

WEAK CONTINUITY AND STRONGLY CLOSED SETS

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(Received May 4, 1984)

ABSTRACT. After demonstrating the usual product theorems for weakly continuous functions, strongly closed and extremely closed subsets are contrasted to support the conjecture that a product of faintly continuous functions need not be faintly continuous. Strongly closed sets are used to characterize Hausdorff spaces and Urysohn spaces, and with these characterizations two results obtained by T. Noiri are obtained by function-theoretic means rather than by point-set method.

KEY WORDS AND PHRASES. *weak continuity, faint continuity, subweak continuity, strongly closed sets.*

1980 MATHEMATICS SUBJECT CLASSIFICATION CODE. 54 C 10.

1. INTRODUCTION.

Takashi Noiri proved: 1) Every weakly continuous function into a Hausdorff space has a closed graph [1] and 2) Every weakly continuous injection into a Urysohn space has a Hausdorff domain [2]. The first of these has been shown true assuming only subweak continuity for the function whereas the second is not generally true for subweakly continuous function [3]. We will obtain Noiri's results by mapping methods via strongly closed sets. But first productivity is discussed for weak continuity, faint continuity and subweak continuity.

2. DEFINITIONS AND NOTATION.

By $f : X \rightarrow Y$ is meant an arbitrary function between arbitrary topological spaces. If $A \subseteq X$, the interior and closure of A are denoted $\text{Int } A$ and $\text{Cl } A$ respectively. The topology for a space X may be written $T(X)$. A subset B of a space Y is \mathcal{O} -open if it contains a closed neighborhood of each of its points. The collection of \mathcal{O} -open sets in Y form a subtopology for Y called the \mathcal{O} -subtopology and denoted $T_{\mathcal{O}}(Y)$. By $Y_{\mathcal{O}}$ is meant the set Y equipped with the \mathcal{O} -subtopology.

Note 1. For any space Y , $Y = Y_{\mathcal{O}}$ if and only if Y is regular. Complements of \mathcal{O} -open sets are called \mathcal{O} -closed and have been studied in connection with H -closed spaces [4], [5], [6]. Finally, for any space X , $D(X)$ denotes the diagonal of $X \times X$.

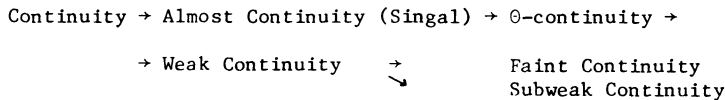
Definition 1. (Levine [7]). A function $f : X \rightarrow Y$ is weakly continuous if $f^{-1}(V) \subseteq \text{Int } f^{-1}(\text{Cl } V)$ for each $V \in T(Y)$.

Definition 2. (Long and Herrington [8]). A function $f : X \rightarrow Y$ is faintly continuous if $f^{-1}(V) \in T(X)$ for each $V \in T_0(Y)$.

Note 2. A function $f : X \rightarrow Y$ is faintly continuous if and only if $f : X \rightarrow Y_0$ is continuous [8].

Definition 3. (Rose [9]). A function $f : X \rightarrow Y$ is subweakly continuous if there is an open basis B for $T(Y)$ such that $\text{Cl } f^{-1}(V) \subseteq f^{-1}(\text{Cl } V)$ for each $V \in B$.

To have a proper perspective of the above defined classes of functions, recall that a function $f : X \rightarrow Y$ is almost continuous (in the sense of Singal and Singal [10]) if $f^{-1}(V) \in T(X)$ for each $V \in T_S(Y)$, where $T_S(Y)$ is the semiregular subtopology of $T(Y)$ generated by the regular open sets in $T(Y)$. Also a function $f : X \rightarrow Y$ is called Θ -continuous [1] if for each $x \in X$ and $V \in T(Y)$ with $f(x) \in V$, there is a neighborhood U of x with $f(\text{Cl } U) \subseteq \text{Cl } V$. The following implication diagram is pieced together from [1], [8], and [9].



In this diagram no implication is reversible in general though for functions into regular spaces, faint continuity implies continuity. Example 2 of [8] actually shows that faint continuity does not imply subweak continuity and the example below will show the impossibility of the reverse implication even if the range space is discrete (and hence regular).

