RANKED SOLUTIONS OF THE MATRIC EQUATION $A_1 X_1 = A_2 X_2$

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ABSTRACT. Let $GF(p^z)$ denote the finite field of p^z elements. Let A_1 be $s \times m$ of rank r_1 and A_2 be $s \times n$ of rank r_2 with elements from $GF(p^z)$. In this paper, formulas are given for finding the number of X_1, X_2 over $GF(p^z)$ which satisfy the matric equation $A_1 X_1 = A_2 X_2$, where X_1 is $m \times t$ of rank k_1, and X_2 is $n \times t$ of rank k_2. These results are then used to find the number of solutions $X_1, \ldots, X_n, Y_1, \ldots, Y_m, m, n > 1$, of the matric equation

$$A_1 X_1 \ldots X_n = A_2 Y_1 \ldots Y_m.$$

KEY WORDS AND PHRASES. Finite Field, Matric equation, Ranked Solutions.

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1. INTRODUCTION. Let GF(q) denote the finite field with $q = p^z$ elements, p

odd. Matrices with elements from GF(q) will be denoted by Roman capitals A,

B, ... A(n,s) will denote a matrix of n rows and s columns, and A(n,s;r)

will denote a matrix of the same dimensions with rank r. I_r denotes the identity

matrix of order r, and I(n,s;r) denotes a matrix of n rows and s columns

having I_r in its upper left hand corner and zeros elsewhere.

In this paper we find the number of solutions $X_1(m,t;k_1)$, $X_2(n,t;k_2)$ to the

matric equation

$$A_1 X_1 = A_2 X_2, \qquad (1.1)$$

where $A_1 = A_1(s,m;r_1)$ and $A_2 = A_2(s,n;r_2)$. Since the ranks of X_1, X_2 are specified,

we call this the ranked case. If the ranks were not specified, we would call it

the unranked case. In section 4 we apply this result to find the number of

solutions $X_1, \ldots, X_n, Y_1, \ldots, Y_m$, $m,n \geq 1$, to the matric equation

$$A_1 X_1 \ \cdots \ X_n = A_2 Y_1 \ \cdots \ Y_m, \qquad (1.2)$$

both in the unranked and ranked cases.

Equation (1.1) is a special case of the matric equation

$$A_1 X_1 + \ldots + A_n X_n = B, \qquad (1.3)$$

and equation (1.2) is a special case of the more general equation

$$A_1 X_{11} \ \cdots \ X_{1m(1)} + \ldots + A_n X_{n1} \ \cdots \ X_{nm(n)} = B. \qquad (1.4)$$

Porter [6] found the number of solutions $X_1, \ldots, X_n$ to (1.3) in the unranked

case. We could find the number of solutions to (1.3) in the ranked case if we

could find the number of ranked solutions to $A_1 X_1 + A_2 X_2 = B$. The number of rank-

ed solutions to $A_1 X_1 + A_2 X_2 = B$ together with the formulae for the number of

solutions to $X_1 \ \cdots \ X_n = B$ would give the number of solutions to (1.4). The

number of unranked solutions to $X_1 \ \cdots \ X_n = B$ is given by Porter in [5]. The

number of ranked solutions to $X_1 \ldots X_n = B$ appears in [9], by the authors.

Presently the authors know of no published results, unranked or ranked, giving the number of solutions to (1.4) except when (1.4) reduces to (1.3). There are partial results in the unranked case to the analogous problem $U_a \ldots U_1 A + B V_1 \ldots V_b = C$. Hodges [1] found the number of unranked solutions with $a = b = 1$. Hodges [2] and Porter [7] found partial results in the unranked case when a, b are arbitrary and Hodges [3] discussed ranked solutions when $a = b = 1$.

2. NOTATION AND PRELIMINARIES. A well known formula due to Landsberg [4] gives the number $g(m, t; s)$ of $m \times t$ matrices of rank s over $GF(q)$.

$$g(m,t;s) = q^{s(s-1)/2} \prod_{i=1}^{s} (q^{m-i+1} - 1)(q^{t-i+1} - 1)/(q^i - 1), \qquad (2.1)$$

for $1 \le s \le \min(m,t)$, $g(m,t;0) = 1$, and $g(m,t;s) = 0$ for $\min(m,t) < s$ or $s < 0$.

