Internat. J. Math. & Math. Sci.
Vol. 8 No. 1 (1985) 69-74

ON LOCALLY CONFORMAL KÄHLER SPACE FORMS

KOJI MATSUMOTO

Department of Mathematics
Faculty of Education
Yamagata University
Yamagata, 990, Japan

(Received May 29, 1984)

ABSTRACT. An m-dimensional locally conformal Kähler manifold (l.c.K-manifold) is characterized as a Hermitian manifold admitting a global closed 1-form α_λ (called the Lee form) whose structure $(F_\mu{}^\lambda, g_{\mu\lambda})$ satisfies

$$\nabla_\nu F_{\mu\lambda} = -\beta_\mu g_{\nu\lambda} + \beta_\lambda g_{\nu\mu} - \alpha_\mu F_{\nu\lambda} + \alpha_\lambda F_{\nu\mu},$$

where ∇_λ denotes the covariant differentiation with respect to the Hermitian metric $g_{\mu\lambda}$, $\beta_\lambda = -F_\lambda{}^\varepsilon \alpha_\varepsilon$, $F_{\mu\lambda} = F_\mu{}^\varepsilon g_{\varepsilon\lambda}$ and the indices $\nu, \mu, \ldots, \lambda$ run over the range $1, 2, \ldots, m$.

For l.c.K-manifolds, I.Vaisman [4] gave a typical example and T.Kashiwada ([1], [2],[3]) gave a lot of interesting properties about such manifolds.

In this paper, we shall study certain properties of l.c.K-space forms. In §2, we shall mainly get the necessary and sufficient condition that an l.c.K-space form is an Einstein one and the Riemannian curvature tensor with respect to $g_{\mu\lambda}$ will be expressed without the tensor field $P_{\mu\lambda}$. In §3, we shall get the necessary and sufficient condition that the length of the Lee form is constant and the sufficient condition that a compact l.c.K-space form becomes a complex space form. In the last §4, we shall prove that there does not exist a non-trivial recurrent l.c.K-space form.

KEY WORDS & PHRASES: L.c.K-manifolds, Lee form, l.c.K-space forms, hybrid, recurrent l.c.K-space form.
1980 MATHEMATICS SUBJECT CLASSIFICATION CODE. 53B25

1. INTRODUCTION.

This paper is directed to specialist readers with background in the area and appreciative of its relation of this area of study.

Let $M(F_\mu{}^\lambda, g_{\mu\lambda}, \alpha_\lambda)$ be an l.c.K-manifold. Then, by the definition, at any point of M there exists a neighborhood in which a conformal metric $g^* = e^{-2\rho} g$ is a Kähler one, i.e.,

$$\nabla^*_\nu (e^{-2\rho} F_{\mu\lambda}) = 0, \qquad d\rho = \alpha,$$

where ∇^*_λ denotes the covariant differentiation with respect to g^*. Then we have

$$\nabla_\nu F_{\mu\lambda} = -\alpha_\mu F_{\nu\lambda} + \alpha^\varepsilon F_{\varepsilon\lambda} g_{\nu\mu} + \alpha_\lambda F_{\nu\mu} + \alpha^\varepsilon F_{\mu\varepsilon} g_{\nu\lambda}. \tag{1.1}$$

The following proposition was proved by T.Kashiwada [1]

PROPOSITION 1.1. A Hermitian manifold $M(F_\mu{}^\lambda, g_{\mu\lambda})$ is an l.c.K-manifold if and only if there exists a global closed 1-form α_λ satisfying (1.1).

In an l.c.K-manifold M, we define a tensor field $P_{\mu\lambda}$ as follows;

$$P_{\mu\lambda} = -\nabla_\mu \alpha_\lambda - \alpha_\mu \alpha_\lambda + \frac{1}{2}\|\alpha\|^2 g_{\mu\lambda}, \tag{1.2}$$

where $\|\alpha\|$ denotes the length of the Lee form α_λ with respect to $g_{\mu\lambda}$.

