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# THE BOOLEAN ALGEBRA AND CENTRAL GALOIS ALGEBRAS

**GEORGE SZETO and LIANYONG XUE**

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ABSTRACT. Let $B$ be a Galois algebra with Galois group $G$, $J_g = \{b \in B \mid bx = g(x)b$ for all $x \in B\}$ for $g \in G$, and $BJ_g = Be_g$ for a central idempotent $e_g$. Then a relation is given between the set of elements in the Boolean algebra $(B_a, \leq)$ generated by $\{0, e_g \mid g \in G\}$ and a set of subgroups of $G$, and a central Galois algebra $Be$ with a Galois subgroup of $G$ is characterized for an $e \in B_a$.

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**1. Introduction.** Galois theory of rings have been intensively studied [1, 3, 4, 5, 6, 7]. Let $B$ be a Galois algebra with Galois group $G$ and $J_g = \{b \in B \mid bx = g(x)b$ for all $x \in B\}$ for each $g \in G$. In [4], it was shown that $BJ_g = Be_g$ for some central idempotent $e_g$ of $B$. Let $B_a$ be the Boolean algebra generated by $\{0, e_g \mid g \in G\}$. In [7], the following structure theorem for $B$ was given: there exist $\{e_i \in B_a \mid i = 1, 2, \ldots, m$ for some integer $m\}$ and some subgroups $H_i$ of G such that $B = \oplus \sum_{i=1}^{m} Be_i \oplus B(1 - \sum_{i=1}^{m} e_i)$ where $Be_i$ is a central Galois algebra with Galois group $H_i$ for each $i = 1, 2, \ldots, m$ and $B(1 - \sum_{i=1}^{m} e_i) = C(1 - \sum_{i=1}^{m} e_i)$ which is a commutative Galois algebra with Galois group induced by and isomorphic with $G$ in case $1 \neq \sum_{i=1}^{m} e_i$, where $C$ is the center of $B$. We observe that (1) $e_i = \Pi_{h \in H_i} e_h$ which is a nonzero monomial in $B_a$ for a maximal subset $H_i$ of $G$, (2) $H_i$ is a subgroup of $G$, and (3) $Be_i$ is a central Galois algebra with Galois group $H_i$. In the present paper, we will discuss a general case: what kind of elements $e$ in $B_a$ and subgroups $H_e$ give a central Galois algebra $Be$ with Galois group $H_e$? We will show that (1) for any nonzero monomial $e = \Pi_{g \in S} e_g$ of $B_a$ for some subset $S$ of $G$, let $H_e = \{g \in G \mid e \leq e_g$, that is, $ee_g = e\}$; then $H_e$ is a subgroup of $G$, (2) when $H_e \neq \{1\}$, $Be$ is a central Galois algebra with Galois group $H_e$ if and only if $e$ is a nonzero minimal element in $B_a$ (i.e., $Be$ is one of the components of $B$ as given in [7, Theorem 3.8]), (3) for a nonzero monomial $e = \Pi_{g \in S} e_g$ of $B_a$ for some subset $S$ of $G$, let $T_e = \{g \in G \mid e = e_g\}$; then $T_e$ is a subgroup of $G$ if and only if $e = 1$, and (4) let $H_1 = \{g \in G \mid e_g = 1\}$. Then $e_g = 0$ for each $g \notin H_1$ if and only if $B$ is either a central Galois algebra with Galois group $H_1$ or a commutative Galois algebra with Galois group $G$. Thus, $\{Be \mid e$ is a nonzero minimal element in $B_a\}$ are the only central Galois algebras with Galois group $H_e$ arising from nonzero monomials $e$ in $B_a$, and when $B_a = \{0, 1\}$, $B$ is a central Galois algebra with Galois group $H_1$ and the center $C$ is a commutative Galois algebra with Galois group $G/H_1$. This fact generalizes the DeMeyer theorem for a Galois algebra with an indecomposable center $C$ (see [1, Theorem 1]).