Example 1. Let X be a T_1 , non- T_2 space, let Y be the set X with discrete topology and let $f : X \rightarrow Y$ be the identity function. Then f is subweakly continuous using the open basis of singleton subsets of Y but not faintly continuous since $Y = Y_0$ and f is not continuous. For later reference, note that f is injective, Y is $T_{2\frac{1}{2}}$, and $D(X) = (f \times f)^{-1}(D(Y))$ is not closed in $X \times X$.

The class of Θ -continuous functions is closed under composition but generally classes of generalized-continuous functions are not closed under composition even when one of the functions being composed is continuous. Noiri [12] gave an example of an almost continuous (Singal) function $f : X \rightarrow Y$ and a continuous function $g : Y \rightarrow Z$ for which $g \circ f$ is not almost continuous (Singal). It is clear, however, that composition of functions from these two classes in the reverse order does produce an almost continuous (Singal) function. Also, compositions, $f \circ g$, of subweakly (faintly) continuous functions are subweakly (faintly) continuous if g is continuous. The following example shows that a composition of two weakly continuous (and hence subweakly continuous) functions may fail to be subweakly continuous, and thus fail to be weakly continuous.

Example 2. Let $X = \{a, b\}$ with $T(X) = \{\emptyset, X, \{b\}\}$, $Y = Z = \{a, b, c\}$ with $T(Y) = \{\emptyset, Y, \{a, c\}, \{b, c\}, \{c\}\}$, and $T(Z) = \{\emptyset, Z, \{a\}, \{b\}, \{a, b\}\}$. Also let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be the inclusion functions. Then f and g are weakly continuous but $g \circ f$ is not even subweakly continuous.

The following result will be useful for investigating properties of products of weakly continuous functions.

Lemma 1. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are weakly continuous and either f or g is continuous, then $g \circ f : X \rightarrow Z$ is weakly continuous.

Proof: Suppose first that $f : X \rightarrow Y$ is continuous and $g : Y \rightarrow Z$ is weakly continuous. For $V \in \mathcal{T}(Z)$, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) \subseteq f^{-1}(\text{Int } g^{-1}(Cl V)) \subseteq \text{Int } f^{-1}(g^{-1}(Cl V)) = \text{Int } (g \circ f)^{-1}(Cl V)$ so that $g \circ f : X \rightarrow Z$ is weakly continuous. Now suppose that $f : X \rightarrow Y$ is weakly continuous and $g : Y \rightarrow Z$ is continuous. If $V \in \mathcal{T}(Z)$, $(g \circ f)^{-1}(V) = (f^{-1}(g^{-1}(V))) \subseteq \text{Int } f^{-1}(Cl g^{-1}(V)) \subseteq \text{Int } f^{-1}(g^{-1}(Cl V)) = \text{Int } (g \circ f)^{-1}(Cl V)$, and hence $g \circ f : X \rightarrow Z$ is weakly continuous.

Corollary 1. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are weakly continuous functions then
1) $g \circ f$ is (weakly) continuous if $(Y)Z$ is regular.

Lemma 2. If $g \circ f$ is weakly continuous and f is an open surjection then g is weakly continuous.

Proof: If $f : X \rightarrow Y$, $g : Y \rightarrow Z$, and if $W \subseteq Z$ is open then by the weak continuity of $g \circ f$, $f^{-1}(g^{-1}(W)) \subseteq \text{Int } f^{-1}(g^{-1}(Cl W)) = U$. Since f is an open surjection, $g^{-1}(W) = f(f^{-1}(g^{-1}(W))) \subseteq f(U) \subseteq \text{Int } g^{-1}(Cl W)$ so that g is weakly continuous.

Lemma 3. If $f : X \rightarrow Y$ and B is an open basis for the topology on Y , f is weakly continuous if and only if $f^{-1}(V) \subseteq \text{Int } f^{-1}(Cl V)$ for each $V \in B$.