In an m-dimensional l.c.K-manifold M, we know the following formula;

$$R_{\mu\varepsilon} F_\lambda{}^\varepsilon + R_{\lambda\varepsilon} F_\mu{}^\varepsilon - (m-2)(P_{\mu\varepsilon} F_\lambda{}^\varepsilon + P_{\lambda\varepsilon} F_\mu{}^\varepsilon) = 0, \tag{1.3}$$

where $R_{\mu\lambda}$ denotes the Ricci tensor with respect to $g_{\mu\lambda}$ [1]. Thus we have

PROPOSITION 1.2. In an m-dimensional ($m \neq 2$) l.c.K-manifold M, the tensor field $P_{\mu\lambda}$ is hybrid, i.e.,

$$P_{\mu\varepsilon} F_\lambda{}^\varepsilon + P_{\lambda\varepsilon} F_\mu{}^\varepsilon = 0, \tag{1.4}$$

if and only if the Ricci tensor $R_{\mu\lambda}$ is hybrid.

From now on in this paper, we assume that the tensor field $P_{\mu\lambda}$ is hybrid.

REMARK. In an m-dimensional ($m \neq 2$) Einstein l.c.K-manifold, the tensor field $P_{\mu\lambda}$ is hybrid, identically.

An l.c.K-manifold M is called an l.c.K-space form if the holomorphic sectional curvature of the section $\{X, FX\}$ at each point of M has the constant value. Let $M(H)$ be an l.c.K-space form with constant holomorphic sectional curvature H. Then the Riemannian curvature tensor $R_{\omega\nu\mu\lambda}$ with respect to $g_{\mu\lambda}$ can be written as

$$\begin{aligned} 4R_{\omega\nu\mu\lambda} &= H(g_{\omega\lambda} g_{\nu\mu} - g_{\omega\mu} g_{\nu\lambda} + F_{\omega\lambda} F_{\nu\mu} - F_{\omega\mu} F_{\nu\lambda} - 2F_{\omega\nu} F_{\mu\lambda}) + 3(P_{\omega\lambda} g_{\nu\mu} - P_{\omega\mu} g_{\nu\lambda} \\ &+ g_{\omega\lambda} P_{\nu\mu} - g_{\omega\mu} P_{\nu\lambda}) - \{\tilde{P}_{\omega\lambda} F_{\nu\mu} - \tilde{P}_{\omega\mu} F_{\nu\lambda} + F_{\omega\lambda} \tilde{P}_{\nu\mu} - F_{\omega\mu} \tilde{P}_{\nu\lambda} - 2(\tilde{P}_{\omega\nu} F_{\mu\lambda} \\ &+ F_{\omega\nu} \tilde{P}_{\mu\lambda})\}, \end{aligned} \tag{1.5}$$

where $\tilde{P}_{\mu\lambda} = P_\mu{}^\varepsilon F_{\varepsilon\lambda}$ [1].

2. L.C.K-SPACE FORMS.

In this section, we shall consider the necessary and sufficient condition that an l.c.K-space form becomes an Einstein one. Next, we shall get an expression of the Riemannian curvature $R_{\omega\nu\mu\lambda}$ that does not include the tensor field $P_{\mu\lambda}$.

Let $M(H)$ ba an m-dimensional l.c.K-space form with constant holomorphic sectional curvature H. Then we have (1.5). Transvecting (1.5) with $g^{\omega\lambda}$, we have from the straightfoward calculation

$$4R_{\mu\lambda} = \{(m+2)H + 3P\} g_{\mu\lambda} + 3(m-4) P_{\mu\lambda}, \tag{2.1}$$

where $P = P_{\mu\lambda} g^{\mu\lambda}$ and it can be written as

$$P = -\nabla_\varepsilon \alpha^\varepsilon + \frac{1}{2}(m-2)\|\alpha\|^2. \tag{2.2}$$

Thus we have

PROPOSITION 2.1. A 4-dimensional l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is an Einstein one and then the scalar field P is constant.

