IJMMS 27:12 (2001) 749–757
PII. S0161171201006627
http://ijmms.hindawi.com

ON Q-ALGEBRAS

JOSEPH NEGGERS, SUN SHIN AHN, and HEE SIK KIM

(Received 29 January 2001)

ABSTRACT. We introduce a new notion, called a Q-algebra, which is a generalization of the idea of $BCH/BCI/BCK$-algebras and we generalize some theorems discussed in BCI-algebras. Moreover, we introduce the notion of "quadratic" Q-algebra, and show that every quadratic Q-algebra $(X;*,e)$, $e \in X$, has a product of the form $x * y = x - y + e$, where $x, y \in X$ when X is a field with $|X| \geq 3$.

2000 Mathematics Subject Classification. 06F35, 03G25.

1. Introduction. Imai and Iséki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras (see [4, 5]). It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [2, 3] Hu and Li introduced a wide class of abstract algebras: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. Neggers and Kim (see [8]) introduced the notion of d-algebras, that is, (I) $x * x = e$; (IX) $e * x = e$; (VI) $x * y = e$ and $y * x = e$ imply $x = y$, which is another useful generalization of BCK-algebras, after which they investigated several relations between d-algebras and BCK-algebras, as well as other relations between d-algebras and oriented digraphs. At the same time, Jun, Roh, and Kim [6] introduced a new notion, called a BH-algebra, that is, (I) $x * x = e$; (II) $x * e = x$; (VI) $x * y = e$ and $y * x = e$ imply $x = y$, which is a generalization of $BCH/BCI/BCK$-algebras, and they showed that there is a maximal ideal in bounded BH-algebras. We introduce a new notion, called a Q-algebra, which is a generalization of $BCH/BCI/BCK$-algebras and generalize some theorems from the theory of BCI-algebras. Moreover, we introduce the notion of "quadratic" Q-algebra, and obtain the result that every quadratic Q-algebra $(X;*,e)$, $e \in X$, is of the form $x * y = x - y + e$, where $x, y \in X$ and X is a field with $|X| \geq 3$, that is, the product is linear in a special way.

2. Q-algebras. A *Q-algebra* is a nonempty set X with a constant 0 and a binary operation "$*$" satisfying axioms:

(I) $x * x = 0$,
(II) $x * 0 = x$,
(III) $(x * y) * z = (x * z) * y$ for all $x, y, z \in X$.

For brevity we also call X a *Q-algebra*. In X we can define a binary relation $\leq$ by $x \leq y$ if and only if $x * y = 0$. Recently, Ahn and Kim [1] introduced the notion of QS-algebras. A Q-algebra X is said to be a *QS-algebra* if it satisfies the additional relation:

(IV) $(x * y) * (x * z) = z * y$, for any $x, y, z \in X$.

Example 2.1. Let $\mathbb{Z}$ be the set of all integers and let $n\mathbb{Z} := \{nz \mid z \in \mathbb{Z}\}$ where $n \in \mathbb{Z}$. Then $(\mathbb{Z}; -, 0)$ and $(n\mathbb{Z}; -, 0)$ are Q-algebras, where "$-$" is the usual subtraction of integers.

Example 2.2. Let $X := \{0,1,2,3\}$ be a set with the following table:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	0	0
2	2	0	0	0
3	3	3	3	0

Then $(X; *, 0)$ is a Q-algebra, which is not a $BCH/BCI/BCK$-algebra.

Neggers and Kim [7] introduced the related notion of B-algebra, that is, algebras $(X; *, 0)$ which satisfy (I) $x * x = 0$; (II) $x * 0 = x$; (V) $(x * y) * z = x * (z * (0 * y))$, for any $x, y, z \in X$. It is easy to see that B-algebras and Q-algebras are different notions. For example, Example 2.2 is a Q-algebra, but not a B-algebra, since $(3 * 2) * 1 = 0 \neq 3 = 3 * (1 * (0 * 2))$. Consider the following example. Let $X := \{0,1,2,3,4,5\}$ be a set with the following table:

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	2	4	5	3
2	2	1	0	5	3	4
3	3	4	5	0	2	1
4	4	5	3	1	0	2
5	5	3	4	2	1	0

Then $(X; *, 0)$ is a B-algebra (see [7]), but not a Q-algebra, since $(5 * 3) * 4 = 3 \neq 4 = (5 * 4) * 3$.

