

SP-CLOSEDNESS IN L -FUZZY TOPOLOGICAL SPACES

BAI SHI-ZHONG

Received 20 February 2002

We introduce and study SP -closedness in L -fuzzy topological spaces, where L is a fuzzy lattice. SP -closedness is defined for arbitrary L -fuzzy subsets.

2000 Mathematics Subject Classification: 54Axx.

1. Introduction. Andrijević [1] introduced the definition of semi-preopen sets in general topological spaces. Thakur and Singh [8] extended this definition to fuzzy topological spaces. In [4], using semi-preopen sets, we have introduced and studied a good definitions of semi-precompactness in L -fuzzy topological spaces.

In this note, along the lines of this semi-precompactness, we introduce a definition of SP -closedness in L -fuzzy topological spaces. Also, we obtain some of its properties. SP -closedness is defined for arbitrary L -fuzzy subsets. It is a weaker form of semi-precompactness, but it is a stronger form of P -closedness [3] and S^* -closedness [7].

2. Preliminaries. Throughout this note, X and Y will be nonempty ordinary sets, and $L = L(\leq, \vee, \wedge, ')$ will denote a fuzzy lattice, that is, a completely distributive lattice with a smallest element 0 and largest element 1 and with an order reversing involution $a \rightarrow a'$ ($a \in L$). We will denote by L^X the lattice of all L -fuzzy subsets of X .

DEFINITION 2.1 (Gierz et al. [6]). An element p of L is called prime if and only if $p \neq 1$, and whenever $a, b \in L$ with $a \wedge b \leq p$, then $a \leq p$ or $b \leq p$. The set of all prime elements of L will be denoted by $\text{pr}(L)$.

DEFINITION 2.2 (Gierz et al. [6]). An element α of L is called union irreducible if and only if whenever $a, b \in L$ with $\alpha \leq a \vee b$, then $\alpha \leq a$ or $\alpha \leq b$. The set of all nonzero union-irreducible elements of L will be denoted by $M(L)$. It is obvious that $p \in \text{pr}(L)$ if and only if $p' \in M(L)$.

Warner [9] has determined the prime element of the fuzzy lattice L^X . We have $\text{pr}(L^X) = \{x_p : x \in X \text{ and } p \in \text{pr}(L)\}$, where, for each $x \in X$ and each $p \in \text{pr}(L)$, $x_p : X \rightarrow L$ is the L -fuzzy set defined by

$$x_p(y) = \begin{cases} p & \text{if } y = x, \\ 1 & \text{otherwise.} \end{cases} \quad (2.1)$$

These x_p are called the L -fuzzy points of X , and we say that x_p is a member of an L -fuzzy set f and write $x_p \in f$ if and only if $f(x) \leq p$.

Thus, the union-irreducible elements of L^X are the function $x_\alpha : X \rightarrow L$ defined by

$$x_\alpha(y) = \begin{cases} \alpha & \text{if } y = x, \\ 0 & \text{otherwise,} \end{cases} \quad (2.2)$$

where $x \in X$ and $\alpha \in M(L)$. Hence, we have $M(L^X) = \{x_\alpha : x \in X \text{ and } \alpha \in M(L)\}$. As these x_α are identified with the L -fuzzy points x_p of X , we will refer to them as fuzzy points. When $x_\alpha \in M(L^X)$, we will call x and α the support of x_α ($x = \text{Supp } x_\alpha$) and the height of x_α ($\alpha = h(x_\alpha)$), respectively. We will denote L -fuzzy topological space by L -fts.

DEFINITION 2.3 (Zhao [10]). Let (X, δ) be an L -fts. A net in (X, δ) is a mapping $S : D \rightarrow M(L^X)$, where D is a directed set. For $m \in D$, we will denote $S(m)$ by S_m , and the net S by $(S_m)_{m \in D}$. If $A \in L^X$ and for each $m \in D$, $S_m \leq A$, then S is called a net in A . A net $(S_m)_{m \in D}$ is called an α -net ($\alpha \in M(L)$) if, for each $\lambda \in \beta^*(\alpha)$ (where $\beta^*(\alpha)$ denotes the union of all minimal sets relative to α), the net $h(S) = (h(S_m))_{m \in D}$ is eventually greater than λ , that is, for each $\lambda \in \beta^*(\alpha)$, there is $m_0 \in D$ such that $h(S_m) \geq \lambda$ whenever $m \geq m_0$, where $h(S_m)$ is the height of L -fuzzy point $S_m \in M(L^X)$. If $h(S_m) = \alpha$ for all $m \in D$, then we will say that $(S_m)_{m \in D}$ is a constant α -net.