If $X = X(e,t)$ and $X = \text{col}[U,Y]$, where U is fixed, $U = U(m,t;s)$ and $Y = Y(e - m, t)$, then the number of ways that Y can be chosen such that X has rank k is given by Porter and Riveland [11] to be

$$G(e,t,m;k,s) = q^{s(e-m)} g(e - m, t - s; k - s). \qquad (2.2)$$

In [9, Theorem 3] the authors found the number of solutions $X(m,f;k)$ to the matric equation $AX = B$, where $A = A(s,m;\rho)$ and $B = B(s,f;\beta)$. This number is given by

$$N(A,B,k) = h(B_0) q^{(m-\rho)\beta} g(m - \rho, f - \beta; k - \beta) = h(B_0) L(m,f;\rho,\beta,k), \qquad (2.3)$$

where $h(B_0)$ is defined as follows. If P, Q are nonsingular matrices such that $PAQ = I(s,m;\rho)$, then $B_0 = PB = (\beta_{ij})$ and $h(B_0) = 1$ if $\beta_{ij} = 0$ for $i > \rho$, $h(B_0) = 0$ otherwise. The number of solutions, when there are any, is denoted by $L(m,f;\rho,\beta,k)$.

Let $A = A(n,s;r)$ and $B = B(n,t;u)$. Then Porter [5] showed that the number

of solution $X_1(s,s_1)$, $X_i(s_{i-1},s_i)$ for $1 < i < a$, $X_a(s_{a-1},t)$ to the matric

equation $AX_1 \ldots X_a = B$. when there are solutions, is given by

$$N(a,s,t,s_i,r,u) = q^{t(s_{a-1}-r)+ss_1+s_1s_2+ \ldots +s_{a-2}s_{a-1}} \sum_{z_{a-1}=0}^{\min(r,t)} H(r,t,y;z_{a-1}).$$

$$.q^{-z_{a-1}s_{a-1}} \prod_{i=2}^{a-1} \sum_{z_{a-1}=0}^{\min(z_{a-i+1},s_{a-i+1})} g(z_{a-i+1},s_{a-i+1};z_{a-i})q^{-z_{a-i}s_{a-i}},$$

(2.4)

where $g(m,t;s)$ is given by (2.1), and $H(s,t,u;z)$ is given in [1] to be

$$H(s,t,u,z) = q^{uz} \sum_{j=0}^{z} (-1)^j q^{j(j-2u-1)/2} \begin{bmatrix} u \\ j \end{bmatrix} g(s - u, t - u; z - j),$$

where the bracket denotes the q-binomial ceefficient defined for non-negative

intergers by

$$\begin{bmatrix} u \\ 0 \end{bmatrix} = 1, \quad \begin{bmatrix} u \\ j \end{bmatrix} = \prod_{i=0}^{j-1} (1 - q^{u-1})/(1-q^{i+1}) \quad \text{if} \quad 1 \le i \le u, \begin{bmatrix} w \\ j \end{bmatrix} = 0$$

if $j > w$. For the purposes of this paper we take $A = I_s$ in (2.4). By [5]

there will always be solutions to $X_1 \ldots X_a = B$, and this number can be repre-

sented by

$$M_a(s,s_1,\ldots,s_{a-1},t,u) = \begin{bmatrix} N(a,s,t,s_i,s,u) & \text{for} & a \ge 2, \\ 1 & \text{for} & a = 1. \end{bmatrix}$$

(2.5)

The number of matrices $D = D(a,b;c)$ such that $D = \text{col}[D_1,D_2]$ where

$D_1 = D_1(d,b)$ and $D_2 = D_2(a - d,b;c -d)$, $d \le \min(a,c)$ is given in [10] to be

$K(a,b,c,d) = q^{(c-d)d} g(d,b + d - c;d)g(a - d,b;c - d)$. The number

$T_n(d_o,\ldots,d_n;k_1,\ldots,k_n,\beta)$ of solutions $X_1(d_o,d_1;k_1),\ldots,X_n(d_{n-1},d_n;k_n)$ to the

matric equation $X_1\ldots X_n = B$, where $B = B(d_o,d_n;\beta)$, is given in [9] by the

following three formulae:

$$T_1(d_o,d_1;k_1,\beta) \quad = \quad \begin{cases} 0 & \text{if } k_1 \neq \beta, \\ \\ 1 & \text{if } k_1 = \beta, \end{cases}$$

$$T_2(d_o,d_1,d_2,;k_1,k_2,\beta) \quad = K(d_o,d_1,k_1,\beta)L(d_1,d_2,;k_1,\beta,k_2), \qquad (2.6)$$

$$T_n(d_o,\ldots,d_n;k_1,\ldots,k_n,\beta) \quad = \sum_{i_n=\beta}^{r_n} \sum_{i_{n-1}=i_n}^{r_{n-1}} \cdots \sum_{i_3=i_4}^{r_3} T_2(d_o,d_i,k_1,k_2,i_3)\cdot$$

$$\cdot \prod_{m=3}^{n} K(d_o,d_{m-1},i_m,i_{m+1})L(d_{m-1},d_m,i_m,i_{m+1},k_m),$$

where $n \geq 3$, $r_j = \min(k_1,\ldots,k_{j-1})$ for $j = 3,\ldots,n$ and $i_{n+1} = \beta$.

3.   THE MAIN RESULT.

THEOREM 1.   If $A = A(s,m;\rho)$, then the number of solutions $X_1(m,t;k_1)\cdot$ $X_2(s,t;k_2)$, for $\rho,k_1 \geq k_2$, to the matric equation

$$AX_1 = X_2, \qquad (3.1)$$

is given by $N(m,t;\rho,k_1,k_2) = g(\rho,t;k_2)L(m,t;\rho,k_2,k_1)$, where $g(\rho,t;k_2)$ is evaluated using (2.1) and $L(m,t;\rho,k_2,k_1)$ is evaluated using (2.3).

PROOF: Let P,Q be nonsingular matrices such that $PAQ = I(s,m;\rho)$. Then (3.1) can be rewritten as

$$I(s,m;\rho)Q^{-1}X_1 = PX_2. \qquad (3.2)$$

The left hand side of (3.2) is of the form $col[Y,0]$ where $Y = Y(\rho,t)$. For

a particular $X_2(s,t;k_2)$, there will be matrices $X_1(m,t;k_1)$ which satisfy

(3.2), and therefore (3.1), provided X_2 is the product of P^{-1} and a matrix of

the form $col[Y,0]$ where $Y = Y(\rho,t;k_2)$. Since P^{-1} is nonsingular there are the

same number of matrices X_2 with this property as there are $\rho \times t$ matrices of

rank k_2. The number of $\rho \times t$ matrices of rank k_2 is given by Landsberg's

formula (2.1) and denoted by $g(\rho,t;k_2)$. For each such X_2 the number of X_1 such

that X_1,X_2 satisfy (3.1) can by represented by $L(m,t;\rho,k_2,k_1)$ as given by

(2.3). Therefore the number of solutions X_1,X_2 to (3.1) is given by

$g(\rho,t;k_2)L(m,t;\rho,k_2,k_1)$, and the theorem is proved.

It should be noted that Theorem 1 is a special case of a theorem of Hodges

[3]. However, our proof, and so the form of the resulting formula, is quite dif-

ferent since Hodges uses exponential sums in his proof and we do not. Our proof

of Theorem 1 is consistent with the methods of proof used in the rest of this

paper.

THEOREM 2. Let $A = A(s,m;\rho) = col[A_1,A_2]$ where $A_1 = A_1(n,m;\alpha_1)$ and

$A_2 = A_2(s - n,m;\alpha_2)$ with $n \le s$. Let P,Q be nonsingular matrices such that

$PA_2Q = I(s - n,m;\alpha_2)$ and $A_1Q = [B_1,B_2]$ where $B_2 = B_2(n,m - \alpha_2;\beta)$. Then

the number of solutions $X_1(m,t;k_1)$, $X_2(n,t;k_2)$ to the matric equation

$$AX_1 = \begin{bmatrix} X_2 \\ 0 \end{bmatrix}, \qquad (3.3)$$

for $\alpha_1 > k_2$ is given by

$$N(m - \alpha_2, t; \beta, k_1, k_2) = g(\beta, t; k_2)L(m - \alpha_2, t; \beta, k_2, k_1),$$

where $g(\beta, t; k_2)$ is given by (2.1) and $L(m - \alpha_2, t; \beta, k_2, k_1)$ is given by (2.3).

PROOF: For A_1, A_2 defined as above we can write (3.3) as the system of equations

$$A_1 X_1 = X_2, \qquad (3.4)$$

$$A_2 X_1 = 0. \qquad (3.5)$$

Substituting $A_2 = P^{-1}I(s - n, m; \alpha_2)Q^{-1}$ into (3.5) and multiplying on the left by P we obtain

$$I(s - n, m; \alpha_2)Q^{-1}X_1 = 0. \qquad (3.6)$$

Let $Q^{-1}X_1 = \text{col}[Y_1, Y_2]$, where $Y_1 = Y_1(\alpha_2, t)$ and $Y_2 = Y_2(m - \alpha_2, t)$. Replacing $Q^{-1}X_1$ in (3.6) by $\text{col}[Y_1, Y_2]$, we have that necessary and sufficient conditions for X_1 to be a solution of (3.6) are that $Y_1 = 0$, rank $Y_2 = k_1$ and $X_1 = Q\text{col}[0, Y_2]$. Using this formulation for X_1 in (3.4) gives

