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EUROPEAN MATHEMATICAL SOCIETY**NEWSLETTER No. 36****June 2000**

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NOTICE FOR MATHEMATICAL SOCIETIES

*Labels for the next issue will be prepared during the second half of August 2000.
 Please send your updated lists before then to Ms Tuulikki Mäkeläinen, Department of Mathematics,
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The Catalan Mathematical Society

Sebastià Xambó-Descamps

One of the 25 societies of the Institute of Catalan Studies (IEC), the Catalan Mathematical Society (SCM) today faces important challenges. Some of these stem from its local contributions to World Mathematical Year 2000, but the main one springs from the EMS Council's decision at Budapest in 1996 to select the SCM's bid to organise the Third European Congress of Mathematics (*3ecm*). This takes place in Barcelona from 10-14 July, in the Palau de Congressos (Congress Palace), with 1500 participants expected.

That the SCM has such international responsibilities should interest mathematicians and professionals who use mathematics, and also most Catalonians, since one of the goals of the Rio de Janeiro declaration was to promote mathematics as a basic element for world development. This fact, a natural consequence of the ingredients that have contributed to the present level of science and technology, is today even more pronounced due to the progressive digitisation of information and knowledge (including fundamental aspects of the financial world), particularly with regard to the operations that regulate its storing, transmission and use.

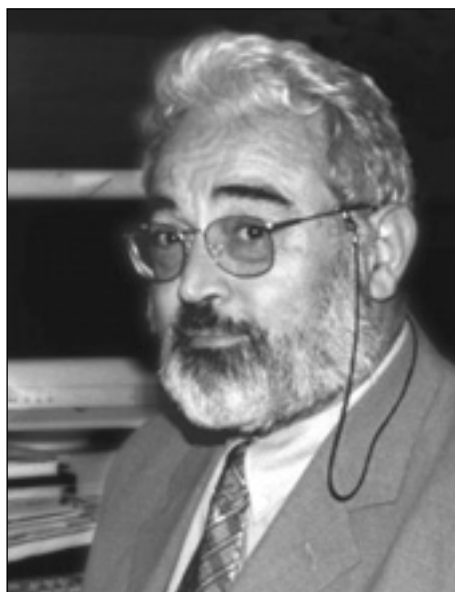
According to its statutes, the SCM's mission includes: "to cultivate the mathematical sciences; to spread its knowledge among the Catalan society; to foster its teaching and research, both pure and applied; and to publish whatsoever works that fit these goals". What are the main activities in which the Society engages in order to fulfil these aims?

The oldest is the organisation in Catalonia of the regional contribution to the Mathematical Olympiad, including the preparation sessions. Since 1996, it has also organised the European 'Kangaroo' or *Cangur*, also called the *Mathematical Feast*. This year, on 16 March, there were over 6000 participants from the top four levels of secondary education, with the collaboration (for the first time) of several city halls.

Last March the *Trobada Matemàtica* (Mathematical meeting) was held for the third time. Taking place once a year, it is based around five or six interdisciplinary lectures by mathematicians from Catalan-speaking lands.

The Society publishes two journals: the **Bulletí** twice a year, and the **SCM Notícies** four times a year. It also publishes books, such as two classics recently translated into Catalan: C. F. Gauss's *Disquisicions aritmè-*

tiques and René Descartes' *La Geometria*. Among high school teachers, the *Recull Cangur* has a growing popularity; this book is a collection of *Cangur* problems, updated every year to prepare entrants for the Mathematical Olympiad.



Another initiative is the Évariste Galois prize for students, awarded each year. In addition, every two years the IEC awards the *Josep Teixidor prize* for the best PhD thesis and the prestigious *Ferran Sunyer i Balaguer international prize* for research memoirs.

The SCM carries out its activities in all the lands of the Catalan language and culture. Thus *Cangur* 1999 and 2000 were organized jointly, for the first time in 1999, in Catalonia, the Balearic Islands and the Valencia Country, while this year the *Trobada* was held in Valencia; the previous two meetings were held in Barcelona and it is hoped that the 2001 meeting will be in Palma de Mallorca. Since mathematics is a universal language, the small linguistic variations between the local tongues, whether in the *Cangur* or the *Trobada*, are very pleasant, when perceptible, for the participants.

The Society also maintains close relations with the institutions on which it depends for its existence. In particular, since most of the courses that are offered need computer rooms and the Society has no such facilities, help from the mathematics schools has been indispensable. Another example is the relationship with the Institut Joan Lluís Vives (a network of the 18 universities in Catalan-speaking lands), which in 1999 housed the

Cangur and awarded special prizes to the twelve best winners of the joint three communities; the award ceremony for this year will be in Barcelona in July, after the *3ecm*.

The SCM is a member of the EMS and has reciprocity agreements with several other societies, among them the Real Sociedad Matemática Española (RSME) and the American Mathematical Society (AMS).

The present SCM, with over 1000 members, was founded in 1986 as one of the four societies resulting from the splitting by the IEC of the *Catalan Society for Physics, Chemistry and Mathematics* (SCCFQM). The SCCFQM was founded in 1931, inside the Science Section of the IEC (which itself dates from 1907), and was initially structured in three branches: Physics and Chemistry; Mathematics and Engineering; and Astronomy, Meteorology and Geophysics. The 1986 SCCFQM resulted from a restructuring in 1968, in which the third branch disappeared and the first was divided into two, Physics and Chemistry, and a second restructuring in 1973, in which the second branch was also split into two, Mathematics and Engineering. In the late 1970s and early 1980s, the Mathematics section experienced considerable growth under the presidency of Manuel Castellet, who is currently President of the IEC and a member of the EMS Council: there was a substantial increase in the membership and of the number of organised activities, such as the 1980 Spanish-Portuguese Meeting.

In the past twenty years, the SCM has been a nexus between researchers in the three mathematics schools (the University of Barcelona, the Universitat Autònoma de Barcelona and the Universitat Politècnica de Catalunya), high school teachers, and mathematicians working in other organisations. An institutional presentation to the general public of World Mathematical Year 2000 was held on 7 March at the University of Barcelona, jointly promoted by the SCM, the three mathematics schools and FEEM-CAT, an organisation of secondary level teachers of mathematics. These same institutions also organised a celebration of WMY2000 at the Parliament of Catalonia in early June.

To conclude, I hope that the Society can continue to play this integrating role, and that more and more people value it as an attractive professional platform to join.

Sebastià Xambó-Descamps is President of the Catalan Mathematical Society

Barcelona Editorial

3ecm: Shaping the 21st century

Marta Sanz-Solé (Barcelona)

In July Barcelona will host the Third European Congress of Mathematics (*3ecm*). As the most visible manifestation of the EMS, the European Congresses of Mathematics aim to present the unity of mathematics in an interdisciplinary, plural, and interactive framework with an innovative format. They provide a forum for the diffusion of mathematical knowledge and for cooperation between mathematicians.

The theme chosen for the *3ecm*, *Shaping the 21st century*, invites mathematicians to think about the main streams that will guide the development of mathematics in the near future. The scientific programme reflects this purpose. Some of the activities may also encourage us to discuss the social role of mathematics and evaluate its impact on the development of new technologies in industry, in education and, more generally, in culture. As in former ECMs there will also be an opportunity to recognise the talent of brilliant young mathematicians through the award of EMS prizes.

By a happy coincidence, the *3ecm* occurs in World Mathematical Year 2000, declared by a resolution of UNESCO in November 1997. In Spain the Congress is considered the most important mathematical activity of the year.

More than eight years ago, the leaders of the Catalan Mathematical Society and their collaborators made a determined bid for the Congress. With effort and keenness, and with a background of self-confidence and hope, our colleagues proposed to host an ECM. Our bid was presented for the first time in Paris in 1992 and for the second time, successfully, in Budapest in 1996. The selection of Barcelona as the site for the *3ecm* is a landmark in our history, the end of a peculiar political situation, and a contribution to the normalisation of the status of mathematics in Catalonia and in Spain compared with that of other European countries with a long tradition in this subject.

Spain, and Catalonia as a part of it, has missed out on the creative progress in science in the modern age. In the early 20th century there were many attempts to create and promote institutions that fostered scientific research – in particular, research in mathematics. These endeavours were suddenly and sadly stopped by a long and cruel civil war. After April 1939 our country was immersed in conditions of material

and cultural poverty in which scientific research was an unattainable luxury. Many intellectual people decided, or were forced, to choose exile as a way to be safe. Those who remained lived in an environment of isolation compared with what was happening beyond the Pyrenees. Our lack of freedom often led to introspective attitudes, largely incompatible with the process of scientific activity.

However, the internal dynamism of a society oppressed by a political regime, but



with well-organised structures in clandestinity, made it possible for us to recover. From a timid attitude towards Europe we are now completely integrated in it. This complex social evolution has resulted in the normalisation of our activities in all fields, and has led in particular to a rapid development in science. Mathematics has been a part of this process. May I take this opportunity to express our gratitude to a generation of mathematicians who succeeded in teaching mathematics to a high level and who trained research fellows under conditions that were not conducive to enthusiasm.

A recent report by the Institute of Catalan Studies makes an accurate quantitative and qualitative analysis of mathematical research in the period 1990-96; its conclusion is that, in terms of production, the scientific level in our country is comparable with that of other European countries with a long tradition in mathematics. Nevertheless, the presence of Catalan

mathematicians in an international scientific forum is very unusual; few of them are members of editorial committees of high-level mathematical journals or take part in scientific committees. Now, for the first time, the Catalan mathematical community is involved in the organisation of a large-scale scientific event. This clearly provides us with a wonderful opportunity to increase the visibility of mathematics in our country and to collaborate to spread the aims of the EMS. This is also both a big challenge for us and a heavy responsibility. More than three years ago we undertook the organisation of the *3ecm*. We started with the Executive Committee as its kernel. Very soon we felt the need to enlarge the team of mathematicians working for the *3ecm*. An organising committee was appointed and support committees for specific tasks were created. In addition we received the valuable contribution of colleagues for special jobs. Many mathematics students have volunteered to assist during the week of the congress.

The organisation of the *3ecm* would have been impossible without substantial backing from a wide variety of public and academic institutions, private foundations and corporations. Thanks to the funds provided by a long list of sponsors we have been able to implement the main objectives of our organisational procedure:

- to provide financial support for young researchers in mathematics, with special attention to their professional situation and their country of origin.
- to afford the scientific recognition programme designed by the EMS for excellent young European researchers in mathematics.
- to offer good facilities for an excellent scientific programme conceived by the Scientific committee and the Round tables committee.

Attending a congress provides the opportunity to meet old friends, make new ones and enjoy some free time. Barcelona, the venue of the *3ecm*, is an attractive destination for cultural tourism. A privileged geographical situation, rich architecture and a large variety of museums focus the attention of a large number of visitors. The diversity of cultural activities is one example of the dynamism of a society known for its open character, vitality and hospitality. We are convinced that the participants of the *3ecm* will enjoy a high-quality scientific event in a splendid environment.

3rd European Congress of Mathematics

Shaping the 21st Century

Barcelona, 10 – 14 July 2000

Summary of main events

(For further details, and information about satellite conferences, see EMS Newsletter 34.)

The plenary lectures will be given by:

Robbert Dijkgraaf (Amsterdam), Hans Föllmer (Berlin), Hendrik Lenstra (Berkeley/Leiden), Yuri Manin (Bonn), Yves Meyer (Cachan), Carles Simó (Barcelona), Marie-France Vignéras (Paris), Oleg Viro (Uppsala/St Petersburg), Andrew Wiles (Princeton)

Parallel lectures will be given by:

Rudolf Ahlswede (Bielefeld), François Baccelli (Paris), Volker Bach (Mainz), Viviane Baladi (Paris), Joaquim Bruna (Barcelona), Xavier Cabré (Barcelona), Peter Cameron (London), Ciro Ciliberto (Rome), Zoé Chatzidakis (Paris), Gianni Dal Maso (Trieste), Jan Denef (Leuven), Barbara Fantechi (Udine), Alexander Givental (California), Alexander Goncharov (Providence), Alexander Grigor'yan (London), Michael Harris (Paris), Kurt Johansson (Stockholm), Konstantin Khanin (Edinburgh/Moscow), Pekka Koskela (Jyväskylä), Steffen Lauritzen (Aalborg), Gilles Lebeau (Palaiseau), Nicholas Manton (Cambridge), Ieke Moerdijk (Utrecht), Eric Opdam (Leiden), Thomas Peternell (Bayreuth), Alexander Reznikov (Durham), Henrik Schlichtkrull (Copenhagen), Bernhard Schmidt (Augsburg), Klaus Schmidt (Vienna), Bálint Tóth (Budapest)

Mini-Symposia:

The speakers at each mini-symposium have been selected by the Chair (listed below).

Quantum chaology [Sir Michael Berry, Bristol], *Computer algebra* [Wolfram Decker, Saarland], *Mathematics in modern genetics* [Peter Donnelly, Oxford], *String theory and M-theory* [Michael Douglas, Rutgers], *Mathematical finance: theory and practice* [Hélyette Geman, Paris], *Quantum computing* [Sandu Popescu, Cambridge], *Free boundary problems* [José Francisco Rodrigues, Lisbon], *Symplectic and contact geometry and Hamiltonian dynamics* [Mikhail Sevryuk, Moscow], *Curves over finite fields and codes* [Gerard van der Geer, Amsterdam], *Wavelet applications in signal processing* [Andrew Walden, London]

Round Tables

Mathematics teaching at the tertiary level

What is mathematics today?

The impact of mathematical research on industry and *vice versa*

The impact of new technologies on mathematical research

Building networks of cooperation in mathematics

How to increase public awareness of mathematics

Shaping the 21st century

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Phone: (+34)-93-270-16-20 Fax: (+34)-93-270-11-80

EMS Council Meeting, 7-8 July 2000

The EMS Council Meeting will take place at the Institut d'Estudis Catalans, Carrer del Carme 47, E-08001 Barcelona (Spain),

starting at 10 a.m. on 7 July

Contact: EMS Secretariat, e-mail: makelain@cc.helsinki.fi

Barcelona accommodation

The Council meeting room is located at the CSIC Residence, where participants can be housed. You can see some information about it in:

http://www.resa.es/home_eng.htm

<http://www.resa.es/csic/castellano/RESIDENCIA.HTM>

The number of reservations is limited.

The address is: Residence Hall for Researchers, C/ Hospital, 64, 08001 Barcelona, Spain.

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e-mail: resa.inv@mx4.redestb.es

Javier Martinez de Albeniz

[albeniz@eco.ub.es],

Treasurer of the SCM

EMS Secretariat

Tuulikki Mäkeläinen

Since the creation of the Society, the Secretariat of the EMS has been at the Department of Mathematics of the University of Helsinki and I have been in charge of it. My main objective has been to try to help the Executive Committee and others working for the EMS by taking care of some of the day-to-day routines.

My activities include preparations with the President, Secretary and Treasurer for the Executive Committee and Council meetings; contacts with member societies; contacts with individual members; daily financial transactions, in consultation with the Treasurer; keeping track of various bureaucratic handlings; replying to queries. Since receiving a M.Sc. degree in Mathematics at the University of Helsinki I have been working with mathematics and mathematicians, with the International Congress of Mathematicians in Helsinki, the International Mathematical Union, with various mathematical journals, and now with the EMS. During these years I have met countless mathematicians. It was once suggested that I write a book *Mathematicians in my life* – I could, with a subtitle 'with love'.

Tuulikki Mäkeläinen, e-mail: tuulikki.makelainen@helsinki.fi



EMS Executive Committee meeting

Bedlewo 24-25 March 2000

Committee members present were: Rolf Jeltsch (President, in the Chair), David Brannan (Secretary), Bodil Branner, Doina Cioranescu, Luc Lemaire (Vice-President), Olli Martio (Treasurer), Andrzej Pelczar (Vice-President) and Marta Sanz-Solé. Apologies were received from Renzo Piccinini and Anatoly Vershik. Also in attendance (by invitation) were Jean-Pierre Bourguignon, Mireille Chaleyat-Maurel, Volker Mehrmann, Tuulikki Makelainen, David Salinger and Bernd Wegner.

Officers' Reports

The President announced that Volker Mehrmann had been elected Editor of the left-hand side of EMIS, and David Salinger had been elected EMS Publicity Officer. He had signed a sponsorship contract with MathDidaktik, made a presentation in Macao, and attended the AMU 2000 Pan-African Congress in Cape Town. J.-P. Bourguignon had attended a meeting in Denmark on digitising journals; it would be good for similar European activities to work together in a co-ordinated way.

The Secretary was looking at remits of committees in cases where existing ones were available, and for possible remits for those that were lacking; he was directed to send the draft remits to Committee Chairs for comment.

The Treasurer reported briefly on the financial statements for the year 1999, which were accepted and signed. Waivers and reductions of fees for certain corporate members who had difficulty in paying their fees were agreed. A proposal for the Society budget for the years 2001 and 2002 was approved and will be submitted to the Society Council meeting in July 2000.

Council Meeting in Barcelona: 7-8 July 2000

The delegates for the individual members for the period 2000-03 will be: Giuseppe Anichini, Vasile Berinde, Giorgio Bolondi, Alberto Conte, C. T. J. Dodson, Jean-Pierre Francoise, Salvador S. Gomis, Laurent Guillope, Klaus Habetha, Willi Jager, Tapani Kuusalo, Laszlo Marki, Andrzej Pelczar, Zeev Rudnick and Gerard Tronel. The terms of office of four existing members of the Executive Committee end at the end of the year 2000, but they are eligible for re-election.

The Executive Committee will propose to Council that Oslo be the site for the Council meeting in 2002, with dates 8-9 June 2002, and will make a recommendation as to the location of the European Mathematical Congress in Summer 2004.

Membership matters

It was agreed to recommend to Council that the European Society for Mathematical and Theoretical Biology (ESMTB) and TIWM

Kaiserslautern be elected corporate members. The European Mathematical Trust had reported its dissolution and thus its withdrawal from EMS membership. It was noted that

the AMS and corporate members of the EMS." It will be proposed to Council that further reciprocity agreements can be approved by the Executive Committee. The first such



Institute of Mathematics, Bedlewo

there is a need for two separate classes for institutional membership, one for academic and one for commercial institutes, and that this should be taken care of whenever the Statutes are revised.

A reciprocity agreement between the EMS and the American Mathematical Society will be recommended to the EMS Council for approval. It will contain the sentence: "This exchange agreement shall in no way interfere with existing reciprocity agreements between

agreements are likely to be between the EMS and the Australian Mathematical Society and the Canadian Mathematical Society.

EMS Committee matters

The procedure to elect members for EMS committees was reviewed: The Executive Committee should elect the Chair, and the Chair suggest committee members for the Executive Committee to approve. When the acceptance of the committee member has



David Brannan, Rolf Jeltsch, Bernd Wegner and Olli Martio

been received, their name and address should be put on EMIS.

The Summer School Committee will now be responsible for the organisation of an annual series of EMS lectures. A call for proposals will



Prof Bojarski, host of the meeting

be put on EMIS and in the *Newsletter*, with a deadline of 1 March in year N for year $N+1$; proposals should contain the name of the person responsible and location(s) for the lectures. Exceptionally, in the year 2000 the deadline should be 30 September for lectures

It was agreed that information on the possibility of obtaining funds for East and Central European mathematicians from the Society's Committee for the Support of East European Mathematicians should be more widely advertised.

Posters produced for the EMS Poster Competition for World Mathematical Year 2000 will be shown in Barcelona during the EMS Congress. An EU-supported 'Raising Awareness in Science Technology Week' will take place throughout Europe from 6-12 November 2000. It was decided to establish an EMS committee to plan the participation of EMS in the 'European Science and Technology Week', partly to replace the WMY2000 committee; and to support a proposal for a competition of papers for Raising Public Awareness in Mathematics.

Alberto Cabada was added to the Committee for Developing Countries; Volker Mehrmann was added to the Electronic Publishing Committee; and Vincenzo Capasso, Jean-Paul Pier and Olli Martio had joined the Group on Relations with European Institutions.

The Executive Committee decided to investigate holding a meeting of editors of non-commercial European mathematics journals



to be given in 2001.

It was agreed to draft an EMS position paper by early May for the French EU Presidency which begins 1 July.

E. Mezzetti had accepted an invitation to chair the Women and Mathematics Committee; her nomination of Polina Agranovich, Nicole Berline, Bodil Branner, Kari Hag, Ina Kersten, Laura Tedeschini Lalli, Irene Sciriha and Tsou Sheung-tsun as Committee members was accepted.

Carles Broto and Rudolf Fritsch were added to the Summer Schools Committee membership. It was agreed that EMS will pay for the production of posters for the EMS Summer Schools, as a standing arrangement. Progress was reported on planning for summer schools in 2001 in St Petersburg and Prague.

I. Diaz, A. Quarteroni and Kjell-Ove Widman have agreed to join the Special Events Committee. The Diderot Forum on 'Mathematics and Music' had been very successful.

during the Barcelona Congress.

The Publicity Officer agreed to update existing EMS publicity material prior to 30cm, and revised arrangements for the production of the annual EMS 'Agenda' poster were approved.

Publication matters

P. Massart has joined the group of Associate Editors of JEMS.

It was agreed that the *EMS Newsletter* will in future provide a limited amount of free space for member societies, for their publications, meetings, etc., with equal rights for the EMS in their Newsletters, and to remind the EMS member societies of the recommendation to use in their letterheads the phrase "member of the European Mathematical Society". Vivette Girault is now in charge of seeking advertising for the *EMS Newsletter*.

The question of publishing a small booklet on 'Early History of EMS' was discussed.

Relations with Mathematical Institutions/Organisations

The 1st EMS-SIAM conference – on *Applied Mathematics in a Changing World* – has been planned to take place in Berlin from 3-6 September 2001; the local organiser is Peter Deuffhard, and Vincenzo Capasso, Heinz Engl, Bjorn Engquist and Stefan Mueller were nominated to represent the EMS on the Programme Committee.

The President is discussing several possible forms of operation with the new President of the African Mathematical Union.

It was agreed that the EMS should apply for membership as a "large associated society" in ICIAM.

Future Executive Committee meetings

6 July 2000, in Barcelona

11-12 November 2000, in London

March 2001, at ITWN in Kaiserslautern

And finally ...

The President thanked the Polish Academy of Sciences, Polish Mathematical Society and Bogdan Bojarski for their assistance in organising and supporting the meeting, and for their tremendous hospitality.

EMS and the International Mathematical Union

The EMS has been granted affiliate status in the IMU.

According to the IMU Statutes, multinational mathematical societies and professional societies can be affiliated with the IMU. The affiliated members have the right to participate in the IMU General Assembly, but have no voting rights. They have a right to submit proposals for joint activities.

For information on the IMU,
see, for instance,
<http://elib.zib.de/IMU>

European Research Conferences (EURESCO)

The European Science Foundation is inviting proposals for new conference series as well as the continuation of established ones. It aims to consolidate at around 50 conferences per annum. The EURESCO programme supports series of top-level meetings devoted to the same general subject. Normally the conferences in a series take place every other year.

If a conference is already supported by the European Commission, it can attract up to an extra 8000 euros. If not so supported, the sum can be up to 25,000 euros.

Deadline: 15 September 2000

Details from: www.esf.org/euresco or from

European Science Foundation,
EURESCO, 1 quai Lezay-Marnèsia, 67080
Strasbourg Cedex, France

Report on the *LIMES-Project*

Michael Jost and Bernd Wegner

LIMES is a project that will be guided by the EMS and funded within the Fifth Framework Programme (FP5) of the European Community. The acronym *LIMES* stands for *Large Infrastructure in Mathematics – Enhanced Services*. It will deal with the enhancement of Zentralblatt MATH to a European database in mathematics. The project fits into the horizontal FP5 programme 'Improving human research potential and the socio-economic knowledge base', providing and improving access to research infrastructures. The duration of the project is from April 2000 to March 2004.

The partners of the *LIMES* project are: FIZ (Fachinformationszentrum Karlsruhe, Germany), MDC (Cellule de Coordination Documentaire Nationale pour les Mathématiques, France), Eidetica (The Netherlands), SIBA (University of Lecce, Italy), DTV (Technical Knowledge Centre and Library of Denmark, Denmark), USDC (University of Santiago de Compostela, Spain), HMS (Hellenic Mathematical Society, Greece), TUB (Technical University of Berlin, Germany), with the EMS as the supervising society. The project coordinator will be FIZ. The contracts for the funding of the projects were signed in March 2000 and the work started on 1 April.

Objectives

The objective of the project is to upgrade the database Zentralblatt MATH into a European-based world-class database for mathematics and its applications, by a process of technical improvement and wide Europeanisation. Upgrading the existing database, improving the present system, and developing a new distributed system, both for the input and output of the data, are necessary to allow Zentralblatt MATH to use the latest developments and to anticipate future developments in electronic technology. This will make Zentralblatt MATH a world reference database, offering full coverage of the mathematics literature world-wide, including bibliographic data, reviews and/or abstracts, indexing, classification, excellent search facilities, and links to offers of articles, with a European basis.

Improvements will be made in three areas:

1. Improvement of content and retrieval facilities through sophisticated further development of the current data sets and retrieval programs.

2. Broader and improved access to the database via national access nodes and new data distribution methods. In particular, improvement of access for isolated universities in regions in economic difficulties and in associated states of Central and Eastern Europe where a mathematical tra-

dition of excellence is under economic threat. Stimulation of usage for all kinds of research, as well as for funding organisations before decision making by initial support of two national test sides.

3. Improved coverage and evaluation of research literature via nationally distributed editorial units, development of technologies for efficient database production, exemplified by two further European member states.

The *LIMES* project sets out to achieve these goals by stimulation of usage (national access nodes and centres for dissemination, including development of adequate licensing models, and links with offers of complete documents); better coverage and more precise evaluation of literature through improved update procedures (for the data gathering, but also for data distribution to access points) via a network of distributed editorial units. Networked access to the database these days is mainly via rapidly evolving WWW technologies, with high potential for further development of innovative access and retrieval methods, and improvements in the quality of data (such as author and source identification through data-mining techniques).

In contrast to the centralised American model, Zentralblatt MATH will be extended to a European research infrastructure with distributed sites (Editorial offices and Access nodes) in the member states. The EMS, representing the scientific community concerned, has promoted these ideas among the national societies and individual members.

Since the EMS is member of Zentralblatt MATH's Editorial Board, it is guaranteed that the results of the project will be available for usage in further research infrastructure partnerships. Tools and methods developed during the project will be usable by project partners on a royalty-free license basis. Further results, documentation and information will be made publicly available. Commercial exploitation of results beyond European research infrastructure services will be considered.

