

## **NONLINEAR INTERACTION OF WAVES IN A HOT INHOMOGENEOUS MAGNETIZED PLASMA**

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**ABSTRACT.** This study is concerned with the theory of parametric coupling of waves in a hot inhomogeneous magnetized plasma in which the temperature gradient has been taken into account. The general dispersion relation and the polarization of the ordinary and the extra-ordinary wave modes are discussed. The eigen-mode solutions of the coupled differential equations for the wave amplitudes are obtained in the terms of the so called three wave interaction matrix elements. The theory of nonlinear wave-wave interactions, which has been extended to the case of an inhomogeneous magnetized plasma, is used to determine the threshold value of the electric field and the frequency shift. The results of this paper are also compared

with the other known results. It is shown that the findings of this study are in excellent agreement with the results of earlier investigators.

KEY WORDS AND PHRASES. *Parametric instability, Waves in inhomogeneous plasma, wave-wave interactions, Threshold electric field and frequency shift.*

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## 1. INTRODUCTION.

In recent years there has been considerable interest in the theory of parametric instability in an inhomogeneous plasma [1-8] because of its fundamental role in the study of weak plasma turbulence, and of many important physical applications. In their recent research-expository and survey article on parametric phenomena in a plasma, Galeev and Sagdeev [9] reviewed the theory of parametric instabilities in an inhomogeneous plasma using the generalized Mathieu equation as a model equation. They have also presented the latest advances of the nonlinear theories of parametric instabilities based on the ideas of weakly turbulent plasma. Perkins and Flick [10] have made an interesting study of parametric instabilities in an inhomogeneous plasmas, and then calculated the threshold electric field. It is shown that the value of the threshold electric field increases in an inhomogeneous plasma because energy propagates away from the unstable region by electron plasma waves. Thus an additional energy loss occurs and is solely responsible for the increase of the threshold electric field not observed in a homogeneous plasma. Eubank [11] experimentally confirmed the theoretical prediction of Perkins and Flick. In a recent paper, Kroll, Ron and Rostoker [12] have suggested that the nonlinear resonance of two transverse electromagnetic waves whose frequencies differ slightly by the electron plasma frequency can be applied to excite longitudinal electron plasma oscillations. Montgomery [13] observed certain mathematical inaccuracy and physical limitations of the work of Kroll, Ron and Rostoker, and then analyzed the problem of nonlinear wave interactions in plasma with laser beams.

Using the perturbation method of Krylov-Bogliubov-Mitropolskii, Montgomery obtained the amplitude-dependent frequency shift and wave number shift with physical significance. It was shown that the resonant excitation of longitudinal plasma oscillations is possible by the transverse electromagnetic waves. Etievant, Ossakow, Ozizmir and Su [14] have investigated the nonlinear wave-wave interactions of electromagnetic waves in an infinite homogeneous plasma. It is interesting and important to take into account the effects of density and temperature gradients on the above problem.

The present study deals with the theory of parametric coupling of waves in a hot inhomogeneous magnetized plasma in which the temperature gradient has been taken into consideration. The general dispersion relation and the polarization of two different wave modes are investigated. The eigen-mode solutions of the coupled equations for the wave amplitudes are obtained in terms of the three wave interaction matrix elements. The theory of nonlinear wave-wave interactions is used to determine the threshold electric field and the frequency shift. The results of this analysis are found to be in excellent agreement with those of earlier workers.

## 2. BASIC EQUATIONS FOR TWO PLASMA MODEL.

In two plasma model, the equations of motion and the continuity equation for each kind of component are in the usual notations given by

$$\frac{Dv_{\alpha}}{Dt} = \frac{Z_{\alpha} e}{m_{\alpha}} \underline{E} - \frac{\kappa}{m_{\alpha}} \frac{\nabla(T_{\alpha} \rho_{\alpha})}{\rho_{\alpha}} - Z_{\alpha} \frac{\Omega_{\alpha}}{\omega_{\alpha}} \times \underline{v}_{\alpha} - \nu_{\alpha} \underline{v}_{\alpha}, \quad (2.1)$$

$$\frac{D\rho_{\alpha}}{Dt} + \rho_{\alpha} \nabla \cdot \underline{v}_{\alpha} = 0, \quad (2.2)$$

where  $\Omega_{\alpha} = \frac{eB_0}{cm_{\alpha}}$  is the cyclotron frequency,  $\nu_{\alpha}$  is the collision frequency of each

component and  $\underline{B} = B_0 \hat{z}$  is the external magnetic field. The subscript  $\alpha$  stands for  $e$  or  $i$  corresponding to electrons and ions respectively,  $Z_\alpha e$  is the charge of an electron or ion and  $\kappa$  is the Boltzmann Constant.