Proof: The necessity is clear. For the sufficiency, let $W \in \mathcal{T}(Y)$. Then $W = \bigcup V_a$ where each $V_a \in B$. So $f^{-1}(W) = \bigcup f^{-1}(V_a) \subseteq \bigcup \text{Int } f^{-1}(Cl V_a) \subseteq \text{Int } \bigcup f^{-1}(Cl V_a) = \text{Int } f^{-1}(Cl W)$.

3. PRODUCT THEOREMS.

In this section, the product theorems proven in [10], [12], and [13], for almost continuous functions (in the sense of Singal and Singal) and in [14] for \mathcal{O} -continuous functions, will be shown to hold for weakly continuous functions. Also a stronger product theorem will be proven for subweakly continuous functions than that found in [3].

Theorem 1. Let $\{f_a : X_a \rightarrow Y_a \mid a \in A\}$ be a family of functions. Then $\prod f_a : \prod X_a \rightarrow \prod Y_a$ is weakly continuous if and only if each f_a is weakly continuous.

Proof: For the sufficiency, suppose each f_a is weakly continuous and let $V = \prod V_a \subseteq \prod Y_a$ be a basic open set. That is, each $V_a \subseteq Y_a$ is open and for all but finitely many a , $V_a = Y_a$. Let $f = \prod f_a$. Then $f^{-1}(V) = \prod f_a^{-1}(V_a) \subseteq \prod \text{Int } f_a^{-1}(Cl V_a) = \text{Int } \prod f_a^{-1}(Cl V_a) = \text{Int } f^{-1}(\prod Cl V_a) = \text{Int } f^{-1}(Cl V)$. By Lemma 3, $f = \prod f_a$ is weakly continuous. For the necessity, suppose that $f = \prod f_a$ is weakly continuous. Then for each a , $f_a \circ P_a = Q_a \circ f$ is weakly continuous by Lemma 1, where $P_a : \prod X_a \rightarrow X_a$ and $Q_a : \prod Y_a \rightarrow Y_a$ are the projections. Since P_a is an open surjection, f_a is weakly continuous by Lemma 2.

Theorem 2. The function $f : X \rightarrow \prod Y_a$ is weakly continuous if and only if each $P_a \circ f$ is weakly continuous.

Proof: The necessity follows from Lemma 1 since the projection $P_a : \prod Y_a \rightarrow Y_a$ is continuous. For the sufficiency let $X_a = X$ for each a and let $d : X \rightarrow \prod X_a$ be the continuous diagonal map. Then $\prod (P_a \circ f)$ is weakly continuous by Theorem 1 so that $f = [\prod (P_a \circ f)] \circ d$ is weakly continuous by Lemma 1.

Corollary 2. Let $\{f_a : X \rightarrow Y_a \mid a \in A\}$ be a family of functions. The function $f : X \rightarrow \prod Y_a$ defined by $f(x) = \{f_a(x)\}$, is weakly continuous if and only if each f_a is weakly continuous.

Proof: For each a , $P_a \circ f = f_a$.

Theorem 3. If $f_a : X_a \rightarrow Y_a$ is subweakly continuous for each $a \in A$ then $f = \prod f_a$ is subweakly continuous.

Proof: Let B_a be an open basis for the topology on Y_a for each $a \in A$. Let $B = \{\prod V_a \mid V_a = Y_a \text{ for all but finitely many } a \in A \text{ and } V_a \in B_a \text{ otherwise}\}$. Then B is an open basis for the product topology on $\prod Y_a$ and for each $\prod V_a \in B$, $Cl f^{-1}(\prod V_a) = Cl \prod f_a^{-1}(V_a) = \prod Cl f_a^{-1}(V_a) \subseteq \prod f_a^{-1}(Cl V_a) = f^{-1}(\prod Cl V_a) = f^{-1}(Cl \prod V_a)$ so that $f = \prod f_a$ is subweakly continuous.

Theorem 4. The function $f : X \rightarrow \prod Y_a$ is subweakly continuous if each $P_a \circ f$ is subweakly continuous.

Proof: Let $X_a = X$ for each a , and let $d : X \rightarrow \prod X_a$ be the continuous diagonal map. Then $\prod (P_a \circ f)$ is subweakly continuous by Theorem 3 so that $f = [\prod (P_a \circ f)] \circ d$ is subweakly continuous.