The role of participants

FIZ operates the central editorial office for Zentralblatt MATH (de/). Founded in 1931, it is today the longest-running international abstracting and reviewing service in the field of pure, applied and industrial mathematics. Zentralblatt MATH covers the entire spectrum of mathematics and theoretical computer science, with special emphasis on areas of applications, and provides about 60,000 reviews per year. As a consequence of this role, FIZ will be engaged in all work packages in the project.

MDC provides mathematics research

libraries and mathematics departments with technical assistance on computerised documentation. In 1999, the MDC released edbm/w3, the first module of the European Database Manager for Mathematics software. In *LIMES* MDC will be in charge of the extensive development of this software-like integration of new tools for search, identification, display, and update, the development and integration of improved access control management, and it will help national access nodes with the development and installation of improved update procedures.

Eidetica is the most recent spin-off company of CWI. It was created by researchers in the Interactive Information Engineering theme, combining expertise in linguistics and mathematical clustering of dependency networks for textual domains. Eidetica will mainly develop new models and tools for identification of author and serial names (two of the most requested features from the user community), based on data-mining techniques, and other improvements of the retrieval process.

SIBA is the structure of the University of Lecce, Italy, that coordinates, manages and develops the Telematics Information System for Research. SIBA coordinates the SINM project, which aims at developing a National Information System for Mathematics which will allow the Italian mathematical community to have easy access to the bibliographic and full-text resources available in the libraries of the field, on-line and on CD-ROM. In *LIMES* SIBA will establish an editorial unit for Zentralblatt MATH in Italy, identifying and covering mathematical publications of the region, according to high reviewing standards and editorial policies.

DTV was established in 1942 as the central library of the Technical University of Denmark (DTU) and the national library for engineering and applied sciences in Denmark. The role of DTV will be twofold: 1. Setting up and running an editorial unit. The unit will be responsible for covering a number of journals and other sources, to be defined and agreed by DTU/MATH/DTV and the coordinator. The unit will be engaged in experimenting with modern electronic-reviewing methods, especially with the results of the SIBA-UNILE system development in this area.

2. Establishing a raw data system for the editorial units. Following the agreement of the publishers, this data will be made available for *LIMES* in two ways:

- for end-user current awareness purposes, through a special *LIMES* gateway added to the Current Awareness System of the EMS and to the Zentralblatt MATH database,
- for cataloguing reuse by the editorial units through the export of raw cataloguing data to the Zentralblatt editorial system.

The role of USDC will be to propagate the use of the database in Spain and to facilitate the access to it, first by installing a national site server and regularly updating the database and new software releases, and second by setting up a task group for

stimulating the usage of the database by Spanish universities, research centres and libraries. A network structure for information in mathematics will be developed, integrating Zentralblatt MATH with national information offers. This work will be undertaken in cooperation mainly with CESGA (Centro de Supercomputacion de Galicia) and RSME (Real Sociedad Matemática Española). HMS, founded in 1918, aims to encourage and contribute to the study and research of mathematics and its applications, as well as to improve mathematical education in Greece. HMS will apply a methodology similar to that of USDC in a different environment.

TUB is one of the largest universities in Germany, offering curricula for the full scale of engineering sciences. The members of its mathematics department have traditionally cooperated with Zentralblatt MATH by serving as consultants for the input to the database. In particular, the Editor-in-chief of Zentralblatt MATH is

member of that department and can look back on a 25-year period of experience in this position. TUB will cooperate with all partners, providing them with continuous advice and caring about the evaluation of results. An important task for TUB will be the development of editorial tools.

As a partner in the Editorial Board of Zentralblatt MATH, the EMS has set up an Innovations Committee whose main task is to define and enact innovative features and procedures in Zentralblatt MATH. The EMS has two roles to play in the project: it represents the end-user, and gives a pan-European framework to the project as a whole. In both roles, it will be part of the Project management.

Michael Jost, Fachinformationszentrum
Karlsruhe, Abteilung Mathematik,
Franklinstrasse 11, D-10587 Berlin
Bernd Wegner, TU Berlin, Fachbereich
Mathematik, Sekr. MA 8-1, Strasse des 17 Juni
135, D-10623 Berlin

EMS 2001 LECTURES - CALL FOR PROPOSALS

The European Mathematical Society has been running a successful series of EMS Lectures for some years. For example, in June this year Professor George Papanicolaou (Stanford, USA) will give series of lectures over a period of four days at ETH in Zurich (Switzerland) on *Financial Mathematics* and over three days at The University of Crete, Heraklion, Crete (Greece) on *Time reversed acoustics*.

The EMS Lectures may be in Pure or Applied Mathematics or may span both areas. With this activity, the Society aims to encourage European mathematicians (especially young mathematicians) to meet and study together current developments in Mathematics and its applications. The lectures should be given over several days in each of at least two locations, in order to give as many as possible the opportunity to attend.

Costs of participation should be kept low, and (if possible) grants should be available to people from countries which cannot afford any financial support. The EMS will guarantee its moral support to the selected lecture series, and will pay for posters advertising the lectures within the European mathematical community; it will also do its best to help the organisers to raise funds, and is likely to offer some financial support to organisers for participants who are young or come from European countries with financial difficulties.

Topics (which may be single or composite) for the lecture series, the sites, and the organisers of the schools will vary from year to year to cover a wide range of the subject.

The Society is now inviting proposals for at least one Lecture Series for 2001. Proposals should contain at least: the topic (title and short description), names of the likely lecturer, the site, the timing, conditions for participants and the name and address of the organiser submitting the proposal.

Please send proposals for series of EMS Lectures in 2001 to:

*Professor D. A. Brannan, Faculty of Mathematics and Computing, The Open University, Walton Hall, Milton Keynes MK7 6AA, United Kingdom
e-mail: ; fax: (+44) 1908 652140
if possible by 30th September 2000.*

The Society would hope to decide on proposals within a month or so.

EMS SUMMER SCHOOLS - CALL FOR PROPOSALS

The European Mathematical Society has been running a successful series of Summer Schools for some years now. Readers of the *Newsletter* will recall, for example, *Newsletter* reports on a 1996 Summer School in Hungary on *Algebraic Geometry* [issue 20] and a 1998 Summer School on *Wavelets in Analysis and Simulation* in France [issue 29].

The series is intended to include at least two summer schools each year – preferably at least one in Pure Mathematics and at least one in Applied Mathematics. With this activity, the Society aims to encourage young European mathematicians to meet and study together current developments in Mathematics and its applications.

The Society's Summer School Committee will also consider sponsoring proposals for summer schools fully organised by other institutions. To meet the EMS expectations, each school should be at pre-doctoral level, should last from 2 to 3 weeks, and have 100-200 participants – mainly graduate students or young mathematicians coming from several European countries. Costs of participation should be kept low, and (if possible) grants should be available to people from countries which cannot afford any financial support. The EMS will guarantee its moral support to the selected

schools, and will pay for posters advertising the summer schools within the European mathematical community; it will also do its best to help the organisers to raise funds.

Topics (which may be single or composite) for summer schools, the sites, and the organisers of the schools are likely to vary from year to year to cover a wide range of the subject.

The Society is now inviting proposals for at least two Summer Schools for 2002. Proposals should contain at least: the topic (title and short description), names of likely lecturers, the site, the timing, anticipated costs, conditions for participants, organising committee membership, and name and address of the organiser submitting the proposal.

Please send proposals for summer schools in 2002 to:

*Professor Renzo Piccinini, Department of Mathematics and its Applications, Università di Milano, Bicocca, Via Bicocca degli Arcimboldi 8, 20126 Milano, Italy;
e-mail: , fax: (+39) 02 6448 7705,
phone: (+39) 02 6448 7751,
if possible by 30 September 2000.*

The Committee would hope to decide on proposals within a month or so.

Correction to Newsletter 35

On page 18, at the end of Tony Gardiner's article on Mathematics in English schools, the final line was inadvertently omitted. The final sentence should have read: In later years (ages 12-15) *Number and algebra* may occupy 55% of the time, *Shape, space and measures* 30%, and *Handling data* 15%.

The Hilbert problems 1900-2000

Jeremy Gray

In 1900 David Hilbert went to the second International Congress of Mathematicians



David Hilbert, around 1900

in Paris to give an invited paper. He spoke on *The Problems of Mathematics*, to such effect that Hermann Weyl later referred to anyone who solved one of the 23 problems that Hilbert presented as entering the honours class of mathematicians. Throughout the 20th century the solution of a problem was the occasion for praise and celebration.

Hilbert in 1900

By 1900 Hilbert had emerged as the leading mathematician in Germany. He was famous for his solution of the major problems of invariant theory, and for his great *Zahlbericht*, or *Report on the theory of numbers*, published in 1896. In 1899, at Klein's request, Hilbert published *The foundations of geometry* as part of the commemorations of Gauss and Weber in Göttingen. Hurwitz saw clearly that the implications of that little book reached far beyond its immediate field. As he put in a letter to Hilbert:

You have opened up an immeasurable field of mathematical investigation which can be called the "mathematics of axioms" and which goes far beyond the domain of geometry.

Hilbert was therefore poised to lead the international community of mathematicians. He consulted with his friends Minkowski and Hurwitz, and Minkowski advised him to seize the moment, writing: *Most alluring would be the attempt to look into the future, in other words, a characterisation of*

Hilbert's Problems

Asterisks denote the ten problems presented during the Paris lecture

1. * Cantor's problem of the cardinal number of the continuum (*continuum hypothesis*)
2. * The compatibility of the arithmetical axioms
3. The equality of two volumes of two tetrahedra with equal bases and equal altitudes
4. The problem of the straight line as the shortest distance between two points (*alternative geometries*)
5. Lie's concept of a continuous group of transformations, without the assumption of the differentiability of the functions defining the group
6. * A mathematical treatment of the axioms of physics
7. * The irrationality and transcendence of certain numbers
8. * Problems of prime numbers (including the *Riemann hypothesis*)
9. A proof of the most general law of reciprocity in any number field
10. The determination of the solvability of a Diophantine equation
11. Quadratic forms with any algebraic numerical coefficients
12. The extension of Kronecker's theorem on abelian fields to any algebraic realm of rationality
13. * A proof of the impossibility of solving any equation of the 7th degree by means of functions of only two arguments
14. A proof of the finiteness of certain complete systems of functions
15. A rigorous foundation for Schubert's enumerative calculus
16. * The problem of the topology of algebraic curves and surfaces
17. The expression of definite forms by squares
18. The building-up of space from congruent polyhedra (*n*-dimensional crystallography groups, etc.)
19. * Determining whether the solutions of regular problems in the calculus of variations are necessarily analytic
20. The general problem of boundary values (variational problems)
21. * A proof of the existence of linear differential equations with a prescribed monodromy group
22. * The uniformisation of analytic relations by means of automorphic functions
23. Further development of the methods of the calculus of variations

the problems to which the mathematicians should turn in the future. With this, you might conceivably have people talking about your speech even decades from now.

And if the Congress itself was something

axiomatisation of physics. But he gave good reasons for caring about his problems, and bound it all up with his inspiring optimism. In opposition to Emil du Bois-Reymond's fashionable academic pes-



Hermann Minkowski



Adolf Hurwitz

of a shambles, Hilbert's written text nonetheless made its impact, just as Minkowski had predicted.

One reason is undoubtedly that, for Hilbert, problem solving and theory formation went hand in hand. Indeed, some of his problems are not problems at all, but whole programmes of research: Hilbert's 6th problem, for example, calls for the

simism, Hilbert insisted that in mathematics: *We can know, and we shall know.*

Hilbert presents his problems

Hilbert's problems came in four groups. In the first group were six foundational ones, starting with an analysis of the real numbers using Cantorian set theory, and including a call for axioms for arithmetic,

COMPTE RENDU
OU
DEUXIÈME CONGRÈS INTERNATIONAL
DES MATHÉMATIENS

TENU A PARIS DU 6 AU 12 AOÛT 1900.

PROCÈS-VERBAUX ET COMMUNICATIONS

Édité à Paris

E. DUPONCEAU,

Ingénieur des Télégraphes, Secrétaire général du Congrès.



PARIS,

GAUTHIER-VILLARS, IMPRIMEUR-LIBRAIRE

DE BUREAU DES LONGITUDES, DE L'ÉCOLE POLYTECHNIQUE,

1902

Réimpression avec accord de l'éditeur

KRAUS REPRINT LIMITED

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1967

Title page of ICM Paris Proceeding

sur les
PROBLÈMES FUTURS DES MATHÉMATIQUES,

PAR M. DAVID HILBERT (Göttingen),

TRADUITE PAR M. L. LAUGEL (*).

Qui ne soulèverait volontiers le voile qui nous cache l'avenir afin de jeter un coup d'œil sur les progrès de notre Science et les secrets de son développement ultérieur durant les siècles futurs? Dans ce champ si fécond et si vaste de la Science mathématique, quels seront les buts particuliers que tenteront d'atteindre les guides de la

Hilbert's lecture from the Paris Proceedings

and the challenge to axiomatise physics. The next six drew on his study of (algebraic) number theory, and culminated with his revival of Kronecker's *Jugendtraum*, and the third set of six were a mixed bag of algebraic and geometric problems covering a variety of topics. In the last group were five problems in analysis – the direction that Hilbert's own interests were going. He asked for a proof that suitably smooth elliptic partial differential equations have the type of solutions that physical intuition (and many a German physics textbook) suggest, even though it had been known since the 1870s that the general problem of that kind does not. He made a specific proposal for advancing the general theory of the calculus of variations.

The Hilbert problems very quickly succeeded in getting young mathematicians to create the future that Hilbert had conjectured – not always accurately, however, as we shall see. The Russian mathematician Serge Bernstein travelled from Paris to Göttingen in 1904 to present his proof that, under the conditions Hilbert had stated, elliptic partial differential equations do have analytic solutions. It would take more than a single book to describe the results produced in this mushrooming field in the 20th century. The opportunistic and power-seeking Bieberbach was another man drawn to the Hilbert problems for the fame that they could confer. In 1908 he showed that there are only a finite number of crystallographic groups in a Euclidean space of any dimension, thus solving part of Hilbert's 18th problem. His results confirmed that there are 17 patterns of this kind for the Euclidean plane, and 219 patterns (or crystal structures) for Euclidean 3-dimensional space.

Hilbert and axiomatisation

Hilbert himself did not work exclusively on the problems, and nor did most of his many students, who were instead often drawn in the early 1900s to the study of 'Hilbert space'. After 1909, when his friend Minkowski died, Hilbert became more and more interested in questions of axiomatisation and the foundations of mathematics. His interest in axiomatising physics was to lead to several lecture courses but few publications, and has accordingly been much misunderstood (see Leo Corry's work for the new picture). Hilbert lectured regularly and well, and he promoted the view that an axiom was a fundamental idea from which many others followed. So axioms played a crucial role in organising theories, as they had done in his *Foundations of geometry*.

As the years went by, the consistency of arithmetic seemed more of a challenge. In 1917 Hilbert wrote:

Since the examination of the consistency is a task that cannot be avoided, it appears necessary to axiomatise logic itself and to prove that number theory and set-theory are only parts of logic.

He imposed high standards on the task. What was needed, he said, were proofs that:

– every mathematical question be solvable in principle;

FEATURE

– every result be checkable;
– every mathematical question be decidable in a finite number of steps: this is the *decision problem* which asks: can a given statement be shown to be provable/refutable in a given theory, or is it independent?

By 1922 Hilbert became locked into a dispute with Brouwer, who believed that



L. E. J. Brouwer

the human mind was strikingly limited in its ability to deal with infinite sets. Hilbert's hopes for the future of mathematics were further darkened by the fact that his best student, Hermann Weyl, also found Brouwer's ideas attractive, and for a time seemed willing to forgo certain mathemat-



Hermann Weyl

ical arguments on philosophical grounds. Hilbert attempted to get round their arguments in 1922 by defining mathematics as a system of signs and distinguishing carefully between mathematics and meta-mathematics. Mathematics was to be identified with the stock of provable formulae, while inference about the content was only admissible at the level of a new meta-math-

ematics. On the basis of this distinction between valid formulae and their interpretations he called for a proof theory – truly a remarkable idea.

In the event Brouwer withdrew from the contest, Weyl found that he needed classical analysis for his work on Lie groups, and the crisis passed, although Hilbert's ideas about proof theory did not convince the experts. But Hilbert continued, with his assistants Ackermann and Bernays, and in 1931 he published a forceful re-statement of his views on the occasion of his becoming an honorary citizen of his native Königsberg. The lecture ends with a moving affirmation of his deepest belief about mathematics:

There are absolutely no unsolvable problems. Instead of the foolish ignorabimus, our answer is on the contrary: We must know, We shall know.

Ironically, the day before Hilbert lectured, the young Austrian logician Kurt Gödel also lectured in Königsberg on his



Kurt Gödel

incompleteness theorem, the work that is popularly said to have killed Hilbert's programme, even though, as Gödel said in the famous paper:

I wish to note expressly that [this theorem does] not contradict Hilbert's formalistic viewpoint.

The problem is that finding such proofs has proved elusive. No agreed place to stand has been found which compels universal assent (such as the elementary rules of logic) and which delivers all of set theory. On the other hand, powerful negative results continued to accumulate. When Alan Turing showed in 1936 that the decision problem is also unsolvable, the original hopes for Hilbert's programme were all in tatters.

The Hilbert problems between the Wars

The Hilbert problems themselves have perhaps proved a more enduring part of his legacy than Hilbert's own work on mathematical logic. Those on number theory have turned out, given Hilbert's own interests, to have been particularly well put. Ironically, Hilbert's own formulation

of Kronecker's *Jugendtraum* was misleading, but the Japanese mathematician Takagi, who had studied under Hilbert in Göttingen in 1900 while writing his thesis for the University of Tokyo, succeeded with the crucial generalisation to the abelian case in 1920. In 1923 Emil Artin bumped into Takagi's work almost by acci-



Emil Artin

dent and the work he did as a result enabled Hasse to solve Hilbert's 9th Problem (calling for a general reciprocity law) in 1927. Hasse had meanwhile also solved Hilbert's 11th problem, on quadratic forms, in 1923. In 1926 Artin solved Hilbert's 17th problem in a restricted, but nonetheless very general, case. Hilbert's 7th Problem (to show that a^b is an irrational transcendental number when a is algebraic and not equal to 0 or 1, and b is irrational and algebraic) was solved by the Soviet mathematician A. O. Gelfond in 1934, and by Th. Schneider independently later the same year.

The Bourbaki connection

After the Second World War the struggle for the heart of mathematics was won by the pure mathematicians. In this context a vigorous contest developed for the mantle of Hilbert. Should it go to mathematical logicians, or to applied mathematicians working in the tradition of Courant and Hilbert, or to the number theorists and algebraists? The most powerful advocate of the last of these views, both by word and deed, was Bourbaki. André Weil and Jean Dieudonné shared a view of mathematics that put Hilbert centre stage, although it was a Hilbert created in their own image. They espoused the axiomatic method, which Dieudonné claimed *has revealed unsuspected analogies and permitted extended generalizations; the origin of the modern developments of algebra, topology and group theory is to be found only in the employment of axiomatic methods.*

André Weil made the connection to Hilbert even more forcefully. In 1947 he quoted Hilbert:

A branch of science is full of life, as long as it offers an abundance of problems; a lack of problems is a sign of death.

Great problems, said Weil, furnish the daily bread on which the mathematician thrives. And turning to Hilbert's famous list of problems he singled out the 5th problem (then still unsolved) on Lie groups, the Riemann hypothesis, and the problem of generalising the theorems of Kronecker's *Jugendtraum* which 'still escapes us, in spite of the conjectures of Hilbert himself and the efforts of his pupils'.

The Hilbert problems after the War

Hilbert's Paris address had brilliantly united problems with their theoretical context and so, for a generation, did Bourbaki. The stock of the Hilbert problems rose with theirs. The remaining ones acquired an extra cachet for having held out, and they too began to fall. The 5th problem, on characterising Lie groups, was solved through the work of Gleason and of Montgomery and Zippin in 1952. The 14th problem on rings of invariants was

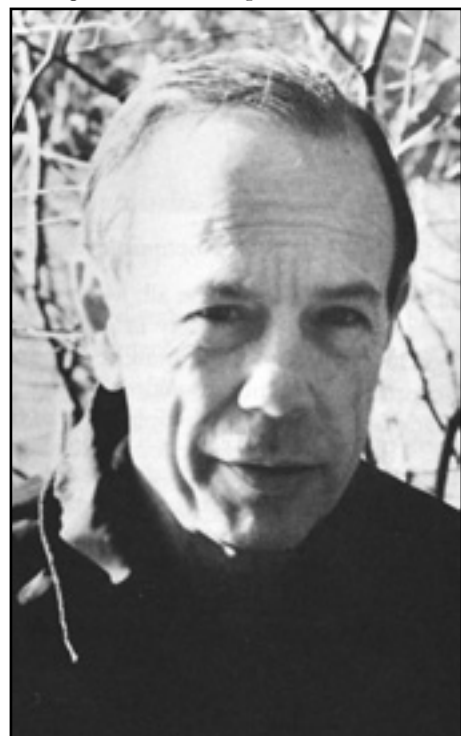
um hypothesis, Cohen did indeed settle Hilbert's 1st problem.

In the early 1970s the Russian mathematician Yuri Matijasevich solved Hilbert's 10th problem (Is there a finite process

problem. ... When I was a young topologist, that was a problem I really wanted to solve – but I could get nowhere. It was Gleason, and Montgomery-Zippin, who solved it, and their solution all but killed the problem. What else is



Martin Davis, Julia Robinson and Yuri Matijasevich



Andrew Gleason

which determines if a polynomial equation is solvable in integers?) in the negative, using earlier papers by the American mathematician Martin Davis, Hilary Putnam, and Julia Robinson. Julia Robinson had been close to solving this problem herself, and a powerful collaboration developed between her and Matijasevich, despite the Cold War. It follows from their work that had Hilbert's 10th problem been answered positively, Goldbach's conjecture (mentioned by Hilbert in Paris) would have been answered in the negative – a connection that Hilbert surely had not suspected.

Few of Hilbert's problems have dwindled with the years. Perhaps the complete solution of the 5th problem is a case in point. In 1986 Jean-Pierre Serre said: *Still, it is true that sometimes a theory can be killed. A well-known example is Hilbert's 5th*

there to find in this direction? I can only think of one question: [but it] seems quite hard – but a solution would have no application whatsoever, as far as I can see.

Yet there are numerous examples where the Hilbert touch has proved beneficial. The great development of algebraic number theory was surely animated by Hilbert's problems. The topic of partial differential equations was re-opened by Hilbert with his 19th problem, and much of that rich theory can be traced back to the work it inspired. Not only are the implications of the solutions and reformulations of the problems still to be worked out, some of the problems are still alive. Russian mathematicians have recently shown that in two cases the original 'solutions' were flawed, and have given different and rigorous accounts. Aspects of both halves of the 16th problem (on real algebraic curves and on vector fields) are still open questions. Hilbert's deepest vision, of the intricate dance of theory and problems in mathematics, is one that all mathematicians share, but few have articulated as well as he.



Paul Cohen

interpreted geometrically by Zariski and then solved in the negative by Nagata in 1959. In the 1960s and 1970s mathematical logicians turned back to the Hilbert problems. A high point was reached with the award of a Fields Medal in 1966 to Paul Cohen for showing that the axiom of choice and the continuum hypothesis are independent of the other axioms of set theory. The axiom of choice is not one of Hilbert's problems, but in calling for a consistent set of axioms for arithmetic, Hilbert had opened the way to similar analyses of all of mathematics, and indeed by establishing the independence of the continu-

Further reading

F. Browder (ed.), *Mathematical Developments Arising from Hilbert's Problems*, Symposia in Pure Mathematics, American Mathematical Society **28**, Providence, Rhode Island, 1976.

Leo Corry, 'David Hilbert and the Axiomatization of Physics (1894–1905)', *Archive for History of Exact Sciences* **51** (1997), 83–198.

Jeremy J. Gray, *We must know, we shall know; a History of the Hilbert Problems*, Oxford University Press, 2000 (to appear).

Jeremy Gray is a Senior Lecturer in the Department of Pure Mathematics, The Open University, UK.

Interview with Peter Deufhard

(President, Konrad Zuse Zentrum, Berlin)

interviewer: Rolf Jeltsch, EMS President (Zurich)

You will be organising the first EMS-SIAM meeting in Berlin from 2-6 September 2001. The main theme is 'Applied Mathematics in our Changing World'. How would you describe what applied mathematics is?

In principle, most substantial mathematics (including pure mathematics) is *applicable*. We now know that Hardy failed in aiming at mathematics that would surely never be applicable – never say 'never'. The question is, whether applicable mathematics, when applied, is already applied mathematics. It is my repeated experience that solving hard real-life problems usually goes far beyond just taking mathematical tools off the shelf. Hence, 'Applied Mathematics', as I understand it, deals with questions arising from quantitative problems outside mathematics and the corresponding development of new mathematics for their solution. This characterisation certainly includes most parts of computational mathematics (such as the design of efficient algorithms in differential equations or discrete mathematics), but also modelling (such as the regularisation of inverse problems or the homogenisation of multi-scale problems), and visualisation, as far as it requires the development of new algorithmic tools (such as feature extraction in fluid dynamics).

Where do you see the differences between applied mathematics, industrial mathematics and computational sciences and engineering?

Even though I have worked quite a bit on really tough industrial problems, I am not too fond of the definition of 'industrial mathematics': the only distinguishing feature in my definition (above) of applied mathematics might be that these problems arise in industry. Computational sciences and engineering, however, deserve some further thoughts: in my understanding, CSE does not say anything on the methods that are used to solve the problems, so this field necessarily includes good, bad and indifferent. I use the term 'scientific computing' to describe the discipline that additionally takes care of the efficiency of methods applied to computational problems from sciences and engineering. There is, however, a close mutual dependence between scientific computing and computational science: only if computational science brings up interesting problems can scientific computing then work on the methods and, in turn, help to raise the complexity barrier of tractable computational problems in science and engineering.

Do you think that applied mathematics has changed in the last 20 years?

Yes, indeed, but this is too long a story. In short: there has been a clear drive from applicable mathematics to applied mathematics. Today mathematics interferes with unexpectedly many facets of our life, partly visible to the public, but mostly invisible.