DEFINITION 2.4 (Thakur and Singh [8]). Let (X, δ) be an L -fts and $f \in L^X$. Then, f is called semi-preopen if and only if there is a preopen set g [3, 5] such that $g \leq f \leq g^-$ and semi-preclosed if and only if f' is semi-preopen. $f_\square = \bigvee \{g : g \text{ is semi-preopen, } g \leq f\}$ and $f_\sim = \bigwedge \{g : g \text{ is semi-preclosed, } g \geq f\}$ are called the semi-preinterior and semi-preclosure of f , respectively.

It is clear that every semi-open L -fuzzy set is semi-preopen and every preopen L -fuzzy set is semi-preopen. None of the converses needs to be true [9].

DEFINITION 2.5 (Aygün [2]). Let (X, δ) be an L -fts and $g \in L^X$, $r \in L$. A collection $\mu = \{f_i\}_{i \in J}$ of L -fuzzy sets is called an r -level cover of g if and only if $(\bigvee_{i \in J} f_i)(x) \not\leq r$ for all $x \in X$ with $g(x) \geq r'$. If each f_i is open, then μ is called an r -level open cover of g . If g is the whole space X , then μ is called an r -level cover of X if and only if $(\bigvee_{i \in J} f_i)(x) \not\leq r$ for all $x \in X$. An r -level cover $\mu = \{f_i\}_{i \in J}$ of g is said to have a finite r -level subcover if there exists a finite subset F of J such that $(\bigvee_{i \in F} f_i)(x) \not\leq r$ for all $x \in X$ with $g(x) \geq r'$.

DEFINITION 2.6 (Bai [4]). Let (X, δ) be an L -fts and $g \in L^X$. We call g semi-precompact if and only if every p -level semi-preopen cover of g has a finite p -level subcover, where $p \in \text{pr}(L)$. If g is the whole space, then we say that the L -fts (X, δ) is semi-precompact.

3. SP-closedness

DEFINITION 3.1. Let (X, δ) be an L -fts and let $g \in L^X$, $r \in L$. An r -level cover $\mu = \{f_i\}_{i \in J}$ of g is said to have a finite r_\sim -level subcover if there exists a finite subset F of J such that $(\bigvee_{i \in F} (f_i)_\sim)(x) \not\leq r$ for all $x \in X$ with $g(x) \geq r'$.

DEFINITION 3.2. Let (X, δ) be an L -fts and let $g \in L^X$. We call g *SP-closed* if and only if every p -level semi-preopen cover of g has a finite p_* -level subcover, where $p \in \text{pr}(L)$. If g is the whole space, then we say that the L -fts (X, δ) is *SP-closed*.

THEOREM 3.3. Every semi-precompact set is *SP-closed* in an L -fts.

PROOF. This immediately follows from Definitions 2.6 and 3.2. \square

THEOREM 3.4. Every *SP-closed* set is not only *P-closed* [3] but also *S*-closed* [7] in an L -fts.

PROOF. Since every preopen L -fuzzy set is semi-preopen and every semiopen L -fuzzy set is semi-preopen, and since for every L -fuzzy set f we have $f_- \leq f^\sim$ and $f_- \leq f_-$, where $f^\sim = \bigwedge \{g : g \text{ is preclosed, } g \geq f\}$ and $f_- = \bigwedge \{g : g \text{ is semiclosed, } g \geq f\}$, this directly follows from the definitions of *SP-closedness*, *P-closedness*, and *S*-closedness*. \square

THEOREM 3.5. Let (X, δ) be an L -fts. Then, $g \in L^X$ is *SP-closed* if and only if, for every $\alpha \in M(L)$ and every collection $(h_i)_{i \in J}$ of semi-preclosed L -fuzzy sets with $(\bigwedge_{i \in J} h_i)(x) \not\geq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, there is a finite subset F of J such that $(\bigwedge_{i \in F} (h_i)^\square)(x) \not\geq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$.

PROOF. This follows immediately from Definition 3.2. \square

DEFINITION 3.6. Let (X, δ) be an L -fts, x_α be an L -fuzzy point in $M(L^X)$, and $S = (S_m)_{m \in D}$ be a net. We call x_α an *SP-cluster point* of S if and only if, for each semi-preclosed L -fuzzy set f with $f(x) \not\geq \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geq n$ and $S_m \not\leq f^\square$, that is, $h(S_m) \not\leq f^\square(\text{Supp } S_m)$.