Proposition 2.3. *If $(X; *, 0)$ is a Q-algebra, then*
(VII) $(x * (x * y)) * y = 0$, *for any $x, y \in X$.*

Proof. By (I) and (III), $(x * (x * y)) * y = (x * y) * (x * y) = 0$. □

We now investigate some relations between Q-algebras and BCH-algebras (also BCK/BCI-algebras). The following theorems are easily proven, and we omit their proofs.

Theorem 2.4. *Every BCH-algebra X is a Q-algebra. Every Q-algebra X satisfying condition (VI) is a BCH-algebra.*

Theorem 2.5. *Every Q-algebra satisfying condition (IV) and (VI) is a BCI-algebra.*

Theorem 2.6. *Every Q-algebra X satisfying conditions (V), (VI), and (VIII) $(x*y)*x = 0$ for any $x,y \in X$, is a BCK-algebra.*

Theorem 2.7. *Every Q-algebra X satisfying $x*(x*y) = x*y$ for all $x,y,z \in X$, is a trivial algebra.*

Proof. Putting $x = y$ in the equation $x*(x*y) = x*y$, we obtain $x*0 = 0$. By (II) $x = 0$. Hence X is a trivial algebra. □

The following example shows that a Q-algebra may not satisfy the associative law.

Example 2.8. (a) Let $X := \{0,1,2\}$ with the table as follows:

$*$	0	1	2
0	0	2	1
1	1	0	2
2	2	1	0

Then X is a Q-algebra, but associativity does not hold, since $(0*1)*2 = 0 \neq 1 = 0*(1*2)$.

(b) Let $\mathbb{Z}$ and $\mathbb{R}$ be the set of all integers and real numbers, respectively. Then $(\mathbb{Z};-,0)$ and $(\mathbb{R};\div,1)$ are nonassociative Q-algebras where "$-$" is the usual subtraction and "$\div$" is the usual division.

Theorem 2.9. *Every Q-algebra $(X;*,0)$ satisfying the associative law is a group under the operation "$*$".*

Proof. Putting $x = y = z$ in the associative law $(x*y)*z = x*(y*z)$ and using (I) and (II), we obtain $0*x = x*0 = x$. This means that 0 is the zero element of X. By (I), every element x of X has as its inverse the element x itself. Therefore $(X;*)$ is a group. □

3. The G-part of Q-algebras.

In this section, we investigate the properties of the G-part in Q-algebras.

Lemma 3.1. *If $(X;*,0)$ is a Q-algebra and $a*b = a*c$, $a,b,c \in X$, then $0*b = 0*c$.*

Proof. By (I) and (II) $(a*b)*a = (a*a)*b = 0*b$ and $(a*c)*a = (a*a)*c = 0*c$. Since $a*b = a*c$, $0*b = 0*c$. □

Definition 3.2. Let $(X;*,0)$ be a Q-algebra. For any nonempty subset S of X, we define

$$G(S) := \{x \in S \mid 0*x = x\}. \tag{3.1}$$

In particular, if $S = X$ then we say that $G(X)$ is the *G-part* of X.

Corollary 3.3. *A left cancellation law holds in $G(X)$.*

Proof. Let $a,b,c \in G(X)$ with $a*b = a*c$. By Lemma 3.1, $0*b = 0*c$. Since $b,c \in G(X)$, we obtain $b = c$. □

Proposition 3.4. *Let* $(X;*,0)$ *be a* Q*-algebra. Then* $x \in G(X)$ *if and only if* $0*x \in G(X)$.