**2. Definitions and notations.** Let $B$ be a ring with 1, $C$ the center of $B$, $G$ an automorphism group of $B$ of order $n$ for some integer $n$, and $B^G$ the set of elements in $B$ fixed under each element in $G$. $B$ is called a Galois extension of $B^G$ with Galois group $G$ if there exist elements $\{a_i, b_i$ in $B$, $i = 1, 2, \ldots, m\}$ for some integer $m$ such that $\sum_{i=1}^{m} a_i g(b_i) = \delta_{1,g}$ for each $g \in G$. $B$ is called a Galois algebra over $R$ if $B$ is a Galois extension of $R$ which is contained in $C$, and $B$ is called a central Galois extension if $B$ is a Galois extension of $C$. Throughout this paper, we assume that $B$ is a Galois algebra with Galois group $G$. Let $J_g = \{b \in B \mid bx = g(x)b$ for all $x \in B\}$ and $J_g^{(A)} = \{b \in A \mid bx = g(x)b$ for all $x \in A\}$ for each $g \in G$, where $A \subset B$. In [4], it was shown that $BJ_g = Be_g$ for some central idempotent $e_g$ of $B$. We denote by $B_a$ the Boolean algebra generated by $\{0, e_g \mid g \in G; \leq\}$, where $e \leq e'$ if $ee' = e$.

**3. The monomials and subgroups.** Let $e$ be a nonzero monomial of $B_a$, $e = \Pi_{g\in S} e_g$ for a subset $S$ of $G$. We have two subsets of $G$, $H_e = \{g \in G \mid e \leq e_g\}$ and $T_e = \{g \in G \mid e = e_g\}$. We are going to show that $H_e$ is a subgroup of $G$, and that $T_e$ is a subgroup of $G$ if and only if $e = 1$. Let $K$ be a subgroup of $G$. Then $K$ is called a nonzero subgroup of $G$ if $\Pi_{k\in K} e_k \neq 0$, and $K$ is called a maximal nonzero subgroup of $G$ if $K \subset K'$, where $K'$ is a nonzero subgroup of $G$ such that $\Pi_{k\in K} e_k = \Pi_{k\in K'} e_k$, then $K = K'$. We note that each nonzero subgroup is contained in a unique maximal nonzero subgroup of $G$. We will show that there exists a one-to-one correspondence between the following three sets: (1) the set of nonzero monomials in $B_a$, (2) the set of maximal nonzero subgroups of $G$, and (3) the set of Galois extensions in $B$ generated by a nonzero monomial $e$ with a maximal Galois subgroup of $G$.

**Lemma 3.1.** *Let $e$ be a nonzero monomial in $B_a$ and $H_e = \{g \in G \mid e \leq e_g\}$. Then $H_e$ is a subgroup of $G$.*

**Proof.** For any $g$, $h \in H_e$, $e \leq e_g$, and $e \leq e_h$. Hence $e \leq e_g e_h$. But $J_g J_h \subset J_{gh}$, so $BJ_g J_h \subset BJ_{gh}$. Therefore $Be_g e_h \subset Be_{gh}$. Thus $e_g e_h \leq e_{gh}$; and so $e \leq e_g e_h \leq e_{gh}$. This implies that $gh \in H_e$. Noting that $G$ is finite, we conclude that $H_e$ is a subgroup of $G$. □

**Theorem 3.2.** *There exists a one-to-one correspondence between the set of nonzero monomials in $B_a$ and the set of maximal nonzero subgroups of $G$.*

**Proof.** Define $f : e \to H_e$ for a nonzero monomial $e$ in $B_a$, where $H_e$ is given in Lemma 3.1. By Lemma 3.1, $H_e$ is a subgroup of $G$. Also, by the definition of $H_e$, it is easy to see that $H_e$ is a maximal nonzero subgroup of $G$. Thus $f$ is well defined. Next we show that $f$ is one to one. Let $e$ and $e'$ be two nonzero monomials in $B_a$ such that $f(e) = f(e')$, that is, $H_e = H_{e'}$. Then $e = \Pi_{h\in H_e} e_h = \Pi_{h\in H_{e'}} e_h = e'$. Thus $f$ is one to one. Moreover, let $K$ be a maximal nonzero subgroup of $G$. Then $e = \Pi_{k\in K} e_k \neq 0$ and $K = \{g \in G \mid e \leq e_g\}$ by the definition of a maximal nonzero subgroup of $G$. Thus $f(e) = K$. Therefore $f$ is a bijection. □

Let $N(H_e)$ be the normalizer of $H_e$ in $G$ for a nonzero monomial $e$ in $B_a$. We next show that $Be$ is a Galois extension with a maximal Galois subgroup $G(e)$ where $G(e) = \{g \in G \mid g(e) = e\}$, and $G(e) = N(H_e)$. Consequently, we can establish a one-to-one correspondence between the set of maximal nonzero subgroups of $G$ and the set of

Galois extensions in $B$ generated by a nonzero monomial $e$ with a maximal Galois subgroup of $N(H_e)$.

**Lemma 3.3.** *For a nonzero monomial $e$ in $B_a$, let $G(e) = \{g \in G \mid g(e) = e\}$. Then,* (1) *$G(e) = N(H_e)$, where $N(H_e)$ is the normalizer of $H_e$ in $G$, and* (2) *$Be$ is a Galois extension with a maximal Galois subgroup of $G(e)|_{Be} \cong G(e)$.*

**Proof.** (1) For any $g \in N(H_e)$, since $Be = B\Pi_{h\in H_e} e_h = B\Pi_{h\in H_e} J_h$, $g(Be) = g(B\Pi_{h\in H_e} J_h)$ $= B\Pi_{h\in H_e} J_{ghg^{-1}} = B\Pi_{h\in gH_eg^{-1}} J_h = B\Pi_{h\in H_e} J_h = Be$ (for $gHg^{-1} = H$). Hence $g(e) = e$; and so $g \in G(e)$. Conversely, for any $g \in G(e)$,

$$Be = g(Be) = g(B\Pi_{h\in H_e} e_h) = g(B\Pi_{h\in H_e} J_h) = B\Pi_{h\in H_e} J_{ghg^{-1}} = B\Pi_{h\in H_e} e_{ghg^{-1}}. \quad (3.1)$$

Thus $e = \Pi_{h\in H_e} e_{ghg^{-1}}$. Therefore $e \le e_{ghg^{-1}}$; and so $ghg^{-1} \in H_e$ for each $h \in H_e$. This implies that $g \in N(H_e)$.

(2) Since $B$ is a Galois algebra with Galois group $G$ and $e \in C^{G(e)}$, $Be$ is a Galois extension with a maximal Galois subgroup of $G(e)|_{Be} \cong G(e)$ (see [7, proof of Lemma 3.7]). Moreover, let $g \in G$ but $g \notin G(e)$. Then $g(e) \neq e$. Thus $g$ is not an automorphism of $Be$; and so $G(e)$ is the maximal Galois group contained in $G$ for $Be$. □

**Theorem 3.4.** *There exists a one-to-one correspondence between the set of maximal nonzero subgroups of $G$ and the set of Galois extensions in $B$ generated by a nonzero monomial $e$ with a maximal Galois subgroup $G(e)|_{Be} \cong G(e)$ such that $G(e) = N(H_e)$.*

**Proof.** Let $\alpha : e \to Be$ for each nonzero monomial $e$ in $B_a$. Then, by Lemma 3.3, $Be$ is a Galois extension in $B$ generated by $e$ with a maximal Galois subgroup $G(e)|_{Be} \cong G(e)$ such that $G(e) = N(H_e)$. Clearly, $\alpha$ is a bijection from the set of nonzero monomials in $B_a$ to the set of Galois extensions $Be$ for a nonzero monomial $e$ in $B_a$ with a maximal Galois subgroup $G(e)|_{Be} \cong G(e)$ which is $N(H_e)$. Thus Theorem 3.4 is an immediate consequence of Theorem 3.2. □

In the following, we show that the set $T_e = \{g \in G \mid e = e_g\}$ for a nonzero monomial $e$ in $B_a$ is not a subgroup of $G$ unless $e = 1$.

**Theorem 3.5.** *Let $e$ be a nonzero monomial in $B_a$ and $T_e = \{g \in G \mid e = e_g\}$. Then $T_e$ is a subgroup of $G$ if and only if $e = 1$.*

**Proof.** Assume $T_e$ is a subgroup of $G$. Then $1 \in T_e$; and so $e = e_1 = 1$. Conversely, assume $e = 1$. Then $T_e = T_1 = \{g \in G \mid 1 = e_g\}$. But the condition that $1 = e_g$ is equivalent to that $1 \le e_g$, so $T_e = T_1 = H_1$ where $H_1$ is given in Lemma 3.1. Hence by Lemma 3.1, $T_e$ is a subgroup of $G$. □

**4. Central Galois algebras.** In Section 3, Lemma 3.1 proves that for a nonzero monomial $e \in B_a$, $H_e$ $(= \{g \in G \mid e \le e_g\})$ is a subgroup of $G$. In [7], it was shown that if $H$ is a maximal subset of $G$ such that $\Pi_{h\in H} J_h \neq \{0\}$, then $H$ is a subgroup of $G$. We will show that the maximal subset $H$ is exactly $H_e$ for a minimal nonzero monomial $e \in B_a$. Thus $Be$ is a central Galois algebra with Galois group $H_e$ (see [7, Theorem 3.6]). Next is a characterization of the central Galois algebra $Be$ with Galois group $H_e$ for a nonzero monomial $e \in B_a$.

**Theorem 4.1.** *Let $e$ be a nonzero monomial in $B_a$ such that $H_e \neq \{1\}$. The following statements are equivalent:*

(1) *$Be$ is a central Galois algebra with Galois group $H_e$.*

(2) *$eJ_g = \{0\}$ for each $g \notin H_e$.*

(3) *$e$ is a minimal nonzero monomial in $B_a$.*

**Proof.** (1)⇒(2). Since $B$ is a Galois algebra over a commutative ring $R$ with Galois group $G$, $B = \oplus\sum_{g\in G} J_g$ (see [4, Theorem 1]). Hence

$$Be = \oplus \sum_{g\in G} eJ_g = \left(\oplus \sum_{h\in H_e} eJ_h\right) \oplus \left(\oplus \sum_{g\notin H_e} eJ_g\right). \tag{4.1}$$

By hypothesis, $Be$ is a central Galois algebra with Galois group $H_e$, so $Be = \oplus\sum_{h\in H_e} J_h^{(Be)}$. But by [7, Lemma 3.3], $J_h^{(Be)} = eJ_h$ for each $h \in H_e$; and so $Be = \oplus\sum_{h\in H_e} eJ_h$. Thus $\oplus\sum_{g\notin H_e} eJ_g = \{0\}$, that is, $eJ_g = \{0\}$ for each $g \notin H_e$.

(2)⇒(1). Since $Be = \oplus\sum_{g\in G} eJ_g = (\oplus\sum_{h\in H_e} eJ_h) \oplus (\oplus\sum_{g\notin H_e} eJ_g)$ and $eJ_g = \{0\}$ for each $g \notin H_e$, $Be = \oplus\sum_{h\in H_e} eJ_h$. By [7, Lemma 3.3] again, $J_h^{(Be)} = eJ_h$ for each $h \in H_e$. Hence $Be = \oplus\sum_{h\in H_e} J_h^{(Be)}$, where $J_h^{(Be)} J_{h^{-1}}^{(Be)} = (eJ_h)(eJ_{h^{-1}}) = eJ_hJ_{h^{-1}} = eC$ which is the center of $Be$. Moreover, $B$ is a Galois $R$-algebra, so it is a separable $R$-algebra. Thus, $Be$ is a separable algebra over $Re$ (see [2, Proposition 1.11, page 46]). Therefore, $Be$ is a central Galois algebra over $Ce$ (see [3, Theorem 1]).

(3)⇒(2). Since $e$ is a minimal nonzero monomial in $B_a$, for each $g \in G$, either $e \le e_g$ or $ee_g = 0$. Since $e \le e_g$ for each $g \in H_e$, we have that $ee_g = 0$ for each $g \notin H_e$. Therefore, $BeJ_g = Bee_g = \{0\}$; and so $eJ_g = \{0\}$ for each $g \notin H_e$.

(2)⇒(3). Suppose $e$ is not a minimal nonzero monomial in $B_a$. Then there exists a $g \in G$ such that $0 < ee_g < e$. By the definition of $H_e$, $e = \Pi_{h\in H_e} e_h$; and so $ee_h = e$ for each $h \in H_e$. Hence $g \notin H_e$. Therefore, $BeJ_g = Bee_g \neq \{0\}$. This implies that $eJ_g \neq \{0\}$ for some $g \notin H_e$. This contradicts hypothesis (2). Thus statement (3) holds. □

When $e$ is a minimal nonzero monomial in $B_a$, Theorem 4.1 shows that $Be$ is a central Galois algebra with Galois group $H_e$. Hence the order of $H_e$ is a unit in $Be$ (see [4, Corollary 3]). Moreover, by Lemma 3.3, $Be$ is a Galois extension with Galois group $G(e)$ which is $N(H_e)$, so we have a structure of $Be$.

**Theorem 4.2.** *For a minimal nonzero monomial $e$ in $B_a$, $Be$ is a central Galois algebra with Galois group $H_e$ and $Ce$ is a commutative Galois algebra with Galois group $G(e)/H_e$.*

**Proof.** Since $e$ is a minimal nonzero monomial in $B_a$, $Be$ is a central Galois algebra with Galois group $H_e$ by Theorem 4.1. Hence $|H_e|$, the order of $H_e$, is a unit in $Ce$. Moreover, by Lemma 3.3, $Be$ is a Galois extension with Galois group $G(e)$ which is $N(H_e)$, so $H_e$ is a normal subgroup of $G(e)$. Let $\{a_i, b_i \mid i = 1, 2, \dots, m\}$ be a $G(e)$-Galois system for $Be$. Then, $\sum_{i=1}^m a_i g(b_i) = \delta_{1,g} e$ for each $g \in G(e)$. Let $x_i = (1/|H_e|)\sum_{h\in H_e} h(a_i)$ and $y_i = \sum_{h\in H_e} h(b_i)$. Then, $x_i$ and $y_i$ are invariant under each element in $H_e$. Hence, $x_i, y_i \in Ce$ since $(Be)^{H_e} = Ce$. It is straightforward to verify that $\{x_i, y_i\}$ is a $G(e)/H_e$-Galois system for $Ce$. □

Theorem 4.1 characterizes a central Galois algebra $Be$ for a minimal nonzero monomial $e \in B_a$. Next we want to characterize a central Galois algebra $B1$ for the maximal monomial 1 in $B_a$.

**Theorem 4.3.** *Let $H_1 = \{h \in G \mid e_h = 1\}$. Then $e_g = 0$ for each $g \notin H_1$ if and only if $B$ is either a central Galois algebra with Galois group $H_1$ or a commutative Galois algebra with Galois group $G$.*

**Proof.** ($\Rightarrow$) Case 1. $H_1 \neq \{1\}$. Since $e_g = 0$ for each $g \notin H_1$, $J_g = \{0\}$ for each $g \notin H_1$. Hence, by (2)$\Rightarrow$(1) in Theorem 4.1, $B$ $(= B1)$ is a central Galois algebra with Galois group $H_1$. Case 2. $H_1 = \{1\}$. By hypothesis, $e_g = 0$ for each $g \neq 1$ in $G$, so $B = \oplus\sum_{g\in G} J_g = J_1 = C$. Thus $B$ is a commutative Galois algebra with Galois group $G$.