THEOREM 3.7. Let (X, δ) be an L -fts. Then, $g \in L^X$ is *SP-closed* if and only if every constant α -net in g , where $\alpha \in M(L)$, has an *SP-cluster point* in g with height α .

PROOF

NECESSITY. Let $\alpha \in M(L)$ and $S = (S_m)_{m \in D}$ be a constant α -net in g without any *SP-cluster point* with height α in g . Then, for each $x \in X$ with $g(x) \geq \alpha$, x_α is not an *SP-cluster point* of S , that is, there are $n_x \in D$ and a semi-preclosed L -fuzzy set f_x with $f_x(x) \not\geq \alpha$ and $S_m \leq (f_x)^\square$ for each $m \geq n_x$. Let x^1, \dots, x^k be elements of X with $g(x^i) \geq \alpha$ for each $i \in \{1, \dots, k\}$. Then, there are $n_{x_1}, \dots, n_{x_k} \in D$, semi-preclosed L -fuzzy set f_{x_i} with $f_{x_i}(x^i) \not\geq \alpha$, and $S_m \leq (f_{x_i})^\square$ for each $m \geq n_{x_i}$ and for each $i \in \{1, \dots, k\}$. Since D is a directed set, there is $n_o \in D$ such that $n_o \geq n_{x_i}$ for each $i \in \{1, \dots, k\}$ and $S_m \leq (f_{x_i})^\square$ for $i \in \{1, \dots, k\}$ and each $m \geq n_o$. Now, consider the family $\mu = \{f_x\}_{x \in X}$ with $g(x) \geq \alpha$. Then, $(\bigwedge_{f_x \in \mu} f_x)(y) \not\geq \alpha$ for all $y \in X$ with $g(y) \geq \alpha$ because $f_y(y) \not\geq \alpha$. Also, for any finite subfamily $\nu = \{f_{x_1}, \dots, f_{x_k}\}$ of μ , there is $y \in X$ with $g(y) \geq \alpha$ and $(\bigwedge_{i=1}^k (f_{x_i})^\square)(y) \geq \alpha$ since $S_m \leq \bigwedge_{i=1}^k (f_{x_i})^\square$ for each $m \geq n_o$ because $S_m \leq (f_{x_i})^\square$ for each $i \in \{1, \dots, k\}$ and for each $m \geq n_o$. Hence, by Theorem 3.5, g is not *SP-closed*.

SUFFICIENCY. Suppose that g is not *SP-closed*. Then by Theorem 3.5, there exist $\alpha \in M(L)$ and a collection $\mu = (f_i)_{i \in J}$ of semi-preclosed L -fuzzy sets with $(\bigwedge_{i \in J} f_i)(x) \not\geq \alpha$ for all $x \in X$ with $g(x) \geq \alpha$, but for any finite subfamily ν of μ , there is $x \in X$ with

$g(x) \geq \alpha$ and $(\bigwedge_{f_i \in \nu} (f_i)_{\square})(x) \geq \alpha$. Consider the family of all finite subsets of μ , $2^{(\mu)}$, with the order $\nu_1 \leq \nu_2$ if and only if $\nu_1 \subset \nu_2$. Then $2^{(\mu)}$ is a directed set. So, writing x_{α} as S_{ν} for every $\nu \in 2^{(\mu)}$, $(S_{\nu})_{\nu \in 2^{(\mu)}}$ is a constant α -net in g because the height of S_{ν} for all $\nu \in 2^{(\mu)}$ is α and $S_{\nu} \leq g$ for all $\nu \in 2^{(\mu)}$, that is, $g(x) \geq \alpha$. Also, $(S_{\nu})_{\nu \in 2^{(\mu)}}$ satisfies the condition that for each semi-preclosed L -fuzzy set $f_i \in \nu$ we have $x_{\alpha} = S_{\nu} \leq (f_i)_{\square}$. Let $y \in X$ with $g(y) \geq \alpha$. Then $(\bigwedge_{i \in J} f_i)(y) \not\geq \alpha$, that is, there exists $j \in J$ with $f_j(y) \not\geq \alpha$. Let $\nu_o = \{f_j\}$. So, for any $\nu \geq \nu_o$,

$$S_{\nu} \leq \bigwedge_{f_i \in \nu} (f_i)_{\square} \leq \bigwedge_{f_i \in \nu_o} (f_i)_{\square} = (f_j)_{\square}. \quad (3.1)$$

