

BOOK REVIEWS

N. Balakrishnan (ed.): ADVANCES IN COMBINATORIAL METHODS AND APPLICATIONS TO PROBABILITY AND STATISTICS. Statistics for Industry and Technology, Birkhäuser, Boston, 1997, xxxiv+562 pages, ISBN 0-8176-3908-X, DM 178,-.

The volume under review, dedicated to Professor Sri Gopal Mohanty, comprises thirty two invited papers selected, as the Editor says in his Preface, so as to review some of the recent developments or to highlight new noteworthy results in the area of combinatorial methods in probability and statistics, to which field S.G. Mohanty has contributed substantially. The book opens with a short biography of Sri Gopal Mohanty and a list of his publications. The articles are organized in seven sections whose titles may indicate the contents of the book: Lattice paths and combinatorial methods, Applications to probability problems, Applications to urn models, Applications to queueing theory, Applications to waiting time problems, Applications to distribution theory, and Applications to nonparametric statistics. The book is amended with detailed author and subject indices.

Ivo Vrkoč, Praha

Pablo Pedregal: PARAMETRIZED MEASURES AND VARIATIONAL PRINCIPLES. Progress in Nonlinear Differential Equations and Their Applications, Vol. 30, Birkhäuser, Basel, 1997, pp. xi+212, ISBN 3-7643-5697-9, DM 98,-.

The book presents a modern approach, based on a deep analysis of Young measures, to problems arising in the calculus of variations and the optimization theory.

The work mainly deals with the weak lower semicontinuity and relaxation of integral functionals. Examples of such functionals are shown in Chapter 2. In Chapters 3 and 4 the author summarizes achievements on the existence of solutions to variational problems under some convexity assumptions and on the relaxation (i.e. some suitable extension) of those problems if they fail to be (quasi)convex.

In Chapter 5 there are two examples of continuum mechanics problems whose solutions lead to the minimization of integral energy functionals which are not sequentially weakly lower semicontinuous. A scalar problem is related to the micromagnetic theory and the vectorial one to phase transitions in nonlinear elasticity.

Chapters 6 and 7 are devoted to an analysis of parametrized (Young) measures and to their relationship to weak, biting and strong convergence of sequences in Lebesgue spaces. An existence theorem for Young measures as well as conditions allowing us to represent weak limits of L^1 -bounded sequences in terms of Young measures are given.

Chapters 8 and 9 analyze parametrized measures generated by sequences of gradients. These measures play a key role in the relaxation of vectorial variational problems. It is shown that the characterization of gradient Young measures is closely related to the notion of quasiconvexity which, up to some growth requirements, represents the necessary and sufficient condition ensuring sequential weak lower semicontinuity of the vectorial variational tasks discussed.

The author studies in detail a subset of gradient Young measures called laminates. A nice construction is made in order to prove that there are gradient parametrized measures which are not laminates.

Finally, the last chapter discusses divergence free Young measures generated by sequences bounded in L^∞ . In particular, it is proved that any Young measure generated by an L^∞ -bounded sequence is also generated by a sequence, say, $\{m_k\}$ such that $\{\operatorname{div} m_k\}$ is compact in $H_{\operatorname{loc}}^{-1}$. This result has an immediate application in the relaxation in micromagnetics.

The book provides a fairly self-contained and interesting presentation of recent results in the calculus of variations and is a useful and rich source of information for researchers and graduate students in mathematical analysis, applied mathematics and material science.

Martin Kružík, Praha

J. F. Jardine: GENERALIZED ÉTALE COHOMOLOGY THEORIES. Progress in Mathematics 146, Birkhäuser Verlag, 1997, ISBN 0-8176-5494-1 (Boston), ISBN 3-7643-5494-1 (Basel), DM 118,-.

The book is a compendium leading to the proof of Thomason's étale cohomological descent theorem for Bott periodic K -theory. The level of generality of the exposition is that of a generalized étale cohomology theory on an étale site for an algebraic variety; étale K -theory and the standard étale cohomology are examples of such a theory.

The first three chapters are devoted to homotopy theory of spectra. In fact, the author works in the category of n -fold spectra, studies various diagonalization functors and related smash products, and analyzes closed model structures on this category.

After presenting some auxiliary results (Chapter 4), the author introduces, in Chapter 5, the K -theory presheaf of spectra. The main technical problem here is that K -theory is not functorial on schemes, but arises rather from a pseudo-functor which must be analyzed with supercoherence theory.

Chapter 6 is devoted to generalized étale cohomology. It contains also a 'naïve' set theoretic technique for replacing big geometric étale sites by small ones, for the purpose of computing étale cohomology.