And even mathematicians recognise that fact. The beauty of a theorem is more and more compared with its use. Fortunately quite often beauty and use go nicely together (I still love beauty). But beauty alone has a short life, as we all know.

Do you expect further changes in the future? In what direction and with what speed?

I hate to answer such questions, which typically are asked at the end of a mathematician's active professional life. I would prefer to continue to be active for a while. I see a lot of changes, but would like to wait with the answer.

You also chair the scientific committee. How are these changes reflected in the programme?

The title 'Applied Mathematics in our Changing World' was deliberately chosen by the initiators from SIAM and the EMS. The key idea is to focus on some (not all!) aspects of mathematics where it interferes with the typically fast changes in our real world. Generally speaking, the fastest changes arise in fields such as finance, network technology, environment and life sciences.

Where would you see the main emphasis in the programme?

Topics to be mentioned (among many others) are medicine (computationally assisted surgery, immune system modelling), biotechnology (density functional theory, drug design – not to be confused with designer drugs!), material sciences (composite materials, crack modelling and simulation), nanoscale technology (fast optical networks, quantum electronics), climate research, communication (network simulation and optimisation), traffic (route guidance and planning), finance (option pricing, derivative trading – of course, not the mathematical derivatives!), speech and image recognition (filter analysis).

How many participants do you expect?

We aim at an attendance of from 500 to 1000 participants.

Do you have any particular message you would like to give to potential participants?

Whatever you have contributed in terms of mathematics or want to contribute to the changes of our world, come to the exciting city of Berlin and contribute your part to the conference community!

To change topic, you are President of the Konrad Zuse Zentrum in Berlin. Some of our readers attended the ICM 1998 organised by Martin Grötschel, also from this institute, but might not know much about it. Could you describe it briefly?

Here I am tempted to refer to a recent article in SIAM NEWS (March 1999) that I wrote for just that purpose. Its title was ZIB – computational mathematics as key to key technology. There you can find a lot of details about ZIB's organisation, mission, funding, and scientific achievements.

You have been director since 1986 when it was founded. What have been the main achievements in this period?

Yes, I founded ZIB in the scientific sense in 1986, and Martin Grötschel joined ZIB in 1991. As for the main achievements in the past: there have been many extremely successful projects in Discrete Mathematics and Indiscrete Mathematics (my subject!) that have been pursued within ZIB. The ZIB story has been a real success story so far, I'm proud to say. Why don't you just give our web address below?

Perhaps the main project structure of our research may be worth mentioning explicitly: we distinguish between long-term projects (usually basic research, oriented towards the treatment of difficult problem classes and funded by ZIB), medium-term projects (usually publicly funded on the basis of competitive proposals), and short-term projects (industrial problems). This span from basic research via algorithm development to real-life applications has turned out to be quite fruitful over the years. Behind this concept stands my firm belief that genuine progress in the solution of hard real-world problems typically requires thorough and creative basic research.

You have almost another decade of directorship ahead of you. In which direction should ZIB evolve?

We are always on our way and have lots of new ideas – enough for the years to come.

ZIB is one of the German supercomputer centres, and may even be THE centre. Will supercomputing increase at ZIB and in Germany?

First of all, supercomputing should be more than just computing on supercomputers. At least, this is the spirit of supercomputing at ZIB, as opposed to other comparable institutes. At ZIB, supercomputing is part of our branch of Computer Science. Secondly, as for the installation of supercomputers, Germany has a governmental road-map for the development of such centres. The trouble is that only very few problems are especially well suited for supercomputers, and very few experts know how to deal efficiently with them. As a consequence, supercomputing does not have a broad lobby, but must be regarded as a strategic issue for any industrial society – spoken in the direction of all the governments of readers of this interview.

Should there be more supercomputer centres in Europe?

There is a natural decay time of supercomputer technology, which more-or-less leads to an optimal number of such centres for each community. It depends on what you define as the community.

Currently most of these centres are funded nationally. Should the European Union go into the business of maintaining such centres?

My personal opinion is that in the long run supercomputers will be a common European strategic issue, but in the short term the organisational details would kill the performance if an EU option were realised too early.

What is the relation between such centres in Europe and in North America, especially the USA?

Like other such centres we have international connections both in research and networking, such as in metacomputing. For example, we did computations about the collision of neutron stars and rotating black holes (the numerical solution of Einstein's equations of general relativity) simultaneously on four supercomputers in San Diego, Urbana, Munich and Berlin.

Let me ask some questions related to your personal research. In your early days in Munich and Heidelberg you worked on ordinary differential equations. You not only did theory but also software and applied examples. How would you position the development of software at universities?

There is no general answer to this kind of question: some university groups take a lot of care in developing software, some don't. However, the short-term restrictions on typical university jobs are a severe obstacle for any software developments. In addition,

this kind of work is still insufficiently valued in terms of a university career. This may partly have to do with the fact that at some places software development is done without any theoretical component – an attempt often prone to lead to rather short-sighted software solutions. Back to your question. In principle, universities could be good places for software design.

I would like to make an additional remark in this context: without any real application work, which will usually include software implementation, university careers have increasingly been slowed down in recent years, and speeded up otherwise. Just look at the many new university openings (and more to come in a few years when the present generation leaves) in the fields of scientific computing or in discrete mathematics. It seems that a good strategy mix is necessary for young people to grade up successfully. By the way, we are always looking for enthusiastic young graduates to join us.



Design of biomolecules in a 3D virtual laboratory at ZIB

How can they maintain the software if PhD students who did it have left the university?

Attempts have been made to establish networks to maintain software, but in my opinion with doubtful success. I do not know a really good remedy for this undesirable effect. On the other hand, redesign of software, which is necessary anyway after a certain period, is strongly supported by some present languages like C++ or Java which partly reduces pressure from this issue.

Have your style of working and the scientific topics chosen changed when moving from a purely university environment in Heidelberg to the ZIB, where you are also attached to the Freie Universität Berlin?

I still hold the Chair of Scientific

Computing at the Free University of Berlin. And I still love teaching in close contact with the next generation. Unfortunately, like other departments world-wide, we do not have enough students. This is in extreme contradiction to the excellent professional perspectives of computational mathematicians that have worked in real applications. As for my personal research topics: with the ZIB as organisational background, I have been able to look at more complex computational problems that play a role in real life and require a larger group. So my activities in Berlin cover all kinds of differential equations, far beyond the Munich or Heidelberg topics of ODEs. To give one example, I worked on adaptive multi-level finite element methods for PDEs, especially in cancer therapy and microwave technology.

What are the research challenges you still want to attack in the future?

I still have a number of personal scientific dreams. Let me just mention the next steps in the development of conformational dynamics, which will play a decisive role in the design of biomolecules, or the further spread of topics in quantitative medicine which opens a new door to computational mathematics. It seems to be a coincidence that some of my favourite topics appear on the list of topics of the SIAM-EMS conference!

I am sure that with you as pivotal person in the organisation of the first EMS-SIAM conference, it will be a success.

For more information on the interviewee, see <http://www.zib.de/deu/flhard/> for more information on the ZIB, see <http://www.zib.de/>

Looking back: Jaroslav Kurzweil

(Prague)

interviewers: Jirí Jarník and Štefan Schwabik

Jaroslav Kurzweil was born on 7 May 1926 in Prague. He graduated in mathematics in 1949 at the Charles University in Prague and started his scientific career as a student of Professor Vojtech Jarník in the metric theory of Diophantine approximations. Until 1951 he was at the Czech Technical University and was then a research student at the Mathematical Institute which became a part of the newly founded Czechoslovak Academy of Sciences in Prague. After a research visit to Poznań with W. Orlicz he studied analytic operations defined in a Banach space with values in another Banach space. He has been fully employed at the Mathematical Institute of the Academy since 1954. In 1955 he received his Candidate of Sciences degree and was appointed Head of the Department of Ordinary Differential Equations at the Institute. At that time his work in ODEs started. His main interest was in stability theory, invariant manifolds and problems concerning the continuous dependence of solutions on a parameter. In this last field he created a new definition of an integral based on Riemann-type integral sums that turned out to be equivalent to the Perron or narrow Denjoy integral. This was independently discovered by Ralph Henstock, and since the early 1960s has been studied intensively. In 1958 Kurzweil received his Doctor of Science degree, and in 1966 was appointed Full Professor of Mathematics.

Soon after the revolution in 1989 Kurzweil was elected Director of the Mathematical Institute of the Academy, and he has held this office till 1996. Since 1990 he has been Chairman of the Board for Accreditation, appointed by the Government of the Czech Republic to give approval to educational activities of all institutions of higher education.

Jaroslav Kurzweil has been Chairman of the Union of Czech Mathematicians and Physicists since 1996. In 1999 he was reelected for the next three years.

The deep trace of Kurzweil's work and personality in Czech mathematics is evident and incontestable to everybody who has met him, either as a mathematician or simply as a man.

What made you choose mathematics as the subject of your studies?

Mathematics has attracted me since my childhood. For example, I remember that when I was a pupil at primary school I asked adults questions of the type: 'how can we calculate the heights of a triangle if we know its sides?'. I wondered how it was possible to construct the triangle and measure its heights, but I could not calculate them. I had no luck and nobody helped me. I was told not to rack my

brains, that all this was well known and that it is impossible to find anything new in this direction. Later, as a student at sec-



ondary school ('gymnasium') I used to solve competition problems published by the Union of Czechoslovak Mathematicians and Physicists in the journal *Rozhledy matematicko-fyzikální* (Horizons in Maths and Physics), intended for secondary school pupils. I read articles appearing in the Journal and also some popular books, such as *On the Apollonius problem*, *On Einstein's theory of relativity and gravitation*, and *Introduction to group theory*. In 1945 I completed my secondary education. I briefly considered the study of medicine, having been influenced by care I received in my childhood during a long hospital stay after an accident. Nevertheless, I enrolled in the Faculty of Science of Charles University in Prague to study mathematics and physics. My decision was also affected by the fact that all secondary school maths appeared quite clear to me without learning it.

You became a university student in 1945, immediately after the end of World War II. How did your studies continue at that time?

The Nazi occupation in 1939-45 resulted in a drastic restriction of Czech school education. All Czech institutions of higher learning were closed from November 1939 till the end of World War II. The number of secondary schools was reduced, the number of students allowed

to continue, and the teaching of some subjects (such as history and literature) was prohibited. Students at higher grades were forced to work in Bohemia or in Germany, mostly in the war industry. The institutions of higher education resumed their activities immediately after the end of the war, but the situation was not normal. First, the number of professors had decreased considerably: some had been executed, some did not survive Nazi concentration camps, others had died by natural causes. Many younger teachers had found other jobs. This caused a generation gap in the community of university teachers, since people younger than 45 years of age were quite an exception. Many former students deprived of the possibility of studying after the abolition of Czech universities in 1939 desired to complete their studies, and others applied who had graduated from secondary schools during the war.

Thus the education in Mathematics I received at the Faculty of Science from 1945 to 1949 was rather unbalanced. I attended excellent lectures and seminars in topology and number theory, but I learnt nothing of differential equations, probability theory or numerical methods (no computers were then available). I graduated with a thesis in metrical number theory; it dealt with Hausdorff measures of sets of real numbers that can be well approximated by rational numbers. I was initiated in this field by Vojtech Jarník. Postgraduate studies had only started in the early 1950s. After 1950 some universities and research institutes were allowed to enrol students in doctoral studies and offer them scholarships. The preparation of the legislative framework only started then, and so the first dissertations might not have been defended until 1956.

After a short period of teaching at the Faculty of Mechanical Engineering of the Czech Technical University I arrived in 1951 as a postgraduate student ('Aspirant') to the institute, which later became the Mathematical Institute of the Academy of Sciences. The generation gap was still strongly felt, there was a lot of improvisation and much was left to the initiative and efforts of individual students. In the initial period of communist rule in Czechoslovakia it was difficult to travel even to politically allied countries, but I was lucky enough to spend several months in 1953 studying in Poland. My stay in Poznań with Professor Władysław

Orlicz was of extraordinary importance for me. I succeeded, among other things, to solve the problem of whether it is possible to uniformly approximate a function defined on a real Banach space by an analytic function (defined by means of a power series). It appeared that in the class of uniformly convex spaces the affirmative answer holds if and only if there exists a polynomial q such that $q(0) = 0$ and $\inf q(x); |x| = 1 > 0$. In the special case of the spaces L^p and ℓ^p this occurs if and only if p is even.

What prospects did you have after completing your doctoral studies?

The institutions of higher learning were then introducing new specialisations and they also started to work with smaller groups of students; they therefore needed younger experts for assistants' jobs. Also research institutes took on qualified specialists, so there was no lack of job offers. In the Fifties our country did not have specialists in some branches of mathematics who would be able to communicate and cooperate with specialists in non-mathematical sciences or with people from 'practical' professions. The Mathematical Institute was supposed to act to improve this situation. This was why the following working groups were founded in it:

- probability theory and mathematical statistics
- theory of PDEs and mathematical theory of elasticity
- theory of ODEs
- numerical methods
- elementary mathematics.

When I finished my doctoral studies, the Director of the Mathematical Institute Vladimír Kníchal offered that I should remain in the Institute as a research worker, allowing me to choose which group I wanted to work in. I chose the group of ODEs, which was gradually increasing and in 1955 was established as a department. Two topics for the start of the research were given by Kníchal – the existence of periodic solutions, and Lyapunov functions and stability.

The work on the first problem was started by a smaller group that ultimately formed an independent department dealing with evolution problems in all their breadth. I started to work on the other topic, and soon succeeded in proving in the case of asymptotic stability that there exists a Lyapunov function with the corresponding properties and is of class C^8 , even if the right-hand side of the differential equation is just continuous. Nice results concerning Lyapunov characterisation of stability under permanently acting perturbations and integral stability were obtained by Ivo Vrkoc, who later started research in stochastic perturbations and gradually formed a small group which now works in the theory of stochastic differential equations in infinite-dimensional spaces.

Thus we have arrived at your research activities ...

I remained, together with several other colleagues, closer to ODEs. We contributed to control theory in its beginnings and to the theory of differential inclusions. The research in the theory of invariant manifolds was based on a geometrical approach; its results were applied to ODEs, and in particular to differential equations with delayed argument. Occasionally I was also engaged in other topics in mathematical analysis. There is not much sense in presenting the details of these results. On the other hand, I would like to mention another branch of research that stemmed from our work in the ODEs.

In classical theorems on the continuous dependence of the solutions of the differential equation $x' = f(x, t, e)$ on a parameter e , it is assumed that f is a continuous function of the triple of variables (x, t, e) . In the Fifties cases were considered in which the right-hand side of the differential equation contains terms of the type $\cos t/e, f(x)/e \cos t/e$, so that with respect to e it is not continuous in a neighbourhood of 0. Nevertheless, the primitive functions with respect to t of such terms are $e \sin t/e, f(x) e^2 \sin t/e$, which are continuous. Thus a natural question arises under what additional conditions the solutions of the equation depend continuously on the parameter e , provided that we assume only that the primitive function to f with respect to t is continuous. When I was looking for general principles governing these problems I was working with Lebesgue integrable functions and Riemann sums. This provided the inspiration for a modification of the Riemann integral; the modification of the usual 'epsilon-delta' definition consists in admitting for d not only positive constants but positive functions (called *gauges*).

I published the modified definition for the first time in 1957 and used it to introduce generalised differential equations and establish results on the continuous dependence of solutions of generalised differential equations on a parameter. In this way the concept of $S(P)$ -integrable functions was obtained.

It was also shown there that a real function g is $S(P)$ -integrable if and only if it is Perron integrable, and that the integral in the sense of $S(P)$ -integrability is the Perron integral of the function g . (In the notation $S(P)$, the symbol S corresponds to the approach by Riemann sums, while P refers to Perron integration.) I regarded the definition as a contribution to the theory of the Perron integral; I did not deal with it for about the next two decades, my main field of interest having been ODEs and related problems.

The same definition was published independently in 1961 by Ralph Henstock. In 1969 McShane published a general theory of integration in abstract spaces, which in the special case of integration on a one-dimensional interval reduces to another modification of my definition. McShane proved that by this modification we obtain exactly the Lebesgue integrable functions.

The above works, including Henstock's 1963 monograph, aroused interest, which led to the publication of several monographs and a long series of papers. Numerous modifications of the original definition were investigated, in particular for integration in multi-dimensional spaces, characterisation of primitives, convergence of sequences of functions. In particular, such modifications of the definition for multi-dimensional integration were found which allow the usual transformation formula and can be used to formulate a general Stokes theorem (for example, by W. Pfeffer in 1993). I joined this trend by a monograph in 1980 and by a series of papers that I published with co-authors in the years 1983-99.

Until recently, in connection with the summation approach to integration, the convergence of sequences of functions was investigated, but no attention was paid to the topologisation of the space of integrable functions. As we know, $S(P)$ integration is Perron integration and, as such, is an extension of Lebesgue integration. Therefore, if for $f: [a, b] \rightarrow \mathbf{R}$ and $g: [a, b] \rightarrow \mathbf{R}$, $f = g$ a.e., and if f is $S(P)$ -integrable, then g is $S(P)$ -integrable and the two functions have a common primitive. It is natural to study both the convergence and the topology in the vector space $\text{Int } S(P)$, which is the space of equivalence classes of $S(P)$ -integrable functions. A convergence of sequences with reasonable properties can be introduced in the space $\text{Int } S(P)$, and we arrive at the following problem: Does there exist a topology $\text{Cal } U$ on $\text{Int } S(P)$ such that $(\text{Int } S(P), \text{Cal } U)$ is a complete topological vector space and every sequence convergent in the above sense is convergent in $(\text{Int } S(P), \text{Cal } U)$ as well?

In view of the first requirement the topology may not be too coarse, by the latter it may not be too fine. I have recently proved that the answer is affirmative. However, if we add the condition that $(\text{Int } S(P), \text{Cal } U)$ is a locally convex space, then the answer is negative.

I am author or co-author of 57 papers which concern the above subjects, which were published between 1957 and 1991.

What brought you to work in mathematics teaching?

I got to mathematics teaching without any effort. In the so-called socialist regime the publishing of textbooks for elementary and secondary schools was directed centrally. There were publishing houses in both Prague and Bratislava, whose task was to publish textbooks which were then used in all schools throughout the country. When the higher authorities decided that new textbooks for primary and secondary schools were to be published, the two publishing houses divided the task of preparing mathematical textbooks for individual grades. Then the publishing houses started to seek for the authors and referees. I do not know what had to be approved 'up there' before the publishers were allowed to appoint the authors and referees. Other institutions involved in

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the process were the Pedagogical Research Institutes, both in Prague and Bratislava. They published, in small printings, the so-called experimental textbooks of various subjects. Those employed in the Institutes were authors of most of these experimental textbooks and were often authors or co-authors of the standard ones also. In the Seventies I was asked by the Director of the Mathematical Institute, Jirí Fábera, to write a review of one of these textbooks. The review was unfavourable, sometimes even biting. I was therefore surprised when the head of the Mathematical Department of the Pedagogical Research Institute, Jana Müllerová, thanked me and asked me to give my critical opinion of future manuscripts. The result was that for several years I served as an advisor to those authors of mathematical textbooks who showed interest.

In the Seventies a very formal approach to elementary mathematics, based on naive set theory, was put through. First-grade pupils in primary schools (6-7 years old) learnt the union and the intersection of sets, and set diagrams became idols. Maybe the authors actually believed that they had discovered how to make maths attractive and accessible, but above all, they had enough influence to enforce this system all over the country. I was co-author of a letter addressed to the Pedagogical Research Institute and signed by about ten outstanding Czech mathematicians working at institutions of higher learning. We pointed out that the process of cognition of children proceeds along other lines, that abstractions have to be prepared by experience, and that they are the top and not the start of the cognitive process. The letter was not published, but was circulated and had a positive effect. New authors were appointed for some textbooks and the whole system was moderated.

The first director of the Mathematical Institute (in the Fifties) was the famous topologist Eduard Čech. He believed that maths could be taught at secondary schools much better and more effectively. He had already organised meetings with teachers before World War II, lectured and discussed with them, and even wrote several secondary school textbooks himself. It was this tradition which led to the foundation of a Department of Didactics of Mathematics in the Mathematical Institute. When the head of this Department Josef Horálek died in 1984, I was put in charge of its activities; of course I did not participate in the proper work of the Department, but I willingly discussed with its members their experience, problems and plans. In 1985 I became Chairman of the Committee for Mathematical Terminology of the Union of Czechoslovak Mathematicians and Physicists. This Committee had earlier prepared for publication a list of mathematical terms and symbols, which went mildly beyond the needs of primary and secondary schools. The aim of the Committee was to prepare a dictionary of

school mathematics with explanations of the terms. The first edition appeared in 1981 and the Committee soon started to prepare an improved edition. It met regularly on seminars, found authors for individual chapters and in 1989 it had prepared about 80 per cent of the manuscript; only two chapters were missing – unfortunately, the key ones concerning geometry and computers. However, after 1989 most of its members were challenged with more urgent tasks and the Committee ceased to work.

In 1987 the Union awarded me its highest distinction and elected me its honorary member.

Institute of the Academy of Sciences, and I remained in this office till 1996. In the same year I was elected the Chairman of the Union of Czech Mathematicians and Physicists.

As a consequence of the new law on institutions of higher studies, passed in 1990, the government appointed the Accreditation Board that was entrusted with important tasks regarding institutions of higher learning. I acted as Chairman of this Board from 1990 to 2000. I have strictly applied the opinion that statistical and scientometrical data have an auxiliary character and that every decision of the Accreditation Board requires expert substantiation. In 1991 I



These stamp were issued in 1987 to commemorate the 125th anniversary of the Union of Czechoslovak Mathematicians and Physicists. The middle one features Kurzweil's teacher Vojtech Jarník (right).

We now come to the year 1989...

One consequence of the generation gap in the mathematical community, which I mentioned in connection with the Nazi occupation, was that I and some of my contemporaries were entrusted very early with organisational and decision-making tasks. I served as a member of numerous expert committees, and committees for academic and scientific degrees. From 1956 to 1970 I was the Chief Editor of the mathematical journal now published under the name *Mathematica Bohemica*.

The year 1989 brought changes and hopes. The system of central directing was to be abandoned and self-government at all levels was to be restored in all areas of society, including education and science. Naturally I desired to participate in bringing about these changes. In 1990 I was elected Director of the Mathematical

was elected a member of the Scientific Board of the Charles University and of the Scientific Board of the Faculty of Mathematics and Physics. In 1994 I was among the founding members of the Learned Society of the Czech Republic, which continues a tradition going back to the second half of the 18th century.

You lived during the periods when your country was governed by Nazis and then by Communists. How do you see the present?

It is indeed a miracle that both regimes, Nazi entirely and Communist except for Belorussia, have disappeared from Europe. Nonetheless, the introduction of each was made possible only due to a failure of Western civilisation and education. This, in my opinion, is a warning for us as well as for the future generations.

Joint BMS-DMV meeting of the Belgian and German Mathematical Societies

8-10 June 2001

The *Belgian Mathematical Society* and the *Deutsche Mathematiker Vereinigung* will hold their first joint meeting at the University of Liege, 8-10 June 2001. It will consist of six 50-minute plenary talks and 10-12 special sessions.

I. Daubechies (Princeton), C. Deninger (Münster) and P. Deuffhard (Berlin) have already agreed to deliver plenary talks:

The following six special sessions (and organisers) have already been fixed:

Arithmetic geometry (G. Cornelissen, Annette Huber, K. Künnemann, W. Veys);

Functional analysis and functional analytic methods in partial differential equations (K. D. Bierstedt, P. Laubin, R. Meise, J. Schmets);

Global analysis (J. Brüning, L. Lemaire);

Optimisation (M. Goemans, M. Grötschel, Ph. Toint, J. Zowe);

Ordinary differential equations and dynamic systems (F. Dumortier, B. Fiedler, J. Mawhin, J. Scheurle);

Representation theory (D. Happel, C. Ringel, F. Van Oystaeyen, A. Verschoren).

Information on the meeting can be found on the homepage:

<http://math-www.uni-paderborn.de/Liege2001/>

Everybody interested in participating to the meeting is asked to preregister by sending an e-mail to , mentioning their name, institution, e-mail address and 2001 BMS-DMV Meeting. All mathematicians who have pre-registered this way will automatically receive the Second Announcement of the meeting in October 2000.

Société Mathématique de France electronic mailing list

The Société Mathématique de France (SMF) maintains an electronic mailing list allowing each member to be periodically and freely informed of the SMF's latest issues.

To sign up, send an e-mail message to sympa@ens.fr without subject or signature and containing exclusively the line:

subscribe publications-smf full name (for example: *subscribe publications-smf Dupont Jean*)

To unsubscribe to the mailing list, send an e-mail message to sympa@ens.fr without subject or signature and containing exclusively the line:

unsubscribe publications-smf

Beware: the body of your message should contain only raw text (neither html nor Word attachment).

ACM (Agenda des conferences mathématiques)

This message is to announce a French initiative on the occasion of World Mathematical Year 2000. It is a web system for the collection of information on all conferences and seminar announcements in all branches of mathematics that anyone with a web access can query in a highly user-friendly way.

It is called the *Agenda des conférences mathématiques*, ACM for short. Its web address is at <http://acm.emath.fr>. It was created by Stephane Cordier (a maitre de conférences at the University of Paris 6 and an active member of the SMAI), has been functioning in France since February 1998 and presently indexes some 100 ongoing seminar series. Its set-up is financially supported by a small grant from the French Ministry of Research, with the understanding that it should be made freely available everywhere, with the full support of the SMF (Société Mathématique de France) and the SMAI (Société de Mathématiques Appliquées et Industrielles). The European Mathematical Society also supports it.

ACM has recently been expanding to other countries (Italy, Germany, Canada, etc.). Its user interface comes in several languages and the aim is to include more, but this will require the help of native speakers of other languages. National and regional correspondents are also needed; some are already available for France, Austria, Belgium, Germany, Italy and Canada. Mirrors will also be welcome everywhere possible, including on EMIS.

At the core of ACM is a large database, but the interesting concept is that the database gets created and maintained automatically, by querying the web pages of the seminar series, conferences or colloquia. Of course, it requires the active cooperation of their organisers, but this is straightforward for them: the URL of the web pages of the announcements have to be indicated to the ACM-master and these pages have to include some simple tags that the ACM engine can recognise. This procedure allows for instantaneous updates. Detailed explanations are available on the ACM server, in English, German, Italian, Spanish and French).