Corollary 3. ([3]) If $f_a : X \rightarrow Y_a$ is subweakly continuous for each $a \in A$ then $f : X \rightarrow \prod Y_a$ defined by $f(x) = \{f_a(x)\}$, is subweakly continuous.

Proof: For each $a \in A$, $P_a \circ f = f_a$.

4. STRONGLY CLOSED SETS.

The notion of a strongly closed subset of a product space will be introduced generalizing the notions of a Θ -closed set [4] and of a strongly closed graph of a function [15]. The notion of an extremely closed set will be introduced generalizing the notion of an extremely closed graph of a function [8]. Using these notions, some product-type Theorems will be obtained for faintly continuous functions.

Definition 4. Let $\{Y_a \mid a \in A\}$ be a family of spaces. A subset $E \subseteq \prod Y_a$ is strongly closed with respect to $B \subseteq A$ (or with respect to the factor $\prod Y_a$, $a \in B$) if for each $y \in (\prod Y_a) - E$, there is a basic open set $\prod V_a$ containing y such that if $W_a = Cl V_a$ for $a \in B$ and $W_a = V_a$ otherwise, then $(\prod W_a) \cap E = \emptyset$. If E is strongly closed with respect to $B = A$, E is said to be totally strongly closed.

Definition 5. Let $\{Y_a \mid a \in A\}$ be a family of spaces. A subset $E \subseteq \prod Y_a$ is extremely closed with respect to $B \subseteq A$ (or with respect to the factor $\prod Y_a$, $a \in B$) if for each $y \in (\prod Y_a) - E$ there is a basic open set $\prod V_a$ containing y with $V_a \in T_\Theta(Y_a)$ for each $a \in A$ and such that $(\prod V_a) \cap E = \emptyset$. If E is extremely closed with respect to $B = A$, E is said to be totally extremely closed.

Note 3. The family of complements of the strongly (extremely) closed sets with respect to $B \subseteq A$ is a subtopology of $T(\prod Y_a)$, the product topology. And extremely closed with respect to $B \subseteq A$ implies strongly closed with respect to $B \subseteq A$.

Note 4. The subtopology of complements of the totally strongly closed sets is $T_\Theta(\prod Y_a)$ and $T(\prod(Y_a)_\Theta)$, the product topology with the Θ -topology on each factor space, is the subtopology of complements of the totally extremely closed sets. Thus, $T(\prod(Y_a)_\Theta) \subseteq T_\Theta(\prod Y_a)$.

Note 5. Since ΠY_a is regular if and only if each Y_a is regular, $T_0(\Pi Y_a) = T(\Pi Y_a)$ implies that $T_0(Y_a) = T(Y_a)$ for each a which in turn implies that $T(\Pi Y_a) = T(\Pi(Y_a)_0) = T_0(\Pi Y_a)$. The next example shows that in general, $T(\Pi(Y_a)_0) \not\subseteq T_0(\Pi Y_a)$. By the above remarks, this proper inclusion requires at least one non-regular Y_a so that $T_0(\Pi Y_a) \not\subseteq T(\Pi Y_a)$.

Example 3. Let R be the set of real numbers, $X = (0, 2) \subseteq R$ and $Y = [0, 2) \subseteq R$. Let $H_r = \bigcup_n \geq r (\frac{1}{2n+1}, \frac{1}{2n})$ for $r \in N = \{1, 2, 3, \dots\}$ and let $G_r = \bigcup_n \geq r (\frac{1}{2n+2}, \frac{1}{2n+1})$ for $r \in N$. Let $T(R)$ be the usual topology and define $T(X)$ to be subgenerated by the subbasic open sets $\{V \subseteq X - \{1\} \mid V \in T(R)\} \cup \{H_r \cup G \mid r \in N \text{ and } 1 \in G \in T(R)\} \cup \{X - (H_r \cup G) \mid r \in N \text{ and } 1 \in G \in T(R)\}$. Let $T(Y)$ have subbasic open sets $\{V \subseteq Y - \{1\} \mid V \in T(R)\} \cup \{G_r \cup \{0\} \mid r \in N\} \cup \{H_r \cup G \mid r \in N \text{ and } 1 \in G \in T(R)\}$. If $f: X \rightarrow Y$ is the inclusion function, the graph of f , $G(f) \subseteq X \times Y$, is totally strongly closed but not totally extremely closed. For $(1, 0) \notin G(f)$ and if $1 \in U \in T(X)$ and if $0 \in V \in T_0(Y)$ then $U \cap V \neq \emptyset$.

