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PARTIAL HENSELIZATIONS

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ABSTRACT. We define and note some properties of k H-pairs (k Henselian pairs), k N-pairs, and k N'-pairs. It is shown that the 2-Henselization and the 3-Henselization of a pair exist. Characterizations of quasi-local 2H-pairs are given, and an equivalence to the chain conjecture is proved.

KEY WORDS AND PHRASES. k Henselian pair, k N-pair, k N'-pair, chain conjecture.

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1. INTRODUCTION.

We define a pair (A,m) to be a k H-pair (a k Henselian pair) in case the ideal m is contained in the Jacobson radical of the commutative ring A and if for every monic polynomial f(X) of degree k in A[X] such that $\bar{f}(X) \in A/m\,[X]$ factors into $\bar{f}(X) = \bar{g}_o(X)\bar{h}_o(X)$ where $\bar{g}_o(X)$ and $\bar{h}_o(X)$ are monic and coprime, there exist monic polynomials g(x), $h(X) \in A[X]$ such that f(X) = g(X)h(X), $\bar{g}(X) = \bar{g}_o(X)$, and $\bar{h}\,(X) = \bar{h}_o(X)$. It is shown that the 2-Henselization and the 3-Henselization of a pair (A,m) exist. Several properties of k H-pairs are noted. And an equivalence to the Chain Conjecture is also given.

2. k H-PAIRS, k N-PAIRS, AND k N'-PAIRS.

In this section we define and give some facts about k H-pairs, k N-pairs, and k N'-pairs. The main result, Theorem (2.10) states that (i) a k H-pair is a k N-pair, (ii) a k N-pair is a k N'-pair, and (iii) an k N'-pair is a j H-pair provided $k \geq \max \{C_{j,n} \mid n = 0,1,\ldots,j\}$.

We begin be stating several definitions. In these definitions and throughout the paper a ring shall mean a commutative ring with an identity element, and J(A) denotes the Jacobson radical of the ring A.

DEFINITION 2.1. (A,m) is a <u>pair</u> in case A is a ring and m is an ideal in A.

DEFINITION 2.2. (A,m) is a <u>k H-pair</u> in case

(i) $m \subseteq J(A)$; and

(ii) for every monic polynomial f(X) of degree k in A[X] such that $\bar{f}(X) \in A/m\,[X]$ factors into $\bar{f}(X) = \bar{g}_o(X)\,\bar{h}_o(X)$ where $\bar{g}_o(X)$ and $\bar{h}_o(X)$ are monic and coprime, there exist monic polynomials g(X), h(X) $\in$ A[X] such that f(X) = g(X)h(X), $\bar{g}(X) = \bar{g}_o(X)$ and $\bar{h}(X) = \bar{h}_o(X)$.

DEFINITION 2.3. Let (A,m) be a pair. A monic polynomial $X^k + a_{k-1}X^{k-1} + \ldots + a_1X + a_o$ of degree k is called a k N-polynomial over (A,m) in case $a_o \in m$ and a_1 is a unit mod m.

DEFINITION 2.4. (A,m) is a k N-pair in case

(i) $m \subseteq J(A)$; and

(ii) every k N-polynomial over (A,m) has a root in m.

The next results give some facts about k N-polynomials and k N-pairs.

LEMMA 2.5. Let f(X) be a k N-polynomial over the pair (A,m). If $m \subseteq J(A)$, then f(X) has at most one root in m.

PROOF. The proof follows from [5, Lemma 1.5], since a k N-polynomial is an N-polynomial.

REMARK. Every k N-polynomial over a k N-pair (A,m) has one and only one root in m.

PROPOSITION 2.6. If (A,m) is a k N-pair, then (A,m) is an j N-pair for 2 $2 \leq j \leq k$.

PROOF. Given a k N-pair (A,m), it suffices to show that (A,m) is a (k-1) N-pair. Let f(X) be a (k-1) N-polynomial over (A,m). Let u be a unit in A and

$g(X) = (X + u)f(X)$. Then $g(X)$ is a k N-polynomial and thus has a root r in m and $0 = g(r) = (r + u)f(r)$. Since $(r + u)$ is a unit, we have $f(r) = 0$. Therefore, (A,m) is a (k - 1) N-pair.