On the user's side, the search engine can be queried in these languages and can be customised to each user's profile or profiles (which can be saved as Bookmarks or Favorites). Simple and complex searches are possible, including geographical regions and time periods. Setting up a profile is a little lengthy the first time, but well worth it because, once bookmarked, it performs a new search in one single step and keeps the date as relative, not absolute. The result of a search is a listing of all the seminar talks, symposia talks and conferences in the database which satisfy the criteria, with their title and a

link to their web sites. One even has the option of asking for a timely e-mail reminder.

As the geographical coverage increases, more numerous and focused regions will be included. Apart from the present ten general subfields of mathematics that can be selected for a focused search, each talk can present a list of MSC2000 codes that can be used in the searches. The ACM database also includes information on mathematical conferences, congresses, workshops and colloquia from the conference calendar maintained by the Atlas Mathematical Conference Abstracts (AMCA) at <http://at.yorku.ca/amca/>. It can also include the information located on the EMIS conference board. For those locations that do not have a web page, it will be possible in France to post the announcements on a special site that will automatically be queried by the ACM robot. This type of procedure should be considered in all other countries at the country level, or by the national Societies of Mathematics. It already exists on the EMIS Conference Board and can be used in European countries.

This an evolving project and the hope of our Societies is that by being a tool for the whole Mathematical community, Europe-wide and World-wide, it can evolve to satisfy the needs of all. The purpose of this message is to inform you of this new service, to suggest that you use it as often as you need it and, most importantly, that you ask the organisers of meetings and seminars to contribute to ACM by using the procedure that allows it to extract the relevant information for its database.

The purpose of this message is also to inform you that the whole concept, and software, is a gift of the French Mathematical Societies and community to their fellow mathematicians around the world; but for ACM to succeed, the contribution of everyone is now needed, and most notably our European fellow Societies.

One final note: ACM does not wish to compete with other sites that propose similar information. On the contrary, it is willing to incorporate all information that resides on these sites, with direct links to them. An example of such cooperation is with ACMA, as noted above.

We sincerely believe that the ACM concept can play an outstanding role in fostering the sense of a world-wide mathematical community. At the dawn of the third

A major challenge for mathematicians

The undervaluing of the role of mathematics in today's society

Jean-Pierre Bourguignon

In today's society, mathematics is present more widely than ever before, but this is rarely acknowledged, even by mathematicians. As a result, citizens need to be comfortable with mathematical situations, so as to be helped with the ever-complex choices they have to make.

This article arose from discussions with many people inside and outside the research mathematical community (in particular, teachers) and the perception I developed from them. For the most part, mathematicians have not appreciated the large demand for mathematics in society and the consequences they must draw from this. The starting point is to analyse what makes mathematics special among the sciences and how it is perceived by those who have to deal with it in some way – parents, professionals, managers, and mathematicians themselves, as teachers or as researchers.

Elaborating on the ordinary perception of mathematics, I present what I see as *a new relationship of mathematics to society*, and consider detailed implications on its internal organisation, together with needs to be addressed in its development and teaching.

1. The ordinary perception of mathematics

Its role in basic teaching Around the world, one's first encounters with mathematics happen in elementary schools. Indeed, all countries consider mathematics to be part of the basic training that children must have. The form it takes varies from one country to the next, but basic operations on numbers are always taught, together with fundamental geometric figures. Mathematics is also viewed everywhere as a subject of its own. From this perspective we cannot say that mathematics lies hidden.

When one digs deeper and considers the actual pedagogical practices, one realises that, although mathematics appears everywhere, the intellectual process on which mathematics is based is not always fully presented. For example, the abstraction process that lies at the heart of mathematics is often hidden behind routines that have value but are only one side of the coin. The same can be said of the research process. Mathematics is sometimes presented as an activity in which one has to

function automatically, when as mathematicians we know how much one has to struggle to solve a problem or break it into simpler pieces. In other words, if a minimum of technique is required to function in a mathematical environment, the mean-



ing of what we manipulate cannot be ignored if we are to ensure a learning process with long-lasting effects.

Some fallacies about mathematics For some people, *mathematics is just the language of the quantitative*. They base their judgement on the fact that mathematics entertains a special relationship with language; this opinion is shared by some of our fellow scientists. We mathematicians know how wrong this is, and how much effort goes into building concepts, establishing facts, and following avenues we once thought plausible but turn out to be dead ends. This widespread belief forces us to consider more carefully how mathematics interacts with other disciplines and what is the exact nature of the mathematical models that appear ever more frequently.

Second, *mathematics is seen as a dead subject*. One of the most frequent questions that professional mathematicians hear from laypersons and from media people, is: "What can you do in a subject where everything is known?" So many examples can be used to prove the contrary that we

too often omit to make this point publicly – in particular, to our students. In my opinion, it is of fundamental importance that, as early as possible (probably even before secondary school), students meet the idea that mathematics is still in the making, and that thousands of mathematicians around the world are working at solving outstanding and extremely varied problems.

These misconceptions are encapsulated in the unfortunate reputation of the word 'abstract', when it is precisely in abstracting from a situation what makes a phenomenon general that mathematics is at its best. Henri Poincaré gave a great definition of mathematics. For him, "faire des mathématiques, c'est donner le même nom à des choses différentes". From that point of view, opposing 'concrete' and 'abstract' can be nonsense. Indeed, extracting from a complex situation what constitutes its core is what makes us really understand how things work. We must therefore spend time in making this process explicit, emphasise the role it plays in the foundation of mathematics. In my opinion, here lies one of the main roots of the *quid pro quo* that currently affects the relationship of mathematics to society.

Some other sources of misunderstanding in connection with education A fact that relates the study of mathematics in school to that of languages is that mathematical knowledge is accumulated, so that a lack of understanding at one stage affects the ability to understand and progress later. This dependence is not as strong in other disciplines such as physics, history or geography.

Another aspect that is largely country dependent is the role that mathematical performance plays in the selection process at school. From this point of view, France used to be an extreme case because of its dual system of higher education, split between the so-called 'Grandes Écoles' and the universities, and the weight attributed to mathematics in the entrance competitions to the 'Grandes Écoles'.

A partial and brief conclusion: *It is clear to me that the image mathematicians give to the general public (an image that is amplified by the media) needs to be worked at to make it more in line with what they actually do. This requires a considerable and sustained effort on the part of mathematicians.*

2. What is the nature of mathematics?

In this section I shall concentrate on a few topics that make mathematics special as a science. It is my opinion that mathematicians have not worked enough on this front and are not always in a good position to 'make a case' for their needs. This implies that more thought needs to be given to the objective basis for the misconceptions discussed in the previous section. Last but not least, we must take a historical perspective when discussing these issues, because mathematics has a long history (maybe the longest of all sciences), and because the situation is changing very rapidly. So what is so special about mathematics?

A special relationship to language To address this question we must realise that the way one expresses mathematics evolves with time. The now well-accepted idea that mathematics can be totally formalised was stated as a general paradigm only at the end of the 19th century. It is largely a philosophical stand and has little in common with the actual work done by most mathematicians or teachers.

We can view this evolution as the culmination of the need to use a precise language. Since mathematicians like to borrow their vocabulary from ordinary language, we must realise that there is a price to pay – namely, that students may resent it as a form of robbery, while we would rather view it as a form of poetry. More attention should be devoted to explaining why we so appropriate the common language, and why we would be in trouble without it. We should behave as responsible citizens and explain what we make of common resources. After all, are we not forcing our students to live double lives, depending on whether we are talking in a mathematical context or not? This is not easy and should be identified as such.

The constraint of expressing mathematics by a well-controlled language is certainly one reason for seeing mathematics *only* as a language. We require so much attention to this matter, and for good reasons, that we should carefully explain why, describe those mathematical activities that are not affected by this practice, and identify times when forcing oneself to stick strictly to a formalised language is a handicap. This particular difficulty appears both for advanced students and for beginners, and may be one of the major hurdles that young researchers must pass.

A special link with truth Confronted with a mathematical statement, a mathematician's goal is to prove that it is 'true'. What does this mean? Of course, now that mathematicians agree to work in the context of a potentially completely formalised theory, its meaning can be none other than 'the statement can be deduced from the basic axioms agreed upon'. In fact, if we are to address this question in the context of the relations of mathematics to society, we are forced to take it in a broader, more philosophical, perspective because we have to confront it with reality. This amounts to deciding whether there is a mathematical reality of which this statement is a part,

and of the status of this reality in relation with ordinary 'sensible' reality. On this broader issue, mathematicians have different views: from the 'Platonists' at one end of the spectrum who believe that they 'discover' new territories and new facts of the mathematical world, to the 'intuitionists' at the other end who view mathematical constructions as purely human scaffoldings based on consensus opinions among a restricted community – in other words, that mathematics is 'invented'. I have no doubt that the vast majority of mathematicians are closer to the Platonist viewpoint than to the other one, or at least that 'they spend a good portion of their professional time behaving as if they were', as André Weil once put it.

The possibility of using today results established centuries ago is another aspect of the special relation that mathematics has with truth. It gives it the possibility of transcending civilisations, and makes its transmission much easier since it is *a priori* less ideology dependent. This founds its *claim for universality*. Of course, this is all based on the role that proofs play in mathematics, in spite of their changing appearance with time. Making this dimension accessible to students is of course the major challenge that teachers face at all levels. This gives us mathematicians a special obligation towards the history of our discipline, and requires that it be properly incorporated in our teaching, both as documented proof of its evolution, and of the progress it has made. Facts accessible in the past to a very restrictive group of great minds can be, and are, correctly assimilated today by large crowds of students, because new concepts and new techniques have been developed in the meantime. Here lies for me the best antidote against the spreading of the devastating belief that mathematics is a dead subject. Let me note in passing that efforts in the direction just mentioned will also give us stronger arguments to support our worldwide struggle for keeping good libraries. To do present-day mathematics, we need access to documents that may be a century old or more, and this need will stay with us in the future.

Another consequence of the relationship of mathematics to truth, also worth pointing out in connection with education, is the following. A person who has understood a mathematical fact becomes able to challenge his or her teacher, and his or her classmates. This experience of winning by exercising the mind is of the utmost importance in training critical minds and giving them access to autonomous thinking. This of course needs to be thought through in relation to the very diverse social environments in which schools are operating.

The unbelievable success story of mathematics throughout history Mathematicians tend to be shy about their field, even though the history of mathematics is one of the most extraordinary success stories of the human adventure. Think that at the end of the 17th century precise mechanisms were designed in order to say something suffi-

ciently pertinent about infinity to be able to handle major questions connected with it. This was the birth of modern analysis. Think that, in spite of formidable evidence imposed on mathematicians by everyday experience, the early part of the 19th century saw the birth of new geometries, generalising the well-established Euclidean geometry, and opening the way to a more radical broadening of the scope that enabled general relativity to take ground. Think that major problems concerning prime numbers and Diophantine equations have been solved one after another, thanks to numerous technical and more conceptual achievements. Think that random quantities can now be properly estimated on the basis of a few precisely analysed assumptions. Many of these revolutions were the basis for major changes in the organisation of society: we come back to this later. Why are we so shy?

3. A new relationship to society

This part is the most controversial, since I dare to offer some perspectives for the future. It may be good to start by taking a last look at the evolution of our discipline in the 20th century, in order to identify the changes that I claim force us to continue.

Mathematics and mathematicians in the 20th century In the 20th century, mathematics, as many other sciences, enjoyed unsurpassed development. Many new results were obtained – some very spectacular, such as the solution of the Fermat problem – but the real change was in the finer web of results and the extraordinary broadening and ramification of areas that transformed our discipline. At the end of the 20th century, the yearly production of research articles had reached the 60,000 mark, when in the 1950s there had been only 5000. This is undoubtedly the mechanical effect of the growth of the mathematical community throughout the past century. Whereas some 250 participants took part in the 1900 Paris Congress, the century ended with the Berlin Congress that hosted 4000. Today, the number of mathematicians actively engaged in research is estimated at about 50,000. At the same time we should keep in mind that the number of biologists is now estimated at about 1 million.

In a more qualitative vein, the 20th century witnessed the generalisation of the use of the axiomatic approach. The distance it allows to take from the immediate reality surrounding us has also some drawbacks – namely, that of providing arguments for people who say that mathematics has deserted the real world. Symmetrically, it also frees mathematicians from relating what they do to real-life problems. Nevertheless, there is no doubt in my mind that this newly gained freedom is a fantastic advantage, provided that we do not isolate ourselves and pretend that we have nothing to say and nothing to learn from other scientists, from engineers, and from society in general.

Before leaving this question of the axiomatic method, it is important to mention that it also had an extraordinary

impact on the teaching of mathematics at an advanced level, allowing us to teach much more quickly to a wider group of students. In that sense, it has greatly contributed to the development of the mathematical enterprise. The impact it has had on the teaching at lower levels has been diversely appreciated but, if some experiences were really negative, the *status quo* was no more acceptable, for the very reasons that we are presently examining.

Coupling mathematics with powerful computing tools Already by the 1970s, new means of computation had appeared in the form of powerful computers, exhibiting first a fantastic number-crunching ability but later also unexpected wonderful visualisation capacities. These developments have continued at an unabated pace. Can one measure the types of impact that the advent of computers has had on mathematics? I propose to organise them in four families:

- with computers, we can immediately deal with huge data sets, with the result that some problems have now become accessible, where earlier mathematical methods were not feasible; this is typical with statistical data. Here, the key question is to learn how to extract relevant information from huge data sets, and this is a mathematical problem.

- these new machines, which handle only finite data, force us to reconsider the relations between the finite and the infinite, about convergence of processes. We can teach a computer how to handle very large numbers, thereby making some areas of mathematics, such as number theory or dynamical systems, amenable to experimentation. Here too, the net result is a considerable broadening of what is accessible to mathematicians; as an example, we can name cryptography, again an activity whose nature is completely mathematical.

- machines have the capacity to repeat a simple algorithm a very large number of times. The example of Julia sets and fractals may have been overused, but we have to learn how to exploit their attractiveness and beauty properly; this has changed the view we have on some structures, transforming their pathological appearance into a real richness that has provided natural scientists with more models relevant to some practical situations.

- computers open new horizons to the numerical treatment of sophisticated equations, a typical example being that of weather forecasting.

By opening new domains to mathematics, changing our views on complexity, and giving new tools to mathematicians to solve their problems, the impact of new computing tools is therefore considerable, even without mentioning the very deep mathematical problems posed by machines and by networks connecting them. This is undoubtedly one of the new frontiers of mathematics. What was once considered by mathematicians as a threat provides a great opportunity to revitalise the discipline and make it relevant to many more situations.

The omnipresence of mathematical

objects in everyday life We now come to what seems the most important aspect of the change occurring at the end of the last century – the omnipresence of mathematical objects in everyday life.

This phenomenon is also connected with the new possibilities offered by computers, but other aspects of modern society that have no relation whatsoever with mathematics play a major role in it. Every day we use many objects that incorporate some part of mathematics, and often recent ones. Most of the time even we mathematicians do not realise it, because the mathematical component is not immediately visible and pinning it down requires some thinking.

The most obvious case is that of the pocket calculator, but computers also exhibit impressive mathematical ability; their use is widespread, and we are confronted with them everywhere. Less evident examples are CD-players, TV sets, cars, etc. This proliferation has been made possible thanks to the widespread use of high-technology products produced cheaply.

The notion of industrial product Today's society is dominated by the notion of industrial product. This means in particular that many objects and structures that surround us have been designed in a well-defined manner. Objects have to fulfil a definite task, and their mere existence is due to the fact that a market for them has been identified. Usually this requires optimising the object or the structure in various ways, a step involving a mathematical process that earlier would have been a purely hardware problem.

This goes further. Even if one is interested in keeping an industrial product longer than was planned by its inceptor, one soon encounters problems because (and this is especially true for high-tech products) they quickly become incompatible with their environment – they cannot be used any more. This compatibility problem poses at a large scale the question of keeping the memory of the past in proper form and, after what we said earlier, this is no minor concern for mathematicians. The notion of an industrial product incorporates its limited life span.

Towards a communications society Claiming that we are entering a society dominated by communications is not original. Here, we consider the consequences of this change from a mathematical point of view.

First, more information is made available to a broader section of the public, leading usually to too much information. To learn how to navigate in such a world is not easy, since one needs in particular to know whether a piece of information is reliable. In view of the quantity it is clear that most of the available information cannot have been properly checked. If one wants citizens to have control of the facts on which to base their judgements, one needs to train them already at school to do that. This makes exposure to critical reasoning more important, and training to distinguish bias a necessity.

If this reliance on communications has the positive aspect of abolishing distance, it also makes us dependent on the quality of information collected by various kinds of sensors that can be taught more easily to send falsified information than our eyes, our fingers, etc.

Where is mathematics around us? We now give some explicit examples of the mathematics around us. We make no claim to be exhaustive, and there is certainly some personal bias.

In modern societies complex systems are everywhere, to the extent that this feature can even be seen as one of their characteristics. One needs to regulate these mechanisms, and this requires a new type of knowledge that is basically mathematical. The management of such systems has important consequences for the development of society.

Biological systems provide many examples of complex systems that we must come to terms with. This domain became one of the great adventures of the second part of the 20th century since the discovery of some fundamental mechanisms of living organisms such as DNA structure. Genomics, which comes out of this, requires handling huge amounts of information, some of it discrete (the digits of the 4-letter alphabet coding of DNA) and some of it geometrical (being able to act on the DNA requires one to know very precisely the geometric accessibility to the proper sequence). Mathematical tools to deal with such problems are not obviously available, and are likely to require new mathematical developments around the question of how to structure large data sets.

Telecommunication systems have already had a wide impact on the way that the whole society functions. Again, when we consider objects through which we communicate, besides the fact that one punches numbers on a keyboard, there is no hint that there is some mathematics behind them. It looks as though the chips, the laser (as in CD-Roms) and the wires are the whole story, when in reality the design of the network plays an equally important role, and this is where mathematics enters. The performance of a telecommunication network is measured through its capacity for not altering the information it carries. This can often be done through error-correcting codes, which encapsulate sophisticated algebraic properties – that is, purely mathematical knowledge.

Another side of telecommunications has to do with the quantity of data to be transmitted. Given a given physical system designed to carry data, it may be necessary to compress the data (typically, images) so as not to exceed the capacity of the network. Designing compression and decompression algorithms is another challenging problem, in which (for example) wavelet theory, a truly mathematical modern adventure, has had an important impact.

The Global Positioning System (GPS) is another example of a modern tool that must incorporate very advanced mathematics in order to reach the accuracy that widens its use. Indeed, in order to improve

precision from 10 metres to 10 centimetres, corrections that take care of effects accounted for by the theory of relativity must be considered. For GPS again, optimising the choice of satellites from which one estimates the position on the earth requires the use of sophisticated mathematics, such as discrete group theory.

Finally, there is still another side to problems posed by transmission of data – their security. In many cases, for one reason or another, one needs to be sure that the information is kept confidential all the way through. Encoding it is the solution, provided that the code cannot be broken by a person different from the one that is supposed to receive it. Here again, one has to deal with discrete data, and discrete mathematics is a part of mathematics that, in many countries, has not yet been given the same priority as the *a priori* more noble part using real numbers.

Data collection and analysis is yet another area where we get used to new ways of functioning and may miss the underlying mathematics. Indeed, at the supermarket, we have got used to no longer seeing the cashier type the price of goods we buy, thanks to the systematic use of barcodes. The net gain for the shop is not only the time saved in typing but the automatic management of the reserves. Here again the recognition process on which the barcode system is based is purely mathematical.

Statistics and polls We read statistics in newspapers every day, and to be able to decipher what they say and what they omit has become of the greatest importance in today's world. The fact that we can collect and deal with large data gives us more opportunities, but citizens need a basic training in this field in order to allow them to detect when they are being manipulated.

Other information that we now find regularly in the news media are polls. We are exposed to many of them, and in this respect we regularly have the feeling that information drawn from them is definitely improper. Since they have become a necessary part of the process by which people form their opinions, they are very important to democracy. Knowing how to estimate the error margin and how polls can be falsified is therefore essential.

Using appropriate rigorous mathematics, it is possible to prove whether a statistical claim is correct or not. It is probably in this area that a contribution towards critical thinking through mathematics is really possible, and at the same time indispensable.

Automatics and robotics Many complex systems are run using control parameters. Automatic pilot systems that run planes are examples of systems where control theory plays an important role.

In fact, we are confronted with many other automated systems in our everyday life: lifts, TV and telephone satellites, cars, etc. Scheduling, the theory that ensures that a given network can provide the service it is designed for, also relies on appropriate mathematical theory. Most trans-

port systems rely on such networks. Indeed, when we are waiting for a bus, a train or a plane, we focus our attention on the material carrier, but the important (but hidden) fact is that this object is part of a whole network. All this relies on sophisticated graph theory.

Optimisation of forms hides a lot of mathematics. We often use objects whose shapes have been designed using advanced mathematical tools, most of the time by extremising some functional – just think of aerodynamic forms of cars or planes to minimise air resistance, and thus to improve fuel efficiency. Many other examples can be given that involve purely design criteria, lower production costs, or improve a material strength.

An interesting example is given by optimising the places where the body of a car is linked to its frame. In choosing them properly (for example, at nodes of the basic harmonic vibrations of the body), one can minimise the transmission to the inside of the car of vibrations caused by bumps on the road.

Yet another example, again taken from the car industry, has to do with the shaping of windshields, which are usually curved surfaces. A difficult question is to determine the shape of the windshield when it is made flat, before one 'forms' it by heating it again to make it deformable.

Banking products and insurance Many of the above examples come from different areas, and point to one fact that makes mathematics different from other sciences – namely, that there is no industrial sector that can easily identify itself with the discipline.

Recently, this may have changed with the new trends taken by the banking and insurance industries. There are several reasons for this change. We have already encountered one of them – the capacity to collect and quickly analyse large data sets. Another is linked to the new telecommunication possibilities, and the fact that today there is really one world financial market. Yet another is deeper, and has to do with the evolution of the banking activity which actually functions more and more like an insurance. A typical example of this change is given by derivatives and options, the promise to buy a certain good at a prize determined by a well-defined mechanism, based on information that will be available later. The key point is to estimate the risks that one takes by making such a deal. One must use sophisticated probabilistic models in order to evaluate the indices on which the price is based and which depend on random parameters. The kind of stochastic processes that are involved turn out not to belong to any of the standard categories that have been extensively studied in connection with processes met in the natural sciences, but rather with the general theory of stochastic processes, a body of knowledge long considered highly theoretical if not esoteric. These developments have led banks and insurance companies to hire mathematicians, and have given birth to a new branch of our discipline, *financial mathematics*, in

which special training is now offered at a number of institutions of higher education.

4. Final words

From all the above challenges concerning the image of mathematics, the most serious seems to me *the need to make it evident that mathematics is a science and alive*. The rest should follow.

In my opinion, *we mathematicians must agree to put more consideration into, and to show more curiosity for, what is happening around us*. We also have to agree to say more about our activities. This requires taking more initiatives towards the general public. We should be helped in that there are many mathematical products around us. It is our duty to exhibit what is mathematical about them.

Secondly, we have many opportunities to make people dream. *Mathematics still remains a great creative adventure*. New challenges are ahead of us. The need in our discipline for great minds remains as pressing as it has been throughout history. To achieve such goals, we need to find appropriate ways to connect ourselves with teachers in schools. In the long run, this is the only channel through which these new trends will be passed on to the new generation. *The future of mathematics lies in the hands of those younger people that our generation will prove able to attract to mathematics*.

A version of this article was presented at a 'Symposium on the Mathematics of the XXth century' held in September 1998 in Luxembourg.

More on Dutch mathematics and π

Peter Stevenhagen

The previous issue of the *Newsletter* had an article about the oldest national mathematical society in existence, the *Wiskundig Genootschap*. As with British stamps, there is no need for an explicit reference to its country of origin.

An editorial error crept into the explanation of *wiskunde*, the Dutch word for mathematics. Many foreigners relate *wiskunde* to the German word *Wissenschaft*, and incorrectly infer that *wiskunde* is Dutch for science, which has the Latin *scire* for *to know* or, in German, *wissen* as its root. The Dutch have *wetenschap* for *Wissenschaft*, and use *weten* for *to know*. *Weten* and *wissen* derive from the same root, which also produced *wits* in English and the Dutch word *wis*, meaning *sure* or *certain*. This is the word that Simon Stevin used in coining the word *wiskunde*. Thus, *wiskunde* literally translates as *surelogy* – a non-existent word coined by the American mathematician Barry Mazur from Harvard.

As the article pointed out, a 'new tombstone' for Ludolph van Ceulen containing 'his' 35 decimals of π will be unveiled in the Pieterskerk in Leiden. For ceremonial reasons, the date of the event has now been advanced to 5 July 2000.

EMS Position paper on the Commission's communication: Towards a European research area

Luc LEMAIRE (Bruxelles)

The following is the EMS's response to the European Commission Communication 'Towards a European research area', issued in January 2000 (see the website <http://www.europa.eu.int/comm/research/area.html>). The text was sent to Commissioner Busquin, who received an EMS delegation (R. Jeltsch, J.-P. Bourguignon and L. Lemaire) on 23 May, for a constructive exchange of ideas on the EMS document.

The European Mathematical Society has analysed the communication and related documents, and welcomes the new orientations that it describes proposing a new stage of development for European Research. It wishes to contribute to the open debate initiated by Commissioner Busquin, bearing in mind its own European mission and its direct connection with research and training activities throughout the countries of the Union, the Associated States and all other countries of the European continent.

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I. Some special features of mathematics

Although the E.M.S. wishes to raise some points pertaining to aspects of research in all fields, it feels the need to point to some particular features of mathematical research, notably in Europe.

Some of these characteristics should have some bearing on the organisation of the E.U. programmes. Let it be clear that we do not call for a blind increase in mathematics funding, or for adopting rules that would favour mathematics, but for an increased flexibility in the definition and the running of the programmes that would leave all opportunities for excellent proposals to be considered in all fields, with best possible efficiency. Different research traditions indeed require different frameworks, and we believe that our comments do not apply only to mathematics.

Mathematical research in Europe is recognised as among the best in the world: several indicators place it ahead of that of North America and of Asia. Other characteristics that distinguish it from sister sciences, and in particular from the experimental sciences, are the following:

(1) A mathematical research team at a given location is generally smaller, and research on a given subject is geographically more dispersed. That the quality of



mathematical research is not necessarily directly dependent on a concentration of heavy resources is certainly an advantage – but requires that this fact be taken into account when setting up programmes. This doesn't mean that no support at all is needed for mathematics, and computational mathematics and scientific computations will request investments – e.g. as large infrastructures.