We have from (2.2) and the Green's theorem [5]

PROPOSITION 2.2. A compact m-dimensional l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid has a non-negative P.

Next, we shall prove the following ;

THEOREM 2.3. An m-dimensional ($m \neq 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is an Einstein one if and only if the tensor field $P_{\mu\lambda}$ is proportional to $g_{\mu\lambda}$.

PROOF. If the tensor field $P_{\mu\lambda}$ is proportional to $g_{\mu\lambda}$, then the tensor field $P_{\mu\lambda}$ can be written as

$$P_{\mu\lambda} = \frac{P}{m} g_{\mu\lambda}. \tag{2.3}$$

Thus we have from (2.1) and (2.3)

$$R_{\mu\lambda} = \{(m+2)H + \frac{6(m-2)}{m}P\}g_{\mu\lambda}.$$

The inverse is trivial, so we omit its proof.

COROLLARY 2.4. An m-dimensional ($m \neq 4$) Einstein l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is a complex space form if $P = 0$.

Transvecting (2.1) with $g^{\mu\lambda}$, we have

$$4R = m(m+2)H + 6(m-2)P, \tag{2.4}$$

where R denotes the scalar curvature with respect to $g_{\mu\lambda}$. By virtue of (2.1) and (2.4), we can easily see that

$$3P_{\mu\lambda} = \frac{4}{m-4}R_{\mu\lambda} - \frac{(m-4)(m+2)H + 4R}{2(m-2)(m-4)}g_{\mu\lambda}, \tag{2.5}$$

$$\tilde{P}_{\mu\lambda} = \frac{4}{3(m-4)}\tilde{R}_{\mu\lambda} - \frac{(m-4)(m+2)H + 4R}{6(m-2)(m-4)}F_{\mu\lambda}, \tag{2.6}$$

where $\tilde{R}_{\mu\lambda} = R_{\mu}{}^{\varepsilon}F_{\varepsilon\lambda}$. Substituting (2.5) and (2.6) into (1.5), we obtain

$$\begin{aligned} R_{\omega\nu\mu\lambda} = &- \frac{(m-4)H + R}{(m-2)(m-4)}(g_{\omega\lambda}g_{\nu\mu} - g_{\omega\mu}g_{\nu\lambda}) + \frac{(m-4)(m-1)H + R}{3(m-2)(m-4)}(F_{\omega\lambda}F_{\nu\mu} \\ &- F_{\omega\mu}F_{\nu\lambda} - 2F_{\omega\nu}F_{\mu\lambda}) + \frac{1}{(m-4)}(R_{\omega\lambda}g_{\nu\mu} - R_{\omega\mu}g_{\nu\lambda} + g_{\omega\lambda}R_{\nu\mu} - g_{\omega\mu}R_{\nu\lambda}) \\ &+ \frac{1}{3(m-4)}\{\tilde{R}_{\omega\lambda}F_{\nu\mu} - \tilde{R}_{\omega\mu}F_{\nu\lambda} + F_{\omega\lambda}\tilde{R}_{\nu\mu} - F_{\omega\mu}\tilde{R}_{\nu\lambda} - 2(\tilde{R}_{\omega\nu}F_{\mu\lambda} + F_{\omega\nu}\tilde{R}_{\mu\lambda})\}. \end{aligned} \tag{2.7}$$

Thus we have

PROPOSITION 2.5. In an m-dimensional ($m \neq 2,4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid, the Riemannian curvature tensor $R_{\omega\nu\mu\lambda}$ can be written as (2.7) without $P_{\mu\lambda}$.

3. COMPACT L.C.K-SPACE FORMS.

In this section, we shall mainly deal with compact l.c.K-space form.