DEFINITION 2.7. Let (A,m) be a pair. A monic polynomial $X^k + d_1 X^{k-1} + d_2 X^{k-2} + \ldots + d_k$ of degree k is called a k N'-polynomial over (A,m) in case d_1 is a unit mod m and $d_2, \ldots, d_k$ belong to m.

DEFINITION 2.8. (A,m) is a k N'-pair in case

(i) $m \subseteq J(A)$; and

(ii) every k N'-polynomial over (A,m) has a root in A, which is a unit.

We note that if (A,m) is a k N'-pair, $f(X) = X^k + d_1X^{k-1} + \ldots + d_k$ is a k N'-polynomial over (A,m) and $r \in A$ is a root of f(X) given by the definition of a k N'-pair, then $\bar{r} = -\bar{d}_1$, and $f'(r)$ is a unit.

PROPOSITION 2.9. Let (A,m) be a k N'-pair, then (A,m) is an j N'-pair for $2 \leq j \leq k$.

PROOF. Given a k N'-pair (A,m), it suffices to show that (A,m) is a (k-1) N'-pair. Let f(X) be a (k-1)N'-polynomial over (A,m). Then Xf(X) is a k N'-polynomial and has a root u, which is a unit. and $uf(u) = 0$ implies that $f(u) = 0$, therefore (A,m) is a (k-1)N'-pair.

THEOREM 2.10. (i) A kH-pair is a kN-pair

(ii) A kN-pair is a kN'-pair

(iii) A kN'-pair is a jH-pair, provided

$$k \geq \max \{C_{j,n} \mid n = 0, 1, \ldots, j\}$$

PROOF. Part (i) follows from the definitions.

The proof of (ii) follows from the proof of [10, Lemma 7]

The proof of (iii) follows from Crèpeaux's proof of [3, Prop. 1]

3. k N-CLOSURE.

In this section we construct the k N-closure for a given pair (A,m). That is, we find the "smallest" k N-pair which "contains" (A,m). The development of this section parallels Greco's development in [5].

In order to construct the k N-closure we need the following definitions.

DEFINITION 3.1. A morphism (of pairs) $\emptyset:(A,m) \to (B,n)$ is a ring homomorphism $\emptyset:A \to B$, such that $\emptyset^{-1}(n) = m$.

DEFINITION 3.2. A morphism (of pairs) $\emptyset:(A,m) \to (B,n)$ is strict in case $n = \emptyset(m)B$ and $\emptyset$ induces an isomorphism $A/m \to B/n$.

DEFINITION 3.3. Let (A,m) be a pair. A k N-pair (B,n) together with a morphism $\emptyset:(A,m) \to (B,n)$ is a k N-closure of (A,m) if for any k N-pair (B',n') and any morphism $\Psi:(A,m) \to (B',n')$ there exists a unique morphism $\Psi':(B,n) \to (B',n')$ such that $\Psi' \circ \emptyset = \Psi$.

DEFINITION 3.4. Let (A,m) be a pair and f(X) a k N-polynomial over (A,m). Let $A[x] = A[X]/(f(X))$, $S = 1 + (m,x)A[x]$ and $B = S^{-1}A[x]$. Then (B,mB) is called a simple k N-extension of (A,m).

DEFINITION 3.5. A k N-extension of (A,m) is a pair obtained from (A,m) by a finite number of simple k N-extensions.

The next two results give some useful properties of simple k N-extensions and k N-extensions.

LEMMA 3.6. Let (B,n) be a simple k N-extension of (A,m). Let $\emptyset:A \to B$ be the canonical morphism. Then:

(i) $x \in n$.

(ii) $\emptyset^{-1}(n) = m$ and $\emptyset:(A,m) \to (B,n)$ is a morphism of pairs.

(iii) $\emptyset:(A,m) \to (B,n)$ is strict.

PROOF. The proof follows from [5, Lemmas 2.3,2.4, and 2.5] since a simple k N-extension is a simple N-extension.

