem, and Applications of Stokes' theorem. Among the topics not mentioned in the titles are a short introduction to Lie groups, Darboux' theorem, elements of pseudo-Riemannian geometry, the Morse lemma for non-degenerate critical points, Sard's theorem, de Rham cohomology, and the residue theorem for functions of complex variables as a non-standard application of Stokes' theorem. The author recommends his text to 'the first year graduate level or advanced undergraduate level'; it is not intended for the first encounter with this topic for an average student. The author offers a rich bibliography of easier texts for beginners or those who prefer more visualisation and less abstraction.

Loyal to the Bourbaki style, the author starts with a paragraph on categories. Thus, his explanation is very precise, with rich formalism and with maximum generality; for example, he always deals with differentiable manifolds of class C^p for sufficiently large positive integers p, and not only with those of class C^∞, as is usual in other texts. Naturally, he must pay a price for this generality, even in such basic concepts as a tangent vector. He also does not exclude a consideration of singularities, for example when treating various versions of Stokes' Theorem, but he does make one concession to the reader: he does not speak about infinite dimensions! In summary, this is an ideal text for people who like a more general and abstract approach to the topic. (ok)

C. Lauro, J. Antoch, V. E. Vinzi and G. Saporta (eds.), ***Multivariate Total Quality Control. Foundation and Recent Advances, Contributions to Statistics, Physica-Verlag (Springer), Heidelberg, 2002, 236 pp., ISBN 3-7908-1383-4***
This very interesting book represents a homogeneous collection of major contributions from three European schools (Naples, Paris and Prague), whose researchers have faced new problems in the field of multivariate statistics for total quality. The discussion covers both theoretical and practical computational aspects of the problem, as well as the necessary mathematical background and several applications.

The first part on off-line control, by J. Antoch, M. Hušková and D. Jarušková, deals with tests on the stability of statistical models.

The problem is formulated in terms of testing hypotheses, and this chapter is the most important part of the book. The next four contributions are closely connected with the on-line process control: J. Antoch and D. Jarušková offer a critical overview of classical methods for detection of a change in a sequence of observations; two contributions of L. Jaupi and N. Niang to multivariate process control refer to the theory of robustness but in different frameworks; and G. Scepi considers situations where control charts for monitoring individual quality characteristics may be not adequate for detecting changes in the overall quality of products. The next two contributions are closely connected with problems of customer satisfaction: V. Esposito Vinzi proposes different strategies for comparative analysis, based on a joint use of non-symmetrical multidimensional data analysis and procrustean rotations, while G. Giordano focuses on multi-attribute preference data that refer to preference judgements expressed with respect to a set of stimuli described by any relevant attributes, in order to perform a conjoint analysis.

This book can be used in courses related to quality control and industrial statistics, and is also a reference book for real-world practitioners in both engineering and business during the phases of planning and checking for quality. (tc)

J. M. Lee, ***Introduction to Smooth Manifolds, Graduate texts in Mathematics 218, Springer, New York, 2002, 628 pp., 84,95 euros, ISBN 0-387-95495-3***

This text provides an elementary introduction to smooth manifolds which can be understood by junior undergraduates. It consists of twenty chapters: Smooth manifolds; Smooth maps; Tangent vectors; Vector fields; Vector bundles; The cotangent bundle; Submersions; Immersions and embeddings; Submanifolds; Lie group actions; Embedding and approximation theorems; Tensors; Differential forms; Orientations; Integration on manifolds; de Rham cohomology; the de Rham theorem; Integral curves and flows; Lie derivatives; Integral manifolds and foliations; Lie groups and their Lie algebras. The book ends with a sixty-page appendix called 'Review of prerequisites' which is a short introduction to general topology, linear algebra and calculus, so the book is self-contained. There are 157 illustrations, which bring much visualisation, and the volume contains many examples and easy exercises, as well as almost 300 'problems' that are more demanding. The subject index contains more than 2700 items!

It is obvious that the 'skeleton' of basic facts has been opulently enhanced by the 'flesh' of comments and explanations, bringing answers to any potential reader's questions before they are asked. The pedagogic mastery, the long-life experience with teaching, and the deep attention to students' demands make this book a real masterpiece that everyone should have in their library. The author's dedication "To my students" could easily be understood as "To my students with love". (ok)

T. Lewinski and J. J. Telega, ***Plates, Laminates and Shells: Asymptotic Analysis and Homogenization, Series on Advances in Mathematics for Applied Sciences 52, World Scientific, Singapore, 2000, 739 pp., US$149, ISBN 981-02-3206-3***

For computations in mechanics, it is important to know which simpler structures provide suitable replacements for the fine microstructure of the material. This can be done by methods of homogenisation. The authors present a thorough study of homogenisation and related methods, with an emphasis on applications to plates, laminates and shells.