(2) The unity of mathematics has always made itself felt with considerable force, and at the end of the 20th century the interplay of mathematical ideas between the different sub-domains had probably reached an all-time high in intensity. There is no doubt that this phenomenon sets mathematics apart from many other branches of science which have split up into multiple specialities, each with its specific culture and sociology. A direct consequence is that the task of a panel of mathematicians having to evaluate research is much easier, the existence of factions and of schools in conflict with one another remaining exceptional.

(3) The distance between fundamental mathematics and applications is relatively short and is getting shorter both in time and in content. In some cases, mathematical subjects developed for reasons purely internal to mathematics have suddenly led to concrete applications, in sharp contrast with the traditional pattern of neighbourhood relationships: pure science – applied science – applications. By way of example, we might mention the use of stochastic processes in finance, or of number theory in cryptography and system security.

(4) In mathematics there is an extremely close relationship between research and training. In fact, most researchers in mathematics are also teachers and

European support for research has an immediate effect on the improvement of mathematical qualifications in the manpower of the future. It is the appropriate moment to point out that the use of people mastering an advanced mathematical training has broadened enormously. Besides finance, we can mention telecommunication as a whole, the drugs industry, material design and so on.

(5) In Central and Eastern Europe and in the Third World the state of research in mathematics is often better than in the other sciences, for relatively low cost initiatives have often allowed the creation of centres of excellence; without being exhaustive, we could name some centres that are at the highest world level: the IMPA in Rio de Janeiro, the Banach Centre in Warsaw, the Tata Institute in Bombay. Some institutes in Vietnam and Africa can also be mentioned. Cooperation in mathematics with such countries should not be seen as a one-way aid to development, but as a true exchange between active centres, from which EU groups can benefit directly.

II. Fundamental research and its role in industrial development

The European Mathematical Society is happy to see the importance of fundamental research underlined at various places of the Communication.

The private sector mostly invests in short-term applied research, and this implies that the public bodies – States and European Union – must bear the major responsibility of long-term investment in fundamental research.

All opinions converge today on the fact that the time-lag before a discovery can be put to practical use is getting shorter and shorter. But the time before commercialisation is getting short because the numerous (and often cheap) innovations that come up everyday from the world of research are more quickly sorted out, and not because the analysis of needs has motivated the discovery. We are in an era where the precise demand is not shaping all the markets.

A very significant example will illustrate this. The necessary scientific basis that gave rise to the explosion of the communication society (the internet, ...) is not tied to discoveries that were motivated by that development, but by very theoretical results, obtained outside the pursuit of commercial aim and following the initiative of the scientists (bottom-up approach).

Indeed, the laser was built as a physical curiosity in the sixties (the production of coherent light). It has now become a necessary ingredient of fast transmission

along optical fibres – as well as being used every day when we listen to compact discs, use code bars and have surgery performed.

Likewise, number theory was studied for centuries as an ultimate amusement of mathematicians – but it led to cryptography and error correcting codes, without which safe transmission on the internet and e-commerce would not exist.

Also, the developments in non-linear analysis, like wavelets in image processing, have preceded the use of sophisticated non-linear models in high technology industry.

In many commercial developments of the future, we should expect to observe the same pattern: scientific discoveries of different fundamental fields being quite unexpectedly transformed into innovative new tools, creating a new market.

What can the E.U. do in this context? Naturally, it must support the development of industrial technology in an integrated way, to avoid negative effects of internal competition.

But it must mostly create a European area of scientific freedom, where the scientists will create new science following their own approach. It must then encourage or produce risk capital sufficiently quickly when a discovery can lead to unexpected applications. In short, Europe has to give a chance to the unexpected. This goes against the long-established idea of very close control on programmes and choices made *a priori*.

III. Human capital

In this age of science and technology, it is obvious that the future of Europe will depend crucially on the level of its researchers, and in fact of all its workforce. Energetic measures are thus required to encourage the pursuit of science, improve the training of the researchers and fight the brain drain to other continents.

III.1 In mathematics, the crux of the matter is probably the question of availability of postdoctoral positions, and the E.M.S. wants to underline the very positive results already achieved by the E.U. through its Network and Fellowship programmes. It calls for a substantial increase of means in these activities (together with other changes described in section III 2 below), which would prevent having to reject excellent proposals, as is now the case in the network category.

Why are postdoctoral positions so important for the future of science in Europe? We all observe that scientists in Europe are often better trained than in the U.S. – up to the doctoral level. This considerable investment in education having been done, Europe often offers no good opportunity for further employment, leaving to the U.S. the benefit of hiring highly competent scientists. As a result, the doctoral and postdoctoral U.S. programmes work with a majority of non-U.S. citizens.

A number of undisputed centres of

excellence do exist in Europe but they have less success, for the simple and basic reason that they do not have enough positions available!

The E.U. can have a strong effect on this problem provided it continues its efforts in the appropriate form. It is in particular extremely important that young researchers after their PhD have the opportunity to enlarge their perspective, by gaining access to centres of high level outside of narrowly defined scientific projects.

Practically, it would first be important to create and impose a tax-free status for the European postdoctoral researchers in mobility paid by the budget of the Commission, with the hope that States would follow that example with their own fellowships. For the moment, the situation varies from country to country and it is ridiculous to see that the same researcher is twice as expensive in Belgium as in the U.K. (in fact, in Belgium the E.U. will have to pay 180.000 Bef per month, the researcher receiving 60.000!). A European tax-free status would overnight double the number of positions offered by the Commission.

Secondly, we propose to define a new (optional) format for a postdoctoral fellow in mobility – namely, two years in another E.U. country, then one year in their country of origin (for the moment, this exists only for researchers from regions in economic difficulties).

Such a move would greatly improve the attractiveness of Europe for the following reasons.

For the moment, the time gap between completion of the PhD and the possibility of getting a permanent position is not easily covered by postdoctoral positions in Europe – and as a result many have to go to the U.S., where they often remain.

The 2+1 positions would be much more attractive: the researcher could completely concentrate on his/her work while abroad, without having to prepare multiple applications for the following year. Back home typically during year 3, he/she would be in a better position to seek permanent employment.

Finally, it happens from time to time that a bright researcher is offered simultaneously a 2-year E.U. fellowship and a permanent job at home, to take effect immediately. The choice is obvious – but not optimal for the development of European science. The employer (e.g., University) will not usually agree to give an immediate leave of absence. If on the other hand, a third year were offered to work in the same place, it could be another matter.

Altogether, these 2+1 positions would not be seen as an obstacle to a local career, but on the contrary could become highly attractive positions, well recognised in a Curriculum.

III.2 The networks are also important components of the postdoctoral programme – well integrated with the effective development of research.

With their present rules, the rate of success has become too small to make choices optimal, and the lack of flexibility make them less adapted to sciences like mathematics, where at each node groups are usually small. Thus, a more flexible rule should allow the inclusion of more small groups, without necessarily increasing the global budget of a network. Moreover, the panel of experts should be given much more freedom, including the possibility to award less money than requested or intervening to suggest the merger of two applications.

The freedom of action of the panel has been abruptly suppressed at the beginning of the Fourth Framework Programme, and in mathematics the rules lead to a dramatic decrease of the cost-efficiency of the programme.

III.3 The demography of researchers and professors in European universities should be studied in detail for every field, to get a fairly precise estimate year by year of future needs. For mathematics, this was done a few years ago by the E.M.S. This study should be updated, and would show some strong variations in the number of retirements over different periods (still following the effect of World War 2, of the growth in the sixties, the crisis in the seventies, and so on).

An ambitious policy is badly needed. It should insure, by an adequate number of doctoral and postdoctoral positions, that the future professorial positions in universities will be awarded to high-class researchers.

III.4 A serious technical problem arises in the definition of Marie Curie training sites. Indeed, they are open only to doctoral students and not postdoctoral ones. In certain fields, this corresponds to well-identified needs, but not in others (like mathematics). We propose to open these sites to both doctoral and postdoctoral researchers, thus allowing for optimal choices by the institutes in each field.

III.5 Finally, anyone who has invited to Europe a scientist from outside the E.U. knows that the visa problem is a major headache and a huge loss of time. In some cases, a foreign scientist invited and paid by a government will not be able to come, because the same government will not provide him/her with a visa.

IV. Centres of excellence

The E.M.S. supports strongly the Commission's view that centres of excellence are an important component of European research, well positioned for efficient and justified E.U. support.

However, those centres cannot be identified (as in the Communication of the Commission) by their capacity to produce industrially applicable discoveries. We note however that the accompanying document: Action for "centres of excellence" with a European dimension gives better criteria, and a list of existing centres showing a broader vision.

As mentioned above, centres of excellence in mathematics exist in Europe at the highest level, and are by no means inferior to U.S. centres like the I.A.S. in Princeton or the M.S.R.I. in Berkeley.

Seven of these centres have in fact already constituted a network of European dimension, pre-tailored for the scope of a European Research Area. The European Post-Doctoral Institute is indeed constituted by the Institut des Hautes Etudes Scientifiques (Bures-sur-Yvette, France), the Isaac Newton Institute for Mathematical Sciences (Cambridge, UK), the Max-Planck Institut für Mathematik (Bonn, Germany), the Max-Planck Institut für Mathematik in den Naturwissenschaften (Leipzig, Germany), the Erwin Schrödinger Institut (Vienna, Austria), the Institut Mittag-Leffler (Djursholm, Sweden) and the Banach Center (Warsaw, Poland).

The aim and function of this E.P.D.I. is so much in line with the E.U. programme that its case should be examined by the appropriate scientific panels. In the past, strict rules on different E.U. programmes made the EPDI not eligible – usually to the regret of scientific panels who rated it as excellent.

Other networks in mathematics have also established themselves at the highest level, either in the E.U. programmes or outside, and we wish to mention here the Network MACSI (Mathematics, computing, simulation in Industry), initiated by ECMI and ECCOMAS.

In order to allow for a choice of optimal centres and networks, the E.M.S. insists once more on the importance of flexibility in the rules: a network of centres of excellence need not be very large, need not be founded on a very specialised topic, and need not consist of large laboratories, although of course it can.

Finally, all calls for proposals should be widely open to competition – science in Europe is not limited to a list of centres fixed once and for all.

V. Infrastructures and electronic databases

The E.M.S. gratefully acknowledges the fact that the definition of a large infrastructure was enlarged in the Fifth Programme, thereby opening the way to a successful application of support of the comprehensive European database of mathematical literature, *Zentralblatt-MATH*.

Further support for electronic database and libraries will be essential for the development of mathematics, and Europe must reach the level of an unavoidable partner in the world competition in this domain.

A special feature of that branch is that documents are never obsolete: a brilliant proof by Bernhard Riemann or Elie Cartan can be of extreme contemporary importance, because ideas remain valid and the tradition of mathematics publication does not include repetition of past results. Thus a present day researcher will

effectively need access to the whole of the literature for his/her work.

On the other hand, the price of subscriptions to journals, in particular published by commercial companies, is rising unreasonably: increases of 10 to 20% per year are the norm, 60% often happens. Thus, the average university library becomes unable to acquire the basic necessary tool for its mathematics researchers.

As a consequence, some U.S. enterprises have embarked on an ambitious programme of digitising the existing scientific literature, in order to distribute it electronically. This effort is funded by private foundations, but access will be reserved to institutes paying a heavy annual fee, likely to remain out of reach for most European universities.

For these reasons, the E.M.S. presses the European Commission to take strong action aiming at developing electronic databases and libraries, including the steps of developing the data and its distribution. A more precise analysis should show whether Europe should catch up with the digitising initiated in the U.S. by competition or by partnership. In the latter case, a sufficiently strong position must be reached before hoping for a real partnership. By strong action, we mean in particular that the Commission support be granted not only for access to infrastructure, or for research related to the development of the infrastructure, but for direct funding of the installation itself.

This does not contradict any principle of subsidiarity, once it is acknowledged that the infrastructure is a truly European one and could just not be conceived at the scale of one state.

We note also that the E.M.S. has the project of establishing itself as a Publishing House, which would work as a non-profit organisation. The European Commission might think of supporting this kind of endeavour at the initial stage, for the long-term benefit of the scientific community.

VI. Central and Eastern Europe and Third World

As indicated above, very high-level mathematics research is pursued both in Central and Eastern Europe and in various developing countries. In Central and Eastern Europe, a high mathematical level has been maintained as a tradition, and using the fact that restricted access to expensive material over a short period of time did not bring irreversible damage.

This tradition is under threat in some countries, simply because economic problems have brought down the salaries of university teachers below a minimal level – thus producing a massive exodus to the west.

It is of the interest of the whole of Europe to preserve its high and long-lasting tradition of mathematical research in all countries. To disrupt it in the East would irreversibly damage a remarkable school of talent, which Europe will need in the near future.

Thus we advocate strong relations with Central and Eastern Europe, on the basis of scientific partnership at high level. The very low cost of living there can be used as an advantage, to organise activities more cost-efficiently.

Concerning the countries of Latin America, Asia and Africa, Europe could at this stage develop strong scientific relations, which would be beneficial in the long run. Indeed, these countries do not usually cherish the idea of being absorbed under the umbrella of powerful neighbours (U.S.A. for Brazil, Japan for Korea, ...). Mathematically, some centres in these countries are at the highest level, and a programme of cooperation would be very efficient. Currently, cooperation is only funded on specific themes defined by the Commission (pollution, forest, health, ...), missing the necessity for these countries to be integrated in the future society of knowledge. Various European states have strong historical and cultural links with developing countries, and these links could form the basis of future cooperation.

VII. E.U. programmes and procedures

Some rules currently enforced by the Commission do not allow for maximal efficiency of the programmes. It seems essential to identify and modify them.

VII.1 The rule of subsidiarity, pushed to some extreme, has sometimes been interpreted as saying that the Commission will not fund research itself, but only European added value. This is a self-defeating interpretation which we strongly regret. The European Space of Research must be in itself a European endeavour, and thus must be funded without administrative restrictions.

In the present programme, examples can be shown in which proposers, panels and administrators of the programme agree that a realisation is worthy of support, yet the rules prevent it.

For infrastructure, such restrictions were described in Section V.

For the programme 'Raising Public Awareness', the description of the programme gives the impression that it will be easier to fund a round table of discussion than an actual concrete realisation (the latter being pushed in the category of accompanying measures).

VII.2 The E.U. procedures, applications and contracts are absurdly complicated, notably because they aim at covering in extreme detail many different situations in the same format. It seems that all documents were drawn up by wary lawyers, and not by scientists. The smallest application is the object of 50 to 80 pages of instructions on the Cordis website. A simple shift of a few months in the execution of a project requires a full amendment to the contract.

Quite often, the questions asked in the application form constitute a barrage between the proposer and the scientific panel. This is in part due to a restrictive

view of subsidiarity: instead of simply saying what research he plans to do, for the panel to read, the proposer will have to answer a series of questions, distinguishing training, research, added European value, externalities, ... It has gone so far that some high-level scientists do not apply any more, whereas other hire the (paid) service of consulting agencies, who specialise in knowing how to transform a scientific project into a 'viable' E.U. application.

VII.3 The scientific panels play, and must play, an essential role in the peer reviewing of proposals. They must examine the proposals and have an actual meeting to exchange information and opinions. To ask for separate marks out of 100 and then to make a simple average (a process unfortunately used by INTAS) transforms the choice into a lottery, different experts having different scales for their marks.

The composition of panels must be handled with great care. At the beginning of the Fifth Programme, a short sentence suddenly appeared in the texts, restricting the choice of panel members by the E.U. staff to scientists who had filled in an application form.

The result could have been expected: in many disciplines, the top-class scientists did not agree to fill in a long form in order to be included in a database of potential experts; on the other hand, 26,000 people applied, producing a list impossible to handle.

We propose to go back to the preceding procedure: the panel secretaries together with the present panel members make proposals, taking into account the level of the experts and their fields. Proposals are also made from the outside, and the panel secretary and the director of the programme make a final choice. They can also call personally some well-known scientists and ask them to take part in the panel.

Anomalies in the way a panel member behaves are monitored by the E.U. staff, and in case a problem arises the panel member is not appointed again.

Our experience of the mathematics and information science panel is that it has evolved well, with competent guidance of panel secretaries at high level.

In case a panel were to concentrate around a specific subfield, it is up to the E.U. administration (directors and panel secretaries) to make the corrections.

VII.4 As already mentioned, panels should be given more freedom by allowing added flexibility to the rules.

VII.5 In some E.U. programmes, part of the first application must be anonymous, in the sense that the scientific programme must be described but the name of scientists in charge must be deleted. This procedure may give an impression of impartiality, but in fact it is absurd. The level of expertise of the researchers is the main factor in the success of a programme, and to delete that information is to deprive

the assessor from a major element of judgement. Moreover, if an assessor is suspected of helping his or her friends, it is clear that these friends will tell him they are behind the application. So nothing is gained but very much is lost in that process.

VII.6 To summarise this section, we reduce our recommendations to three simple mottoes: more flexibility, fewer rules, less paperwork.

VIII. Benchmarking

The E.M.S. certainly approves of benchmarking exercises, but not to the extent that it uses up a substantial part of E.U. funding and of scientists' time.

Moreover, the benchmarking described compares best practice in various E.U. countries, whereas the initial aim of the programme seems to be to catch up on U.S. efficiency, where organisation of research is concerned.

Maybe a benchmarking of practice with the U.S. would simply confirm that their efficiency is related to more freedom, less control, less planning, more facilities to patent a discovery, and simplified procedures to start a company, and these could be major guidelines for Europe.

IX. Summary of recommendations

The European Mathematical Society recommends that

1. the Commission take strong initiatives to develop in Europe a large area of fundamental research, preserving a 'bottom-up' approach leaving the door open for the unexpected discoveries that could shape tomorrow's society. Concurrently, administrative simplifications and encouragement of risk capital should allow for quick exploitation of these discoveries.
2. more flexibility be introduced in the definition of E.U. programmes, so that sciences with different traditions and format can be efficiently supported.
3. the Improving Human Capital programme be maintained and amplified, with the following specific suggestions:
 - the creation of a tax-free status for doctoral and post-doctoral European researchers;
 - the creation of '2+1 fellowships', with two years in another country and a one-year return fellowship;
 - a study of the demography of university professors in each field, and adequate measures to insure a supply of high-level researchers when necessary;
 - the development of the idea of centres of excellence and networks of such centres, with flexible rules and definitions, allowing for the different ways of different fields.
4. the Commission firmly support the development of European electronic databases and libraries, to allow access to the necessary scientific information all over the continent.
5. the Central and Eastern European mathematicians be integrated rapidly into all the Commission programmes,

as equal scientific partners,

6. E.U. programmes include mathematical collaboration with researchers of developing countries, where a good mathematical level has been reached.
7. all Commission procedures be enormously simplified, to avoid possible neglect of programmes by the best scientists; this applies to all steps of the process, from the initial applications to the final reports.
8. the scientific panels be given more freedom to judge, amend or make suggestions about the proposals.

Moreover, it would be extremely useful to:

- allow both doctoral and postdoctoral researchers in the Marie Curie training sites;
- go back to the previous system of appointments for panel members, to be chosen jointly by the present members and the Commission staff;
- suppress the idea of evaluating anonymous proposals;
- stress the importance of easy visa procedures for recognised researchers.

World Mathematical Year Stamps

In EMS Newsletter 35 we featured the Belgian stamp issued to commemorate World Mathematical Year 2000. Further stamps have since been issued by Luxembourg, Slovakia and Argentina. Two of these are shown below: the Luxembourg stamp features the WMY 2000 logo, Fermat's last theorem, Stokes' theorem, the Riemann zeta-function and the decimal expansion of p , while the Argentinian stamp depicts the infinity sign.

If any reader can send the Slovakian stamp to the Editor, it will appear in the next issue.



Forthcoming conferences

compiled by Kathleen Quinn

Please e-mail announcements of European conferences, workshops and mathematical meetings of interest to EMS members, to k.a.s.quinn@open.ac.uk. Announcements should be written in a style similar to those here, and sent as Microsoft Word files or as text files (but not as TeX input files). Space permitting, each announcement will appear in detail in the next issue of the Newsletter to go to press, and thereafter will be briefly noted in each new issue until the meeting takes place, with a reference to the issue in which the detailed announcement appeared

July 2000

2-7: 6th International Conference on p -Adic Analysis, Ioannina, Greece

Information: contact A. K. Katsaras, Dept. of Math., Univ. of Ioannina, 45110, Ioannina, Greece,
tel: (+30)-651-98289, fax: (+30)-651-46361
e-mail: akatsar@cc.uoi.gr
Web site: http://www.uoi.gr/conf_sem/p-adic
[For details, see EMS Newsletter 34]

2-15: NATO Advanced Study Institute 20th Century Harmonic Analysis – a Celebration, Tuscany, Italy

Information:
Web site: <http://www.cs.umb.edu/~asi/analysis2000>

3-5: International Conference on Monte Carlo and Probabilistic Methods for Partial Differential Equations, Monte Carlo, Monaco

Information:
e-mail: Monique.Simonetti@sophia.inria.fr

3-5: Rencontre de Probabilistes, Lectures on Probabilistic Topics in 3D Fluids, Barcelona, Spain

Speaker: Franco Flandoli (Università di Pisa)
Aim: the meeting will be devoted to some advanced lectures on probabilistic methods for 3D fluids. These will present the state of the art in a subject that lies at the interface between fluid dynamics and probability theory. Two points of view will be considered: the study of stochastic Navier-Stokes equation and the modelling of vortex structures by probabilistic methods

Organisers: the probability teams of the University Paris 13 and the University of Barcelona

Information:

Web site: <http://orfeu.mat.ub.es/~gaesto/fluid.htm>

3-7: ALHAMBRA 2000, Granada, Spain

Information: contact ALHAMBRA 2000 Conference eurocongres Avda. Constitución, 18 - Blq.4 E-18012 - Granada, Spain
tel: (+34)-958-209-361, fax: (+34)-958-209-400
e-mail: alhambra2000@ugr.es, eurocongres@mx3.redestb.es
Web site: <http://www.ugr.es/local/alhambra2000>
[For details, see EMS Newsletter 34]

3-7: ANTS IV Algorithmic Number Theory Symposium, Leiden, the Netherlands

Information:
e-mail: ants4@wins.uva.nl
Web site: <http://www.math.leidenuniv.nl/ants4/>
[For details, see EMS Newsletter 34]

3-7: Functional Analysis Valencia 2000, Valencia, Spain

Information: contact: K. D. Bierstedt or J. Bonet, Univ. Paderborn, FB 17, Math., D-33095 Paderborn, Germany or Universidad Politécnica de Valencia, Departamento de Matemática Aplicada, E-46071 Valencia, Spain
e-mail: VLC2000@uni-paderborn.de
Web site: <http://math-www.uni-paderborn.de/VLC2000>
[For details, see EMS Newsletter 32]

3-9: Euro-Summer School on Mathematical Aspects of Evolving Interfaces, Madeira, Portugal

Information:
e-mail: maei2000@lmc.fc.ul.pt
Web site: <http://maei.lmc.fc.ul.pt>
[For details, see EMS Newsletter 34]

4-6: Catop 2000, Fribourg, Switzerland

Scope: categorical topological methods
Information:
Web site: <http://www.unifr.ch/math/catop2000/>
[For details, see EMS Newsletter 34]

4-7: 2nd International Conference on Mathematical Methods in Reliability, Bordeaux, France

Information: contact Dr. Valentina Nikoulina, Université Victor Segalen - Bordeaux 2, Statistique Mathématique, UFR MI2S, B.P. 69 33076 Bordeaux Cedex, FRANCE,
tel: (+33) 5 57 57 10 70 & 5 57 57 14 25,
ax: (+33) 5 56 98 57 36
& (+33) 5 57 57 12 63
e-mail: vnikou@mi2s.u-bordeaux2.fr, Nikolaos.Limmios@utc.fr
Web site: <http://www.mass.u-bordeaux2.fr/MI2S/MMR2000/>
[For details, see EMS Newsletter 34]

5-7: Scandinavian Workshop on Algorithm Theory, Bergen, Norway

Information:
e-mail: telle@ii.uib.no
Web site: <http://www.ii.uib.no/swat2000>

5-8: Ordinal and Symbolic Data Analysis (OSDA 2000), Brussels, Belgium

Information:
Web site: <http://www.ulb.ac.be/sciences/ulbmth/osda2000>
[For details, see EMS Newsletter 35]

6-8: 6th Barcelona Logic Meeting, Barcelona, Spain

**17–20: IUTAM Symposium 2000/10
Diffraction and Scattering in Fluid Mechanics
and Elasticity, Manchester, UK**

Information: contact Professor David Abrahams, Department of Mathematics, University of Manchester, Oxford Road, Manchester M13 9PL, UK,
tel: (+44)-161-275-5901,
fax: (+44)-161-275-5819
e-mail: i.d.abrahams@ma.man.ac.uk
Web site: <http://www.keele.ac.uk/depts/ma/iutam/>
[For details, see EMS Newsletter 33]

17–21: Colloquium on Semigroups, Szeged, Hungary

Information:
e-mail: algebra@math.u-szeged.hu

**17–21: 9th International Conference on
Fibonacci Numbers and their Applications,
Luxembourg-City, Luxembourg**

Information:
e-mail: howard@mthsc.wfu.edu

**17–22: Colloquium on Lie Theory and
Applications, Vigo, Spain**

Information: contact I Colloquium on Lie Theory and Applications, E. T. S. I. Telecomunicación, Universidad de Vigo, 36280 Vigo, Spain
tel: (+86) 81 21 52 // (+86) 81 24 45,
fax: (+86) 81 21 16 // (+86) 81 24 01
e-mail: clieta@dma.uvigo.es
Web site: <http://www.dma.uvigo.es/~clieta/>
[For details, see EMS Newsletter 33]

**17–22: International Congress on
Mathematical Physics, London, UK**

Information:
Web site: <http://icmp2000.ma.ic.ac.uk/>

**19–26: 3rd World Congress of Non-linear
Analysts (WCNA-2000), Catania, Italy**

**22–28: New Mathematical Methods in
Continuum Mechanics, Anogia, Crete**

Information:
e-mail: ball@maths.ox.ac.uk
[For details, see EMS Newsletter 35]

**23–31: ASL European Summer Meeting
(Logic Colloquium 2000), Paris, France**

Information:
e-mail: asl@math.uiuc.edu
Web site: <http://lc2000.logique.jussieu.fr>
[For details, see EMS Newsletter 33]