COROLLARY 3.7. If (B,n) is a k N-extension of (A,m), then the canonical morphism $\emptyset:(A,m) \to (B,n)$ is strict.

We note that a k N-extension of a quasi-local ring (A,m) is a quasi-local ring.

The following lemma is used to show that the partial order defined in Definition (3.9) is well defined.

LEMMA 3.8. Let (A',m') be a k N-extension of (A,m) and let (B,n) be a pair with $n \subseteq J(B)$. Let $\emptyset:(A,m) \to (A',m')$ be the canonical morphism. Then for any morphism $\Psi:(A,m) \to (B,n)$ there is at most one morphism $\Psi':(A',m') \to (B,n)$ such that $\Psi' \circ \emptyset = \Psi$.

PROOF. The proof follows from [5, Lemma 3.1] since a k N-extension is an N-extension.

In particular, the above lemma holds when (B,n) is a k N-extension of (A,m).

DEFINITION 3.9. Define a partial order on the set of k N-extensions of (A,m) as follows: If (A',m') and (A",m") are two k N-extensions of (A,m), then $(A',m') \leq (A'',m'')$ if and only if there is a morphism $\Psi:(A',m') \to (A'',m'')$ such that $\Psi \circ \emptyset = \emptyset''$, where $\emptyset:(A,m) \to (A',m')$ and $\emptyset'':(A,m) \to (A'',m'')$ are the canonical morphisms.

PROPOSITION 3.10. Let (A,m) be a pair. Then the k N-extensions of (A,m) form a directed set with the order relation and the morphisms defined above.

PROOF. The proof is analogous to [5, Prop. 3.3].

LEMMA 3.11 Let (A',m') be a k N-extension of (A,m) and let $\emptyset:(A,m) \to (A',m')$ be the canonical morphism. Let (B,n) be a k N-pair and let $\Psi:(A,m)\to(B,n)$ be a morphism. Then there is a unique morphism $\Psi':(A',m')\to(B,n)$ such that $\Psi = \Psi' \circ \emptyset$.

PROOF. The proof is analogous to [5, Prop. 3.4].

THEOREM 3.12. Let (A,m) be a pair and let (A^{kN},m^{kN}) be the direct limit of the set of all k N-extensions. Then (A^{kN},m^{kN}) with the canonical morphism $(A,m) \to (A^{kN},m^{kN})$ is a k N-closure of (A,m).

PROOF. The proof is analogous to [5, Thm. 3.5].

We note that if (A,m) is a quasi-local ring; then a k N-closure (A^{kN},m^{kN}) of (A,m) is quasi-local, since the direct limit of quasi-local rings is quasi-local.

4. k H-CLOSURES AND AN EQUIVALENCE TO THE CHAIN CONJECTURE.

In this section, we note the existence of a 2H-closure and of a 3H-closure, we give some characterization of a quasi-local 2H-pair, and we observe that the H-closure (or Henselization) of a pair (A,m) can be written as the direct limit or union of k H-pairs, k = 2,3,4,... . We also give an equivalence to the Chain Conjecture.

DEFINITION 4.1. Let (A,m) be a pair. A k H-pair (B,n), together with a morphism $\emptyset:(A,m)\to(B,n)$ is a k H-closure of (A,m) if for any k H-pair (B',n') and any morphism $\Psi:(A,m)\to(B'n')$, there exists a unique morphism $\Psi':(B,n)\to(B',n')$ such that $\Psi' \circ \emptyset = \Psi$.

THEOREM 4.2. Let (A,m) be a pair. Then:

(i) a 2 H-closure of (A,m) is (A^{2N}, m^{2N}).

(ii) a 3 H-closure of (A,m) is (A^{3N}, m^{3N}).

PROOF. It suffices to show that a k N-closure ($k = 2,3$) is a k H-pair. And by Theorem 2.10, we have that a 2N-pair is a 2H-pair, and that a 3N-pair is a 3H-pair.

DEFINITION 4.3. If $\emptyset:A\to B$ is a ring homomorphism, then B is said to be k-integral over A in case each $b \in B$ satisfies a monic polynomial of degree k over $\emptyset(A)$.