Thus, we get a semi-preclosed L -fuzzy set f_j with $f_j(y) \geq \alpha$ and $\nu_o \in 2^{(\mu)}$ such that for any $\nu \geq \nu_o$, $S_{\nu} \leq (f_j)_{\square}$. That means that $y_{\alpha} \in M(L^X)$ is not an SP -cluster point $(S_{\nu})_{\nu \in 2^{(\mu)}}$ for all $y \in X$ with $g(y) \geq \alpha$. Hence, the constant α -net $(S_{\nu})_{\nu \in 2^{(\mu)}}$ has no SP -cluster point in g with height α . \square

COROLLARY 3.8. *An L -fts (X, δ) is SP -closed if and only if every constant α -net in (X, δ) has an SP -cluster point with height α , where $\alpha \in M(L)$.*

THEOREM 3.9. *Let (X, δ) be an L -fts and $g, h \in L^X$. If g and h are SP -closed, then $g \vee h$ is SP -closed as well.*

PROOF. Let $\{f_i\}_{i \in J}$ be a p -level semi-preopen cover of $g \vee h$, where $p \in \text{pr}(L)$. Then, $(\bigvee_{i \in J} f_i)(x) \not\leq p$ for all $x \in X$ with $(g \vee h)(x) \geq p'$. Since p is prime, we have $(g \vee h)(x) \geq p'$ if and only if $g(x) \geq p'$ or $h(x) \geq p'$. So, by the SP -closedness of g and h , there are finite subsets E, F of J such that $(\bigvee_{i \in E} (f_i)_{\sim})(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$ and $(\bigvee_{i \in F} (f_i)_{\sim})(x) \not\leq p$ for all $x \in X$ with $h(x) \geq p'$. Then, $(\bigvee_{i \in E \cup F} (f_i)_{\sim})(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$ or $h(x) \geq p'$, that is, $(\bigvee_{i \in E \cup F} (f_i)_{\sim})(x) \not\leq p$ for all $x \in X$ with $(g \vee h)(x) \geq p'$. Thus, $g \vee h$ is SP -closed. \square

THEOREM 3.10. *Let (X, δ) be an L -fts and $g, h \in L^X$. If g is SP -closed and h is semi-preclopen, then $g \wedge h$ is SP -closed.*

PROOF. Let $\{f_i\}_{i \in J}$ be a p -level semi-preopen cover of $g \wedge h$, where $p \in \text{pr}(L)$. Then, $(\bigvee_{i \in J} f_i)(x) \not\leq p$ for all $x \in X$ with $(g \wedge h)(x) \geq p'$. Thus, $\mu = \{f_i\}_{i \in J} \cup \{h'\}$ is a p -level semi-preopen cover of g . In fact, for each $x \in X$ with $g(x) \geq p'$, if $h(x) \geq p'$, then $(g \wedge h)(x) \geq p'$, which implies that $(\bigvee_{i \in J} f_i)(x) \not\leq p$, thus $(\bigvee_{k \in \mu} k)(x) \not\leq p$. If $h(x) \not\geq p'$ then $h'(x) \not\leq p$ which implies $(\bigvee_{k \in \mu} k)(x) \not\leq p$. From the SP -closedness of g , there is a finite subfamily ν of μ , say $\nu = \{f_1, \dots, f_n, h'\}$ with $(\bigvee_{k \in \nu} k_{\sim})(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. Then, $(\bigvee_{i=1}^n (f_i)_{\sim})(x) \not\leq p$ for all $x \in X$ with $(g \wedge h)(x) \geq p'$. In fact, if $(g \wedge h)(x) \geq p'$, then $g(x) \geq p'$, hence $(\bigvee_{k \in \nu} k_{\sim})(x) \not\leq p$. So, there is $k \in \nu$ such that $k_{\sim}(x) \not\leq p$. Moreover, $h(x) \geq p'$ as well, that is, $h'(x) \leq p$. Since h is semi-preopen, then h' is semi-preclosed, that is, $h' = (h')_{\sim}$. So, $h'(x) \leq p$ implies that $(h')_{\sim}(x) \leq p'$. Consequently, $(\bigvee_{i=1}^n (f_i)_{\sim})(x) \not\leq p$ for all $x \in X$ with $(g \wedge h)(x) \geq p'$. Hence, $g \wedge h$ is SP -closed. \square