The Maxwell equations are

$$\text{curl } \underline{E} = -\frac{1}{c} \frac{\partial \underline{B}}{\partial t}, \quad \text{div } \underline{E} = -4\pi e(n-n_0), \quad (2.3ab)$$

$$\text{curl } \underline{B} = \frac{1}{c} \frac{\partial \underline{E}}{\partial t} - \frac{4\pi}{c} ne v_\alpha, \quad \text{div } \underline{B} = 0, \quad (2.4ab)$$

We consider a plasma model whose density distribution varies as  $\rho_\alpha = \rho_{\alpha 0}(1-x\delta)$  with the density gradient  $\delta = \frac{1}{\rho_{\alpha 0}} \left( \frac{d\rho_\alpha}{dx} \right)$  and neglect the time dependence of  $\rho_{\alpha 0}$  due to any external electric field. The zeroth order solution of (2.1) gives

$$\langle v_{ox}(t) \rangle_e = \frac{\kappa T_e}{m_e} \frac{\delta v_e}{(\Omega_e^2 - v_e^2)}, \quad (2.5)$$

$$\langle v_{oy}(t) \rangle_e = \frac{\kappa T_e}{m_e} \frac{\delta \Omega_e Z_e}{(\Omega_e^2 - v_e^2)}, \quad (2.6)$$

with 
$$\underline{\delta} = \delta \hat{x} = \left( \frac{\nabla \rho_0}{\rho_0} + \frac{\nabla T_0}{T_0} \right) \quad (2.7)$$

### 3. THE FIRST ORDER EQUATIONS.

In the first order approximation, equations (2.1) - (2.2) can be written as, dropping the subscript  $\alpha$ ,

$$\frac{\partial v_1}{\partial t} + \underline{v}_0 \cdot \nabla v_1 + \underline{v}_1 \cdot \nabla v_0 = \frac{Ze}{m} \underline{E}_0 - Z\Omega_0 \times \underline{v}_1 - \frac{\kappa}{m} \frac{\nabla(T\rho)}{\rho} - Z\Omega_1 \times \underline{v}_0 - \underline{v}v_1, \quad (3.1)$$

where  $\left( \frac{\nabla T\rho}{\rho} \right)_1$  represents the first order terms in  $\left( \frac{\nabla T\rho}{\rho} \right)$ .

Assuming the density gradient is small, we take the space and time dependent of the first order quantities as  $\exp[i(\underline{k} \cdot \underline{r} - \omega t)]$ . The equation (3.1) can then readily be solved to obtain

$$\underline{v}_1 = \underline{\mu} \cdot \underline{E}_1, \tag{3.2}$$

where  $\underline{\mu}$  is the mobility tensor given by

$$\underline{\mu}_\alpha = \frac{Ze}{m} \underline{\mu}_c \cdot \left[ \left(1 - \frac{\underline{k} \cdot \underline{v}_0}{\bar{\omega}}\right) \mathbf{I} + \frac{\underline{k} \cdot \underline{v}_0}{\omega} \right], \tag{3.3}$$

when  $\mathbf{I}$  is the unit dyadic and  $\underline{\mu}_c$  is

$$\underline{\mu}_c = \frac{1}{\Delta} \left[ (\bar{\omega} - \underline{k} \cdot \underline{v}_0) (\hat{x}\hat{x} + \hat{y}\hat{y}) - \tilde{G}\hat{y}\hat{y} \right] + \frac{\Delta\hat{z}\hat{z}}{(\bar{\omega} - \underline{k} \cdot \underline{v}_0)} + iz\Omega_0\hat{x}\hat{y} - iz\Omega_0\hat{y}\hat{x}, \tag{3.4}$$

with

$$\tilde{G} = \frac{\kappa T}{m(\bar{\omega} - \underline{k} \cdot \underline{v}_0)} \left\{ \gamma k^2 + 2i(1-\gamma)k \delta \right\}, \tag{3.5}$$

$$\Delta = (\bar{\omega} - \underline{k} \cdot \underline{v}_0)^2 - (\bar{\omega} - \underline{k} \cdot \underline{v}_0)\tilde{G} - \Omega_0^2, \tag{3.6}$$

$$\bar{\omega} = \omega + i\nu$$

and  $\gamma$  is the ratio of the two specific heats.