REMARK. If B is k-integral over A, then B is also j-integral over A for all $j \geq k$.

In the next three items we give examples of rings and elements which are k-integral over a given ring A.

LEMMA 4.4. If A is an integrally closed domain and $f(X) \in A[X]$ is a monic polynomial of degree k, then $A[X]/(f(X))$ is k-integral over A.

PROOF. Let $A[x] = A[X]/(f(X))$ and let L be the quotient field of A. Then $[L(x):L] \leq k$ and thus each $\alpha \in A[x]$ satisfies a monic polynomial $g(X) \in L[X]$ of degree $\leq k$. Since α is integral over A and A is integrally closed, it follows that $g(X) \in A[X]$. Therefore $A[x]$ is k-integral over A.

LEMMA 4.5. Let A be a ring and let $f(X) = X^2 + \alpha X + \beta \in A[X]$. Then $A[X]/(f(X))$ is 2-integral over A.

PROOF. Let $A[x] = A[X]/(f(X))$ and then all of the elements of $A[X]$ are of the form $ax + b$ where $a,b \in A$. To show that $A[x]$ is 2-integral over A, we need to find $F, G \in A$ such that

$$(ax + b)^2 + F(ax + b) + G = 0.$$

By expanding the left side, we see that $F = a\alpha - 2b$ and $G = a^2\beta - b^2 - Fb = a^2\beta + b^2 - ab\alpha$ are the needed values. Therefore $A[X]$ is 2-integral over A.

EXAMPLE 4.6. Each element of $\mathrm{End}_A(A^k)$ is k-integral over A by [1, Proposition 2.4].

In fact, if M is any A-module generated by k elements, each element of $\mathrm{End}_A(M)$ is k-integral over A.

DEFINITION 4.7. (A,m) is a <u>(≤k)H-pair</u> in case (A,m) is a j H-pair for $2 \le j \le k$.

It follows by Theorem 2.10 that if (A,m) is a j N-pair (or j H-pair), then (A,m) is a (≤k)H-pair provided $j \ge \max\{C_{k,n} \mid n = 0, 1, \ldots k\}$. In particular we have that for k = 2,3, or 4, a k H-pair is also a (≤k)H-pair.

LEMMA 4.8. Let (A,m) be a quasi-local domain which is a (≤k)H-pair. Then every k-integral extension domain of A is quasi-local.

PROOF. The proof is analogus to [6, (30.5)]

DEFINITION 4.9. A ring A is <u>decomposed</u> if A is the product of finitely many quasi local rings.

THEOREM 4.10. Let (A,m) be a quasi local ring. Then the following statements are equivalent.

(i) Every finite 2-integral A-algebra B is decomposed.

(ii) Every finite free 2-integral A-algebra B is decomposed.

(iii) Every A-algebra of the form A[X]/(f(X)), where $f(X) \in A[X]$ is monic and of degree 2, is decomposed.

(iv) (A,m) is a 2 H-pair.

PROOF. (i)$\Rightarrow$(ii) is clear. (ii)$\Rightarrow$(iii) is clear by (4.5). The proofs that (iii)$\Rightarrow$(i) and that (iii)$\Leftrightarrow$(iv) follow classical lines; for example, see [9, Prop. 5, p.2].

THEOREM 4.11. A quasi local domain (A,m) is a 2H-pair if and only if every 2-integral extension domain A' of A is quasi-local.

PROOF. ($\Rightarrow$) is true by (4.8).