The final chapter combines the methods developed in the book to a proof of Thomason's theorem and also of some related results, as the Nisnevich descent theorem. Also the Lichtenbaum-Quillen conjecture—an approximation to Thomason's theorem without Bott periodicity—is discussed.

The book is addressed to students and researchers interested in algebraic K -theory and related fields. It assumes some preliminary knowledge of stable homotopy theory and algebraic geometry.

Martin Markl, Praha

H. Triebel: FRACTALS AND SPECTRA RELATED TO FOURIER ANALYSIS AND FUNCTION SPACES. Monographs in Mathematics, Vol. 91, Birkhäuser Verlag, Basel, 1997, viii+271 pp., ISBN 3-7643-5776-2 (Basel), 0-8176-5776-2 (Boston), DM 148,-.

The book is devoted to a study of those aspects of fractal geometry in $\mathbb{R}_s p n$, which are connected to the Fourier analysis, function spaces, and pseudodifferential operators. In an earlier book [D. E. Edmunds and H. Triebel, Function spaces, entropy numbers, differential operators, Cambridge, 1996] the authors successfully applied estimates of entropy numbers of compact embeddings between function spaces to the spectral theory of degenerate pseudodifferential operators on bounded domains and on \mathbb{R}^n . A good part of the book under review is based on similar techniques, but this time in the context of fractals.

The exposition departs from some basic material on fractals with special emphasis on the d -sets. One of the central aims of the book is to introduce and study function spaces on d -sets. Let Γ be a d -set. The $L^p(\Gamma)$ spaces are relatively easy to define since the measure on

Γ is more or less uniquely determined, but their structure and relations to other function spaces are very complicated. This together with the introduction and study of the $B_{p,q}^s(\Gamma)$ spaces is treated in detail in Chapter 4, and needs a lot of deep preliminary material. This is contained in Chapters 2 and 3, and includes entropy numbers on weighted ℓ_p spaces with a dyadic block structure, and a new approach to the atomic decomposition of spaces $B_{p,q}^s$ and $F_{p,q}^s$ on \mathbb{R}^n , consisting in further atomization of the atoms, which results in subatomic (or quarkonial) decomposition. A thorough study of asymptotic behaviour of entropy numbers of embedding between these function spaces is carried out next. It is needless to say that there is virtually no literature on this topic and hence most of the material presented is published here for the first time.

The final Chapter 5 deals with spectra of pseudodifferential operators with fractal coefficients. On suitable function spaces, these operators are compact, and estimates for distribution of their eigenvalues and counting function can be obtained. Particular attention is paid to n -dimensional drums with a compact fractal layer.

Luboš Pick, Praha

A. I. Saichev, W. A. Woyczynski: DISTRIBUTIONS IN THE PHYSICAL AND ENGINEERING SCIENCES—VOLUME 1: DISTRIBUTIONAL AND FRACTAL CALCULUS, INTEGRAL TRANSFORMS AND WAVELETS. Birkhäuser, Basel, 1997, 346 pages, DM 78,—.

The book is a modern version of a graduate course for physical sciences and engineering which is usually labeled *Advanced Mathematics for Engineers and Scientists*. It is written from the unifying viewpoint of distribution theory and enriched by such modern topics as wavelets, nonlinear phenomena and white noise theory, which have become very important in the practice of physical scientists. Major topics included in the book are split in two parts: Part 1: *Distributions and their basic physical applications*, containing the basic formalism and generic examples, and Part 2: *Integral transforms and divergent series*, which contains chapters on Fourier, Hilbert and wavelet transforms and an analysis of the uncertainty principle, divergent series and singular integrals.

The book is intended for graduate students and researchers in applied mathematics, physical sciences and engineering. For reading it, no special prerequisites are necessary: some basic definitions and facts which are needed as elements on differential equations, Fourier series, complex variables and linear algebra are reviewed in the text. In solving some problems, familiarity with basic computer programming methods is necessary—mastering a symbolic manipulation language such as *Mathematica*, *MATLAB* or *Maple* would suffice. Even though the book is **not** addressed to pure mathematicians who plan to pursue the research in distribution theory the exposition is mathematically rigorous, results are proved and assumptions are formulated explicitly and in such a way that the resulting proofs are as simple as possible.

Finally, I would like to express my opinion that the way in which the book is written may help to improve communication between applied scientists on the one hand, and mathematicians on the other. The first group is often only vaguely aware of the modern mathematical tools that can be applied to physical problems, while the latter is often ignorant of how physicists and engineers reason about their problems and how they adapt pure mathematical theories to become effective tools.

Emil Vitásek, Praha

I. Kuzin, S. Pohozaev: ENTIRE SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATIONS. Progress in Nonlinear Differential Equations and Their Applications 33, Birkhäuser, Basel, 1997, 250 pages, ISBN 3-7643-5323-6, DM 148,-.