By Note 2, $f_a: X_a \rightarrow Y_a$ is faintly continuous for each $a \in A$ if and only if each $f_a: X_a \rightarrow (Y_a)_0$ is continuous which holds if and only if $f = \Pi f_a: \Pi X_a \rightarrow \Pi (Y_a)_0$ is continuous. But for a family of functions $\{f_a: X_a \rightarrow Y_a \mid a \in A\}$, $f = \Pi f_a$ is faintly continuous if and only if $f: \Pi X_a \rightarrow (\Pi Y_a)_0$ is continuous. These remarks establish the following.

Theorem 5. If $\Pi f_a: \Pi X_a \rightarrow \Pi Y_a$ is faintly continuous then each $f_a: X_a \rightarrow Y_a$ is faintly continuous.

Example 3 strongly suggests that the converse of Theorem 5 does not hold.

Theorem 6. If $f: X \rightarrow \Pi Y_a$ is faintly continuous then each $P_a \circ f$ is faintly continuous.

Proof: If $f: X \rightarrow (\Pi Y_a)_0$ is continuous then $f: X \rightarrow \Pi (Y_a)_0$ is continuous which holds if and only if $P_a \circ f: X \rightarrow (Y_a)_0$ is continuous and this holds if and only if $P_a \circ f: X \rightarrow Y_a$ is faintly continuous.

Corollary 4. If $\{f_a: X \rightarrow Y_a \mid a \in A\}$ is a family of functions and if $f: X \rightarrow \Pi Y_a$, defined by $f(x) = \{f_a(x)\}$, is faintly continuous, then each f_a is faintly continuous.

Proof: For each $a \in A$, $f_a = P_a \circ f$.

5. APPLICATIONS.

Firstly, Hausdorff and Urysohn spaces, Y , are characterized in terms of the diagonal set, $D(Y) \subseteq Y \times Y$.

Proposition 1. A space Y is T_2 if and only if $D(Y)$ is strongly closed with respect to each factor.

Proof: If Y is T_2 and $(y_1, y_2) \in (Y \times Y) - D(Y)$ then $y_1 \neq y_2$ and there are disjoint open sets V_1 and V_2 containing y_1 and y_2 respectively. Thus $Cl V_1 \cap V_2 = \emptyset = V_1 \cap Cl V_2$ so that $(Cl V_1 \times V_2) \cap D(Y) = \emptyset = (V_1 \times Cl V_2) \cap D(Y)$, showing that $D(Y)$ is strongly closed with respect to each factor of $Y \times Y$. For the sufficiency, if $D(Y)$ is strongly closed, $D(Y)$ is closed so that Y is T_2 .

Proposition 2. A space Y is $T_{2\frac{1}{2}}$ if and only if $D(Y)$ is totally strongly closed.

Proof: Suppose first that Y is $T_{2\frac{1}{2}}$ and let $(y_1, y_2) \in (Y \times Y) - D(Y)$. Then $y_1 \neq y_2$ and there exist open sets V_1 and V_2 containing y_1 and y_2 respectively with $\text{Cl } V_1 \cap \text{Cl } V_2 = \emptyset$. Thus $(\text{Cl } V_1 \times \text{Cl } V_2) \cap D(Y) = \emptyset$ showing that $D(Y)$ is totally strongly closed. Conversely, if $D(Y)$ is totally strongly closed and $y_1, y_2 \in Y$ with $y_1 \neq y_2$ then $(y_1, y_2) \notin D(Y)$ so that for some open sets V_1 and V_2 containing y_1 and y_2 respectively, $(\text{Cl } V_1 \times \text{Cl } V_2) \cap D(Y) = \emptyset$. Thus $\text{Cl } V_1 \cap \text{Cl } V_2 = \emptyset$ and Y is $T_{2\frac{1}{2}}$.