Let $M(H)$ be an m-dimensional l.c.K-space form with constant holomorphic sectional curvature H. If we assume that the scalar curvature R is constant, then by virtue of (2.4) all of the scalar fields R,H and P are constant. Under this assumption, differentiating (2.1) covariantly, we get

$$4\nabla_{\omega}R_{\nu\mu} = 3(m-4)\nabla_{\omega}P_{\nu\mu}. \tag{3.1}$$

Substituting (1.2) into the above equation, we have

$$4\nabla_{\omega}R_{\nu\mu} = 3(m-4)\{-\nabla_{\omega}\nabla_{\nu}\alpha_{\mu} - (\nabla_{\omega}\alpha_{\nu})\alpha_{\mu} - \alpha_{\nu}\nabla_{\omega}\alpha_{\mu} + \frac{1}{2}(\nabla_{\omega}\|\alpha\|^2)g_{\nu\mu}\}. \tag{3.2}$$

By virtue of the Ricci identity [5] and the assumption $\nabla_{\mu}\alpha_{\lambda} = \nabla_{\lambda}\alpha_{\mu}$, the equation (3.2) implies

$$4(\nabla_\omega R_{\nu\mu} - \nabla_\nu R_{\omega\mu}) = 3(m-4)\{R_{\omega\nu\mu}{}^{\varepsilon}\alpha_\varepsilon + \alpha_\omega(\nabla_\nu\alpha_\mu) - \alpha_\nu(\nabla_\omega\alpha_\mu)$$
$$+ \frac{1}{2}(\nabla_\omega\|\alpha\|^2 g_{\nu\mu} - \nabla_\nu\|\alpha\|^2 g_{\omega\mu})\}.$$

Transvecting the above equation with $g^{\nu\mu}$ and taking account of the formula $2\nabla_\varepsilon R_\lambda{}^\varepsilon = \nabla_\lambda R$ [5], we obtain

$$R_\omega{}^\varepsilon\alpha_\varepsilon + (\nabla_\varepsilon\alpha^\varepsilon)\alpha_\omega + \frac{1}{2}(m-2)\nabla_\omega\|\alpha\|^2 = 0. \tag{3.3}$$

Substituting (2.1) into (3.3), we obtain

$$\{(m+2)H + 3\|\alpha\|^2 + \nabla_\varepsilon\alpha^\varepsilon\}\alpha_\omega + \frac{m-4}{2}\nabla_\omega\|\alpha\|^2 = 0. \tag{3.4}$$

Thus we have

THEOREM 3.1. In an m-dimensional ($m \neq 2,4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid and the scalar curvature R is constant, the length $\|\alpha\|$ of the Lee form α_λ is non-zero constant if and only if

$$(m+2)H + 3\|\alpha\|^2 + \nabla_\varepsilon\alpha^\varepsilon = 0. \tag{3.5}$$

By virtue of (3.5) and the Green's theorem, we have

COROLLARY 3.2. In a compact m-dimensional ($m \neq 2,4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid and the scalar curvature R is constant, if the length $\|\alpha\|$ of the Lee form α_λ is non-zero constant, then there exists the following relation between the holomorphic sectional curvature H and the length $\|\alpha\|$ of the Lee form α_λ;

$$(m+2)H + 3\|\alpha\|^2 = 0. \tag{3.6}$$

COROLLARY 3.3. There does not exist a compact m-dimensional ($m \neq 2,4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid and the holomorphic sectional curvature H is positive if the length $\|\alpha\|$ of the Lee form α_λ and the scalar curvature R are constant. Especially, if $H = 0$, then the manifold M must be locally Euclidean, that is, the Riemannian curvature tensor $R_{\omega\nu\mu\lambda}$ is identically zero.