The book is devoted to the study of elliptic semilinear partial differential equations of the type $-\Delta u + f(x, u) = 0$ in N space dimensions with a function f of appropriate regularity. It is not intended to be a monograph but includes a representative part of the subject characterizing the up-to-date state of the theory of this important problem.

As illustrated in the introduction, many models of mathematical physics lead to the above semilinear equation. It is used for example in the investigation of standing wave type solutions to nonlinear Schrödinger equation, travelling waves for a nonlinear Klein-Gordon equation, etc. The presentation is accessible to all audience with rudiments of classical functional analysis, function spaces theory and basics of partial differential equations.

After a necessary list of definitions, Chapter 1 starts with the classical variational method in an abstract form and a simple application of the classical monotonicity method to an elliptic problem on R^N . Next, in Chapter 2, more complicated so-called noncoercive problems with an eigenvalue as a parameter are considered. A series of abstract theorems is proved which are then applied to some elliptic problems. Also, a first example of the application of the so-called *concentration compactness principle*, introduced by P. L. Lions, is presented. In Chapter 3, especially noncoercive problems are treated by such advanced methods as the well known *mountain pass theorem* of Ambrosetti and Rabinowitz and, again, the concentration compactness method. Chapter 4 is devoted to the problems reducible to ordinary differential equations such as the problem with radial symmetry or just the problem in one space variable. Then the attention is paid to autonomous equations with the demonstration of the simplest ideas of phase plane and dynamic systems application to the problem in question. Finally, Chapter 5 exhibits other modern methods: use of *upper and lower solutions*, illustration of the *Leray-Schauder method* applied to the specific problems arising in R^N , *fibering method* in an abstract form making it possible, in particular, to prove the existence of infinitely many solutions to the corresponding elliptic problems, and also provides examples of nonexistence theorems. The authors also list methods that could not be included in the book with detailed references.

The ideas presented in the book are easy to follow and with respect to its remarkable contents it should be recommended to specialists in partial differential equations, PhD students as well as to post doctoral courses.

Ivan Straškraba, Praha

J.-M. Muller: ELEMENTARY FUNCTIONS—ALGORITHMS AND IMPLEMENTATION. Birkhäuser, Basel, 1997, 220 pages, DM 118,-.

The book under review deals with the problem how to compute the elementary functions (as sine, cosine, exponentials, logarithms etc.) on a modern computer quickly and accurately. It gives the concepts and background necessary to understand and build algorithms for computing these functions, presenting and structuring the algorithms (hardware-oriented as well as software-oriented), and discusses issues related to the accurate floating-point implementation. The purpose is not to give “cookbook recipes” that allow one to implement some given functions, but to provide the reader with the knowledge that is necessary to build, or adapt, algorithms to their specific computing environment.

The book is intended for two different audiences: *specialists*, who have to design floating-point systems (hardware or software parts) or to do research on algorithms, and *inquiring minds*, who just want to know what kind of methods are used to compute mathematical functions in current computers or pocket calculators. Because of this, it will be helpful

for postgraduate and advanced undergraduate students in computer science or applied mathematics as well as for professionals engaged in the design of algorithms, programs or circuits that implement floating-point arithmetic, or simply for engineers or scientists who want to improve their culture in that domain. Much of the book can be understood with only a basic grounding in computer science and mathematics. Moreover, the basic notions from computer arithmetic that are necessary to understand are recalled in the first chapter.

In the conclusion, let us briefly sketch the contents of the book. After the preliminary chapter mentioned above the main part of the book is divided into three major parts. The first part consists of two chapters and is devoted to algorithms using polynomial or rational approximations of elementary functions and, possibly, tables. The second part consists of three chapters, and deals with „shift-and-add“ algorithms, i.e. hardware-oriented algorithms that use additions and shifts only. The last part consists of three chapters. It discusses issues that are important when accuracy is the major goal.

Emil Vitásek, Praha

D. Alpay, A. Dijkma, J. Rovnyak, H. de Snoo: SCHUR FUNCTIONS, OPERATOR COLLIGATIONS, AND REPRODUCING KERNEL PONTRYAGIN SPACES. Operator Theory: Advances and Applications, vol.96. Birkhäuser, Basel, 1997, 248 pages, hardcover, ISBN 3-7643-5763-0, DM 148,-.