Proposition 3. If Y is T_3 then $D(Y)$ is totally extremely closed.

Proof: If Y is T_3 thus Y is regular and T_2 so that $Y = Y_0$ and $D(Y) = D(Y_0)$ is closed in $Y \times Y = Y_0 \times Y_0$ and hence totally extremely closed.

Since there are Hausdorff non-Urysohn spaces, Propositions 1 and 2 show that strongly closed with respect to each $a \in B$ does not imply strongly closed with respect to B .

Theorem 7. If $f_a : X_a \rightarrow Y_a$ is weakly (faintly) continuous for $a \in B \subseteq A$ and continuous for $a \in A - B$ and if $E \subseteq \prod Y_a$ is strongly (extremely) closed with respect to B , then $(\prod f_a)^{-1}(E)$ is closed.

Proof: Suppose first that E is extremely closed with respect to $B \subseteq A$ and that f_a is faintly continuous for $a \in B$, and continuous otherwise. Then $\prod f_a : \prod X_a \rightarrow \prod Y_a$ is continuous where $Z_a = (Y_a)_0$ for $a \in B$ and $Z_a = Y_a$ otherwise. Further, E is closed in $\prod Z_a$ so that $(\prod f_a)^{-1}(E)$ is closed. Now, suppose that f_a is weakly continuous for $a \in B$ and continuous otherwise and that E is strongly closed with respect to B . Then if $x = \{x_a\} \notin (\prod f_a)^{-1}(E)$ then $y = \{f_a(x_a)\} \notin E$. Thus there exists a basic open set $\prod V_a$ containing y such that $\prod W_a \cap E = \emptyset$ where $W_a = \text{Cl } V_a$ for $a \in B$ and $W_a = V_a$ otherwise. Hence $x \in (\prod f_a)^{-1}(\prod V_a) = \prod (f_a^{-1}(V_a)) \subseteq \prod \text{Int } f_a^{-1}(W_a) = \text{Int } \prod (f_a^{-1}(W_a)) = \text{Int } (\prod f_a)^{-1}(\prod W_a) \subseteq (\prod f_a)^{-1}(\prod W_a) \subseteq \prod X_a - (\prod f_a)^{-1}(E)$. Thus $(\prod f_a)^{-1}(E)$ is closed.

If, in Theorem 7, $B = A$, the second part of the proof is simpler since E is totally strongly closed if and only if E is \emptyset -closed in which case $(\prod f_a)^{-1}(E)$ is closed if $\prod f_a$ is only faintly continuous but by Theorem 1, $\prod f_a$ is weakly continuous if each f_a is weakly continuous. In fact, one might question whether the weak continuity of f_a for $a \in B$ might be replaced with faint continuity. But recall the Example 3 indicates the likelihood that in that case $\prod f_a$ may fail to be faintly continuous.

Corollary 4. [1] If Y is Hausdorff and $f : X \rightarrow Y$ is weakly continuous then f has a closed graph, $G(f) \subseteq X \times Y$.

Proof: Let $i : Y \rightarrow Y$ be the continuous identity function. If Y is T_2 , $D(Y) \subseteq Y \times Y$ is strongly closed with respect to the first factor so that by Theorem 7, if f is weakly continuous, $G(f) = (f \times i)^{-1}(D(Y))$ is closed.

Corollary 5. [2] If Y is Urysohn and $f : X \rightarrow Y$ is a weakly continuous injection then X is Hausdorff.

Proof: By Proposition 2, $D(Y)$ is totally strongly closed if Y is Urysohn and by Theorem 7, $(f \times f)^{-1}(D(Y))$ is closed if f is weakly continuous. If also f is injective, $(f \times f)^{-1}(D(Y)) = D(X)$ is closed so that X is Hausdorff. Baker [3] noted by Example 1 that Corollary 5 is not true if weak continuity is reduced to subweak continuity. In particular, this shows that Theorem 7 does not hold with weak continuity replaced by

subweak continuity. However, Baker [3] proved that Corollary 4 can be strengthened by replacing weak continuity with subweak continuity. This is subweakly continuous functions into Hausdorff spaces have closed graphs. Apparently, Theorem 7 cannot be used to obtain Baker's improved version of Corollary 4, so that results obtained through Theorem 7 may not be "sharp" or "optimum". Some consequences of the improved version of Corollary 4 follow.