Proof. If $x \in G(X)$, then $0*x = x$ and $0*(0*x) = 0*x$. Hence $0*x \in G(X)$.
Conversely, if $0*x \in G(x)$, then $0*(0*x) = 0*x$. By applying Corollary 3.3, we obtain $0*x = x$. Therefore $x \in G(X)$. □

For any Q-algebra $(X;*,0)$, the set

$$B(X) := \{x \in X \mid 0*x = 0\} \tag{3.2}$$

is called the *p-radical* of X. If $B(X) = \{0\}$, then we say that X is a *p-semisimple* Q-algebra. The following property is obvious.
(IX) $G(X) \cap B(X) = \{0\}$.

Proposition 3.5. *If* $(X;*,0)$ *is a* Q*-algebra and* $x, y \in X$*, then*

$$y \in B(X) \Longleftrightarrow (x*y)*x = 0. \tag{3.3}$$

Proof. By (I) and (III) $(x*y)*x = (x*x)*y = 0*y = 0$ if and only if $y \in B(X)$. □

Definition 3.6. Let $(X;*,0)$ be a Q-algebra and $I(\neq \emptyset) \subseteq X$. The set I is called an *ideal* of X if for any $x, y, z \in X$,
(1) $0 \in I$,
(2) $x*y \in I$ and $y \in I$ imply $x \in I$.
Obviously, $\{0\}$ and X are ideals of X. We call $\{0\}$ and X the *zero ideal* and the *trivial ideal* of X, respectively. An ideal I is said to be *proper* if $I \neq X$.

In Example 2.2 the set $I := \{0,1,2\}$ is an ideal of X.

Proposition 3.7. *Let* $(X;*,0)$ *be a* Q*-algebra. Then* $B(X)$ *is an ideal of* X.

Proof. Since $(0*0)*0 = 0$, by Proposition 3.5, $0 \in B(X)$. Let $x*y \in B(X)$ and $y \in B(X)$. Then by Proposition 3.5, $((x*y)*x)*(x*y) = 0$. By (III), $((x*y)*(x*y))*x = 0*x = 0$. Hence $x \in B(X)$. Therefore $B(X)$ is an ideal of X. □

Proposition 3.8. *If* S *is a subalgebra of a* Q*-algebra* $(X;*,0)$*, then* $G(X) \cap S = G(S)$.

Proof. It is obvious that $G(X) \cap S \subseteq G(S)$. If $x \in G(S)$, then $0*x = x$ and $x \in S \subseteq X$. Then $x \in G(X)$ and so $x \in G(X) \cap S$, which proves the proposition. □

Theorem 3.9. *Let* $(X;*,0)$ *be a* Q*-algebra. If* $G(X) = X$*, then* X *is* p*-semisimple.*

Proof. Assume that $G(X) = X$. By (X), $\{0\} = G(X) \cap B(X) = X \cap B(X) = B(X)$. Hence X is p-semisimple. □

Theorem 3.10. *If* $(X;*,0)$ *is a* Q*-algebra of order* 3*, then* $|G(X)| \neq 3$*, that is,* $G(X) \neq X$.

Proof. For the sake of convenience, let $X = \{0,a,b\}$ be a Q-algebra. Assume that $|G(X)| = 3$, that is, $G(X) = X$. Then $0*0 = 0$, $0*a = a$, and $0*b = b$. From $x*x = 0$ and $x*0 = x$, it follows that $a*a = 0$, $b*b = 0$, $a*0 = a$, and $b*0 = b$. Now let $a*b = 0$. Then 0, a, and b are candidates of the computation. If $b*a = 0$, then

$a*b=0=b*a$ and so $(a*b)*a=(b*a)*a$. By (III), $(a*a)*b=(b*a)*a$. Hence $0*b=0*a$. By the cancellation law in $G(X)$, $b=a$, a contradiction. If $b*a=a$, then $a=b*a=(0*b)*a=(0*a)*b=a*b=0$, a contradiction. For the case $b*a=b$, we have $b=b*a=(0*b)*a=(0*a)*b=a*b=0$, which is also a contradiction. Next, if $a*b=a$, then $(a*(a*b))*b=(a*a)*b=0*b=b\neq 0$. This leads to the conclusion that Proposition 2.3 does not hold, a contradiction. Finally, let $a*b=b$. If $b*a=0$, then $b=a*b=(0*a)*b=(0*b)*a=b*a=0$, a contradiction. If $b*a=a$, $b=a*b=(0*a)*b=(0*b)*a=b*a=0$, a contradiction. For the case $b*a=b$, we have $a=0*a=(b*b)*a=(b*a)*b=b*b=0$, which is again a contradiction. This completes the proof. □

Proposition 3.11. *If $(X;*,0)$ is a Q-algebra of order 2, then in every case the G-part $G(X)$ of X is an ideal of X.*

Proof. Let $|X|=2$. Then either $G(X)=\{0\}$ or $G(X)=X$. In either case, $G(X)$ is an ideal of X. □

Theorem 3.12. *Let $(X;*,0)$ be a Q-algebra of order 3. Then $G(X)$ is an ideal of X if and only if $|G(X)|=1$.*

Proof. Let $X:=\{0,a,b\}$ be a Q-algebra. If $|G(X)|=1$, then $G(X)=\{0\}$ is the trivial ideal of X.

Conversely, assume that $G(X)$ is an ideal of X. By Theorem 3.10, we know that either $|G(X)|=1$ or $|G(X)|=2$. Suppose that $|G(X)|=2$. Then either $G(X)=\{0,a\}$ or $G(X)=\{0,b\}$. If $G(X)=\{0,a\}$, then $b*a\notin G(X)$ because $G(X)$ is an ideal of X. Hence $b*a=b$. Then $a=0*a=(b*b)*a=(b*a)*b=b*b=0$, which is a contradiction. Similarly, $G(X)=\{0,b\}$ leads to a contradiction. Therefore $|G(X)|\neq 2$ and so $|G(X)|=1$. □

Definition 3.13. An ideal I of a Q-algebra $(X;*,0)$ is said to be *implicative* if $(x*y)*z\in I$ and $y*z\in I$, then $x*z\in I$, for any $x,y,z\in X$.

Theorem 3.14. *Let $(X;*,0)$ be a Q-algebra and let I be an implicative ideal of X. Then I contains the G-part $G(X)$ of X.*

Proof. If $x\in G(X)$, then $(0*x)*x=x*x=0\in I$ and $x*x=0\in I$. Since I is implicative, it follows that $x=0*x\in I$. Hence $G(X)\subseteq I$. □

Definition 3.15. Let X and Y be Q-algebras. A mapping $f:X\to Y$ is called a *homomorphism* if

$$f(x*y)=f(x)*f(y),\quad \forall x,y\in X. \tag{3.4}$$

A homomorphism f is called a *monomorphism* (resp., *epimorphism*) if it is injective (resp., surjective). A bijective homomorphism is called an *isomorphism*. Two Q-algebras X and Y are said to be *isomorphic*, written by $X\cong Y$, if there exists an isomorphism $f:X\to Y$. For any homomorphism $f:X\to Y$, the set $\{x\in X\mid f(x)=0\}$ is called the *kernel* of f, denoted by $\operatorname{Ker}(f)$ and the set $\{f(x)\mid x\in X\}$ is called the *image* of f, denoted by $\operatorname{Im}(f)$. We denote by $\operatorname{Hom}(X,Y)$ the set of all homomorphisms of Q-algebras from X to Y.