The following proposition was proved by T.Kashiwada [1];

PROPOSITION 3.4. In a compact m-dimensional ($m \neq 2$) l.c.K-manifold M, if

$$\tilde{H}_\varepsilon{}^\varepsilon - R \geqslant 0 \tag{3.7}$$

holds good, then the manifold M is a Kähler manifold, where $\tilde{H}_{\mu\lambda} = \frac{1}{2}R_\mu{}^\varepsilon{}_{\delta\gamma}F^{\delta\gamma}F_{\varepsilon\lambda}$. The inequality $\geqslant$ in this case is naturally reduced to $=$.

Now, let $M(H)$ be a compact m-dimensional ($m \neq 2,4$) l.c.K-space form. Then transvecting (2.5) with $F^{\omega\nu}F^{\mu\lambda}$, we get

$$\frac{1}{2}R_{\omega\nu\mu\lambda}F^{\omega\nu}F^{\mu\lambda} = \frac{-m(m+2)H + R}{3}. \tag{3.8}$$

By virtue of (2.4) and (3.8), we obtain

$$H_\varepsilon{}^\varepsilon - R = \frac{m(m+2)H - 4R}{3}. \tag{3.9}$$

Thus we have from PROPOSITION 3.4 and (3.9)

THEOREM 3.5. In a compact m-dimensional ($m \neq 2,4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid, if the inequlity $m(m+2)H \geqslant 4R$ holds good, then the manifold M is a complex space form.

4. RECURRENT L.C.K-SPACE FORMS.

A Riemannian manifold M is said to be recurrent if the Riemannian curvature tensor

$R_{\omega\nu\mu\lambda}$ satisfies

$$\nabla_\kappa R_{\omega\nu\mu\lambda} = \theta_\kappa R_{\omega\nu\mu\lambda} \tag{4.1}$$

for a certain non-zero vector field θ_κ. For a recurrent Riemannian manifold, it is trivial that

$$\nabla_\nu R_{\mu\lambda} = \theta_\nu R_{\mu\lambda}, \qquad \nabla_\lambda R = \theta_\lambda R. \tag{4.2}$$

Now, let $M(H)$ be an m-dimensional ($m \neq 2,4$) recurrent l.c.K-space form which the tensor field $P_{\mu\lambda}$ is hybrid. Then we have (2.7) and (4.1). Differentiating (2.7) covariantly and taking account of (4.1) and (4.2), we have