**24–29: SIAG OP-SF Summer School 2000,
Laredo, Spain**

Theme: orthogonal polynomials and special functions
Information:
e-mail: ran@cica.es or pacomarc@ing.uc3m.es

**24–3 August: EMS Summer School, New
Analytic and Geometric Methods in Inverse
Problems, Edinburgh, UK**

Information: contact Erkki Somersalo, Helsinki University of Technology, Finland
e-mail: esomersa@dopey.hut.fi

**25–30: Colloquium on Differential Geometry
and its Applications, Debrecen, Hungary**

Information:
Web site: <http://pc121.math.klte.hu/diffgeo>

**29–4 August: Curves and Abelian Varieties
over Finite Fields and their Applications,
Anogia, Crete**

[For details, see EMS Newsletter 35]

30–5 August: ICOR 2000, Innsbruck, Austria

**31–3 August: 3rd Conference of Balkan
Society of Geometers, Bucharest, Romania**
Information: contact V. Balan, University Politehnica of Bucharest, Department of Mathematics I, Splaiul Independentei 313, RO-77206, Bucharest, Romania,
fax: (+401) 411.53.65
e-mail: vbalan@mathem.pub.ro
[For details, see EMS Newsletter 33]

**31–4 August: Numerical Modelling In
Continuum Mechanics (Theory, Algorithms,
Applications), Prague, Czech Republic**

Information:
e-mail: nmicm@karlin.mff.cuni.cz
Web site: <http://www.karlin.mff.cuni.cz/katedry/knm/nmicm2000>
[For details, see EMS Newsletter 35]

**31–4 August: Workshop on Partial
Differential Equations: Thermo, Visco and
Elasticity, Konstanz, Germany**

Information: local organiser:
reinhard.racke@uni-konstanz.de
Web site: <http://www.mathe.uni-konstanz.de/~racke/announ/ws2000.html>
[For details, see EMS Newsletter 35]

August 2000

**2–9: Summer School on Mathematical
Physics (emphasis on Quantum Field
Theory), Sandbjerg Manor, Denmark**

Information:
Web site: <http://www.maphysto.dk/events/>

**2–18: Rings, Modules and Representations –
Constanta 2000, Constanta, Romania**

Information:
Address: University of Stuttgart, Mathematisches Institut B/3, Pfaffenwaldring 57, 70550 Stuttgart, Germany
fax: (+49)-711-685-5322
e-mail: buro@poolb.mathematik.uni-stuttgart.de
Web site: <http://web.mathematik.uni-stuttgart.de/~ovid>
[For details, see EMS Newsletter 35]

**3–5: Recent Development in the Wave Field
and Diffuse Tomographic Inverse Problems,
Edinburgh,**

CONFERENCES

algorithms, Concurrent and parallel computations, Computer networks, Discrete mathematics in relation to computer science, Computer aided design, Theory of data, Programming, Image processing, Pattern recognition, Communication systems, Information systems, Manufacturing systems, Data base, Software technologies, Software engineering, Applications in mechanics, physics, chemistry, biology, technology and economics

Chairman of the organising committee:

Professor Drumi Bainov, P.O. Box 45, Sofia 1504, Bulgaria

tel: (+359) 2437343,

fax: (+359) 29879874

e-mail: dbainov@mbox.pharmfac.acad.bg

17–3 September: EMS Summer School in Probability Theory, Saint-Flour, Cantal, France

Information: contact: P. Bernard, Laboratoire de Mathématiques Appliquées, Univ. Blaise Pascal, F-63177 Aubiere,
tel/fax: (+33) 4 73 40 70 64

e-mail: bernard@ucfma.univ-bpclermint.fr

[For details, see EMS Newsletter 34]

19–25: Discrete and Algorithmic Geometry, Anogia, Crete

Information:

e-mail: ziegler@math.tu-berlin.de

[For details, see EMS Newsletter 35]

20–23: 3rd International Workshop on Scientific Computing in Electrical Engineering SCEE–2000, Warnemünde, Germany

Information:

Web site: <http://www.SCEE-2000.uni-rostock.de>

21–25: International Association for Mathematics and Computers World Congress (IMACS 2000), Lausanne, Switzerland

Information: contact Prof. Robert Owens, IMACS Congress 2000, DGM-IMHEF-LMF, Swiss Federal Institute of Technology, CH-1015 Lausanne, Switzerland

tel: (+41)-21-693.35.89,

fax: (+41)-21-693.36.46

e-mail: robert.owens@epfl.ch

Web site: <http://imacs2000.epfl.ch>

[For details, see EMS Newsletter 32]

27–1 September: 9th Summer St Petersburg Meeting in Mathematical Analysis, St Petersburg, Russia

Information:

Web site: www.pdmi.ras.ru/EIMI/2000/analysis9/index.html

30–2 September: Innovations in Higher Education 2000, Helsinki, Finland

Information:

e-mail: sari.lindblom-ylanne@helsinki.fi

Web site: <http://www.helsinki.fi/inno2000>

2–9: International Conference on Topology and its Applications, Ohrid, Macedonia

Invited speakers: P. J. Collins (University of Oxford), J. Dydak (University of Tennessee, Knoxville), A. Koyama (Osaka Kyoiku University), Y. Lisica (Russian University of Peoples Friendship, Moscow), L. Rubin (University of Oklahoma, Norman), J. M. R. Sanjurjo (Universidad Complutense, Madrid), M. Scheepers (Boise State University, Idaho)

Programme committee: Prof. Yuri Lisica

(Moscow), Prof. Jose Sanjurjo (Madrid), Prof. Nikita Shekutkovski

Local organising committee: Nikita

Shekutkovski (president), Biljana Krsteska, Liljana Babinkostova

Call for papers: abstracts of 300–500 words should be sent no later than 1 July to the mailing address of the conference: ICTA2000, Institute of Mathematics, Faculty of Natural Sciences and Mathematics St. Cyril and Methodius University, Skopje, Macedonia, or by e-mail to:

icta2000@iunona.pmf.ukim.edu.mk. If the abstract is sent electronically in TeX format, then it should also be sent in compiled form (as a .jep file ready for printing) or printed

Sponsor: Faculty of Natural Sciences and Mathematics at St. Cyril and Methodius University, Skopje, Macedonia

Information:

Web site: <http://www.pmf.ukim.edu.mk/mathematics/icta2000.html>

3–6: 31st European Mathematical Psychology Group Meeting (EMPG 2000), Graz, Austria

Information:

e-mail: empg2000@psyserver.kfunigraz.ac.at

Web site:

<http://psyserver.kfunigraz.ac.at/empg2000/>

Vosmanský

Proceedings: to be published in 2001

Abstracts: will be distributed at the beginning of the Colloquium

Site: Santon hotel in the beautiful area of the Brno dam lake, available by city transport

Conference fee: USD 170 (before 31 July), USD 190 (later); this covers the Registration fee (usual conference expenses, proceedings and social programme) and the package of board and lodging for the whole period of the conference

Deadline: for abstracts and registration, 15 June 2000 (later applications may be accepted)

Information:

e-mail: cdde@math.muni.cz

Web site: <http://www.math.muni.cz/cdde>

5–16: Advanced Course on Algebraic Quantum Groups, Bellaterra, Spain

Information:

e-mail: quantum@crm.es

Web site: <http://crm.es/quantum>

10–17: Summer School on Geometry of Quiver-Representations and Preprojective Algebras, Isle of Thorn, UK

Information: contact Karin Erdmann, Mathematical Institute, University of Oxford, Oxford OX1 3LB, UK

e-mail: erdmann@maths.ox.ac.uk

Web site: <http://www.mathematik.uni-bielefeld.de/~sek/summerseries.html>

[For details, see EMS Newsletter 34]

11–13: Optimal Discrete Structures and Algorithms – ODSA 2000, Rostock, Germany

Scope: the emphasis will be on the interactions between several aspects of discrete mathematics, mathematical optimisation, and theoretical computer science

Topics: combinatorial optimisation and algorithms on discrete structures, extremal problems in posets, design theory, coding theory, cryptography, graph theory, complexity theory

Invited speakers: C. J. Colbourn (Burlington), P. Frankl (Tokyo), J. R. Griggs (Columbia), C. T. Hoang (Thunder Bay), D. Kreher (Houghton), A. Pott (Magdeburg), J. P. Spinrad (Nashville), I. Wegener (Dortmund), G. Ziegler (Berlin)

Programme: nine invited plenary lectures (55 minutes) and contributed talks (25 minutes) in parallel sessions

Organisers: A. Brandstädt, K. Engel, H.-D. Gronau, R. Labahn (Rostock)

Proceedings: a special issue of *Discrete Applied Mathematics* with papers related to the conference and refereed according to the usual procedures

Information:

e-mail: odsa@mathematik.uni-rostock.de

Web site: <http://www.math.uni-rostock.de/odsa>

11–14: International Colloquium in Honour of Professor Michel Mendès, Bordeaux, France

Information:

Web site: <http://www.math.u-bordeaux.fr/~stan/Colloque/MMF.html>

[For details, see EMS Newsletter 35]

11–15: Boundary Integral Methods: Theory and Applications, Bath, UK

Information:

Web site: <http://www.ima.org.uk/mathematics/conferences.htm>

[For details, see EMS Newsletter 34]

12–15: Imaging and Digital Image Processing: Mathematical Methods, Algorithms and Applications, Leicester, UK

Information:

Web site: <http://www.ima.org.uk/mathematics/conferences.htm>

[For details, see EMS Newsletter 34]

12–15: IWOTA – Portugal 2000 International Workshop on Operator Theory and Applications, Faro, Portugal

Information: contact F.-O. Speck,

Departamento de Matematica, Instituto Superior Tecnico, U.T.L., 1049-001 Lisboa, Portugal

tel: +351-21-8417095,

fax: +351-21-8417598,

e-mail: fspeck@math.ist.utl.pt

Web site: <http://www.ualg.pt/cma/iwota>

[For details, see EMS Newsletter 35]

13–15: International Conference of the Royal Statistical Society, Reading, UK

Information:

e-mail: rss2000@reading.ac.uk

15–18: Physical Interpretations of Relativity Theory, London, UK

Information:

CONFERENCES

Advisory committee: R. Brown (Bangor), J. Bochnak (Amsterdam), C.-P. Bruter (Paris 11), M. Chaves (Porto), M. Emmer (Roma 1), T. C. Kuo (Sydney), Richard Palais (Brandeis), V. Poenaru (Paris 11)

Proceedings: to be published

Information: contact C. P. Bruter,

fax: 01 60 92 51 43

e-mail: bruter@univ-paris12.fr

22–27: EURESCO Conference on Geometry, Analysis and Mathematical Physics: Analysis and Spectral Theory, San Feliu de Guixols, Spain

Information:

<http://www.esf.org/euresco/00/pc00127a.htm>

[For details, see EMS Newsletter 35]

22–27: EURESCO Conference on Number Theory and Arithmetical Geometry: Motives and Arithmetic, Obernai (near Strasbourg), France

Organiser: U. Jannsen (Regensburg)

Information: available from

Web site: <http://www.esf.org/euresco>

22–28: 4th International Conference on Functional Analysis and Approximation Theory (4th FAAT), Acquafredda di Maratea, Potenza, Italy

Information: contact any of the organising committee or visit the website

Web site:

<http://www.dmi.uniba.it/maratea/FAAT2000.htm>

[For details, see EMS Newsletter 35]

25–27: International Workshop on Automated Deduction in Geometry, Zurich, Switzerland

Information:

e-mail: wang@calfor.lip6.fr

26–30: XI Biannual Conference of ECMI The European Consortium for Mathematics for Industry, Palermo, Italy

Information:

Web site: <http://www.ecmi.dk/>

29–2 Oct: IXth Oporto Meeting On Geometry, Topology And Physics, Oporto, Portugal

[series formerly known as the Oporto Meetings on Knot Theory and Physics]

Aim: to bring together mathematicians and physicists interested in the inter-relation between geometry, topology and physics and to provide them with a pleasant and informal environment for scientific interchange

Themes: this year the focus will be on differential geometry and algebraic geometry

Programme: mainly short courses, of approximately three lectures each, given by the main speakers, and a limited number of seminars (more details later). The talks are at the advanced graduate or postdoctoral level, and should be of interest to all researchers wishing to learn about recent developments in the overlap between geometry, topology and physics

Main speakers: Maks. A. Akiyis (Jerusalem) and Vladislav V. Goldberg (Newark), *The Geometry of Submanifolds with Degenerate Gauss Mappings* (joint course); Joseph Landsberg (Toulouse), *Representation Theory and the Geometry of Projective Varieties*; Dietmar Salamon (Zurich), *Floer Homology*; Simon Salamon

(Oxford); Rick W. Sharpe (Toronto), *A Survey of Infinite Dimensional Lie Groups*

Information:

Web site:

<http://fisica.ist.utl.pt/~jmourao/om/omix/om00b.html>

October 2000

7–10: International Conference on Mathematical Modelling and Computational Experiments (ICMCE), Dushanbe, Tajikistan

Information:

Web site: <http://www.tajnet.com/>

15–21: DMV Seminar on the Riemann Zeta Function and Random Matrix Theory, Mathematisches Institut Oberwolfach, Germany

Information: Prof. Dr Matthias Kreck, Universität Heidelberg, Mathematisches Institut, Im Neuenheimer Feld 288, 69120 Heidelberg, Germany

15–21: DMV Seminar on Motion by Curvature, Mathematisches Institut Oberwolfach, Germany

Information: Prof. Dr Matthias Kreck, Universität Heidelberg, Mathematisches Institut, Im Neuenheimer Feld 288, 69120 Heidelberg, Germany

20–22: One Hundred Years of the Journal L'Enseignement Mathématique, Geneva, Switzerland

Topics: on the occasion of the centennial of the journal *L'Enseignement Mathématique* (a leading journal in the internationalization of mathematics education at the beginning of the 20th century), it was felt appropriate to hold a symposium with the aims of looking at the evolution of mathematics education over the last century and identifying some guidelines and trends for the future, taking into account, among other sources, the documents, debates and related papers that appeared in *L'Enseignement Mathématique*. The emphasis of the symposium is on secondary education (students in the age range of about 12 to 18 or 19 years). In addition to proposing a reflection on the history of mathematics education and the evolution of mathematics and its teaching and learning in the 20th century, the symposium also gives the opportunity of a gathering of some of the main actors, during the last decades, in mathematics education as considered from an international perspective

Organizers: ICMI, University of Geneva

Programme committee: Daniel Coray (Switzerland), Fulvia Furinghetti (Italy), Hélène G

12–18: DMV Seminar on Characteristic Classes of Connections, Riemann-Roch Theorems, Analogies with e-Factors, Mathematisches Institut Oberwolfach, Germany

Organisers: Spencer Bloch (Chicago) and Helene Esnault (Essen)

Information: Prof. Dr Matthias Kreck, Universität Heidelberg, Mathematisches Institut, Im Neuenheimer Feld 288, 69120 Heidelberg, Germany

27–31 December: Foundations of Probability and Physics, Vaxjo, Sweden

Aim: to reconsider foundations of probability theory in connection with foundations of physics (quantum as well as statistical)

Topics: the following problems will be discussed:

(A) Creation of the conventional probability theory: Kolmogorov's axiomatics, 1933 (Why and How?). Is probability a measure or a frequency? A role of ergodicity. Why did Kolmogorov's model become dominating in mathematics and physics?

(B) Creation of probabilistic foundations of quantum mechanics: Is quantum probability a measure, frequency, potentiality or degree of my belief? The great diversity of opinions. Why does everybody use finally Kolmogorov's measure-theoretical approach?

(C) Probability in statistical physics, The role of ergodicity, Individual and statistical ergodicity.

(D) Einstein-Podolsky-Rosen paradox, Bell's inequality, locality, reality, probability. Conventional probabilities and Bell's inequality (via Bell, Clauser, Horne, Shimony, Holt, Aspect, Home, Selleri, d'Espagnat, Fine, Rastal, Greenberger, Zeilinger,...). Thermodynamical approach (via de Broglie, Lochak, de Muynck). Quantum fluctuations, stochastic dynamics. Von Mises frequency approach and Bell's inequality. Fluctuating probabilities, modified Bell's inequality. Quantum probabilities. Negative probabilities. p -adic probabilities. p -adic model of fluctuating reality. (E) Quantum computers and information and probability.

(F) Paradoxes of probability theory (in particular, violations of the law of large numbers or ergodicity). Randomness, complexity and foundations of probability theory and physics

Invited Speakers: S. Albeverio (Bonn Univ.), H. Amannschpacher (Inst. of Psychology, Freiburg), L. Ballentine (Simon Fraser), J. Bricmont (FYMA), A. Holevo (Steklov Math. Inst.), S. Gudder (Denver Univ.), P. Lahti (Turku Univ.), T. Maudlin (Rutgers), A. Shimony (Boston Univ.), J. Summhammer (Atominst. Wien)

Organisers: L. Accardi (Rome), W. De Muynck (Eindhoven), T. Hida (Meijo University, Japan), A. Khrennikov (Sweden), V. Maximov (Belostok Univ., Poland), L. Accardi (Univ. Rome-2, Italy), W. De Muynck (Univ. of Eindhoven, the Netherlands), T. Hida (Meijo University, Japan), A. Khrennikov (Växjö University, Sweden), V. Maximov (Belostok Univ., Poland)

Organising institution: International Center for Mathematical Modeling [in physics, technique and cognitive sciences], Vaxjo University, Sweden

Deadline: 1 October 2000

Information:

e-mail: send abstracts and application forms to

Karl-Olof.Lindahl" @msi.vxu.se
Web site: <http://www.msi.vxu.se/aktuell/konferenser.html>

December 2000

18–20: 5th International Conference on Mathematics in Signal Processing, Coventry, UK

Information: contact Pamela Bye, The Institute of Mathematics and its Applications, Catherine Richards House, 16 Nelson Street, Southend-on-Sea, Essex SS1 1EF, England

fax: (+44) 1702 354111

e-mail: conferences@ima.org.ac.uk

[For details, see EMS Newsletter 34]

January 2001

28–3 February: 2001 XXI International Seminar on Stability Problems for Stochastic Models, Eger, Hungary

[Dedicated to the 70th jubilees of M. Arató (Debrecen, Hungary) and V. M. Zolotarev (Moscow, Russia), Honorary Chairmen of the Seminar]

Topics: limit theorems in probability theory; characterisations of probability distributions and their stability; theory of probability metrics; stochastic processes and queueing theory; actuarial and financial mathematics; applications in informatics and computer sciences; applied statistics

Language: English

Programme committee: Chairmen: Z. Daróczy and G. Pap (Debrecen, Hungary), V. Yu. Korolev (Moscow, Russia). Members: A. Balkema (Amsterdam), A. Benczúr (Moscow), V. E. Bening (Moscow), A. V. Bulinski (Moscow), L. Csöke (Eger), S. Csörgö (Szeged), I. Fazekas (Debrecen), J. Fritz (Budapest), V. V. Kalashnikov (Moscow), J. Kormos (Debrecen), V. M. Kruglov (Moscow), P. Major (Debrecen), G. Michaletzky (Budapest), J. Mijneer (Leiden), T. Móri (Budapest), E. Omey (Brussels), A. Plucinska (Warsaw), L. Zeidler (Pécs), M. van Zuijlen (Nijmegen)

Secretaries of the organising committee: A. V. Kolchin (Moscow), S. Baran (Debrecen)

Sponsors (provisional): Steklov Mathematical Institute of the Russian Academy of Sciences (Moscow), Institute of Mathematics and Informatics of the University of Debrecen (Hungary), Károly Eszterházy College of Education (Eger)

Proceedings: to be published in the *Journal of Mathematical Sciences*

Deadline: for registration and for abstracts, 15 October 2000

Information:

e-mail: stabil@math.klte.hu or

Recent books

edited by Ivan Netuka and Vladimír Souček

Books submitted for review should be sent to the following address:

Ivan Netuka, MÚUK, Sokolovská 83, 186 75 Praha 8, Czech Republic.

A. Balog, G. O. H. Katona, A. Recki and D. Szász (eds.), *European Congress of Mathematics, Budapest, 1996, Vols. I, II*, Progress in Mathematics 168, 169, Birkhäuser, Basel, 1998, 334 and 402 pp., each DM159, ISBN 3-7643-5497-6 and 3-7643-5496-8

The tradition of European Congresses of Mathematics started in 1992 in Paris and from the beginning they have been major events in European mathematical life. These volumes present the second one in Budapest in 1996. The main lectures were given by N. Alon, *Randomness in discrete mathematics*; D. McDuff, *Symplectic topology*; J. Kollár, *Low degree polynomial equations*; A. S. Merkurjev, *K-theory and algebraic groups*; V. Milman, *High-dimensional convexity theory*; and St. Müller, *Microstructures, geometry and the calculus of variations*. There were ten prize winners announced at the Congress, some of whose lectures can be found in these volumes: W. T. Gowers, *Geometry of Banach spaces*; A. Huber, *Extensions of motives*; D. Kramkov, *Statistics and mathematics of finance*; J. Matoušek, *Geometric set systems*; and L. Polterovich, *Symplectic geometry*. The volumes conclude with records of five round tables during the Congress (electronic literature; mathematical games; women and mathematics; public image of mathematics; education). Written versions of invited lectures in both volumes provide a very good overview of recent substantial progress in a broad variety of mathematical fields and should be of interest to any working mathematician. (vs)

R. Becker, *Cônes convexes en analyse*, Travaux en cours 59, Hermann, Paris, 1999, 245 pp., ISBN 2-7056-6384-3

At the end of the 1950s, Gustav Choquet developed his theory of integral representations for compact convex subsets in locally convex spaces, and also for locally compact convex cones. Later he published several papers generalising his theory to the case of weakly complete convex cones. The book contains a postlude where the master Choquet himself explains the history and the value of the theory he introduced.

The author starts with basic properties of weakly complete convex cones and introduces the main objects (extremal points, hats, topologies on certain cones, conic measures, maximal measures, cones bien-coiffé, the Cartier-Fell-Meyer theorem and its generalisation). He then introduces several applications of the theory (proofs of the Bochner-Weil theorem and of Bernstein's theorem based on the Choquet theory) and presents a short out-

line of the Brelot and Bauer axiomatic potential theory. Further chapters are devoted to a deeper study of conic measures, zonofoms, adapted spaces and other subjects.

It would have been useful if the author had also mentioned the notable results of N. Boboc and A. Cornea from the theory of lower semicontinuous cones, or the results of J. Bliedtner and W. Hansen concerning the simpliciality of certain cones of superharmonic functions in potential theory.

This book is readable by anybody with a basic knowledge of functional analysis. (jl)

M. Berger, *Riemannian Geometry During the Second Half of the Twentieth Century*, University Lecture Series 17, American Mathematical Society, Providence, 2000, 182 pp., US\$34, ISBN 0-8218-2052-4

This is a new edition of the survey which appeared for the first time as *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Band 100, Heft 2, 1998. It starts with a historical review devoted to Riemannian geometry up to 1950 'from Gauss and Riemann to Chern'. The next six chapters cover the following basic topics: pinching problems, curvature (sectional, Ricci, scalar) of a given sign, finiteness, compactness, collapsing, space forms, Einstein manifolds, the Yamabe problem, spectra and eigenfunctions, period geodesics, geodesic flow, volumes, systols, holonomy groups, Kähler manifolds, spinors, harmonic maps, generalisations of Riemannian geometry, submanifolds and geometric measure theory.

Because the survey is devoted almost exclusively to global aspects of Riemannian geometry, the Riemannian manifolds are usually supposed to be compact, or at least complete. The exposition is systematic and careful, yet also elegant and charming, written with a pedagogical mastery that is characteristic of the author. The book includes a short introduction, comments on the main topics, 33 pages of bibliography, and subject and author indices. (ok)

E. Berlekamp and T. Rodgers (eds.), *The Mathematician and Pied Puzzler*, A. K. Peters, Natick, 1999, 266 pp., £22, ISBN 1-56881-075-X

This book is a collection of nearly forty articles from recreational mathematics, written by foremost puzzlers, magicians and mathematicians. It contains many interesting puzzles, games, and mathematical problems (for example, is it possible to fit unit squares on an $n \times n$ sheet of graph paper labelled with 0s and 1s so that every neighbourhood is odd). What most of these articles have in common is that they show the 'magic face' of mathematics, in

contrast to the mathematics that the reader may have learnt at school.

The book is dedicated to Martin Gardner, the famous author of the *Mathematical Games* column in *Scientific American* for many years. The name of this mathematician and pied puzzler guarantees that the book will bring enjoyment to a wide range of readers. (ec)

C. Cercignani and D. H. Sattinger, *Scaling Limits and Models in Physical Processes*, DMW Seminar 28, Birkhäuser, Basel, 1998, 190 pp., DM58, ISBN 3-7643-5985-4 and 0-8176-5985-4

This book, written by experts on their research areas, is in two parts. In the first part, *Scaling and Mathematical Models in Kinetic Theory*, Carlo Cercignani provides a well-written quick introduction to perturbation and scaling methods in the mathematical theory of the Boltzmann equation. The second part, *Scaling, Mathematical Modelling, and Integrable Systems*, was written by David Sattinger, and presents an introduction to dispersive waves and a derivation of the associated completely integrable systems, as the Korteweg de Vries and non-linear Schrödinger equations, via a careful treatment of the inverse scattering theory for the Schrödinger operator.

Both parts contain interesting historical comments and a huge bibliography. The book can be recommended to non-specialists in mathematics, physics and engineering who desire a quick insight into these areas. (jmal)

S. B. Cooper and J. K. Truss (eds.), *Sets and Proofs and Models and Computability*, London Mathematical Society Lecture Notes Series 258, 259, Cambridge University Press, Cambridge, 1999, 436 and 419 pp., each £29.95, ISBN 0-521-63549-7 and 0-521-63550-0

These two volumes contain articles by invited speakers at *Logic Colloquium '97* (the major international meeting of the Association of Symbolic Logic); all authors are specialists in their respective fields. In these volumes, the reader can obtain comprehensive information about the current state of research in almost all branches of mathematical logic.

Mathematics **179**, Springer, New York, 1998, 194 pp., DM98, ISBN 0-387-98377-5

This book is the second edition of the 1972 original version and remains unchanged, except that several mistakes and typographical errors have been corrected. There is also a brief report with references on the current state of the double-starred open problems.