Using equations (3.2) and taking the Maxwell equations (2.3ab) - (2.4ab), it turns out that

$$\mathbf{D} \cdot \underline{E}_1 = 0, \tag{3.7}$$

with

$$\mathbf{D} = \left( k^2 - \frac{\omega^2}{c^2} \right) \mathbf{I} - \hat{k}\hat{k} - \frac{i\omega}{c^2} \sum \omega_\alpha^2 \left( \mathbf{I} + \frac{\underline{v}_{\alpha 0} \underline{k}}{\omega} \right) \cdot \underline{\mu}_c, \tag{3.8}$$

where

$$\omega_\alpha^2 = \frac{4\pi e p_\alpha}{m_a}, \tag{3.9}$$

In the case of wave propagation along with x-direction ( $\underline{k} = k\hat{x}$ ), we obtain

the dispersion relation

$$\text{Det } D(\mathbf{k}, \omega) = 0, \tag{3.10}$$

There are two independent solutions of equation (3.10) in the form

$$D_{zz} = 0, \tag{3.11}$$

which corresponds to the ordinary wave, and

$$D_{xx} D_{yy} - D_{xy} D_{yx} = 0, \tag{3.12}$$

which corresponds to the extra-ordinary wave, where  $D_{xx}, D_{yy}, \dots$  are the elements of the matrix  $D$ .

The first order fields for the different modes (ordinary and extra-ordinary) are obtained as

$$\underline{E}_o = \hat{a}_o A_o \exp[i(\underline{k}_o \cdot \underline{r} - \omega_o t)], \tag{3.13}$$

$$\underline{E}_e = \hat{a}_e A_e \exp[i(\underline{k}_e \cdot \underline{r} - \omega_e t)], \tag{3.14}$$

with 
$$a_o = \hat{z}, \quad a_e = (i b \hat{x} + \hat{y}) / (1 + b^2)^{\frac{1}{2}}, \tag{3.15ab}$$

and 
$$b = -i \frac{E_x}{E_y} = \frac{i D_{yy}}{D_{yx}}, \tag{3.16}$$

where

$$D_{yy} = \left( k^2 - \frac{\omega^2}{c^2} \right) + \sum_{\alpha} \frac{\omega_{\alpha}^2 \omega}{c^2 \Delta} \left[ (\bar{\omega} - \underline{k} \cdot \underline{v}_o - \tilde{G}) + \frac{iz\Omega_o k v_{oy}}{\omega} \right] \tag{3.17}$$

and 
$$D_{yx} = \sum_{\alpha} \frac{\omega_{\alpha}^2}{c^2} \left[ \frac{(\bar{\omega} - \underline{k} \cdot \underline{v}_o) k v_{oy}}{\omega \Delta} - z\Omega_o \right] \tag{3.18}$$

4. THE SECOND ORDER EQUATIONS.

In the second order approximation, we obtain from equations (2.1) - (2.2)

$$\frac{\partial \underline{v}_2}{\partial t} + \underline{v}_0 \cdot \nabla \underline{v}_2 + \underline{v}_1 \cdot \nabla \underline{v}_1 + \underline{v}_2 \cdot \nabla \underline{v}_0 = \frac{Ze}{m} \underline{E}_2 - \underline{v} \underline{v}_2 - Z \underline{\Omega}_0 \times \underline{v}_2 - Z \underline{\Omega}_1 \times \underline{v}_1 - Z \underline{\Omega}_2 \times \underline{v}_0 + \frac{\kappa}{m} \left( \frac{\nabla T \rho}{\rho} \right)_2, \tag{4.1}$$

with

$$\frac{\kappa}{m} \left( \frac{\nabla T \rho}{\rho} \right)_2 = \frac{i \kappa T}{m} \left[ \{ \gamma \underline{k} - i(\gamma - 2) \underline{\delta} \} \frac{\rho_2}{\rho_0} + \frac{\gamma - 1}{2} \{ \gamma \underline{k} - i(\gamma - 2) \underline{\delta} \} \frac{\rho_1^2}{\rho_0^2} + \frac{i \gamma \rho_1 \nabla \rho_1}{\rho_0^2} \right], \tag{4.2}$$

$$\frac{\rho_2}{\rho_0} = \left[ \frac{\rho_0}{\rho_1} (\underline{k} \cdot \underline{v}_1) + (\underline{k} - i \underline{\delta}) \cdot \underline{v}_2 \right] \div (\omega - \underline{k} \cdot \underline{v}_0), \tag{4.3}$$