COROLLARY 3.11. *Let (X, δ) be an SP -closed space and g be a semi-preclopen L -fuzzy set. Then g is SP -closed.*

DEFINITION 3.12. Let (X, δ) and (Y, τ) be L -fts's. A function $f : (X, \delta) \rightarrow (Y, \tau)$ is called

- (1) semi-preirresolute if and only if $f^{-1}(g)$ is semi-preopen in (X, δ) for each semi-preopen L -fuzzy set g in (Y, τ) ;
- (2) weakly semi-preirresolute if and only if $f^{-1}(g) \leq (f^{-1}(g_{\sim}))_{\square}$ for each semi-preopen L -fuzzy set g in (Y, τ) .

THEOREM 3.13. Let $f : (X, \delta) \rightarrow (Y, \tau)$ be a semi-preirresolute mapping with $f^{-1}(y)$ is finite for every $y \in Y$. If $g \in L^X$ is SP-closed in (X, δ) , then $f(g)$ is SP-closed in (Y, τ) as well.

PROOF. Let $\{f_i\}_{i \in J}$ be a p -level semi-preopen cover of $f(g)$, where $p \in \text{pr}(L)$. Because f is semi-preirresolute, $\{f^{-1}(f_i)\}_{i \in J}$ is a p -level semi-preopen cover of g . By the SP-closedness of g , $\{f^{-1}(f_i)\}_{i \in J}$ has a finite p_{\sim} -level subcover, that is, there is a finite subset F of J such that $(\bigvee_{i \in F} (f^{-1}(f_i)_{\sim}))_{\sim}(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$. We are going to show that $\{f_i\}_{i \in J}$ has a finite p_{\sim} -level subcover of $f(g)$, that is, $(\bigvee_{i \in F} (f_i)_{\sim})(y) \not\leq p$ for all $y \in Y$ with $f(g)(y) \geq p'$. Since $f^{-1}(y)$ is finite for every $y \in Y$, $f(g)(y) \geq p'$ implies that there is $x \in X$ with $g(x) \geq p'$ and $f(x) = y$. Again, f is semi-preirresolute. Thus, we have

$$\begin{aligned} \left(\bigvee_{i \in F} (f_i)_{\sim} \right) (y) &= \left(\bigvee_{i \in F} (f_i)_{\sim} \right) (f(x)) = \left(\bigvee_{i \in F} f^{-1}((f_i)_{\sim}) \right) (x) \\ &= \left(\bigvee_{i \in F} (f^{-1}((f_i)_{\sim}))_{\sim} \right) (x) \geq \left(\bigvee_{i \in F} (f^{-1}(f_i))_{\sim} \right) (x) \not\leq p. \end{aligned} \quad (3.2)$$

This has proved that $\{f_i\}_{i \in J}$ has a finite p_{\sim} -level subcover of $f(g)$. Hence, $f(g)$ is SP-closed. \square

THEOREM 3.14. Let $f : (X, \delta) \rightarrow (Y, \tau)$ be a weakly semi-preirresolute mapping with $f^{-1}(y)$ is finite for every $y \in Y$. If $g \in L^X$ is semi-precompact in (X, δ) , then $f(g)$ is SP-closed in (Y, τ) .

PROOF. Let $\{f_i\}_{i \in J}$ be a p -level semi-preopen cover of $f(g)$, where $p \in \text{pr}(L)$. Because f is weakly semi-preirresolute, for every $i \in J$, $f^{-1}(f_i) \leq (f^{-1}((f_i)_{\sim}))_{\square}$. Then, $\{(f^{-1}((f_i)_{\sim}))_{\square}\}_{i \in J}$ is a p -level semi-preopen cover of g . By the semi-precompactness of g , $\{(f^{-1}((f_i)_{\sim}))_{\square}\}_{i \in J}$ has a finite p -level subcover, that is, there is a finite subset F of J such that $(\bigvee_{i \in F} (f^{-1}((f_i)_{\sim}))_{\square})(x) \not\leq p$ for all $x \in X$ with $g(x) \geq p'$.