The book begins with basic results in Banach space theory (main examples, linear functionals, weak topologies, the Hahn-Banach and open mapping theorems, Lebesgue and Hardy spaces). The second chapter is devoted to Banach algebras (also the Gelfand-Mazur theorem and the Gelfand transform). Further chapters explain topics in the geometry of Hilbert spaces and operators on Hilbert spaces and C^* -algebras (normal and self-adjoint operators, the Gelfand-Naimark theorem, the spectral theorem and functional calculus, weak and strong operator topologies, W^* -algebras, the Fuglede theorem). One chapter is devoted to compact and Fredholm operators and index theory (the Calkin algebra, the Fredholm alternative, representations of the C^* -algebra of compact operators). The last two chapters deal with Hardy spaces (Beurling's theorem, the F. and M. Riesz theorem, the maximal ideal space of H^8 , the Gleason-Whitney theorem, abstract harmonic extensions, maximal ideal spaces) and with Toeplitz operators (the spectrum and invertibility of Toeplitz operators, localisation, Widom's result on the connectedness of the spectrum). At the end of each chapter there are notes suggesting additional reading and a large number of problems of various difficulty.

The book is accessible to anybody with a basic knowledge of analysis, general topology, measure theory and algebra. (jl)

P. Drábek, P. Krejčí and P. Takáč, *Nonlinear differential equations*, *Research Notes in Mathematics* **404**, Chapman & Hall/CRC, Boca Raton, 1999, 196 pp., £30, ISBN 1-58488-036-8

The three papers in this volume are adapted versions of a series of lectures given by the editors at the Seminar in Differential Equations, Chvalatice, Czech Republic, in 1998. They present the authors' recent research in a way that is also comprehensible to PhD students working in differential equations. The presentations are based on recent papers and preprints and also contain new results. The first contribution, on non-linear eigenvalue problems and the Fredholm alternative (P. Drábek), studies the eigenvalues of the p -Laplacian, solvability of the elliptic boundary value problem with right-hand side, and points to the connection with the optimal Poincaré inequality. Most of this paper deals with the one-dimensional case.

The second text, on evolution variational inequalities and multi-dimensional hysteresis operators (P. Krejčí), is devoted to a discussion on the influence of the geometry of the convex constraint on analytical properties of the solution operator of a

variational inequality. The physical motivation in plasticity is explained in the text. The last contribution, on degenerate elliptic equations in ordered Banach spaces and applications (P. Takáč), is concerned with quasi-linear elliptic boundary value problems with Dirichlet boundary conditions in bounded domains in R^n , including the p -Laplacian. The questions of existence and uniqueness of solutions, Fredholm alternatives, and comparison principles are studied and compared with the linear ($p = 2$) case. Systems of equations are also studied. In the last section the abstract part-metric method in an ordered topological cone is introduced and illustrated on an elliptic system. (efas)

J. von zur Gathen and J. Gerhard, *Modern Computer Algebra*, Cambridge University Press, Cambridge, 1999, 753 pp., £29.95, ISBN 0-521-64176-4

This monograph is devoted to modern algorithms for solving classical algebraic problems. These algorithms form the basis of various contemporary computer algebra systems, and are presented, together with their complexity analysis, with the necessary theoretical background and accompanied by numerous applications. This approach makes it possible for the reader to comprehend even the rather sophisticated recent methods. Readers can also improve their understanding by solving some of the numerous exercises and research problems, following each chapter. The text is divided into five parts, named after the founders of the respective areas. In Part I (Euclid), the Euclidean algorithm, its modular forms, and the subresultant theory for polynomials are examined. Applications include the Chinese Remainder Algorithm, Diophantine approximations, and decoding of BCH codes. Part II (Newton) deals mainly with the fast arithmetic based on the Fast Fourier Transform (FFT). The Karatsuba and Strassen algorithms, as well as fast Chinese remaindering and the fast Euclidean algorithm, are presented. There is a special chapter on the applications of FFT to image compression. Part III (Gauss) deals with polynomial factorisation. Various algorithms are presented in full detail (such as the Zassenhaus one and a polynomial-time algorithm for factorisation in $Q[x]$). The main ingredients are Hensel lifting and short vectors in lattices. Part IV (Fermat) presents the two basic tasks of algorithmic number theory: testing for primality and the factoring of integers. There is a special chapter on applications to cryptography, such as the RSA cryptosystem. Part V (Hilbert) deals with three more advanced topics: the Gröbner bases of ideals in polynomial algebras (including complexity analysis of the Buchberger algorithm), symbolic integration of rational functions, and symbolic summation. As applications, Gröbner proof systems and Petri nets are considered, as well as the determination of all spatial conformations of cyclohexane.

The monograph has an appendix summarising the necessary background mater-

ial from algebra, linear algebra and complexity theory. There is a 30-page list of references, a list of notation and an index. The monograph is a welcome survey of a fast-growing and topical area of modern mathematics and computer science. (jtrl)

E. A. Gavosto, S. G. Krantz and W. McCallum (eds.), *Contemporary Issues in Mathematics Education*, *Mathematical Sciences Research Institute Publications* **36**, Cambridge University Press, Cambridge, 1999, 174 pp., £12.95, ISBN 0-521-65471-8 and 0-521-65255-3

The problem of how to teach mathematics has become extremely important recently. Many things have changed. Undoubtedly, computers have had a serious influence on the teaching process. Further, most of our students major in non-mathematical fields such as biology, medicine, econometrics, technology, etc. To understand their main subject, students must sometimes know some advanced parts of mathematics, such as partial differential equations or the fast Fourier transform. The usual courses of mathematics are limited and the teacher cannot present the necessary background with complete proofs. Even in calculus courses for students of mathematics it may not be possible to include all proofs at the usual level of rigour.

This booklet contains writings from the Conference on the Future of Mathematics Education at Research Universities held in Berkeley in 1996, and is divided into four parts: *Mathematics education at the university*, *Case studies in mathematics education*, *The debate over school mathematics education*, and *Reports from the working groups*. The papers stress the importance of motivation in teaching mathematics (including its historical background) and in the presentation of surveys of advanced topics. It is also important to consult teachers of other subjects; for example, it is quite surprising to learn of the level of interest in mathematics among some biology professors, who say that they never met a differential equation in their research that could be solved analytically. The mathematical community polarises on what is the appropriate balance between theory, technique and applications. This book presents the views of several mathematics instructors and professors on effective mathematics teaching. Some authors include useful examples to demonstrate their ideas.

This book is valuable and can be recommended to mathematicians who are willing to use the experience and advice of their colleagues for improving the teaching process. (ja)

R. E. Gompf and A. I. Stipsicz, *4-Manifolds and Kirby Calculus*, *Graduate Studies in Mathematics* **20**, American Mathematical Society, Providence, 1999, 558 pp., US\$65.00, ISBN 0-8218-0994-6

RECENT BOOKS

ial, and the prerequisites are not too high. A postgraduate student will probably not have to look very often to other sources. The exposition contains many really pretty and interesting examples and a great number of exercises; the final chapter is then devoted to solutions of some of these. I think that this type of presentation makes the subject more attractive and its study easier. We add for young mathematicians that the theory of 4-manifolds has developed very quickly in the last two decades, and can currently be considered as a very promising field of study. (jiva)

A. Hajnal and P. Hamburger, *Set Theory*, London Mathematical Society Student Texts 48, Cambridge University Press, Cambridge, 1999, 316 pp., £16.95, ISBN 0-521-59344-1 and 0-521-59667-X

This textbook on set theory is based on courses given by András Hajnal in Eötvös Loránd University in Budapest. It divides into two parts.

Part I, *Introduction to set theory*, explains the basics in an almost naive way, but with enough rigour to give a proof, not only a claim, of (say) the result that there is no set of all sets. All standard material for an undergraduate course is here. This part ends with cardinal exponentiation, König's theorem and the Hausdorff formula. A 40-page appendix follows, where an axiomatic treatment is developed, with emphasis on the logical background (interpretation of the language, absoluteness of formulas and relative consistency).

Part II, the major part of the book, contains Hajnal's beloved subject, infinitary combinatorics. Stationary sets, Δ -systems, Ramsey's theorem, partition calculus, set mappings, square-bracket arrows, large cardinals, saturated ideals – all these are presented in considerable detail. The culmination is Shelah's theorem on the power $2^{\aleph_1}$. Although this is no longer a text for undergraduates, the clarity of presentation and the choice of the easiest proofs continues. A very fine feature of the book is its exercises. Each chapter contains about ten of them, usually demanding more than a mechanical application of theorems proved in the text. Give this book fifteen minutes' attention – you will find that you must buy it! (ps)

F. Hirsch and G. Lacombe, *Elements of Functional Analysis*, Graduate Texts in Mathematics 192, Springer, New York, 1999, 393 pp., DM98.00, ISBN 0-387-98254-7

This book concerns that part of functional analysis that does not require advanced topological tools and does not cover topics like locally convex spaces or cornerstones of functional analysis (the open mapping and closed graph theorems and the Hahn-Banach theorem). The first part of the book is devoted to function spaces and their duals. It includes chapters on the space of continuous functions on a compact set (the Stone-Weierstrass and Ascoli theorems), on locally compact spaces and Radon measures (Daniell's approach to integration theory, Radon measures), rudiments from the theory of Hilbert spaces

(the Riesz representation theorem, duals of Hilbert spaces, weak convergence and Hilbert bases) and a brief theory of L^p -spaces (also duals, the Radon-Nikodým theorem, weak convergence and convolution). The second chapter is dedicated to operator theory (operators on Banach and Hilbert spaces and their spectra, operational calculus on Hermitian operators and compact self-adjoint operators, Fredholm theorems, kernel operators). A comprehensive final part of the book is an introduction to the theory of distributions. Multiplication, differentiation and convolution of distributions are studied in a fair amount of detail. There are chapters on the fundamental solutions of a differential operator, with classical examples of the Laplacian, the heat and wave operators and the Cauchy-Riemann operator. As applications, several regularity results – in particular, those concerning the Sobolev spaces $W^{1,p}(R^n)$ – are proved. In the final part the Laplacian on an open subset of R^n is studied (the Dirichlet problem, classical and variational formulations, eigenvalues of the Dirichlet Laplacian, and the heat and the wave problems).

The book is equipped with a rich collection of exercises, with notes containing examples and counter-examples, applications to test the reader's understanding of the text, and digressions to introduce new concepts. Answers are presented at the end. It is readable by anybody with a basic knowledge of analysis (differential calculus in several variables, integration theory with respect to a measure, the basics of Banach space theory and the topology of metric spaces). It can be recommended to graduate students and will help more advanced students and researchers improve their knowledge of some parts of functional analysis. (jl)

F. den Hollander, *Large Deviations*, Fields Institute Monographs 14, American Mathematical Society, Providence, 2000, 143 pp., US\$49, ISBN 0-8218-1989-5

The subject of this book is large deviation theory. It corresponds to a graduate course taught by the author at the Fields Institute for Research in Mathematical Sciences in Toronto in 1998. The book divides into two parts.

Part A (Chapters I–V) describes the theory, while Part B (Chapters VI–X) presents applications. Chapters I and II contain the classical basic large deviation theory for i.i.d. random variables. Chapter III presents general definitions and theorems in a more abstract context. Chapter IV contains a discussion of large deviations for Markov chains, while Chapter V deals with large deviations for random sequences with moderate dependence.

A number of interesting applications of the above general theory are presented in Part B; for example, there are applications to statistical hypothesis testing (Chapter VI), random walk environment (Chapter VII), heat conduction with random sources and sinks (Chapter VIII), polymer chains (Chapter IX) and interacting diffusions (Chapter X). There are many exercises,

with solutions at the end of the book.

This is a useful book on large deviations. It can be used as a text for advanced PhD students with a really good background in mathematical analysis and probability theory. (mhusk)

D. H. Hyers, G. Isac and T. M. Rassias, *Stability of Functional Equations in Several Variables*, Progress in Nonlinear Equations and Their Applications 34, Birkhäuser, Boston, 1998, 313 pp., DM178, ISBN 0-8176-4024-X and 3-7643-4024-X

This book presents the current state of solutions of one of Ulam's problems: is an ε -homomorphism of metric groups close to some homomorphism? The affirmative answer for the Cauchy functional equation on a Banach space has been well known for a long time (Hyers, 1941). This question can be extended in various ways – for example, how does the answer depend on the structure of the group (without commutativity the result is not true)?, under what conditions is an ε -homomorphism a homomorphism (so-called superstability) or a homomorphism plus something 'small' (stability modulo ...)? Similar questions can be asked for multiplicative functionals on a Banach algebra, for polynomial functionals on a Banach space ($\Delta_h^n f = 0$), for inequalities instead of equalities (e.g. ε -convex functions), for set-valued maps, etc. Recent results on Ulam's other problems concerning the stability of stationary points or local minimum points are also described.

The authors present both classical and recent results clearly and in a self-contained fashion. Since only a basic knowledge of algebra and functional analysis is required for understanding much of this text, it is accessible for graduate students. (jmil)

V. Ivrii, *Microlocal Analysis and Precise Spectral Asymptotics*, Springer Monographs in Mathematics, Springer, Berlin, 1998, 731 pp., DM178, ISBN 3-540-62780-4

The main topic of this comprehensive monograph (more than 700 pages) concerns asymptotic formulae for the number $N(\lambda)$ of eigenvalues of an operator A , which do not exceed λ , and corresponding precise remainder estimates.

The first part of the monograph develops the necessary tools of micro-local analysis for h pseudo-differential and h -Fourier integral operators, and contains the main theorems on propagation of singularities. The second part contains semiclassical spectral asympt

a huge amount of unpublished new material. The reader is expected to have a good background in the theory of pseudo-differential and Fourier integral operators. (vs)

J. Jost, *Riemannian Geometry and Geometric Analysis*, Universitext, Springer, Berlin, 1998, 455 pp., DM 78, ISBN 3-540-63654-4

This is the second edition of a textbook based on the material of a three-semester course given by the author at the University of Bochum. It includes material needed for the study of geometrical and analytic properties of Riemannian manifolds.

At the beginning is a nice introduction to Riemannian geometry. In the first four chapters, all basic material for the subject is collected and presented in an understandable way. The fifth chapter develops Morse theory and the sixth chapter contains a description of symmetric spaces, offering a rich source of important examples in Riemannian geometry. In the second part of the book, the author returns to the structure of geodesics on Riemannian manifolds using analytical tools (geodesics are treated as critical points of the energy functional). In addition, existence and further properties of harmonic maps between Riemannian manifolds are intensively studied, with special attention to the two-dimensional case.

In this second edition, some new and very interesting material is added. Seiberg-Witten theory has recently found many spectacular applications in geometry and the topology of manifolds. The author first collects basic material for this theory (spin geometry, the Dirac operator), and then treats Seiberg-Witten equations as an analogue of the Ginsberg-Landau equations. Each chapter ends with appendices called 'Perspectives', containing further results and directions for possible research. The book will be very useful for readers from many areas of mathematics. (jbu)

R. Knobel, *An Introduction to the Mathematical Theory of Waves*, Student Mathematical Library 3, American Mathematical Society, Providence, 1999, 196 pp., US\$23, ISBN 0-8218-2039-7

This book provides an introduction to the basic terminology and methods for solving and analysing partial differential equations that describe wave phenomena – travelling and standing waves, the Fourier method, the method of characteristics, shock and rarefaction waves, and the viscosity method for conservation laws. Deeper mathematical concepts are mentioned without proof, but are well illustrated with simple models (such as the superposition principle on the one-dimensional wave equation, and shock waves and viscosity solution on the traffic-flow problem). The derivation of models from physical laws is also given in some cases (the sine-Gordon equation, vibrating string, conservation law). The book also contains elementary computer experiments (such as graphing the analytically obtained solution) and exercises. To understand the text only a

knowledge of calculus is required. This book is based on an undergraduate summer course. It may provide an interesting first reading on high analysis at an elementary level. (efas)

V. Lakshmikantham and S. G. Deo: *Method of Variation of Parameters for Dynamic Systems*, Series in Mathematical Analysis and Applications 1, Gordon and Breach Publishers, New Delhi, 1998, 317 pp., £54, ISBN 90-5699-160-4

Many results on differential equations whose linear parts are perturbed by (small) non-linearities are based on so-called variation of parameters formulae. These formulae can also be extended to certain cases where there are no adequate linear parts. In this way, the method of variation of parameters is closely related to modifications of the Lyapunov second method (Lyapunov-like functions), comparison principles and monotone iteration techniques. This book brings together various aspects of these procedures for ordinary differential, integro-differential, delay, difference, impulse differential, and stochastic differential equations. Applications are given to stability, controllability, periodic solutions, and more generally to boundary value problems. There are also brief sections devoted to abstract differential equations and more special equations. This book partly overlaps other books written by the first author, but has the advantage of collecting similar ideas together. (jml)

J. D. Lamb and D. A. Preece (eds.), *Surveys in Combinatorics, 1999*, London Mathematical Society Lecture Note Series 267, Cambridge University Press, Cambridge, 1999, 298 pp., £24.95, ISBN 0-521-65376-2

This volume contains the nine invited talks presented at the 17th British Combinatorial Conference, held at the University of Kent in 1999. A special feature is W. T. Tutte's article, *The coming of matroids*, which covers the development of the theory of matroids from the point of view of one of the masters.

The remaining eight survey articles deal with graph theory (4), finite geometries (2), probability and complexity (1) and combinatorial designs (1). C. Thomassen surveys parity arguments and problems in graph theory and their connection to the cycle space, and proves a new result on odd K_4 -subdivisions in 4-connected graphs. R. Thomas discusses recent and not-so-recent excluded minor theorems for graphs, connected to linkless embeddings, Hadwiger's conjecture, Pfaffian orientations, and other topics. N. C. Wormald surveys results and methods for random graphs with restricted vertex degrees, stressing results for regular graphs. J. Pach describes some results from geometrical graph theory where edges are represented in the plane by straight-line segments, and considers some generalisations to hypergraphs (here simplices replace segments). S. Ball gives an overview of the recent method in finite geometry where objects are represented by polynomials (problems concern nuclei,

affine blocking sets, maximal arcs, and unitals). K. Metsch's article is on Bose-Burton type theorems for finite projective, affine and polar spaces. M. Dyer and C. Greenhill focus on approximate sampling from combinatorially defined sets and approximate counting using Markov chain Monte-Carlo methods, viewed from the perspective of combinatorial algorithms. C. J. Colbourn, J. H. Dinitz and D. R. Stinson are concerned with applications of combinatorial designs to communication, cryptography and networking; their extensive bibliography has 159 items. (jnes)

G. F. Lawler and C. N. Lester, *Lectures on Contemporary Probability*, Student Mathematical Library 2, American Mathematical Society, Providence, 1999, 95 pp., US\$19, ISBN 0-8218-2029-X

This well-written booklet is based on lectures and computer labs for undergraduates, held by the authors at the IAS/park City Mathematics Institute. The subjects include random walks (simple and self-avoiding), Brownian motion, card shuffling, Markov chains, Markov chain Monte-Carlo, spanning trees, computer simulation of random walks and models in finance. The booklet is divided into 13 chapters (lectures) and includes a number of exercises. The reader is assumed to be familiar with the basic elements of probability. The authors try to present topics that are accessible to advanced undergraduates and to show the appeal of parts of modern probability, and make the lectures very attractive. To avoid long calculations they rely either on intuition or on results of simulations. The emphasis is on a probabilistic style of thinking. (mhusk)

Tan Lei (ed.), *The Mandelbrot Set, Theme and Variations*, London Mathematical Society Lecture Notes Series 274, Cambridge University Press, Cambridge, 2000, 365 pp., ISBN 0-521-77476-4

This volume provides a systematic exposition of current knowledge concerning the Mandelbrot set, and presents the latest research in complex dynamics. Topics discussed include the universality and local connectivity of the Mandelbrot set, parabolic bifurcations, critical circle homeomorphisms, absolutely continuous invariant measures, and matings of polynomials, along with the geometry, dimension and local connectivity of Julia sets. In addition to presenting new work, this collection presents important results hitherto unpublished or difficult to find in the literature.

The book contains a collection of sixteen papers (four in French). The preface presents an excellent historical background of the topic. The papers are grouped according to the properties discussed (universality of the Mandelbrot set, quadratic Julia sets and the Mandelbrot set, Julia set of rational maps, foundational results). The book will be of interest to graduate students in mathematics, physics and mathematical biology, as well as to researchers in dynamical systems and Kleinian groups. (pp)

RECENT BOOKS

H. van Maldeghem: *Generalized Polygons, Monographs in Mathematics* 93, Birkhäuser, Basel, 1998, 502 pp., DM218, ISBN 3-7643-5864-5 and 3-8176-5864-5

This book gives a contemporary survey of results in the theory of generalised polygons. The approach used is mostly geometrical in nature, and several algebraic results are also proved in a geometric way. A *generalised polygon* (the notion is due to Tits) is a geometry (an incidence structure) that generalises the notion of an ordinary polygon. This notion is then related to another combinatorial structure called a *building* and also to more general geometries, such as partial geometries, partial quadrangles, near polygons, etc. Another type of structure used to describe generalised polygons is a finite group of a special type.

Each chapter starts with an introduction, containing motivation and history related to its content. We find a classification and description of the classical and mixed quadrangles, classical hexagons, and non-classical hexagons and octagons. There is a chapter in which the coordinatisation of polygons is studied, and another chapter contains a description of homomorphisms of polygons and their automorphisms groups. Moufang polygons are investigated in great detail in several chapters. The final chapter is devoted to topological polygons and contains results from algebraic topology. The book contains a lot of material and is complements the monograph by Payne and Thas, *Finite Generalized Quadrangles*, Pitman Research Notes in Mathematics **110**, 1984. (jbu)

J. T. McClave and T. Sincich, *Statistics* 8e, Prentice Hall, New Jersey, 2000, 848 pp., £25.99, ISBN 0-13-022329-8 and 0-13-022574-6

This is an undergraduate introductory text in probability and statistics. The chapters devoted to probability contain such classical elementary topics as events, sample spaces, conditional probability, and discrete and continuous random variables. Statistical chapters describe confidence intervals, tests of hypotheses, analysis of variance, linear regression and model building, contingency tables, and non-parametric statistics. The book contains plenty of examples and exercises based on very rich collections of real data. Numerical processing of data is illustrated on the TI-83 Graphic Calculator with references to the well-known statistical software packages SPSS, Minitab and SAS.

This textbook has become very popular. Its eighth edition includes a new chapter on multiple regression, and two other chapters have been rewritten. The authors include relevant recent results, such as the Wilson adjusted confidence interval for a population proportion, which is based on a paper published in 1998. On the other hand, the authors could have mentioned that the Tukey method for multiple comparisons was generalised for unequal sample sizes, since the generalisation is available in many statistical packages and belongs to the most sensitive methods.

This book contains no mathematical derivations of statistical methods, but it helps the reader to understand statistical methodology and carry out necessary calculations. The data used in the book is available on a diskette in ASCII format. There also exist supplements for the instructor and supplements available for purchase by students. (ja)

A. N. Parshin and I. R. Shafarevich (eds.), *Algebraic Geometry V, Encyclopaedia of Mathematical Sciences* 47, Springer, Berlin, 1999, 247 pp., DM159, ISBN 3-540-61468-0

The main topic of this volume is the structure theory of Fano varieties. A smooth projective variety is a *Fano variety* if its anticanonical divisor is ample. Examples of Fano varieties in dimension 2 are the so-called del Pezzo surfaces, and many other examples of Fano varieties can be found here.

In the first part of the book, the basic properties of Fano varieties and results concerning defining equations of varieties related with Fano varieties are presented. The classification problem for algebraic varieties was recently developed thanks to the Mori theory of minimal models and its extension from surfaces to higher-dimensional varieties. A minimal model is defined as a normal projective variety with a numerically effective canonical divisor. The Mori conjecture says that any irreducible algebraic variety over an algebraically closed field of characteristic 0 is birationally equivalent either to a minimal model or to a fibration over a variety of smaller dimension with the general fibre being a Fano variety. This book gives a very good survey of known results on Fano varieties, and also includes contemporary results and possible generalisations. There is a list of open questions and problems. (jbu)

M. S. Petković and L. D. Petković, *Complex Interval Arithmetic and its Applications, Mathematical Research* 105, Wiley-VCH, Berlin, 1999, 284 pp., DM 198, ISBN 3-527-40134-2

Real interval arithmetic was introduced in the 1960s as a tool for handling uncertainties in the data, round-off errors and computations in finite precision arithmetic. Complex interval arithmetic, introduced ten years later, is still much less known. The present book is the first monograph devoted entirely to this subject and its various applications.

The two basic types of complex interval arithmetic (rectangular and circular) are introduced in Chapter 1. In Chapter 2, methods for estimating range of complex functions are studied with help of so-called circular centred forms. Chapter 3 is devoted to best approximations of the range of complex functions by discs; in particular, the functions e^Z , Z^n and $Z^{1/k}$ are studied in detail. A typical application of complex arithmetic is the inclusion of polynomial zeros. This problem is first handled in Chapter 4 for single zeros, using interval slope methods, and its exposition continues in Chapter 5 (simultaneous inclusion

of complex zeros) and Chapter 6 (improved inclusion methods for polynomial complex zeros). Various possibilities for parallel implementations of inclusion methods are presented in Chapter 7. Numerical stability of iterative processes is discussed in Chapter 8, and the book concludes in Chapter 9 with numerical computation of curvilinear integrals. The bibliography has 183 items.

This book demonstrates the usefulness of complex interval arithmetic for solving numerical problems arising in complex analysis. It may be of interest both to specialists and graduate students. In particular, the part of the book that discusses inclusion of polynomial zeros (Chapters 4-6) may be found useful by specialists in control theory. (jroh)

M. Picardello and W. Woess (eds.), *Random Walks and Discrete Potential Theory, Symposia Mathematica* 39, Cambridge University Press, Cambridge, 1999, 361 pp., £16.95, ISBN 0-521-77312-1

This book contains the proceedings of the conference on 'Random walks and discrete potential theory', held in Crotona (Italy) in June 1997. It covers the interplay between the behaviour of a class of stochastic processes (random walks) and structure theory. Written by leading researchers, this collection of invited papers presents links with spectral theory and discrete potential theory, and includes probabilistic and structure theoretic aspects. Its interdisciplinary approach spans several areas of mathematics, including geometrical group theory, discrete geometry and harmonic analysis. The book will be of interest to researchers and postgraduate students, both in mathematics and statistical physics. (pp)

R. C. Read and R. J. Wilson, *An Atlas of Graphs*, Oxford Science Publications, Clarendon Press, Oxford, 1998, 454 pp., £75, ISBN 0-19-853289-X

This is an unusual book, somewhat in the spirit of the *Atlas of finite simple groups*, although on a simpler and less technical level (and in a smaller format). It contains some 10000 pictures of graphs, tables of the number of graphs with given properties, and extensive tables of properties of depicted graphs. Apart from its artistic value it will be of use to graph theory specialists, chemists and combinatorialists in general. It is certainly the most massive amount of graph data in existence.