Proposition 3.16. *Suppose that $f : X \to X'$ is a homomorphism of Q-algebras. Then*
(1) $f(0) = 0'$,
(2) f *is isotone, that is, if $x * y = 0$, $x, y \in X$, then $f(x) * f(y) = 0'$.*

Proof. Since $f(0) = f(0*0) = f(0)*f(0) = 0'$, (1) holds. If $x, y \in X$ and $x \le y$, that is, $x * y = 0$, then by (1), $f(x)*f(y) = f(x*y) = f(0) = 0'$. Hence $f(x) \le f(y)$, proving (2). □

Theorem 3.17. *Let $(X;*,0)$ and $(X;*',0')$ be Q-algebras and let B be an ideal of Y. Then for any $f \in \mathrm{Hom}(X,Y)$, $f^{-1}(B)$ is an ideal of X.*

Proof. By Proposition 3.16(1), $0 \in f^{-1}(B)$. Assume that $x * y \in f^{-1}(B)$ and $y \in f^{-1}(B)$. Then $f(x)*f(y) = f(x*y) \in B$. It follows from the fact that B is an ideal of Y that $f(x) \in B$, that is, $x \in f^{-1}(B)$. This means that $f^{-1}(B)$ is an ideal of X. The proof is complete. □

Since $\{0'\}$ is an ideal of X', $\mathrm{Ker}(f) = f^{-1}(\{0'\})$ for any $f \in \mathrm{Hom}(X,Y)$. Hence we obtain the following corollary.

Corollary 3.18. *The kernel* $\mathrm{Ker}(f)$ *is an ideal of X.*

4. The quadratic Q-algebras. Let X be a field with $|X| \ge 3$. An algebra $(X;*)$ is said to be *quadratic* if $x * y$ is defined by $x * y := a_1x^2 + a_2xy + a_3y^2 + a_4x + a_5y + a_6$, where $a_1, \ldots, a_6 \in X$, for any $x, y \in X$. A quadratic algebra $(X;*)$ is said to be *quadratic Q-algebra* (resp., *QS-algebra*) if it satisfies conditions (I), (II), and (III) (resp., (IV)).

Theorem 4.1. *Let X be a field with $|X| \ge 3$. Then every quadratic Q-algebra $(X;*,e)$, $e \in X$, has the form $x * y = x - y + e$ where $x, y \in X$.*

Proof. Define

$$x * y := Ax^2 + Bxy + Cy^2 + Dx + Ey + F. \tag{4.1}$$

Consider (I).

$$e = x * x = (A+B+C)x^2 + (D+E)x + F. \tag{4.2}$$

Let $x := 0$ in (4.2). Then we obtain $F = e$. Hence (4.1) turns out to be

$$x * y = Ax^2 + Bxy + Cy^2 + Dx + Ey + e. \tag{4.3}$$

If $y := x$ in (4.3), then

$$e = x * x = (A+B+C)x^2 + (D+E)x + e, \tag{4.4}$$

for any $x \in X$, and hence we obtain $A + B + C = 0 = D + E$, that is, $E = -D$ and $B = -A - C$. Hence (4.3) turns out to be

$$x * y = (x-y)(Ax - Cy + D) + e. \tag{4.5}$$

Let $y := e$ in (4.5). Then by (II) we have

$$x = x * e = (x-e)(Ax - Ce + D) + e, \tag{4.6}$$

that is, $(Ax - Ce + D - 1)(x - e) = 0$. Since X is a field, either $x - e = 0$ or $Ax - Ce + D - 1 = 0$. Since $|X| \geq 3$, we have $Ax - Ce + D - 1 = 0$, for any $x \neq e$ in X. This means that $A = 0, 1 - D + Ce = 0$. Thus (4.5) turns out to be

$$x * y = (x - y) + C(x - y)(e - y) + e. \tag{4.7}$$

To satisfy condition (III) we consider $(x * y) * z$ and $(x * z) * y$.

$$\begin{aligned}(x * y) * z &= (x * y - z) + C(x * y - z)(e - z) + e \\ &= (x - y - z) + C(x - y)(e - z) + 2e \\ &\quad + C[(x - y) + C(x - y)(e - y) + (e - z)](e - z) \\ &= (x - y - z) + C(x - y)(2e - y - z) + 2e \\ &\quad + C^2(x - y)(e - y)(e - z) + C(e - z)^2.\end{aligned} \tag{4.8}$$

Interchange y with z in (4.8). Then

$$\begin{aligned}(x * z) * y = (x - z - y) &+ C(x - z)(2e - z - y) + 2e \\ &+ C^2(x - z)(e - z)(e - y) + C(e - y)^2.\end{aligned} \tag{4.9}$$

By (4.8) and (4.9) we obtain

$$0 = (x * y) * z - (x * z) * y = C^2(e - y)(e - z)(z - y). \tag{4.10}$$

Since X is a field with $|X| \geq 3$, we obtain $C = 0$. This means that every quadratic Q-algebra $(X; *, e)$, has the form $x * y = x - y + e$ where $x, y \in X$, completing the proof. □

Example 4.2. Let $\mathbb{R}$ be the set of all real numbers. Define $x * y := x - y + \sqrt{2}$. Then $(\mathbb{R}; *, \sqrt{2})$ is a quadratic Q-algebra.

Example 4.3. Let $\mathcal{K} := \mathrm{GF}(p^n)$ be a Galois field. Define $x * y := x - y + e$, $e \in \mathcal{K}$. Then $(\mathcal{K}; *, e)$ is a quadratic Q-algebra.

Theorem 4.4. *Let X be a field with $|X| \geq 3$. Then every quadratic Q-algebra on X is a (quadratic) QS-algebra.*

Proof. Let $(X; *, e)$ be a quadratic Q-algebra. Then $x * y = x - y + e$ for any $x, y \in X$, and hence

$$\begin{aligned}(x * y) * (x * z) &= (x - y + e) * (x - z + e) \\ &= (x - y + e) - (x - z + e) + e \\ &= z - y + e = z * y,\end{aligned} \tag{4.11}$$

completing the proof. □

Remark 4.5. Usually a nonquadratic Q-algebra need not be a QS-algebra. See the following example.

Example 4.6. Consider the Q-algebra $(X;*,0)$ in Example 2.2. This algebra is not a QS-algebra, since $(3*1)*(3*2) = 3 \neq 0 = 2*1$.

Corollary 4.7. *Let X be a field with $|X| \geq 3$. Then every quadratic Q-algebra on X is a BCI-algebra.*

Proof. It is an immediate consequences of Theorems 2.5 and 4.4. □

Theorem 4.8. *Let X be a field with $|X| \geq 3$. Then every quadratic Q-algebra $(X;*,e)$ is p-semisimple. Furthermore, if $\operatorname{char}(X) \neq 2$, then $G(X) = B(X)$.*

Proof. Notice that $B(X) = \{x \in X \mid e*x = e\} = \{x \in X \mid e-x+e = e\} = \{x \in X \mid e-x = 0\} = \{e\}$, that is, $(X;*,e)$ is p-semisimple. Also, if $\operatorname{char}(X) \neq 2$, then 2 is invertible in X and $G(X) = \{x \in X \mid e*x = x\} = \{x \in X \mid e-x+e = x\} = \{x \in X \mid 2e = 2x\} = \{x \in X \mid e = x\} = \{e\}$. Of course, if $\operatorname{char}(X) = 2$, then $2e = 2x = 0$ for all $x \in X$, whence $G(X) = X$. □

This shows that there is a large class of examples of p-semisimple QS-algebras obtained as quadratic Q-algebras.

Theorem 4.9. *Let X be a field with $|X| \geq 3$. Then every quadratic Q-algebra on X is isomorphic to every other such algebra defined on X.*

Proof. Let $x*y := x-y+e_1$ and $x*'y := x-y+e_2$, where $e_1,e_2 \in X$. Let $\pi(x) := x+(e_2-e_1)$, for all $x \in X$. Then $\pi(x*y) = [(x-y)+e_1]+(e_2-e_1) = (x-y)+e_2 = (x+(e_2-e_1))+(y+(e_2-e_1))+e_2 = \pi(x)*'\pi(y)$, whence the fact that $\pi^{-1}(x) = x+(e_1-e_2)$ yields the conclusion that π is an isomorphism of Q-algebras. □

Theorem 4.10. *Let X be a field with $|X| \geq 3$. Then every quadratic Q-algebra $(X;*,e)$ determines the abelian group $(X,+)$ via the definition $x+y = x*(e-y)$.*

Proof. Note that $x*(e-y) = x-(e-y)+e = x+y$ returns the additive operation of the field X, which is an abelian group. □

Not every quadratic Q-algebra $(X;*,e)$, $e \in X$, on a field X with $|X| \geq 3$ need be a BCK-algebra, since $((x*y)*(x*z))*(z*y) = e+(y-z) \neq e$ in general.