Using the space and time dependence of all second order quantities in the form  $\exp[i(\underline{k} \cdot \underline{r} - \omega t)]$ , we find

$$\underline{v}_2 = \frac{Ze}{m} \mu_c \cdot \left[ \left( 1 - \frac{\underline{k} \cdot \underline{v}_0}{\omega} \right) \underline{I} + \frac{\underline{k} \cdot \underline{v}_0}{\omega} \right] \cdot \underline{E}_2 + \mu_c \cdot \underline{H}, \tag{4.4}$$

with  $\underline{H} = -\underline{v}_1 \cdot \nabla \underline{v}_1 - \underline{\Omega}_1 \times \underline{v}_1 - i Q,$  (4.5)

$$Q = \frac{\kappa T}{m} \left[ \frac{\rho_1 (\underline{k} \cdot \underline{v}_1) (\gamma \underline{k} - i(\gamma - 2) \underline{\delta})}{\rho_0 (\omega - \underline{k} \cdot \underline{v}_0)} + \frac{\gamma - 1}{2} \{ \gamma \underline{k} - i(\gamma - 2) \underline{\delta} \} \frac{\rho_1^2}{\rho_0^2} + i \gamma \rho_1 \frac{\nabla \rho_1}{\rho_0} \right], \tag{4.6}$$

Using the Maxwell equations and eliminating the second order magnetic field  $B_2$ , it follows that

$$\nabla \times (\nabla \times \underline{E}_2) = \frac{1}{c^2} \frac{\partial^2 \underline{E}_2}{\partial t^2} = - \frac{4\pi}{c^2} \frac{\partial \underline{J}_2}{\partial t}, \tag{4.7}$$

whence  $\underline{J}_2 = \rho_0 \underline{v}_2 + \rho_2 \underline{v}_0 + \rho_1 \underline{v}_1,$  (4.8)

Substituting the values of  $\underline{v}_2$  in equation (4.4), we obtain

$$\begin{aligned} \nabla \times (\nabla \times \underline{E}_2) + \frac{1}{c^2} \frac{\partial^2 \underline{E}_2}{\partial t^2} + \frac{4\pi}{c^2} \sum_{\alpha} \left( \rho_{\alpha 0} \mu_{\alpha} + \frac{\rho_{\alpha 0} \underline{v}_{\alpha 0} \underline{k} \cdot \mu_{\alpha}}{\omega - \underline{k} \cdot \underline{v}_{\alpha 0}} \right) \frac{\partial \underline{E}_2}{\partial t} \\ = \frac{4\pi i \omega}{c^2} \sum_{\alpha} \rho_{\alpha 0} \left( \underline{I} + \frac{\underline{v}_{\alpha 0} \underline{k}}{\omega} \right) \cdot \left( \mu_c \cdot \underline{H}_{\alpha} + \frac{\underline{v}_{\alpha 1} \rho_{\alpha 1}}{\rho_{\alpha 0}} \right), \end{aligned} \tag{4.9}$$

## 5. EQUATION FOR THE INTERACTING MODES.

We assume a plane wave solution with an amplitude varying in the direction of propagation and write

$$\underline{E}_2 = \hat{a}_\omega A_\omega(\mathbf{x}, t) \exp[i(\mathbf{k} \cdot \underline{\mathbf{r}} - \omega t)] , \quad (5.1)$$

where  $A_\omega$  is a slowly varying function of  $\mathbf{x}$  and  $t$  because the nonlinearity is assumed to be weak.