Chapter 1 summarises the mathematical background, including quasiconvexity, Γ convergence, G convergence and H convergence. Chapter 2 studies elastic plates constructed by a periodic repeating of three-dimensional cells in two independent directions. The formal asymptotic expansion of the equilibrium solution is justified by Γ convergence methods, and the effect of bending and stretching of plates is investigated in Kirchhoff's model. The non-linear model governed by the Von Kármán equation is studied, including bifurcation phenomena. The Reissner-Hencky model is suitable for moderately thick plates; Hoff's approach can be applied to sandwich plates with soft core. In Chapter 3, the authors study elastic plates with cracks, and compute stiffness loss of cracked laminates. In Chapter 4, elastic perfectly plastic material which is subject to a loading is considered. The corresponding functional has linear growth and thus its analysis leads to non-reflexive spaces, such as the space of functions with bounded variation. Chapter 5 is devoted to elastic and perfectly plastic shells. In Chapter 6, homogenisation methods are applied to the optimum design of plates and shells. This requires further mathematical methods, such as Y-transformation and its Fourier analysis, quasiconvexity or quasiaffinity of some concrete expressions, and Young measures. The problems of optimal design studied here have the feature that optimising sequences increases the fineness of partitioning. The minimum compliance problem of thin elastic plates is a representative of issues of this chapter.

This book contains a great amount of material. As always in mathematical modelling, the problems studied here are fairly complicated. Despite this, the level of mathematical accuracy is very high. The authors present a representative selection of known results, including some of their extensive research, and experts in the field will find a lot of information. However, the methods used here are of broader significance and thus may provide inspiration for readers interested in quite distant fields of applied mathematics. (jama)

A. L. Lewis, ***Option Valuation under Stochastic Volatility with Mathematica Code, Finance Press, Newport Beach, 2000, 350 pp., USD$97,50, ISBN 0-9676372-0-1***

In this book option pricing is studied in the case when the price of a security and its volatility both follow diffusion processes. The theory and methods presented here provide solutions for many option valuation problems that are not captured by the Black and Scholes theory, such as smile and term structure of implied volatility. A generalised Fourier transform approach is used to solve PDEs that determine the option price; this approach plays an important role throughout the book, and the author presents new and general results. Volatility series expansions are also used to solve the corresponding PDE, and this technique is applied to obtain explicit formulas for the smile. The term structure of implied volatility is considered, and methods for computing asymptotic implied volatility are presented. Various types of stochastic processes for volatility are treated, especially, GARCH diffusion models of stochastic volatility, and diffusion limits of well-known discrete time GARCH models are considered whose parameters are easily estimated. Many examples are given that illustrate the theory and many Mathematica routines for evaluation formulas and solving the PDE are included. This makes the book very interesting and useful for both financial academics and traders. (zp)

H. Li and F. van Oystaeyen, ***A Primer of Algebraic Geometry. Constructive Computational Methods, Pure and Applied Mathematics 227, Marcel Dekker, New York, 2000, 370 pp., US$165, ISBN 0-8247-0374-X***

This book can be used as a first course in algebraic geometry for students and researchers who are not primarily pure mathematicians. It is also useful for applications in computer algebra, robotics and computational geometry and mathematical methods in technology. Only a basic knowledge of linear algebra, basic algebraic structures and elementary topology are needed as prerequisites.

Chapter 1 describes affine algebraic sets and presents the Nullstellensatz, together with its applications. Chapter 2 is devoted to a study of polynomial and rational functions on algebraic sets, and the structure of coordinate rings of algebraic sets. Chapter 3 introduces projective algebraic sets in projective spaces and studies their structure. Relations between affine and corresponding projective notions are discussed, and there are a definition and characterisations of multiprojective spaces, Segre spaces and the Veronese variety. Chapter 4 introduces the Gröbner basis for an ideal in $K = k[x_1, \ldots, x_n]$ as a combination of an ordered structure of ideals in K and the division algorithm; some applications of Gröbner bases to basic questions in algebraic geometry are given. The dimension of an algebraic set and its geometric and algebraic role in projective geometry are explained in Chapter 5, while Chapter 6 presents basic facts from the theory of quasi-varieties (local theory). The final two chapters develop the theory of plane algebraic curves (including intersection theory), the theory of divisors and the Riemann-Roch theorem, and the class of elliptic curves is studied in detail. Some additional results from algebra are given in two appendices, and the book contains many examples. I wish to recommend this well-written book to anyone interested in applied algebraic geometry. (jbu)