A small remark: for such an exhaustive atlas perhaps the list of 'special graphs' (listed on pp. 263-265) is somewhat too short. (jnes)

J. Robertson and W. Webb, *Cake-Cutting Algorithms: Be Fair*

piece received.

We can trace the roots of the problem to the Polish group (Banach, Knaster), and the problem was presented by H. Steinhaus in 1947. Since then the cake-cutting problem has blossomed and has been widely popularised. The book also includes notes on other similar problems (dividing beach-front property, the trimming algorithm, dividing an estate, a risk-free bet) while other well-known ones (the problem of the Nil, the problem of similar regions, the bisection problem, the ham sandwich problem, the border problem) are left out.

The purpose of the book is to discuss the cake-cutting problem, to explain the meaning of a 'fair share', and to offer algorithms for its solution. The first few chapters provide a survey of the problem while later ones contain proofs of previously introduced problems. Topics in the book include fairly dividing a cake (two and more persons), pieces or crumbs, how many cuts are needed?, unequal shares, moving knife algorithms, the Stromquist algorithm, disagreement, strong fair division, envy-free division, exact division, applying graph theory, impossibility theorems, Ramsey partitions and fair division. Exercises are included at the end of each chapter. The book is readable by anybody with a basic mathematical knowledge and can be enjoyed by both students and researchers. (jl)

T. Roubíček: *Relaxation in Optimization Theory and Variational Calculus*, de Gruyter Series in Nonlinear Analysis and Applications 4, Walter de Gruyter, Berlin, 1997, 474 pp., DM298, ISBN 3-11-014542-1

Having an integral functional as the argument of 'direct methods' that prove existence of the minimum relies on lower semi-continuity of this functional, which is closely related to convexity properties of the integrand. In many cases, a lack of convexity results in oscillating minimising sequences (or their gradients), whose weak limits are not minimisers of the problem. In the late 1930s, L. C. Young showed that the non-existence of minima of some functionals can be overcome by enlarging the set of competing curves. These 'generalised curves' can be described by means of probability measures, now widely known as 'Young measures' and the procedure (enlarging) known as 'relaxation'.

In Chapter 2 the author deals with an abstract and original theory of suitable convex and (σ) -compact envelopes of topological spaces. This is made specific in Chapter 3 where the main emphasis is on convex σ -compactifications of Lebesgue spaces. It is shown that Young measures and their generalisations can be viewed as special cases of these compactifications, and various topological properties of particular compactifications are studied in detail. The rest of the chapter is mainly devoted to approximation theory and numerical analysis of general convex compactifications, enabling efficient computer implementation of relaxed problems.

Optimal control theory is treated in

Chapter 4, starting with an abstract framework and continuing with optimisation problems with controls on Lebesgue spaces. Much attention is paid to first-order optimality conditions for the relaxed problem and to integral and localised (Pontryagin-type) maximum principles. Four examples are analysed: the optimal control of non-linear dynamical systems, elliptic and parabolic partial differential equations and the optimal control of the Hammerstein integral equation. The treatment of the first example (dynamical systems) also includes its numerical approximation and computational results.

The programme of Chapters 5 and 6 is the relaxation in the variational calculus in the scalar (Chapter 5) and the vectorial case (Chapter 6). The author begins with convex σ -compactifications of Sobolev spaces $W^{1,p}$ and constructs relaxation schemes for the reflexive case (e.g., problems of martensitic microstructure in elasticity, $p > 1$), as well as the non-reflexive case (e.g., the non-parametric minimal surface problem, $p = 1$). Weierstrass-type optimality conditions are derived for the relaxed problem, and its stability, correctness and well-posedness are investigated. The theoretical results for $p > 1$ are accompanied with a numerical approximation theory, including optimality conditions for approximate solutions and sample calculations.

The full strength of convex compactifications is explored in Chapter 7, dealing with the relaxation in game theory, where both convexity and compactness are needed to ensure the existence of solutions. The chapter contains two examples: games with dynamical systems and elliptic games, the former also with some numerical results.

The book, containing many original results, has a large bibliography and commentary, as well as reviews of classical and standard facts from topology, optimisation, functional and mathematical analysis (Chapter 1) which makes it fairly self-contained. It is a comprehensive and useful source of information for researchers and for graduate students working in optimal control theory, the calculus of variations or the optimisation of distributed parameter systems. (mkr)

R. Roussarie, *Bifurcations of Planar Vector Fields and Hilbert's Sixteenth Problem*, Progress in Mathematics 164, Birkhäuser, Basel, 1998, 204 pp., DM118, ISBN 3-7643-5900-5

In this book the author studies bifurcations of limit periodic sets in families X_λ of planar vector fields depending on a parameter λ in R^k , where the phase space is R^2 or in general a surface of genus 0. As an example, polynomial vector fields

$X_\lambda^n(x, y) = P(\lambda, x, y) \delta/\delta x + Q(\lambda, x, y) \delta/\delta y$ with polynomials P, Q of degree $\leq n$ are considered. The problem under consideration is the change of behaviour of a vector field when λ varies X_λ – more precisely, the change of singular elements (singular points and isolated periodic orbits – limit cycles), when λ crosses a bifurcation set

$\Sigma \subset R^k$. The number of limit cycles that bifurcate from a periodic limit set Γ , called the cyclicity of Γ in X_λ , plays a crucial role. The cyclicity depends only on the germ (unfolding) of X_λ along Γ and a general conjecture is that it is finite for any analytic unfolding. Hilbert's 16th problem is associated with the question: 'Prove that for any $n \geq 2$, there exists a finite number $H(n)$ such that any polynomial vector field of degree $\leq n$ has less than $H(n)$ limit cycles'. There is a positive answer to this problem in Chapter 2, but – as the author adds – an algorithm for finding the bound $H(n)$ is unknown.

The author recalls in Chapter 1 the Poincaré-Bendixson theory, and in Chapter 2 studies limit periodic sets and their cyclicity, concentrating on quadratic vector fields. The single vector fields are studied in Chapter 3, using the desingularisation theory. In Chapter 4 regular limit cycles, in Chapter 5 elementary graphics, and in Chapter 6 non-elementary graphics, are considered and studied by methods of differential and analytic geometry, using the Bautin ideal and asymptotic methods. (jeis)

I. P. Stavroulakis and S. A. Tersian, *Partial Differential Equations. An Introduction with Mathematica and Maple*, World Scientific, Singapore, 1999, 297 pp., £23, ISBN 9-8102-3891-6

This book provides an elementary introduction to the classical theory of first and second-order partial differential equations. Linear problems are studied and the explicit formulae for problems with homogeneous and non-homogeneous boundary conditions are provided. The contents of the book coincide, for example, with the first part of the monograph by L. Ewans, *Partial Differential Equations*, AMS, 1998 (where, however, a more detailed discussion and comments are given); the separate chapters of the book can also be found in other (cited) textbooks. Here, perhaps, more convenient examples for students is given.

Although the book has a promising subtitle, the usage of *Mathematica* consists only of drawing graphs (of insufficient

groups of a manifold (with values in R) can be represented by means of the de Rham complex and harmonic forms. Similar facts for relative cohomology groups are less standard. This book offers a nice systematic treatment of these questions from an analytic point of view. The main tools used are some PDE techniques, with an additional insight given by interpretation in terms of Brownian motion and probability theory. The first four chapters are preparatory and contain basic facts needed from differential geometry, functional analysis, Markov processes, Fourier integral operators and spaces of currents. The calculus of pseudo-differential operators is used in the main part of the book for a treatment of interior elliptic boundary value problems and for proofs of the main index theorems. The book is well organised and is written in an understandable way with many intuitive explanations and remarks. (vs)

K. Thomsen, *Limits of Certain Subhomogeneous C^* -Algebras*, *Mémoires de la Société Mathématique de France* 71, Société Mathématique de France, Paris, 1997, 125 pp., ISBN 2-85629-064-7

The main topic of this booklet is the structure of a certain type of simple unital C^* -algebras. Typical C^* -algebras considered here are algebras of matrix-valued continuous maps defined on the interval $[0, 1]$ or on the unit circle T , with special properties in a finite sequence of distinguished points as well as inductive limits of a sequence of finite direct sums of such algebras. The main result is a classification theorem for a special type of such algebras and its extension to the non-unital case. Proofs of the main theorems use several different previously developed methods and are quite technical. The book is suitable for specialists in the field. (vs)

J. Winkelmann, *Complex Analytic Geometry of Complex Parallelizable Manifolds*, *Mémoires de la Société Mathématique de France* 72-73, Société Mathématique de France, Paris, 1998, 219 pp., ISBN 2-85629-070-1

The main subject of this book is the study of complex parallelisable manifolds – complex manifolds that are quotients of complex Lie groups by discrete subgroups. Special attention is paid to quotients by discrete subgroups that are cocompact or at least of finite covolume. As a special case, arithmetic groups are studied and a criterion for cocompactness of arithmetic subgroups is presented.

The corresponding lattices in complex Lie groups are discussed and their basic properties are presented. The special cases studied here are complex tori, and it is proved that they are the only quotients admitting a Kähler metric. Complex analytic subspaces of quotients and their Kodaira dimension are discussed. Properties of holomorphic mappings and automorphism groups of parallelisable manifolds are investigated; it seems that holomorphic mappings are often automatically equivariant.

An important part of the book is devoted to the study of homogeneous vector bundles. Some conditions for their flatness are proved, and the classification of flat bundles over parallelisable manifold is given. The author also treats deformations of compact parallelisable manifolds and the structure of the related cohomology group. As an important example, complex nilmanifolds are studied and it is shown that they are always built from complex tori admitting a certain kind of complex multiplication. The author also presents algebraic groups for which any invariant holomorphic function invariant with respect to Zariski dense discrete subgroups is constant.

This book can be recommended to all readers interested in the topics covered. (jbu)

W. Woess, *Random Walks on Infinite Graphs and Groups*, *Cambridge Tracts in Mathematics and its Applications* 78, Cambridge University Press, Cambridge, 2000, 334 pp., £40, ISBN 0-521-55292-3

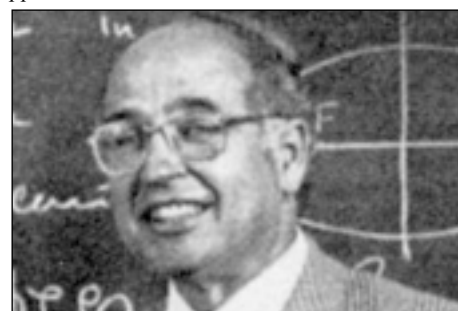
The topic of random walks is situated somewhere between probability, potential theory, harmonic analysis, geometry, graph theory and algebra. The beauty of the subject stems from linking, both in the way of thinking and in the methods employed, of different fields. The main theme of this book is the interplay between the behaviour of a class of stochastic processes (random walks) and discrete structure theory. The author considers Markov chains whose state space is equipped with the structure of an infinite locally finite graph, or as a particular case, of a finitely generated group. The transition probabilities are assumed to be adapted to the underlying structure in some way that must be specified precisely in each case.

From a probabilistic viewpoint, the question is what impact the particular type of structure has on various aspects of the behaviour of the random walk. Conversely, random walks may also be seen as a useful tool for classifying, or at least describing, the structure of graphs and groups. Links with spectral theory and discrete potential theory are also discussed.

This book will be essential reading for all researchers in stochastic processes and related topics. According to the author, this book is *not* self-contained, by the nature of its topic. Another question may be whether it is usable by graduate students. Here let us quote the author: 'It has been my experience that usually, this must be taken with caution, and is mostly true only in the presence of a guiding hand that is acquainted with the topic. Proofs are sometimes a bit condensed, and it may be that even students above the student level will need pen and paper when they want to work through them seriously – in particular because of the variety of different methods and techniques that I have tried to unite in this text. This does not mean, however, that parts of this book could not be used for graduate or even undergraduate courses.' (jant)

Book Review by Sir Michael Atiyah

Conversation on Mind, Matter and Mathematics by Jean-Pierre Changeux and Alain Connes edited and translated by M B De Bevoise Princeton University Press, 1995 260 pp., £19.95, ISBN: 0 691 08759 8



Does mathematics have an existence independent of our physical world? Do mathematicians discover theorems, rather than invent them? Such questions have exercised the minds of philosophers and mathematicians since the time of Plato, and many books have addressed the issues. What is unusual and topical about this book is that it records a dialogue between a mathematician, Alain Connes, and a biologist, Jean-Pierre Changeux. Each is an international authority in his field and both are Professors at the Collège de France, so that they speak from first-hand experience and the intellectual level is high. As a neuro-physiologist Changeux is particularly interested in the structure and organisation of the brain. He is intrigued by the capacity of the brain to handle the kind of sophisticated intellectual abstractions represented by mathematics, and his dialogue with Connes is an attempt to get to grips with this problem.

Most mathematicians would agree that their subject has evolved from the traditional disciplines of arithmetic and geometry. We have built an elaborate superstructure on these twin foundations and so the nature of 'mathematical reality' can be adequately dealt with by posing basic questions about the integers 1, 2, 3 ... and about the Euclidean geometry of triangles, circles, etc. Connes is an unabashed Platonist. For him there is an 'archaic mathematical reality' represented by objects such as circles or integers, which has an existence, independent of experience or of the human mind. One practical consequence of such a belief, already adopted by NASA, is that mathematics provides the best form of potential communication with any extra-terrestrial intelligence.

Any mathematician must sympathize with Connes. We all feel that integers, or circles, really exist in some abstract sense and the Platonist view is extremely seductive. But can we really defend it? Had the universe been one-dimensional or even discrete it is difficult to see how geometry could have evolved. It might seem that with the integers we are on firmer ground, and that counting is really a primordial notion. But let us imagine that intelligence had resided, not in mankind, but in some

vast solitary and isolated jelly-fish, buried deep in the depths of the Pacific Ocean. It would have no experience of individual objects, only with the surrounding water. Motion, temperature and pressure would provide its basic sensory data. In such a pure continuum the discrete would not arise and there would be nothing to count.

Even more fundamentally, in a purely static universe without the notion of time, causality would disappear and with it that of logical implication and of mathematical proof. Connes actually alludes to this philosophical dilemma in the context of relativistic cosmology.

It may be argued that such "gedanken universes" are not to be taken seriously. Our actual universe is a given datum and the inevitable background of all intelligent discussion. But this is tantamount to conceding that mathematics has evolved from the human experience. Man has created mathematics by idealising or abstracting elements of the physical world. The number 2 for example represents the common attribute of all pairs of objects that we have encountered, in the same way as the word "chair" represents what is common to all the individual items of furniture that we sit on. Admittedly chairs are not so precisely and unambiguously defined as numbers: is a three-legged stool a chair? Language is inherently more fuzzy or open-ended than mathematics, but mathematics can properly be viewed as a special kind of language. This applies not only to the nouns or objects but also to the verbs or processes such as addition and subtraction.

Connes will protest that there are thousands of different languages reflecting the history and culture of a particular people, whereas mathematics is universal and unique. Surely this gives it a special status? This can be debated at length, but a short answer is that the diversity of languages disguises a fundamental structural similarity, which is why dictionaries help us to translate. Moreover different mathematical notations, such as the Roman numeral system, do exist but western civilisation has produced a uniformity which was not inevitable.

In describing mathematics as a language it is important to emphasise that a language is not merely a set of words and grammatical rules for producing coherent sentences. Words mean something and they relate to our experience. In a similar way a mathematical statement has a meaning and one which, I would argue, rests ultimately on our experience.

For a Platonist like Connes mathematics lives in some ideal world. I find this a difficult notion to grasp and prefer to say more pragmatically that mathematics lives in the collective consciousness of mankind. I am not here embracing any obscure metaphysical notion, but merely observing that there are two essential components of mathematics. In the first place it deals with concepts and abstract processes which live in the conscious mind of the individual mathematician. Secondly, it must be communicable to other mathematicians. The famous Indian genius Ramanujan fre-

quently produced marvellous formulae by some unknown mental process which could neither be described nor repeated. I think Connes would accept that such formulae do not become an accepted part of mathematics until others have understood them and verified their validity.

Our collective consciousness may not be sufficiently ideal for Connes but it is the world where all ideas live, not just mathematical ones.

Where does this point of view leave the dichotomy between creation or discovery in mathematics? By resisting the embrace of the Platonic world have we lost the possibility of making "discoveries"? Is every theorem man-made? Not at all. Since our concepts are based on the physical world we can discover facts about these concepts experimentally. For example the formula

$$3 \times 5 = 5 \times 3$$

can be discovered by setting out 15 objects in a rectangular array. One can then move on to the more general algebraic formula

$$a \times b = b \times a,$$

where a and b represent arbitrary integers. In a similar way the famous formula of Pythagoras

$$a^2 + b^2 = c^2$$

relating the length of the sides of any right-angled triangle was no doubt discovered experimentally. So man creates the concepts of mathematics but he discovers the subsequent connections between them. The reason he can have it both ways is that mathematics is firmly rooted in the real world.

The legally minded reader may object that I have slurred the difference between an empirical discovery, always concerned with the specific, and a proof, concerned with the general. I plead guilty, but only on a technicality and I hope the judge will be lenient. Sometimes a discovery will carry with it the seeds of a formal proof, as with the example that $3 \times 5 = 5 \times 3$. Sometimes one has to work harder as Pythagoras had to do. But if possession is nine-tenths of the law, discovery is nine-tenths of the proof.

In his dialogue with Connes, Changeux keeps hitting the Platonist rock. As a hard-headed experimental scientist Changeux wants to identify mathematics with what actually goes on in the brain. For him this is the only reality and the only place where mathematics exists. Connes disputes this extreme attitude and prefers to say that mathematical reality (which exists elsewhere) is reflected in the neurological processes of the brain. To confuse the two is like identifying a piece of literature or music with the ink and paper on which it is recorded. It is hard to disagree, but fascinating questions remain.

The brain is frequently compared to an electronic computer and artificial intelligence is now a branch of computer science. But, as the dialogue brings out, this analogy has serious limitations and may in fact be obscuring some of the essential features. As Connes notes, the speed of transmission of signals in the brain is vastly less than the corresponding speeds in modern computers. As a result computers are much better

and faster than humans at certain kinds of calculation, but in most important respects they are still a long way behind. For example, computers are not yet serious contributors to mathematical theory. Perhaps by analysing the structure of mathematics we may learn something about the way the brain operates. In particular a key feature of mathematics is its hierarchical nature. Examples of patients with specific brain damage show that particular levels of reasoning are associated with definite parts of the brain.

For Changeux, the comparison between the structure of mathematics and the structure of the brain can be looked at both from the evolutionary aspect and in terms of function. In evolutionary terms, the brain has had to create a hierarchy of levels that adequately reflect the physical environment and the challenges it poses. Man has been the ultimate winner of the evolutionary process and his brain has the structure needed to produce mathematics. Would a different neurological solution have led to a different kind of mathematics, or does mathematics depend only on the functional capacity of the brain, not on its biological mechanism?

If one views the brain in its evolutionary context then the mysterious success of mathematics in the physical sciences is at least partially explained. The brain evolved in order to deal with the physical world, so it should not be too surprising that it has developed a language, mathematics, that is well suited for the purpose. The sceptic can point out that the struggle for survival only requires us to cope with physical phenomena at the human scale, yet mathematical theory appears to deal successfully with all scales from the atomic to the galactic. Perhaps the explanation lies in the abstract hierarchical nature of mathematics which enables us to move up and down the world scale with comparative ease.

The internal workings of the mathematical mind were well described by Jaques Hadamard who distinguished three stages in attacking a problem: preparation, incubation and illumination. What these correspond to neurologically is a challenging question which generates a lively discussion between Connes and Changeux. Is there for example a Darwinian element in the search for successful ideas? Henri Poincaré argued that the subconscious mind generates random ideas and "illumination" occurs when one of these is selected.

Personal Column

We list below information about some appointments, awards and deaths that have occurred during late 1999 and early 2000. Since this list is inevitably incomplete, we invite you to send appropriate information from other countries to Barbara Maenhaut [b.m.maenhaut@open.ac.uk] or to your Country representative (see Issue 34) for inclusion in the next issue. Please also send any items you feel should be included in future Personal Columns.

Appointments

Paul Garthwaite (Aberdeen) and **Chris Jones** have been appointed Professors in the Statistics Department at the Open University, and **Pia Larsen**, **Karen Vines** and **Catriona Queen** have been appointed Lecturers.

Robert Hunt (Cambridge) has been appointed Deputy Director of the Isaac Newton Institute for Mathematical Sciences.

Robert MacKay and **Andrew Stuart** have been appointed Professors at Warwick University; **John Rawnsley** and **Colin Rourke** have been promoted to Personal Chairs; **Claude Baesens**, **Peter Topping** and **Balasz Szendroi** have been appointed Lecturers.

Awards

Luigi Ambrosio (Scuola Normale Superiore of Pisa) was awarded the Renato Caccioppoli Prize (1998) by the Italian Mathematical Union for his contributions to the Calculus of Variations and Partial Differential Equations.

F. Bethuel and **F. Helein** have been awarded the 1999 Fermat Prize in Mathematics for 'several important contributions to the theory of variational calculus which have consequences in physics and geometry.'

The French Académie des Sciences has awarded special mathematical prizes to **Laurent Clozel** (Orsay), **Pierre Colmez** (Paris), **Sylvestre Gallot** (Grenoble), **Laurent Manivel** (Saint Martin d'Hères) and **Wandelin Werner** (Orsay).

Yves Colin de Verdière (Grenoble) has been awarded the Prix Ampère de l'Electricité de France by the French Académie des Sciences, for fundamental contributions to spectral theory.

Joachim Cuntz (Münster) was awarded the 1999 Gottfried Leibniz Prize, given by the Deutsche Forschungsgemeinschaft, for work in functional analysis.

Riccardo De Arcangelis (Napoli) was awarded the Calogero Vinti

Prize (1998) by the Italian Mathematical Union in recognition of his theoretical research in the Calculus of Variations.

Persi Diaconis (Stanford) was elected to hold the first UK Hardy Fellowship from May-August 2001, at Queen Mary and Westfield College, London.

Matthias R. Gaberdiel has been awarded a Royal Society University Research Fellowship to work in the Department of Applied Mathematics and Theoretical Physics, Cambridge.

Mikhail Gromov (Paris and New York) was awarded the 1999 Balzan Prize for Mathematics, awarded by the Fondazione Internazionale Premio E. Balzan.

Alice Guionnet (Orsay) has received the 1999 Oberwolfach Prize for outstanding research in stochastics.

Nicholas J. Higham (Manchester) has been awarded a Royal Society Senior Research Fellowship.

Tom Hoeholdt (Technical University of Denmark) has been elected an IEEE Fellow, from 1 January 2000, for "fundamental contributions to the theory, analysis and decoding algorithms of algebraic geometry codes".

Peter Hydon (Guildford, Surrey) has been promoted to Reader in mathematics.

Kurt Johansson (Royal Institute of Technology, Stockholm) and **David Wilson** (Microsoft Research) have been awarded the Rollo Davidson Prizes for 2000.

Marco Manetti (University of Roma "La Sapienza") has been awarded the Giuseppe Bartolozzi Prize (1999) by the Italian Mathematical Union for his contributions to the theory of algebraic surfaces.

Bernard Maurey (Paris) has been awarded a State Prize by the French Académie des Sciences for fundamental work on Banach spaces.

Alun Morris (Aberystwyth) received an OBE and **Andrew Wiles** FRS (Princeton) received a knighthood in the British New Years Honours List. **Sir Roger Penrose** FRS (Oxford) has been appointed to the Order of Merit.

Stefan Müller (Leipzig) and **Dieter Lüst** (Potsdam) have been awarded the 1999 Gottfried Leibniz Prize, awarded by the Deutsche Forschungsgemeinschaft, for work in functional analysis.

Erich Novak (Erlangen-Nürnberg) is the first winner of the Prize for Achievement in Information-Based Complexity.

Juan-Pablo Ortega and **Tudor Ratiu** (Lausanne) have been awarded the 1999 *Ferran Sunyer i Balaguer Prize* by the Institut d'Estudis Catalans for their monograph entitled *Hamiltonian Singular Reduction*. The awards will be presented on 10 July during the *3ecm* in Barcelona.

James C. Robinson has been awarded a Royal Society University Research Fellowship to work in the Mathematics Institute, Warwick.

Guido Sanguinetti (Genova) was awarded the Franco Tricerri Prize (1999) by the Italian Mathematical Union for his doctoral work in differential geometry.

Jean-Pierre Serre (Paris) has been awarded the 2000 Wolf Prize in Mathematics, for his many fundamental contributions to topology, algebraic geometry, algebra and number theory; he shares this prize with **Raoul Bott**.

Volker Strassen (Konstanz) has been awarded the 1999 Georg Cantor Medal by the Deutsche Mathematiker Vereinigung.

Alain-Sol Sznitman (Zürich) has been awarded the 1999 Line and Michel Loève International Prize in Probability.

John Toland, FRS (Bath) has been awarded an honorary degree from

POSTDOCTORAL POSITIONS IN MATHEMATICS IN THE UNIVERSITY OF AVEIRO

The Research unit Mathematics and its Applications of the University of Aveiro accepts applications for post-doctoral research positions in the area of Functional Analysis until June 30th, 2000. Each position has a duration of at most one year and shall start no sooner than September 1st, 2000, at a date convenient for both parties.

The areas of research are

- Non-Standard Methods in Analysis and Topology;
- Function Spaces on Irregular Domains;
- Convolution Type Operators and Diffraction Theory.

The successful applicant is supposed to work as a member of one of the teams of the Group of Functional Analysis in one or more of these topics.

Candidates should have a PhD – preferably obtained within the last 5 years – in an area relevant to at least one of the subjects mentioned above and shall be selected on the basis of the following documents:

1. a letter of intent,
2. curriculum vitae,
3. official proof of PhD award,
4. copy of two research works
(preferably in Portuguese, English, French or Spanish)
5. copy of passport.

The above documents must be sent, by registered mail, to
Secretaria do Departamento de Matemática
(c/o Prof. António Caetano)
Universidade de Aveiro
3810-193 AVEIRO
Portugal

For more information, please contact acaetano@mat.ua.pt