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The determination of the homomorphisms of some representation of the group L_2^2

Let us consider a group G and its two arbitrary representations

$$(1) \qquad \bar{x} = f(x, a)\,, \quad x, \bar{x} \in X\,, \quad a \in G\,,$$

$$(2) \qquad \bar{y} = g(y, a)\,, \quad y, \bar{y} \in Y\,, \quad a \in G\,.$$

A function $\varphi\colon X \to Y$ of the property that

$$(3) \qquad \varphi\big(f(x, a)\big) = g\big(\varphi(x), a\big)\,, \quad x \in X\,, \quad a \in G$$

is named the homomorphism of the algebraic structure (X, f) to the structure (Y, g).

The problem is to solve equation (3) with the following representation $(\alpha, \beta, \gamma, \delta, \ldots, k = 1, 2)$

$$a \in G \Leftrightarrow a = (L^\alpha_\beta, L^\alpha_{\beta\gamma}) \in L^2_2\,,$$

$$x \in X \Leftrightarrow x = (\underset{k}{x_\alpha}, \underset{k}{x_{\alpha\beta}}) \in R^{12} \text{ and } \det \|\underset{k}{x_\alpha}\| \neq 0$$

$$(4) \qquad \bar{x} = f(x, a) \Leftrightarrow \begin{cases} \underset{k}{\bar{x}_\alpha} = L^\gamma_\alpha \underset{k}{x_\gamma} \\ \underset{k}{\bar{x}_{\alpha\beta}} = L^\gamma_{\alpha\beta} \underset{k}{x_\gamma} + L^\gamma_\alpha L^\delta_\beta \underset{k}{x_{\gamma\delta}} \end{cases} \quad x, \bar{x} \in X\,, \quad L \in L^2_2\,.$$

L^2_2 denotes the differential group of order 2 in two dimensional space. The function g may be arbitrary.

By the above assumptions equation (3) takes the form

$$(5) \qquad \varphi(\underset{k}{\bar{x}_\alpha}, \underset{k}{\bar{x}_{\alpha\beta}}) = g\big(\varphi(\underset{k}{x_\alpha}, \underset{k}{x_{\alpha\beta}}), L^\alpha_\beta, L^\alpha_{\beta\gamma}\big)\,.$$

A function φ, which satisfies (3), may be also called the „generalized homogeneous function" with respect to the functions f and g (see [1]). In [1] there is given a method of solving the functional equations of the type (3). The equation (5) will be solved here by using this method (see also [2]).

The domains of transitivity of points $x \in X$ with respect to the representation (4) are 10-dimensional surfaces in 12-dimensional space $R^{12} \supset X$. One of the „generators" of the familly of these surfaces is of the form

$$(6) \qquad \begin{cases} \underset{k}{x}_\gamma = \underset{k}{\delta}_\gamma , \\ \underset{k}{x}_{\gamma\delta} = \delta^1_\gamma \delta^2_\delta \underset{k}{\delta}^\varkappa T_\varkappa , \qquad T = (T_1, T_2) \in R^2 . \end{cases}$$

If we put the right-hand-side expressions of (6) in the formulae (4) instead of x, then we obtain the one-to-one transformation of the form

$$(7) \qquad \begin{cases} \underset{k}{\overline{x}}_\alpha = L^\gamma_\alpha \underset{k}{\delta}_\gamma , \\ \underset{k}{\overline{x}}_{\alpha\beta} = L^\gamma_{\alpha\beta} \underset{k}{\delta}_\gamma + L^\gamma_\alpha L^\delta_\beta \delta^1_\gamma \delta^2_\delta \underset{k}{\delta}^\varkappa T_\varkappa , \end{cases} \qquad L \in L^2_2 , \ T \in R^2 , \ \overline{x} \in X .$$

The inverse transformation of (7) is the following

$$(8) \qquad \begin{cases} T_a = \frac{1}{\Delta}[(\underset{2}{\overline{x}}_{12} - \underset{2}{\overline{x}}_{21}) \delta^2_a - (\underset{1}{\overline{x}}_{12} - \underset{1}{\overline{x}}_{21}) \delta^1_a] , \qquad \Delta = \det \|\underset{k}{\overline{x}}_a\| , \\ L^a_\beta = \delta^k_\beta \underset{k}{\overline{x}}_a , \\ L^a_{\beta\gamma} = (\underset{k}{\overline{x}}_{\beta\gamma} - \underset{1}{\overline{x}}_{\beta\gamma} \underset{2}{\overline{x}}_{\beta\gamma} T_k) \delta^{ka} . \end{cases}$$

Now we put the right-hand-side expressions of (6) in (5) instead of x and we obtain

$$(9) \qquad \varphi(\underset{k}{\overline{x}}_a , \underset{k}{\overline{x}}_{a\beta}) = g\big(\Phi(T_a), L^a_\beta , L^a_{\beta\gamma}\big)$$

where

$$\Phi(T_a) \overset{df}{=} \varphi(\underset{k}{\delta}_a , \delta^1_\gamma \delta^2_\delta \underset{k}{\delta}^\varkappa T_\varkappa) .$$

If we were to put the right-hand-side expressions of (8) in (9) instead of $T_a, L^a_\beta, L^a_{\beta\gamma}$, then the formula for the general solution of the equation (5) would be obtained. The function Φ: $R^2 \to Y$ may be arbitrary.

REFERENCES

[1] S. Topa, *On a generalization of homogeneous functions*, Publ. Math. Debrecen, 13 (1966), 1-4, 289-300.

[2] S. Topa, *Determination of differential concomitants of the first class of a pair of covariant vectors in a two-dimensional space*, Ann. Polon. Math. XIX (1967), 337-341.