($\Leftarrow$). We will show that (A,m) is a 2H-pair by showing that every finite free 2-integral A-algebra is decomposed. Let B be a finite free 2-integral A-algebra. Since B is decomposed if and only if B/nil rad B is decomposed, we may assume that B is reduced. Since B is flat over A, regular elements of A are also regular in B. Thus the minimal primes of B contract to {0} in A. Let $\{P_i\}_{i\in I}$ be the minimal primes of B. Then for each $i \in I$, B/P_i is a 2-integral extension domain of A and is quasi local by the hypothesis. Thus each minimal prime P_i is contained in a unique maximal ideal. By [2, Proposition 3, p. 329], the set of minimal primes of B is finite. Let $I_j = \cap_{P_i \subseteq M_j} P_i$ where M_j, j=1,..., n, are the maximal ideals of B. Then the I_j are coprime, and $\cap_{j=1}^{n} I_j = 0$ since B is reduced. So by the Chinese Remainder Theorem $B \cong \pi_{j=1}^{n} B/I_j$ and each B/I_j is quasi local. Thus B is decomposed and therefore (A,m) is a 2H-pair.

COROLLARY 4.12. Let (A,m) be a quasi local domain which is 2H-pair. Let A' be an integral extension domain of A. If $b \in A'$ is 2-integral over A, then $b \in J(A')$ or b is a unit.

PROOF. A[b] is a 2-integral extension domain of A and is thus quasi local. The result follows since all the maximal ideals of A' contract to the unique maximal ideal of A[b].

We will now show that the N-closure of a pair (A,m) is the direct limit of the k N-closures of (A,m). It will follow from this result that the H-closure of (A,m) can be written as the direct limit of k H-pairs.

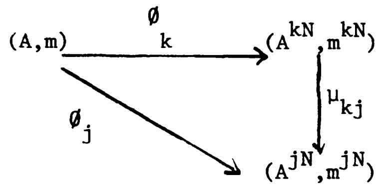
DEFINITION 4.13. Let (A,m) be a pair. Then (A,m) is an N-pair (respectively, a H-pair) in case (A,m) is a k N-pair (respectively, a k H-pair) for k = 2,3,... .

DEFINITION 4.14. Let (A,m) be a pair. An N-pair (respectively, an H-pair) (B,n), together with a morphism $\emptyset:(A,m)\to(B,n)$ is an N-closure (respectively, an H-closure) of (A,m) if for any N-pair (respectively, any H-pair) (B',n'), and any morphism Ψ: $(A,m)\to(B',n')$, there exists a unique morphism Ψ': $(B,n)\to(B',n')$ such that $\Psi' \circ \emptyset = \Psi$.

THEOREM 4.15. Let (A,m) be a pair. Then the H-closure of (A,m) is isomorphic to the N-closure.

PROOF. See [5, Lemma 1.4 and Theorem 5.10].

PROPOSITION 4.16. Let (A^N,m^N) be an N-closure of (A,m). Then $(A^N,m^N) \cong \text{dir lim } (A^{kN},m^{kN})$, where the directed system $\{(A^{kN},m^{kN}),\mu_{kj}\}$ of k N-closures of (A,m), k=2,3,4,..., is ordered by $(A^{kN},m^{kN})\leq(A^{jN},m^{jN})$ iff $k \leq j$ and if $k \leq j$, then μ_{kj}: $(A^{kN},m^{kN})\to(A^{jN},m^{jN})$ is the unique morphism which makes the following diagram commute:



where $\emptyset_j$ and $\emptyset_k$ are the canonical morphisms.

PROOF. The proof follows immediately from Definitions (3.3) and (4.14) and the definition of a direct limit.

COROLLARY 4.17. Let (A^H,m^H) be the H-closure of (A,m). Then $(A^H,m^H) \cong$ dir lim (A_i,M_i) where (A_i,m_i) is an i <u>H-pair</u> for i = 2,3,... .

PROOF. For a given i, let $(A_i,m_i) = (A^{kN},m^{kN})$ where $k = \max' \{C_{j,n} | n=0,1,\ldots,j\}$. Then the corollary follows by results (2.10), (4.15) and (4.16).

We now give an equivalence to the Chain Conjecture. The terminology used is the same as in [8] or [10].

THEOREM 4.18. The following statements are equivalent:

(i) The Chain Conjecture holds.

(ii) Every 2 Henselian local domain A, such that the integral closure of A is quasi-local, is catenary.

PROOF. (i)$\Rightarrow$(ii). This follows by [8, Thm. 2.4].