($\Leftarrow$) Assume $B$ is a central Galois algebra with Galois group $H_1$. Then $H_1 \neq \{1\}$. Hence, by (1)$\Rightarrow$(2) in Theorem 4.1, $J_g = 1J_g = \{0\}$ for each $g \notin H_1$. Thus $e_g = 0$ for each $g \notin H_1$. Next, assume $B$ is a commutative Galois algebra with Galois group $G$. Then $J_g = \{0\}$ for each $g \neq 1$ in $G$ (see [3, Proposition 2]). Hence $e_g = 0$ for each $g \neq 1$ in $G$. Therefore $H_1 = \{1\}$ and $e_g = 0$ for each $g \notin H_1$. □

As a consequence of Theorem 4.3, the DeMeyer theorem (see [1, Theorem 1]) for central Galois algebras with a connected center is generalized.

**Corollary 4.4.** *Let $B$ be a Galois algebra with Galois group $G$. If $B_a = \{0,1\}$, then $B$ is a central Galois algebra with Galois group $H_1$ and $C$ is a commutative Galois algebra with Galois group $G/H_1$.*

**Proof.** Since $B_a = \{0,1\}$, $e_g = 0$ for each $g \notin H_1$; and so the corollary holds. □

We conclude the present paper with an example of a Galois algebra $B$ such that $B_a = \{0,1\}$, but its center $C$ is not indecomposable.

**Example 4.5.** Let $R[i,j,k]$ be the quaternion algebra over the real field $R$, $B = R[i,j,k] \oplus R[i,j,k]$, and $G = \{1, g_i, g_j, g_k, g, gg_i, gg_j, gg_k\}$, where $g_i(a_1,a_2) = (ia_1i^{-1}, ia_2i^{-1})$, $g_j(a_1,a_2) = (ja_1j^{-1}, ja_2j^{-1})$, $g_k(a_1,a_2) = (ka_1k^{-1}, ka_2k^{-1})$, and $g(a_1,a_2) = (a_2,a_1)$ for all $(a_1,a_2)$ in $B$. Then,

(1) $B$ is a Galois extension with a $G$-Galois system: $\{a_1 = (1,0),\ a_2 = (i,0),\ a_3 = (j,0),\ a_4 = (k,0),\ a_5 = (0,1),\ a_6 = (0,i),\ a_7 = (0,j),\ a_8 = (0,k);\ b_1 = (1/4)(1,0),\ b_2 = -(1/4)(i,0),\ b_3 = -(1/4)(j,0),\ b_4 = -(1/4)(k,0),\ b_5 = (1/4)(0,1),\ b_6 = -(1/4)(0,i),\ b_7 = -(1/4)(0,j),\ b_8 = -(1/4)(0,k)\}$.

(2) $B^G = \{(r,r) \mid r \in R\} \cong R$.

(3) By (1) and (2), $B$ is a Galois algebra over $R$ with Galois group $G$.

(4) $J_1 = C = R \oplus R$, $J_{g_i} = (Ri) \oplus (Ri)$, $J_{g_j} = (Rj) \oplus (Rj)$, $J_{g_k} = (Rk) \oplus (Rk)$, and $J_g = J_{gg_i} = J_{gg_j} = J_{gg_k} = \{0\}$.

(5) $BJ_1 = BJ_{g_i} = BJ_{g_j} = BJ_{g_k} = B1$ and $BJ_g = BJ_{gg_i} = BJ_{gg_j} = BJ_{gg_k} = \{0\}$. Hence $e_1 = e_{g_i} = e_{g_j} = e_{g_k} = 1$ and $e_g = e_{gg_i} = e_{gg_j} = e_{gg_k} = 0$. Thus $B_a = \{0,1\}$.

(6) $H_1 = \{1, g_i, g_j, g_k\}$ and $B$ is a central Galois algebra with Galois group $H_1$.

(7) $C = R \oplus R$ which is a commutative Galois algebra with Galois group $G/H_1 \cong \{1,g\}$.

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George Szeto: Department of Mathematics, Bradley University, Peoria, IL 61625, USA
*E-mail address*: szeto@hilltop.bradley.edu

Lianyong Xue: Department of Mathematics, Bradley University, Peoria, IL 61625, USA
*E-mail address*: lxue@hilltop.bradley.edu

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