Proposition 4. If X is compact, Y is Hausdorff and $f : X \rightarrow Y$ is subweakly continuous (and bijective) then f is closed (and open).

Proof: Let X be compact, Y be Hausdorff and $f : X \rightarrow Y$ be subweakly continuous. Then closed subsets of X are compact and have closed images under f since the graph of f is closed. If f also preserves complements, f is also open.

Proposition 5. If Y is Hausdorff and $f : X \rightarrow Y$ is a subweakly continuous bijection then f^{-1} is c -continuous [16] (i.e. $f(V)$ is open if $X-V$ is closed and compact).

Proof: Long and Hendrix [17] showed that every closed graph function is c -continuous. If f is bijective with a closed graph then the graph of f^{-1} is closed. Proposition 5 improves a result of [17] by replacing almost continuity (in the sense of Singal and Singal) with subweak continuity.

Proposition 6. [3] If Y is Hausdorff and $f : X \rightarrow Y$ is a subweakly continuous injection then X is T_1 .

Proof: In the graph of f , $G(f) \subseteq X \times Y$, is closed and f is injective then for each $x \in X$, $\{x\} = f^{-1}(f(x))$ is closed since $\{f(x)\}$ is compact.

Since subweakly continuous functions into Hausdorff spaces have closed graphs, they are c -continuous. (Inverse images of open complements of compact sets are open.) Thus Proposition 6 follows from the result of Long and Hendrix [17] that c -continuous injections into a T_1 space must have a T_1 domain.

Proposition 7. If $f : X \rightarrow X$ has a closed graph, $G(f)$, then $F = \{x \in X \mid f(x) = x\}$, the set of fixed points for f , is closed.

Proof: Let $x \in \text{Cl } F$ and let $\{x_a\}$ be a net in F with $x_a \rightarrow x$. Then $\{(x_a, x_a)\}$ is a net in $G(f)$ and $(x_a, x_a) \rightarrow (x, x)$. Thus $x \in F$ if $G(f)$ is closed.

Corollary 6. [3] If A is a subweakly continuous retract of a Hausdorff space then A is closed.

Proof: A retract is the set of fixed points for the retraction.

Corollary 7. If A is a c -continuous retract of a locally compact Hausdorff space, then A is closed.

Proof: From [17], c -continuous functions into locally compact Hausdorff spaces have closed graphs.

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Space dynamics is a very general title that can accommodate a long list of activities. This kind of research started with the study of the motion of the stars and the planets back to the origin of astronomy, and nowadays it has a large list of topics. It is possible to make a division in two main categories: astronomy and astrodynamics. By astronomy, we can relate topics that deal with the motion of the planets, natural satellites, comets, and so forth. Many important topics of research nowadays are related to those subjects. By astrodynamics, we mean topics related to spaceflight dynamics.

It means topics where a satellite, a rocket, or any kind of man-made object is travelling in space governed by the gravitational forces of celestial bodies and/or forces generated by propulsion systems that are available in those objects. Many topics are related to orbit determination, propagation, and orbital maneuvers related to those spacecrafts. Several other topics that are related to this subject are numerical methods, nonlinear dynamics, chaos, and control.

The main objective of this Special Issue is to publish topics that are under study in one of those lines. The idea is to get the most recent researches and published them in a very short time, so we can give a step in order to help scientists and engineers that work in this field to be aware of actual research. All the published papers have to be peer reviewed, but in a fast and accurate way so that the topics are not outdated by the large speed that the information flows nowadays.

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|------------------------|-----------------|
| Manuscript Due | July 1, 2009 |
| First Round of Reviews | October 1, 2009 |
| Publication Date | January 1, 2010 |

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