$$A_1 Q \begin{bmatrix} 0 \\ Y_2 \end{bmatrix} = X_2. \qquad (3.7)$$

Let $A_1 Q = [B_1 \ B_2]$, where $B_1(n, \alpha_2)$ and $B_2 = B_2(n, m - \alpha_2, \beta)$ in (3.7) we then obtain

$$B_2 Y_2 = X_2. \tag{3.8}$$

By Theorem 1 there are $N(m - \alpha_2, t; \beta, k_1, k_2)$ pairs Y_2, X_2 which satisfy (3.8).

Since Q is nonsingular there are the same number of pairs X_1, X_2 which satisfy

(3.3).

Equations (3.4) and (3.5) represent a special system of two simultaneous

equations in the two matrices X_1, X_2. Very few results seem to exist for such

systems. The authors are unable to find the number of solutions to the general

system of two simultaneous equations in X_1, X_2. Such information would allow us

to enumerate the ranked solutions to $A_1 X_1 + A_2 X_2 = B$.

THEOREM 3. Let $A_1 = A_1(s, m; r_1)$ and $A_2 = A_2(s, n; r_2)$. Let P_1, Q_1 be non-

singular matrices such that $P_1 A_1 Q_1 = I(s, m; r_1)$. Define $P_1 A_2 = \mathrm{col}[A_{21}, A_{22}]$,

where $A_{21} = A_{21}(r, n; \alpha_1)$ and $A_{22} = A_{22}(s - r_1, n; \alpha_2)$. Let P_2, Q_2 be nonsingular

matrices such that $P_2 A_{22} Q_2 = I(s - r_1, n; \alpha_2)$. Define $A_{21} Q_2 = [B_1, B_2]$, where

$B_2 = B_2(r_1, n - \alpha_2; \beta)$. Then the number of solutions $X_1(m, t; k_1)$, $X_2(n, t; k_2)$ to

the matric equation

$$A_1 X_1 = A_2 X_2 , \tag{3.9}$$

is given by

$$N(m, n, t; r_1, r_2, k_1, k_2, \alpha_1, \alpha_2, \beta)$$

$$= \sum_{k_{11}=0}^{\min(\alpha_1, k_1)} G(m, t, r_1; k_1, k_{11}) g(\beta, t; k_{11}) L(n - \alpha_2, t; \beta, k_{11}, k_2),$$

where $G(m, t, r_1; k_1, k_{11})$ can be evaluated using (2.2), $g(\beta, t; k_{11})$ is given by

(2.1), and $L(n - \alpha_2, t; \beta, k_{11}, k_2)$ is given by (2.3).

PROOF: The number of solutions to (3.9) is the same as the number of solutions

to

$$I(s, m; r_1) X_1 = P_1 A_2 X_2. \tag{3.10}$$

Letting $X_1 = \text{col}[X_{11}, X_{12}]$, $X_{11} = X_{11}(r_1, t; k_{11})$, $X_{12} = X_{12}(m - r, t; k_{12})$ and

$A = P_1 A_2$, (3.10) becomes

$$AX_2 = \begin{bmatrix} X_{11} \\ \\ 0 \end{bmatrix} .$$

(3.11)

Theorem 2 gives the number of pairs X_{11}, X_2 that satisfy (3.11). For each

$X_{11}(r_1, t; k_{11})$ the number of $X_{12}(m - r_1, t; k_{12})$ such that $X_1(m, t; k_1) = \text{col}[X_{11}, X_{12}]$

is given by (2.2), denoted by $G(m, t, r_1; k_1, k_{11})$. Therefore the number of solutions

X_1, X_2 to (3.9) is given by the product $G(m, t, r_1; k_1, k_{11}) \mathbf{s}(\mathbf{\Phi}, t; k_{11}) L(n - \alpha_2, t; \beta, k_{11} k_2)$

summed over the possible values of K_{11} where $K_{11} \leq \alpha_1$ by the hypothesis of

Theorem 2.

4. SOME APPLICATIONS.

We can now use Theorem 3 together with some other known results to find the

number of solutions $X_1, \ldots, X_n, Y_1, \ldots, Y_m$ to (1.2) in both the unranked and

unranked cases.

THEOREM 4. Let $A_1 = A_1(m, s_0; r_1)$ and $A_2 = A_2(m, t; r_2)$. Let P_1, Q_1 be non-

singular matrices such that $P_1 A_1 Q_1 = I(m, s_0; r_1)$. Define $P_1 A_2 = \text{col}[A_{21}, A_{22}]$,

where $A_{21} = A_{21}(r_1, t_0; \alpha_1)$ and $A_{22} = A_{22}(m - r_1, t_0; \alpha_2)$. Let P_2, Q_2 be non-

singular matrices such that $P_2 A_{22} Q_2 = I(m - r_1, t_0; \alpha_2)$. Define $A_{21} Q_2 = [B_1, B_2]$,

where $B_2 = B_2(r_1, t_0 - \alpha_2; \beta)$. Then the number of solutions $X_1(s_0, s_1), \ldots,$

$X_n(s_{n-1}, s_n)$, $Y_1(t_0, t_1), \ldots, Y_m(t_{m-1}, t_m)$, $m, n \geq 1$, $s_n = t_m$ to (1.2) is given

by

$$\sum_{i_1 = 0}^{\min(s_0, \ldots, s_n)} \sum_{i_2 = 0}^{\min(t_0, \ldots, t_m)} N(s_0, t_0, s_n; r_1, r_2, i_1, i_2, \alpha_1, \alpha_2, \beta) .$$

$$. M_n(s_o, \ldots, s_n, i_1) M_m(t_o, \ldots, t_m, i_2),$$

where $N(s_o, t_o, s_n; r_1, r_2, i_1, i_2, \alpha_1, \alpha_2, \beta)$ can be evaluated using (3.9),

$M_n(s_o, \ldots, s_n, i_1)$ and $M_m(t_o, \ldots, t_m, i_2)$ can be evaluated using (2.5).

PROOF: Consider the equations

$$A_1 U = A_2 V, \tag{4.1}$$

$$U = X_1 \ldots X_n, \tag{4.2}$$

$$V = Y_1 \ldots Y_m, \tag{4.3}$$

where $U = U(s_o, s_n; i_1)$, $V = V(t_o, t_m; i_2)$, $0 \le i_1 \le \min(s_o, \ldots, s_n)$ and

$0 \le i_2 \le \min(t_o, \ldots, t_m)$. The number of solutions U, V to (4.1) is given by (3.9)

and is represented by $N(s_o, t_o, s_n; r_1, r_2, i_1, i_2, \alpha_1, \alpha_2, \beta)$. The numbers $M_n(s_o, \ldots s_n, i_1)$

and $M_m(t_o, \ldots, t_m, i_2)$ represent the number of solutions to (4.2) and (4.3),

respectively, for a fixed U or V. M_n and M_m can be evaluated using (2.5).

The product $N M_n M_m$ summed over the possible ranks of U and V gives the number

of solutions to (1.2) in the unranked case.

The next theorem is proved in the same way that Theorem 4 is proved except

that we use (2.6) to obtain the number of ranked solutions $X_1, \ldots, X_n, Y_1, \ldots, Y_m$

to the matric equations (4.2) and (4.3) .

THEOREM 5. Let $A_1, A_2, P_1, Q_1, P_2, Q_2, A_{21}, A_{22}, B_1, B_2, B_{11}, B_{12}, \alpha_1, \alpha_2,$ and β be

as in Theorem 4. Then the number of solutions

$X_1(s_o, s_1; j_1), \ldots, X_n(s_{n-1}, s_n; j_n), Y_1(t_o, t_1; k_1), \ldots, Y_m(t_{m-1}, t_m; k_m), m, n \ge 1,$

$s_n = t_m$ to (1.2) is given by

$$\sum_{i_1=0}^{\min(j_1,\ldots,j_n)} \sum_{i_2=0}^{\min(k_1,\ldots,k_m)} N(s_o,t_o,s_n;r_1,r_2,i_1,i_2,\alpha_1,\alpha_2,\beta).$$

$$\cdot\, T_n(s_o,\ldots,s_n;j_1,\ldots,j_n,i_1) T_m(t_o,\ldots,t_m;k_1,\ldots,k_m,i_2),$$

where $N(s_o,t_o,s_n;r_1,r_2,i_1,i_2,\alpha_1,\alpha_2,\beta)$ is evaluated using (3.9) and

$T_n(s_o,\ldots,s_n;j_1,\ldots,j_n,i_1)$ and $T_m(t_o,\ldots,t_m;k_1,\ldots,k_m,i_2)$ are evaluated using

(2.6).

NOTE: This paper was written while the second named author was on leave at the

University of Wyoming.

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