H. Okamoto and M. Shoji, ***The Mathematical Theory of Permanent Progressive Water-Waves, Advanced Series in Nonlinear Dynamics 20, World Scientific Publishing Co., New Jersey, 2001, 229 pp., £29, ISBN 981-02-4449-5 and 981-02-4450-9***

This small book is an analytical and numerical overview of solutions to equations representing water-waves in a two-dimensional region of finite or infinite depth, following the research interests and results of the authors. It contains a brief mathematical derivation of various forms of the governing equations: the waves under consideration move with constant speed and do not change their profile during the motion. Different kind of waves, such as pure capillary (Crapper's), pure gravity (Stokes' highest wave), rotational (Gerstner's trochoidal wave) and solitary waves are treated, and further capillary-gravity waves, interfacial progressive waves and waves with negative surface tension are considered. The existence of solutions is formulated as a bifurcation problem. The authors consider various bifurcation parameters (gravity, surface tension), and a normal form analysis is used for describing the bifurcation modes. There is an account of results from numerical experiments.

The book is aimed at readers desiring a first insight to water-wave theory, but with previous experience in bifurcation theory and numerical analysis. Looking for more sophisticated results and verifications, they will find many references here. (ef)

J.-P. Otal, ***The Hyperbolization Theorem for Fibered 3-Manifolds, SMF/AMS Texts and Monographs 7, American Mathematical Society, Providence, 2001, 126 pp., US$39, ISBN 0-8218-2153-9***
This book aims to present a full and systematic proof of the Thurston hyperbolisation theorem in the special case of fibred 3-manifolds. The 3-dimensional manifolds studied here are manifolds that are fibre bundles over the circle with a compact surface *S* as a model fibre: such manifolds *M* are described by the isotopy class of their monodromy – that is, by a diffeomorphism of *S*. It is proved here that a complete hyperbolic metric of finite volume can be constructed on such manifolds: the main ingredients for the proof are geodesic laminations, the double limit theorem and the Sullivan theorem. The proof is based on Thurston's original (unpublished) paper with a new approach to the double limit theorem. This very nice book is the English translation of the French original, published by the Société Mathématique de France. It can be recommended to all mathematicians interested in the fascinating field of low-dimensional geometry. (jbu)

J.-P. Pier (ed.), ***Development of Mathematics 1950-2000, Birkhäuser, Basel, 2000, 1372 pp., DM 298, ISBN 3-7643-6280-4***
To write comments on the history of mathematics in the second half of the 20th century is a difficult task, due to the enormous growth in the number of working mathematicians. The aim of this book is to contribute to such a project with a collection of essays on evolution in certain fields.

A few contributions are devoted to the foundation of mathematics, set theory, logic and topology and their applications in various parts of mathematics (J.-Y. Girard, J.-P. Ressayre, B. Poizat, F. W. Lawvere and S. Sorin). Papers by E. Fouvry, M. Waldschmidt, J.-L. Nicolas and C. Ciliberto discuss topics from number theory and its applications. Graphs and their applications in other fields are discussed by C. Berge. Riemannian geometry is the topic of a paper by M. Berger. A paper by M.-F. Roy surveys results in real algebraic geometry. Various versions of *K*-theory are discussed by M. Karoubi. Two papers by V. Arnold are devoted to the local theory of critical points of functions and to dynamical systems. Relations between thermodynamics and chaotic dynamical systems are studied by V. Baladi. B. Mandelbrot discusses questions in fractal geometry. A review by J. Dieudonné is devoted to mathematics during the last 50 years in France, while one by V. M. Tikhomirov discusses Moscow mathematics in this period. A paper by M. Smorodinsky treats questions in information theory, and I. Chalendar and J. Esterle discuss some questions in functional analysis. Fourier analysis is treated by P. L. Butzer, J. R. Higgins and R. L. Sten, and wavelets by S. Jaffard. PDEs are discussed by J. Sjöstrand and R. Temam (microlocal analysis, Navier-Stokes equations). The calculus of variation is the topic of a paper by F. H. Clarke. Several complex variable theory and plurisubharmonic functions are discussed by P. Dolbeault and C. Kiselman. Random processes, branching processes, Brownian motion, random walks, statistics and stochastic analysis are treated by P.-A. Meyer, M. Yor, J.-F. Le Gall, Y. Guivarc'h, K. D. Elworthy, G. R. Grimmett, L. Le Cam and B. Prum. The book concludes with interviews with A. Douady, M. Gromov and F. Hirzebruch, and various bibliographic and other data. It is important to know our own history, and this book contributes very well to this aim. (vs)

C. C. Pugh, ***Real Mathematical Analysis, Undergraduate Texts in Mathematics, Springer, New York, 2002, 437 pp., EUR 59,95, ISBN 0-387-95297-7***
This introduction to undergraduate real analysis is based on a course taught many times by the author over the last thirty-five years at the University of California, Berkeley. It starts with the definition of real numbers by cuts and continues with basic facts about metric and topological spaces. A clear exposition of differentiation and Riemann integration of a function of real variable appears in Chapter 3, while Chapter 4 presents basic facts about the space of continuous functions. Multivariable calculus, including a general Stokes' formula, is studied in Chapter 5 and the book ends with Lebesgue integration theory. The exposition is informal and relaxed, with a number of pictures. The emphasis is on understanding the theory rather than on formal proofs. The text is accompanied by very many exercises, and the students are strongly encouraged to try them. (sh)

W. Rossmann, ***Lie Groups: An Introduction Through Linear Groups, Oxford Graduate Texts in Mathematics 5, Oxford University Press, 2002, 265 pp., £35, ISBN 0-10-859683-9***
A general theory of simple Lie groups and their finite-dimensional representations is usually left for courses at graduate level. To present the basic ideas of the theory to undergraduates is much easier and transparent if it is done for Lie groups of matrices. This is the approach chosen in this book. Basic facts of the Lie theory for linear groups (exponential map, Campbell-Baker-Hausdorff series, Lie algebras of linear groups, homomorphisms and coverings, roots and weights, topology of classical groups) are explained in the first three chapters. The next short chapter introduces basic properties of manifolds, Lie groups and their homogeneous spaces. Chapter 5 describes integration on linear groups and their homogeneous spaces, and the Weyl integration formula. The final chapter describes finite-dimensional representations of classical groups (including Schur's lemma, the Peter-Weyl theorem, characters, Weyl character and Weyl dimension formulas and the Borel-Weil theorem). Each chapter contains many useful exercises, very suitable for problem sets.

The book is very well written, covering essential facts needed for understanding the theory, and is easy to use when preparing courses on Lie groups. The chosen approach follows Weyl's original ideas and helps one develop a suitable intuitive background for the general theory. Historical remarks scattered through the book are interesting and useful. This is an excellent addition to the existing literature and should be useful for teachers and students, and for mathematicians and mathematical physicists. (vs)

A. Shen and N. K. Vereschagin, ***Basic Set Theory, Student Mathematical Library 17, American Mathematical Society, Providence, 2002, 116 pp., US$21, ISBN 0-8218-2731-6***
The contents of this book are based on lectures given in the authors' undergraduate courses at the Moscow State University. It gives the reader a survey of 'naïve' set theory, set theory without an axiomatic approach. The text is full of interesting challenges to the reader – exercises with a wide range of difficulty.

Chapter 1 introduces the notions of a set and its cardinality, and present the Cantor-Bernstein theorem, Cantor Theorem, etc. The final two paragraphs give the set-theoretical definition of a function and operations on cardinal numbers, with emphasis on injections, surjections and bijections and their role in comparing the cardinalities of sets. The style of problems to be solved is rather combinatorial. Chapter 2 focuses on ordered sets and different types of orderings. Under transfinite induction and recursive definition, Zorn's lemma and Zermelo's theorem appear, followed by an application to the existence of the Hamel basis. Ordinals and ordinal arithmetics are presented here, followed by an application to Borel sets. The book concludes with a short glossary and a bibliographical index. The book should be understandable to undergraduate students, while some parts should also be of interest to others. (lpo)

H. Sohr, ***The Navier-Stokes Equations: An Elementary Functional Analytic Approach, Birkhäuser Advanced Texts, Birkhäuser, Basel, 2001, 367 pp., 104 euros, ISBN 3-7643-6545-5***
This book is devoted to the system of Navier-Stokes equations for a velocity vector field u with pressure π. The first n equations of the system express the law of balance of momentum, while the final equation is the continuity equation for incompressible fluid with constant density and viscosity 1. As a special case, we get the system describing Newtonian flow. The author uses a functional-analytic approach to study the existence of a weak solution u, the existence of the associated pressure π and additional regularity of u and its uniqueness. The method is based on reformulating these problems in terms of the Stokes operator A, its fractional powers, and the semigroup generated by A. Although this approach is well known, various applications to the system of Navier-Stokes equations are dispersed throughout the literature.

