ZESZYTY NAUKOWE UNIWERSYTETU JAGIELLOŃSKIEGO

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I. Kotlarski

The generalized Laplace convolution

Let f_0, f_1, f_2 be three real functions, locally integrable over $[0, \infty)$. Their generalized convolution is defined as

$$(*) \qquad (f_0, f_1, f_2)(u_1, u_2) = \int_0^{\min(u_1,u_2)} f_0(t) f_1(u_1-t) f_2(u_2-t)\, dt\,, \quad u_1 \geqslant 0\,,\ u_2 \geqslant 0\,.$$

The following theorem is established. Let f_0, f_1, f_2 be real functions defined on $[0, \infty)$, non equivalent to zero, for which the integrals $\int_0^\infty e^{-st}|f_k(t)|\,dt\,(\mathrm{Re}\, s > s_k)$ converge for some real s_k, $(k = 0, 1, 2)$.

Then the generalized convolution $(*)$ determines all the functions f_0, f_1, f_2 up to a set of zero measure, up to a shift and up to non-zero constant factors.