We next drop the second derivatives of  $A$  in comparison to  $k^2 A$  and  $\omega^2 A$ , and then use the linear dispersion relation  $D \cdot \underline{E}_2 = 0$  to obtain the final result

$$(u_1 \frac{\partial}{\partial \mathbf{x}} + \frac{\partial}{\partial t}) A_1 = A_3 A_2 V_{123} (\omega_1 |\omega_2| \omega_3) \exp\{i(\mathbf{x} \Delta \underline{\mathbf{k}} - t \Delta \omega)\} , \quad (5.2)$$

$$(u_2 \frac{\partial}{\partial \mathbf{x}} + \frac{\partial}{\partial t}) A_2 = A_1 A_3 V_{213} (-\omega_2 |-\omega_1| \omega_3) \exp\{i(-\mathbf{x} \Delta \underline{\mathbf{k}} + t \Delta \omega)\} , \quad (5.3)$$

$$(u_3 \frac{\partial}{\partial \mathbf{x}} + \frac{\partial}{\partial t}) A_3 = A_1 A_2 V_{321} (-\omega_3 |\omega_2| -\omega_1) \exp\{i(-\mathbf{x} \Delta \underline{\mathbf{k}} + t \Delta \omega)\} , \quad (5.4)$$

$$\text{where } \Delta \underline{\mathbf{k}} = \underline{\mathbf{k}}_3 - (\underline{\mathbf{k}}_2 + \underline{\mathbf{k}}_1) \quad \text{and} \quad \Delta \omega = \omega_3 - (\omega_1 + \omega_2) , \quad (5.5ab)$$

We write the matrix elements for  $\hat{a}_1 = \hat{a}_3 = a_o$  and  $\hat{a}_2 = a_e$  where  $a_o$  and  $a_e$  are defined in (3.15ab).

$$\begin{aligned} V_{123} &= 2\pi i \int_{\alpha} \rho_{\alpha o} \{-i(\hat{a}_1^* \cdot \mu_c' \cdot \mu_\alpha^3 \cdot \hat{a}_3)(\hat{k}_3 \cdot \mu_\alpha^2 \cdot \hat{a}_2) - \frac{Ze}{m_\alpha \omega_3} \\ &\quad \cdot (\hat{a}_1^* \cdot \mu_c' \cdot \hat{a}_3)(\underline{\mathbf{k}}_3 \cdot \mu_\alpha^2 \cdot \hat{a}_2) + \frac{(\hat{a}_1^* \cdot \mu_\alpha^3 \cdot \hat{a}_3)(\underline{\mathbf{k}}_2 - i\delta) \cdot \mu_\alpha^2 \cdot \hat{a}_2}{(\omega_2 - \underline{\mathbf{k}}_2 \cdot \underline{\mathbf{v}}_{\alpha o})} \\ &= \frac{1}{2} \int_{\alpha} \frac{Ze \omega_\alpha^2 (k_2 - i\delta)}{m_\alpha (\omega_3 - \underline{\mathbf{k}}_3 \cdot \underline{\mathbf{v}}_{\alpha o}) (\omega_2 - \underline{\mathbf{k}}_2 \cdot \underline{\mathbf{v}}_{\alpha o}) \Delta_2} \left[ \left(1 + \frac{k_2 v_{\alpha o x}}{\omega_2}\right) (\bar{\omega}_2 - k_2 v_{\alpha o x}) \frac{b_2}{\sqrt{1+b_2^2}} \right. \\ &\quad \left. + \left(Z \Omega_o + \frac{(\omega_2 - \underline{\mathbf{k}}_2 \cdot \underline{\mathbf{v}}_{\alpha o}) k_2 v_{\alpha o y}}{\omega_2}\right) \frac{1}{\sqrt{1+b_2^2}} \right] , \quad (5.6) \end{aligned}$$

$V_{321}$  is obtained by interchanging  $\omega_1$  and  $\omega_3$  in  $V_{123}$  and

$$\begin{aligned}
 V_{231} &= 2\pi i \int_{\alpha} \frac{\rho_{\alpha 0} Ze}{m_{\alpha}} \left[ (\hat{a}_2^* + \frac{\hat{a}_2^* \cdot V_{\alpha 0} k_2}{\omega_2}) \cdot \mu_c^2 \left\{ \frac{k_3}{\omega_3} (\hat{a}_3 \cdot \mu_{\alpha}' \cdot \hat{a}_1) + \frac{k_1}{\omega_1} (\hat{a}_1 \cdot \mu_{\alpha}^3 \cdot \hat{a}_3) \right\} \right] \\
 &= \frac{1}{2} \int_{\alpha} \frac{\omega_{\alpha}^2}{\Delta_2} \frac{Ze}{m_{\alpha}} \left( \frac{(\bar{\omega}_2 - k_2 V_{\alpha 0}) b_2 + Z\Omega_0}{\sqrt{1 + b_2^2}} \right) \left[ \frac{k_3}{\omega_3 (\bar{\omega}_1 - k_1 V_{\alpha 0})} + \frac{k_1}{\omega_1 (\bar{\omega}_3 - k_3 V_{\alpha 0})} \right] \quad (5.7)
 \end{aligned}$$