The author assumes a basic knowledge of functional analysis tools in Hilbert and Banach spaces, but most of the important properties of Sobolev spaces, distributions, operators, etc., are collected in the first two chapters of the book. Properties of the Stokes operator are studied in detail in Chapter III, where the stationary variant of the system is also considered. The system is non-linear due to the convective term, which appears from the 'convective derivative of momentum' and is responsible for most of the troubles with existence, regularity and uniqueness of solution. Thus, the author drops the convective term first and investigates the linearised variant of the system in Chapter IV. It is interesting that problems with regularity of pressure appear already. The full non-linear system is considered in the last chapter.

The reader will find a very interesting, clear, concise and self-contained approach to the mathematical theory of the Navier-Stokes equations. The book will be of interest to applied and pure mathematicians, as well as to postgraduate students working in the field of fluid mechanics. (pka)

F. G. Timmesfeld, ***Abstract Root Subgroups and Simple Groups of Lie-Type, Monographs in Mathematics 95, Birkhäuser, Basel, 2001, 389 pp., DM 220, ISBN 3-7643-6532-3***
The main topic treated in this book is the theory of groups generated by abstract root subgroups. Their description (starting with the special case of rank 1 groups) can be found in the first two chapters. The main part of the book is Chapter 3, describing the classification of groups generated by a class of abstract root subgroups. Chapter 4 describes a reduction of the classification of finite groups generated by a class of root involutions to the classification of groups generated by abstract root subgroups. The final chapter contains some applications, and in this part proofs are merely sketched. The book will be of interest to mathematicians interested in finite group theory. (vs)

G. Wüstholz, ***A Panorama of Number Theory or The Ìiew from Baker's Garden, Cambridge University Press, 2002, 356 pp., £55, ISBN 0-521-80799-9***
These conference proceedings commemorating Alan Baker's 60th birthday have a two-fold purpose: to document the continuing influence of Baker's seminal work on transcendence theory and diophantine approximations from the 1960s and 1970s, and to take stock of the most recent developments in these (and related) areas. The contributors include E. Bombieri, J.-H. Evertse, G. Faltings, D. Goldfeld, G. Margulis, D. W. Masser, Yu. V. Nesterenko, P. Sarnak and G. Wüstholz. (jnek)

A. Zygmund, ***Trigonometric Series, Vols. I, II, 3rd edition, Cambridge University Press, 2003, 364 pp., £39.95, ISBN 0-521-89053-5***
The third edition of this well-known and popular book appears with a foreword by Robert Fefferman as a paperback in the Cambridge Mathematical Library. Both volumes are included here. Seventeen chapters contain all the classical related results: while Volume I deals with basic results on Fourier series (with the exception of Carleson's and Hunt's results on the convergence almost everywhere of Fourier series of functions from L^p, $p > 1$), Volume II is devoted to such topics as trigonometric interpolation, differentiation of series, generalised derivatives and the Fourier integral. Instead of listing the missing results, which would probably be simpler than making an account of those which are included, I will borrow the last few lines from Fefferman's foreword: 'In fact, what is surprising about the current volume is not what is missing. What is surprising is that a single person could write such an extraordinary comprehensive and masterful presentation of such a vast field. This volume is a text of historic proportion, having influenced several generations of some of the greatest analysts of the twentieth. It holds every promise to do the same in the twenty-first.' The text can be recommended for libraries, students and anyone wanting a good reference book on the field. (jive)

Interdisciplinary research opportunity for mathematicians under the 6th Framework Programme

The European Commission recently published the first call for proposals for research on 'new and emerging science and technology' (NEST), under the 6th EU Research Framework Programme (2003-2006). The 215 million euro NEST action opens the possibility for the Commission to finance research at the frontiers of knowledge in areas suggested by researchers themselves, with an emphasis on multi-disciplinary research. Research linking advanced mathematics with other disciplines is eligible for funding under NEST

The present call for proposals involves an indicative budget of 28 million euro, but the level of funding for NEST will increase substantially over the four years of the programme. It concerns two action lines:

- ADVENTURE projects will feature novel and interdisciplinary research areas, ambitious and tangible objectives, with a potential for high impact, but also imply a high risk of failure.
- INSIGHT projects will investigate discoveries that might hamper our health and quality of life, such as for instance a new pathogen or environmental contaminant, a potential failure in financial and other markets, new forms of crime etc.

Further information can be obtained from the European Commission at *www. cordis.lu/nest*
NEST mailbox: *rtd-nest@cec.eu.int*