A Schur function is a function holomorphic in the unit disc and bounded by one there; equivalently, it is a holomorphic function in the unit disc such that a certain kernel associated to it is positive semidefinite. These functions have received considerable attention in the past, both on their own merit and due to their prominent occurrence in approximation theory, invariant subspace theory and application areas. It is safe to say that they are very well understood nowadays, and a lot is also known about their operator-valued generalizations, when their values are contractions from one Hilbert space into another. The main goal of this monograph is to develop the theory of operator-valued Schur functions in the indefinite inner product space setting. The indefiniteness can even enter in two ways: either the values are permitted to be operators acting between two indefinite inner product (Krein or Pontryagin) spaces, or the associated kernels are allowed to have a certain finite number of negative squares. The authors use an elegant systems-theoretic approach, based on viewing the generalized Schur functions as the characteristic functions of operator colligations (isometric, coisometric or unitary) whose state spaces are reproducing kernel Pontryagin spaces; this makes it possible to give a unified treatment of both the Hilbert space and the indefinite inner product space case. The necessary background material on Krein spaces, reproducing kernels, etc. is also included to make the book self-contained. The presentation is very nice and the book will make an excellent reading for anyone interested in the area.

Miroslav Engliš, Praha

C. E. D'Attelis, Elena M. Fernández-Berdaquer (eds.): WAVELET THEORY AND HARMONIC ANALYSIS IN APPLIED SCIENCES. Birkhäuser, Boston 1997, 358 pages, ISBN 3-7643-3953-5, DM 158,-.

The book appeared in the series Applied and Numerical Harmonic Analysis. It is a collection of 14 chapters by different authors that cover the theory, computation, and applications in the field of wavelet and discrete Fourier analysis, and interactions among these topics.

The subject is grouped in three parts: Theory and implementation, Applications to biomedical sciences, and Applications in physical sciences.

Part 1 is aimed rather at the recent theoretical development than at an elementary exposition to wavelet theory. Part 2 presents applications to ECG, EEG, and cardiorespiratory signal analysis. Part 3 is concerned with wavelet networks for modeling nonlinear processes, semiconductor device modeling, estimating wave attenuation and dispersion in rocks, and solving the Maxwell equations in magnetotellurics. Most chapters of applied character provide the reader also with numerical results for the problems solved, and tables and figures.

The book is interesting for a wide interdisciplinary readership who work in applied mathematics, electrical engineering, biomedical research, physics, and geophysics. It brings new ideas and techniques that can be applied in many branches of science.

Karel Segeth, Praha

Abraham Boyarski, Paweł Góra: LAWS OF CHAOS. Invariant measures and dynamical systems in one dimension. Probability and Its Applications, Birkhäuser, Boston, 1997, xv+399 pages, ISBN 0-8176-4003-7, DM 128,-.

Recently, several excellent books dealing with one-dimensional dynamical systems have appeared (see, for example, the monograph *One-dimensional dynamics* by W. de Melo and S. van Strien, Springer-Verlag, 1993). The book under review, however, differs from them noticeably: First, it focuses on a particular yet sufficiently deep topic, namely, on dynamical systems defined by piecewise monotonic mappings of an interval into itself. The ergodic theoretic point of view is adopted, results and methods connected with existence of absolutely continuous invariant measures being stressed. Second, the Laws of Chaos are intended as a rather elementary textbook. The reader is assumed to have only moderate preliminary knowledge of real analysis and measure theory, proofs are given in detail, many examples are provided, and every chapter is amended with problems. (Sometimes also very important results, as is Lasota-Yorke's method of lower functions, are deferred to exercises. The reader, however, may find detailed solutions to selected problems at the end of the book.)

Let me describe briefly the contents of the book. In Chapters 2 and 3, basic notions and results of measure and ergodic theory which are needed in the sequel are recalled, with a particular emphasis paid to nonsingular transformations. In the fourth chapter, the Frobenius-Perron operator that plays a basic role in the authors' approach is investigated. Several theorems on existence of absolutely continuous invariant measures, including the famous one due to A. Lasota and J. Yorke, are presented in Chapters 5 and 6, while in the next chapter the quasi-compactness of the Frobenius-Perron operator and its consequences are discussed. Chapter 8 is devoted to the properties of invariant measures: for example, supports of invariant measures and smoothness of their densities are studied and a central limit theorem is established. These chapters may be viewed as the core of the book, more special topics being treated in the remaining parts (Markov transformations in Chapter 9, approximation of invariant densities in Chapter 10, stability of invariant measures with respect to deterministic and stochastic perturbations of the transformation in Chapter 11 and the inverse problem for the invariant density in Chapter 12). Finally, various applications of the theory developed to both mathematical and physical problems are discussed in the last, thirteenth chapter.

Both authors have contributed considerably to the ergodic theory of one-dimensional dynamical systems and their book is a valuable source of information on this interesting topic.

Jan Seidler, Praha