Problem 4.11. Construct a cubic Q-algebra which is not quadratic. Verify that among such cubic Q-algebras there are examples which are not QS-algebras. Furthermore, the question whether there are non-p-semisimple cubic Q-algebras is also of interest.

References

[1] S. S. Ahn and H. S. Kim, *On QS-algebras*, J. Chungcheong Math. Soc. **12** (1999), 33–41.
[2] Q. P. Hu and X. Li, *On BCH-algebras*, Math. Sem. Notes Kobe Univ. **11** (1983), no. 2, part 2, 313–320. MR 86a:06016. Zbl 579.03047.
[3] ______, *On proper BCH-algebras*, Math. Japon. **30** (1985), no. 4, 659–661. MR 87d:06042. Zbl 583.03050.
[4] K. Iséki, *On BCI-algebras*, Math. Sem. Notes Kobe Univ. **8** (1980), no. 1, 125–130. MR 81k:06018a. Zbl 0434.03049.

[5] K. Iséki and S. Tanaka, *An introduction to the theory of BCK-algebras*, Math. Japon. **23** (1978), no. 1, 1–26. MR 80a:03081. Zbl 385.03051.

[6] Y. B. Jun, E. H. Roh, and H. S. Kim, *On BH-algebras*, Sci. Math. **1** (1998), no. 3, 347–354. MR 2000d:06026. Zbl 928.06013.

[7] J. Neggers and H. S. Kim, *On B-algebras*, in preparation.

[8] ______, *On d-algebras*, Math. Slovaca **49** (1999), no. 1, 19–26. CMP 1 804 469. Zbl 943.06012.

Joseph Neggers: Department of Mathematics, University of Alabama, Tuscaloosa, AL 35487-0350, USA

E-mail address: jneggers@gp.as.ua.edu

Sun Shin Ahn: Department of Mathematics Education, Dongguk University, Seoul 100-715, Korea

E-mail address: sunshine@dgu.ac.kr

Hee Sik Kim: Department of Mathematics, Hanyang National University, Seoul 133-791, Korea

E-mail address: heekim@hanyang.ac.kr

Special Issue on
Singular Boundary Value Problems for Ordinary Differential Equations

Call for Papers

The purpose of this special issue is to study singular boundary value problems arising in differential equations and dynamical systems. Survey articles dealing with interactions between different fields, applications, and approaches of boundary value problems and singular problems are welcome.

This Special Issue will focus on any type of singularities that appear in the study of boundary value problems. It includes:

- Theory and methods
- Mathematical Models
- Engineering applications
- Biological applications
- Medical Applications
- Finance applications
- Numerical and simulation applications

Before submission authors should carefully read over the journal's Author Guidelines, which are located at http://www.hindawi.com/journals/bvp/guidelines.html. Authors should follow the Boundary Value Problems manuscript format described at the journal site http://www.hindawi.com/journals/bvp/. Articles published in this Special Issue shall be subject to a reduced Article Processing Charge of €200 per article. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	May 1, 2009
First Round of Reviews	August 1, 2009
Publication Date	November 1, 2009

Lead Guest Editor

Juan J. Nieto, Departamento de Análisis Matemático, Facultad de Matemáticas, Universidad de Santiago de Compostela, Santiago de Compostela 15782, Spain; juanjose.nieto.roig@usc.es

Guest Editor

Donal O'Regan, Department of Mathematics, National University of Ireland, Galway, Ireland; donal.oregan@nuigalway.ie