

A NOTE ON A PAPER BY BRENNER

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We note that a result of Brenner (1962) follows from a theorem of Lerch (1896) which also extends it.

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Let m and n be relatively prime integers with $n \geq 2$. Let \sim be the equivalence relation on the set $S = (\mathbb{Z}/n\mathbb{Z}) \setminus \{0\}$ given by $t_1 \sim t_2$ if and only if there exists an integer k such that $m^k t_1 = t_2$. Denote by N the number of equivalence classes. Brenner proved the following result [1].

THEOREM 1. *If n is odd, then $(-1)^N$ equals the Jacobi symbol (m/n) .*

The purpose of this note is to point out that the above result is a consequence of a theorem of Lerch [3] dating back to 1896, which, moreover, extends [Theorem 1](#) to the case of even n .

THEOREM 2 (Lerch). *For relatively prime integers m and n , with $n \geq 2$, the sign of the permutation π induced by multiplication by m on $(\mathbb{Z}/n\mathbb{Z}) \setminus \{0\}$ equals*

- (a) *the Jacobi symbol (m/n) if n is odd;*
- (b) *1 if n is even and not divisible by 4;*
- (c) *$(-1)^{(m-1)/2}$ if n is divisible by 4.*

Observe that N is the number of cycles τ_1, \dots, τ_N in the decomposition of π into a product of disjoint cycles (1-cycles need to be included). Now if l_i is the length of τ_i , then the sign of τ_i equals $(-1)^{l_i-1}$, so, if n is odd, the sign of π equals

$$(-1)^{\sum_{i=1}^N (l_i-1)} = (-1)^{n-1-N} = (-1)^N. \quad (1)$$

Thus [Theorem 1](#) follows from [Theorem 2](#), as does the following extension.

COROLLARY 3. *For n even $(-1)^N$ equals -1 , if $n \equiv 2 \pmod{4}$, and $(-1)^{(m+1)/2}$, if $n \equiv 0 \pmod{4}$.*

Lerch's theorem, which generalizes a result of Zolotareff [4] on the Legendre symbol, considerably simplifies the theory of quadratic residues (see, e.g., [2]) and deserves to be more widely known.

REFERENCES

- [1] J. L. Brenner, *A new property of the Jacobi symbol*, Duke Math. J. 29 (1962), 29-31.
- [2] F. Hirzebruch and D. Zagier, *The Atiyah-Singer Theorem and Elementary Number Theory*, Mathematics Lecture Series, no. 3, Publish or Perish, Massachusetts, 1974.

- [3] M. Lerch, *Sur un théorème de Zolotarev*, Bull. Intern. de l'Acad. François Joseph 3 (1896), 34–37 (French).
- [4] G. Zolotareff, *Nouvelle démonstration de la loi de réciprocité de Legendre*, Nouvelles Annales de Math. **11** (1872), no. 2, 354–362 (French).

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