(ii)$\Rightarrow$(i). By [8, Thm. 2.4] it suffices to show that every Henselian local domain is catenary. Let A be a Henselian local domain. Then A is also 2 Henselian and the integral closure of A is quasi-local by [6, (43.12)]. Thus by the hypothesis A is catenary.

5. <u>EXAMPLES</u>.

In this section we show that there exist k N-pairs which are not N-pairs and there exist k H-pairs which are not H-pairs. More precisely, for each prime number p we give an example of a pair which is not a p N-pair but is a k N-pair for $2 \le k < p$. This example also shows that for any integer $k \ge 2$, there exists a k H-pair which is not a p H-pair for some sufficiently large prime number p.

Let $p > 2$ be a prime number. Let (R,q) be a normal quasi-local domain such that there exists an $f(X) = X^p + \ldots + a_1X + a_o \in R[X]$, where $a_1 \notin q$, $a_o \in q$ and f(X) is irreducible over R[X].

In particular, let $R = Z_{(2)}$ and let $f(X) = X^p + 3X + 6$. Then by Eisenstein's Criterior, f(X) is irreducible in Q[X], and thus irreducible in $Z_{(2)}[X]$ since f(X) has content 1.

Let K be the quotient field of R and let $\overline{K}$ be an algebraic closure of K. Let R' be the integral closure of R in $\overline{K}$ and P' any maximal ideal in R'. Now f(X) as an element of R'[X] factors completely, and since $P' \cap R = q$, f(X) has a unique root $\alpha \in P'$. Let L be the least normal extension of K containing α. Then $p \mid [L:K]$ and by [7, Thm. 6] there is a maximal field M without α of exponent p with $K \subseteq M \subset \overline{K}$. Let $A = R' \cap M$ and let $m = P' \cap A$.

Now (A,m) is not a p N-pair since f(X) is a p N-polynomial over (A,m) which does not have a root in m. But (A,m) is a k N-pair for $2 \leq k < p$. For, let g(X) be a (p - 1)N-polynomial over (A,m). Then g(X) as an element of R'[X] has a unique root $\beta \in P'$. Now $[M(\beta):M] \leq p - 1$, but by [7, Thm. 2], $[M(\beta):M] = p^i$ for some $i \geq 0$. So $[M(\beta):M] = 1$ and $\beta \in M$. Thus $\beta \in m = P' \cap A$ and (A,m) is a (p - 1)N-pair. It follows by (2.6) that (A,m) is a k N-pair for $2 \leq k < p$.

<u>REMARK</u>. If j and the prime number p are closen such that $p > \max \{C_{j,n} | n=0,1,\ldots,j\}$, then by Theorem 2.10, the above example is an example of a pair (A,m) such that (A,m) is not a p H-pair, but (A,m) is a k H-pair for $2 \leq k \leq j$.

Let the notation be as in the above example. Then (A_m, mA_m) is as an example of a normal quasi-local domain which is not a p N-pair, but is a k N-pair for $2 \leq k < p$.

6. <u>PROPERTIES OF k N-PAIRS</u>.

We conclude this paper by noting that many of the properties of the Hensilization or N-closure of a pair which S. Greco proved in [5] also hold for a k N-closure and thus also for a 2 H-closure and a 3 H-closure. Some of these results are: direct limits commute with k N-closures, cf. [5, Cor. 3.6]; a k N-closure of (A,m) is flat over A and is faithfully flat over A iff $m \subseteq J(A)$, cf. [5, Thm. 6.5]; a k N-closure of a noetherian ring is noetherian, and if a k N-closure of (A,m) is Noetherian and $m \subseteq J(A)$, then A is Noetherian, cf. [5, Cor. 6.9]; if A is Noetherian and A has one of the properties R_k, S_k, regular, or Cohen-Macaulay, then a k N-closure of (A,m) also has that property, and the converse is also true provided $m \subseteq J(A)$, cf. [5, Cor. 7.7]; a k N-closure preserves locally normal, cf. [5, Thm. 9.7]; and a k N-closure of a reduced ring is reduced, cf. [5, Thm. 8.7].

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Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:

- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
- Loading orders
- Selection of routing and service

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