We consider  $\omega, k, \Delta k$  real and  $A_3$  is the pump of fixed amplitude. The small amplitudes are described by equations (5.2) - (5.4). We take  $A$ 's to be space independent so that

$$A_1 = \left[ \exp \left\{ i \int_0^t \frac{1}{2} \Delta \omega dt \right\} \right] \psi^* \quad (5.8)$$

and then we find

$$\left[ \frac{\partial^2 \psi}{\partial t^2} + i \Delta \omega \frac{\partial \psi}{\partial t} + \frac{(\Delta \omega)^2 \psi}{4} \right] = v_{\alpha}^2 \psi \quad (5.9)$$

where 
$$v_{\alpha}^2 = (V_{123} V_{213} A_3^2) \quad (5.10)$$

Neglecting  $i \Delta \omega \frac{\partial \psi}{\partial t}$ , it follows that the solution of (5.9) by W.K.B. method assumes the form

$$\psi = \frac{1}{\sqrt{g}} \left[ A_1 \exp \left\{ i \int_0^t g(t) dt \right\} + B_2 \exp \left\{ -i \int_0^t g(t) dt \right\} \right] \quad (5.11)$$

with

$$g = \left\{ V_{123} V_{213} A_3^2 - \frac{(\Delta \omega)^2}{4} \right\}^{\frac{1}{2}} \quad (5.12)$$

When the wave packet drifts along  $x$  the time increment  $dt$  can be written as  $dt = (dx) / \left( \frac{\partial \omega}{\partial k} \right)$  to obtain

$$A_1 \sim \left[ \exp \int_0^{x_t} g(x) dx \right] / \left( \frac{\partial \omega}{\partial k} \right) \quad (5.13)$$

where the integration is limited to the instability zone. It is interesting to note that the threshold value of the wave can be calculated when  $g < 0$

$$A_3 = \frac{\Delta\omega}{2\sqrt{V_{123}V_{213}}} \quad , \quad (5.14)$$

Thus the value of the frequency shift can be obtained from (5.14) as

$$\Delta\omega = 2A_3 \sqrt{V_{123}V_{213}} \approx \frac{A_3 \omega_e^2 e}{m\Delta_2 \sqrt{1+b_2^2}} \{(\omega_2 - k_2 V_{ox}) b_2 - \Omega_0\} \left[ 1 + \frac{k_2 (\omega_2 - k_2 V_{ox}) (b_2 V_{ox} + V_{oy})}{\omega_2 \{(\omega_2 - k_2 V_{ox}) b_2 - \Omega_0\}} \right]^{\frac{1}{2}} \\ \times \left[ \frac{1}{(\omega_3 - k_3 V_{ox}) (\omega_2 - k_2 V_{ox})} \left\{ \frac{k_3}{\omega_3 (\omega_1 - k_1 V_{ox})} + \frac{k_1}{\omega_1 (\omega_3 - k_3 V_{ox})} \right\} \right]^{\frac{1}{2}} \quad (5.15)$$

where the assumption  $\frac{\omega_e^2}{m} \gg \frac{\omega_i^2}{\mu}$  is invoked.

With the following numerical values

$$\begin{aligned} \delta &= 0 & \Omega_0 &= 0 \\ \omega_1 = \omega_3 &= 2.7 \times 10^{15} \text{ sec}^{-1} \\ \omega_2 = \omega_e &= 5.6 \times 10^{11} \text{ sec}^{-1} \\ k_3 - k_2 &= k_1 = 19 \text{ cm}^{-1} \\ a_1, a_3 &\approx \frac{10^6}{3} \text{ e. s. u/cm} ; \end{aligned}$$

it turns out that  $\Delta\omega \approx 10^8 \text{ sec}^{-1}$  which is in good agreement with that of Montgomery [13]

Similarly, considering the space dependence only, one can calculate the wave number shift from equations (5.2) - (5.4).

## 6. DISCUSSION.

The general features of density gradient and magnetic field in the nonlinear interactions of plasma oscillations have been investigated. This is important in connection with its use as an optical density probe suggested by Kroll, Ron and

Rostoker [13] or as a controlled source of plasma oscillations conceived by Montgomery [13] the frequency shift will deviate with the increased value of the density gradient. However, strong magnetic field will have only influence on the frequency shift.

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