We are going to show that $\{f_i\}_{i \in J}$ has a finite p_{\sim} -level subcover of $f(g)$, that is, $(\bigvee_{i \in F} (f_i)_{\sim})(y) \not\leq p$ for all $y \in Y$ with $f(g)(y) \geq p'$. In fact, if $f(g)(y) \geq p'$ and since $f^{-1}(y)$ is finite for every $y \in Y$, there is $x \in X$ with $g(x) \geq p'$ and $f(x) = y$. So,

$$\begin{aligned} \left(\bigvee_{i \in F} (f_i)_{\sim} \right) (y) &= \left(\bigvee_{i \in F} (f_i)_{\sim} \right) (f(x)) = \left(\bigvee_{i \in F} f^{-1}((f_i)_{\sim}) \right) (x) \\ &\geq \left(\bigvee_{i \in F} (f^{-1}((f_i)_{\sim}))_{\square} \right) (x) \not\leq p. \end{aligned} \quad (3.3)$$

Hence, $f(g)$ is SP-closed. \square

ACKNOWLEDGMENT. This work is supported by the National Natural Science Foundation of China and the Provincial Natural Science Foundation of Guangdong.

REFERENCES

- [1] D. Andrijević, *Semipreopen sets*, Mat. Vesnik **38** (1986), no. 1, 24–32.
- [2] H. Akgün, α -*compactness in L-fuzzy topological spaces*, Fuzzy Sets and Systems **116** (2000), no. 3, 317–324.
- [3] H. Akgün and S. R. T. Kudri, *P-closedness in L-fuzzy topological spaces*, Fuzzy Sets and Systems **109** (2000), no. 2, 277–283.
- [4] S.-Z. Bai, *Semi-precompactness in L-fuzzy topological spaces*, in press.
- [5] ———, *The SR-compactness in L-fuzzy topological spaces*, Fuzzy Sets and Systems **87** (1997), no. 2, 219–225.
- [6] G. Gierz, K. H. Hofmann, K. Keimel, J. D. Lawson, M. W. Mislove, and D. S. Scott, *A Compendium of Continuous Lattices*, Springer-Verlag, Berlin, 1980.
- [7] S. R. T. Kudri, *Semicompactness and S^* -closedness in L-fuzzy topological spaces*, Fuzzy Sets and Systems **109** (2000), no. 2, 223–231.
- [8] S. S. Thakur and S. Singh, *On fuzzy semi-preopen sets and fuzzy semi-precontinuity*, Fuzzy Sets and Systems **98** (1998), no. 3, 383–391.
- [9] M. W. Warner, *Frame-fuzzy points and membership*, Fuzzy Sets and Systems **42** (1991), no. 3, 335–344.
- [10] D. S. Zhao, *The N-compactness in L-fuzzy topological spaces*, J. Math. Anal. Appl. **128** (1987), no. 1, 64–79.

BAI SHI-ZHONG: DEPARTMENT OF MATHEMATICS, WUYI UNIVERSITY, JIANGMEN GUANGDONG 529020, CHINA

Special Issue on Boundary Value Problems on Time Scales

Call for Papers

The study of dynamic equations on a time scale goes back to its founder Stefan Hilger (1988), and is a new area of still fairly theoretical exploration in mathematics. Motivating the subject is the notion that dynamic equations on time scales can build bridges between continuous and discrete mathematics; moreover, it often reveals the reasons for the discrepancies between two theories.

In recent years, the study of dynamic equations has led to several important applications, for example, in the study of insect population models, neural network, heat transfer, and epidemic models. This special issue will contain new researches and survey articles on Boundary Value Problems on Time Scales. In particular, it will focus on the following topics:

- Existence, uniqueness, and multiplicity of solutions
- Comparison principles
- Variational methods
- Mathematical models
- Biological and medical applications
- Numerical and simulation applications

Before submission authors should carefully read over the journal's Author Guidelines, which are located at <http://www.hindawi.com/journals/ade/guidelines.html>. Authors should follow the Advances in Difference Equations manuscript format described at the journal site <http://www.hindawi.com/journals/ade/>. Articles published in this Special Issue shall be subject to a reduced Article Processing Charge of €200 per article. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at <http://mts.hindawi.com/> according to the following timetable:

Manuscript Due	April 1, 2009
First Round of Reviews	July 1, 2009
Publication Date	October 1, 2009

Lead Guest Editor

Alberto Cabada, Departamento de Análise Matemática, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain; alberto.cabada@usc.es

Guest Editor

Victoria Otero-Espinar, Departamento de Análise Matemática, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain; mvictoria.oter@usc.es