$$\begin{aligned}
&\frac{H}{m-2}\theta_\kappa(g_{\omega\lambda}g_{\nu\mu} - g_{\omega\mu}g_{\nu\lambda}) - \frac{(m-1)H}{3(m-2)}\theta_\kappa(F_{\omega\lambda}F_{\nu\mu} - F_{\omega\mu}F_{\nu\lambda} - 2F_{\omega\nu}F_{\mu\lambda})\\
&+ \frac{(m-4)(m-1)H + R}{3(m-2)(m-4)}\{(g_{\kappa\mu}F_{\nu\lambda} - g_{\kappa\lambda}F_{\nu\mu} + 2g_{\kappa\nu}F_{\mu\lambda})\beta_\omega + (g_{\kappa\lambda}F_{\omega\mu} - g_{\kappa\mu}F_{\omega\lambda}\\
&- 2g_{\kappa\omega}F_{\mu\lambda})\beta_\nu + (g_{\kappa\nu}F_{\omega\lambda} - g_{\kappa\omega}F_{\nu\lambda} + 2g_{\kappa\lambda}F_{\omega\nu})\beta_\mu + (g_{\kappa\omega}F_{\nu\mu} - g_{\kappa\nu}F_{\omega\mu} - 2g_{\kappa\mu}F_{\omega\nu})\beta_\lambda\\
&+ (F_{\kappa\mu}F_{\nu\lambda} - F_{\kappa\lambda}F_{\nu\mu} + 2F_{\kappa\nu}F_{\mu\lambda})\alpha_\omega + (F_{\kappa\lambda}F_{\omega\mu} - F_{\kappa\mu}F_{\omega\lambda} - 2F_{\kappa\omega}F_{\mu\lambda})\alpha_\nu\\
&+ (F_{\kappa\nu}F_{\omega\lambda} - F_{\kappa\omega}F_{\nu\lambda} + 2F_{\kappa\lambda}F_{\kappa\nu})\alpha_\mu + (F_{\kappa\omega}F_{\nu\mu} - F_{\kappa\nu}F_{\omega\mu} - 2F_{\omega\nu}F_{\kappa\mu})\alpha_\lambda\}\\
&- \frac{1}{3(m-4)}[\{R_\omega{}^\varepsilon g_{\kappa\mu}F_{\nu\lambda} - R_\omega{}^\varepsilon g_{\kappa\lambda}F_{\nu\mu} - R_\nu{}^\varepsilon g_{\kappa\mu}F_{\omega\lambda} + R_\nu{}^\varepsilon g_{\kappa\lambda}F_{\omega\mu} + 2(R_\omega{}^\varepsilon g_{\kappa\nu}F_{\mu\lambda}\\
&+ R_\mu{}^\varepsilon g_{\kappa\lambda}F_{\omega\mu})\}\beta_\varepsilon + \{R_\omega{}^\varepsilon F_{\kappa\mu}F_{\nu\lambda} - R_\omega{}^\varepsilon F_{\kappa\lambda}F_{\nu\mu} - R_\nu{}^\varepsilon F_{\kappa\mu}F_{\omega\lambda} + R_\nu{}^\varepsilon F_{\kappa\lambda}F_{\omega\mu}\\
&+ 2(R_\omega F_{\kappa\nu}F_{\mu\lambda} + R_\mu F_{\kappa\lambda}F_{\omega\nu})\}\alpha_\varepsilon + (g_{\kappa\mu}\tilde{R}_{\nu\lambda} - g_{\kappa\lambda}\tilde{R}_{\nu\mu} + 2g_{\kappa\nu}\tilde{R}_{\mu\lambda})\beta_\omega + \{g_{\kappa\lambda}\tilde{R}_{\omega\mu}\\
&- g_{\kappa\mu}\tilde{R}_{\omega\lambda} - 2(F_{\mu\lambda}R_{\omega\kappa} + g_{\kappa\omega}\tilde{R}_{\mu\lambda})\}\beta_\nu + (g_{\kappa\nu}\tilde{R}_{\omega\lambda} - F_{\nu\lambda}R_{\omega\kappa} + F_{\omega\lambda}R_{\kappa\nu} - g_{\kappa\omega}\tilde{R}_{\nu\lambda}\\
&+ 2g_{\kappa\lambda}\tilde{R}_{\omega\nu})\beta_\mu + \{F_{\nu\mu}R_{\omega\kappa} - g_{\kappa\nu}\tilde{R}_{\omega\mu} + g_{\kappa\omega}\tilde{R}_{\nu\mu} - F_{\omega\mu}R_{\kappa\nu} - 2(g_{\omega\nu}\tilde{R}_{\kappa\mu} + F_{\omega\nu}R_{\kappa\mu})\}\beta_\lambda\\
&+ (F_{\kappa\mu}\tilde{R}_{\nu\lambda} - F_{\kappa\lambda}\tilde{R}_{\nu\mu} + 2F_{\kappa\nu}\tilde{R}_{\mu\lambda})\alpha_\omega + \{F_{\kappa\lambda}\tilde{R}_{\omega\mu} - F_{\kappa\mu}\tilde{R}_{\omega\lambda} - 2(F_{\mu\lambda}\tilde{R}_{\kappa\omega} + F_{\kappa\omega}\tilde{R}_{\mu\lambda})\}\alpha_\nu\\
&+ (F_{\kappa\nu}\tilde{R}_{\omega\lambda} - F_{\nu\lambda}\tilde{R}_{\kappa\omega} + F_{\omega\lambda}\tilde{R}_{\kappa\nu} - F_{\kappa\omega}\tilde{R}_{\nu\lambda} + 2F_{\kappa\lambda}\tilde{R}_{\omega\nu})\alpha_\mu + \{F_{\nu\mu}\tilde{R}_{\kappa\omega} - F_{\kappa\nu}\tilde{R}_{\omega\mu}\\
&+ F_{\kappa\omega}\tilde{R}_{\nu\mu} - F_{\omega\mu}\tilde{R}_{\kappa\nu} - 2(F_{\kappa\mu}\tilde{R}_{\omega\nu} + F_{\omega\nu}\tilde{R}_{\kappa\mu})\}\alpha_\lambda] = 0.
\end{aligned} \tag{4.3}$$

Transvecting (4.3) with $F^{\omega\lambda}$, we get

$$\begin{aligned}
&\frac{(m+2)H}{3}\theta_\kappa F_{\nu\mu} = \frac{(m+2)\{(m-4)(m-1)H + R\}}{3(m-4)(m-2)}(g_{\kappa\nu}\beta_\mu - g_{\kappa\mu}\beta_\nu - F_{\kappa\mu}\alpha_\nu + F_{\kappa\nu}\alpha_\mu)\\
&\frac{1}{3(m-4)}[\{(m-1)R_\nu{}^\varepsilon F_{\kappa\mu} - 5R_\mu{}^\varepsilon F_{\kappa\nu}\}\alpha_\varepsilon + \{(m-1)R_\nu{}^\varepsilon g_{\kappa\mu} - 5R_\mu{}^\varepsilon g_{\kappa\nu}\}\beta_\varepsilon\\
&+ (RF_{\kappa\mu} + 5R_{\kappa\mu})\alpha_\nu - \{RF_{\kappa\nu} + (m-1)R_{\kappa\nu}\}\alpha_\mu + (Rg_{\kappa\mu} + 5R_{\kappa\mu})\beta_\nu\\
&- \{Rg_{\kappa\nu} + (m-1)R_{\kappa\nu}\}\beta_\mu].
\end{aligned}$$

From this, we obtain

$$H\theta_\kappa = 0. \tag{4.4}$$

Thus we have

THEOREM 4.1. An m-dimensional ($m \neq 2,4$) recurrent l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is trivial, that is, the manifold is locally symmetric or of zero holomorphic sectional curvature.

Let $M(H)$ be a 4-dimensional recurrent l.c.K-space form. Then, by virtue of PROPOSITION 2.1, the manifold is Einstein. Thus we have from (2.1) and (4.2)

$$(2H + P)\theta_\kappa = 0. \tag{4.5}$$

Thus we have

THEOREM 4.2. A 4-dimensional recurrent l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is trivial or the manifold has a property $2H + P = 0$.

REFERENCES

1. VAISMAN, I. On Locally Conformal Almost Kähler Manifolds, Isurael J. Math., 24 (1979), 338-351.

2. KASHIWADA, T. Some Properties of Locally Conformal Kähler Manifolds, Hokkaido Math. J., 8 (1970), 191-198.

3. KASHIWADA, T. On V-Killing Forms in a Locally Conformal Kähler Manifold with Parallel Lee Form, preprint.

4. KASHIWADA, T. On V-harmonic Forms in compact Locally Conformal Kähler Manifolds with Parallel Lee Form, preprint.

5. YANO, K. Differential Geometry on Complex and Almost Complex Spaces, Pergamon Press, 1965

Mathematical Problems in Engineering

Special Issue on Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	March 1, 2009
First Round of Reviews	June 1, 2009
Publication Date	September 1, 2009

Guest Editors

Edson Denis Leonel, Department of Statistics, Applied Mathematics and Computing, Institute of Geosciences and Exact Sciences, State University of São Paulo at Rio Claro, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru