

ON CHARACTERIZATIONS OF A CENTER GALOIS EXTENSION

GEORGE SZETO and LIANYONG XUE

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ABSTRACT. Let B be a ring with 1 , C the center of B , G a finite automorphism group of B , and B^G the set of elements in B fixed under each element in G . Then, it is shown that B is a center Galois extension of B^G (that is, C is a Galois algebra over C^G with Galois group $G|_C \cong G$) if and only if the ideal of B generated by $\{c - g(c) \mid c \in C\}$ is B for each $g \neq 1$ in G . This generalizes the well known characterization of a commutative Galois extension C that C is a Galois extension of C^G with Galois group G if and only if the ideal generated by $\{c - g(c) \mid c \in C\}$ is C for each $g \neq 1$ in G . Some more characterizations of a center Galois extension B are also given.

Keywords and phrases. Galois extensions, center Galois extensions, central extensions, Galois central extensions, Azumaya algebras, separable extensions, H -separable extensions.

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1. Introduction. Let C be a commutative ring with 1 , G a finite automorphism group of C and C^G the set of elements in C fixed under each element in G . It is well known that a commutative Galois extension C is characterized in terms of the ideals generated by $\{c - g(c) \mid c \in C\}$ for $g \neq 1$ in G , that is C is a Galois extension with Galois group G if and only if the ideal generated by $\{c - g(c) \mid c \in C\}$ is C for each $g \neq 1$ in G (see [3, Proposition 1.2, page 80]). A natural generalization of a commutative Galois extension is the notion of a center Galois extension, that is, a noncommutative ring B with a finite automorphism group G and center C is called a center Galois extension of B^G with Galois group G if C is a Galois extension of C^G with Galois group $G|_C \cong G$. Ikehata (see [4, 5]) characterized a center Galois extension with a cyclic Galois group G of prime order in terms of a skew polynomial ring. Then, the present authors generalized the Ikehata characterization to center Galois extensions with Galois group G of any cyclic order [7] and to center Galois extensions with any finite Galois group G [8]. The purpose of the present paper is to generalize the above characterization of a commutative Galois extension to a center Galois extension. We shall show that B is a center Galois extension of B^G if and only if the ideal of B generated by $\{c - g(c) \mid c \in C\}$ is B for each $g \neq 1$ in G . A center Galois extension B is also equivalent to each of the following statements:

- (i) B is a Galois central extension of B^G , that is, $B = B^G C$ which is G -Galois extension of B^G .
- (ii) B is a Galois extension of B^G with a Galois system $\{b_i \in B, c_i \in C, i = 1, 2, \dots, m\}$ for some integer m .
- (iii) the ideal of the subring $B^G C$ generated by $\{c - g(c) \mid c \in C\}$ is $B^G C$ for each $g \neq 1$ in G .

2. Definitions and notations. Throughout this paper, B will represent a ring with 1 , $G = \{g_1 = 1, g_2, \dots, g_n\}$ an automorphism group of B of order n for some integer n , C the center of B , B^G the set of elements in B fixed under each element in G , and $B * G$ a skew group ring in which the multiplication is given by $gb = g(b)g$ for $b \in B$ and $g \in G$.

B is called a G -Galois extension of B^G if there exist elements $\{a_i, b_i \in B, i = 1, 2, \dots, m\}$ for some integer m such that $\sum_{i=1}^m a_i g(b_i) = \delta_{1,g}$. Such a set $\{a_i, b_i\}$ is called a G -Galois system for B . B is called a center Galois extension of B^G if C is a Galois algebra over C^G with Galois group $G|_C \cong G$. B is called a central extension of B^G if $B = B^G C$, and B is called a Galois central extension of B^G if $B = B^G C$ is a Galois extension of B^G with Galois group G .

Let A be a subring of a ring B with the same identity 1. We denote $V_B(A)$ the commutator subring of A in B . We call B a separable extension of A if there exist $\{a_i, b_i \in B, i = 1, 2, \dots, m\}$ for some integer m such that $\sum a_i b_i = 1$, and $\sum b a_i \otimes b_i = \sum a_i \otimes b_i b$ for all $b \in B$ where \otimes is over A . B is called H -separable extension of A if $B \otimes_A B$ is isomorphic to a direct summand of a finite direct sum of B as a B -bimodule. B is called centrally projective over A if B is a direct summand of a finite direct sum of A as a A -bimodule.

3. The characterizations. In this section, we denote $J_j^{(C)} = \{c - g_j(c) \mid c \in C\}$. We shall show that B is a center Galois extension of B^G if and only if $B = B J_j^{(C)}$, the ideal of B generated by $J_j^{(C)}$, for each $g_j \neq 1$ in G . Some more characterizations of a center Galois extension B are also given. We begin with a lemma.

LEMMA 3.1. *If $B = B J_j^{(C)}$ for each $g_j \neq 1$ in G (that is, $j \neq 1$), then*

- (1) *B is a Galois extension of B^G with Galois group G and a Galois system $\{b_i \in B; c_i \in C, i = 1, 2, \dots, m\}$ for some integer m .*
- (2) *B is a centrally projective over B^G .*
- (3) *$B * G$ is H -separable over B .*
- (4) *$V_{B * G}(B) = C$.*

PROOF. (1) Since $B = B J_j^{(C)}$ for each $j \neq 1$, there exist $\{b_i^{(j)} \in B, c_i^{(j)} \in C, i = 1, 2, \dots, m_j\}$ for some integer m_j , $j = 2, 3, \dots, n$ such that $\sum_{i=1}^{m_j} b_i^{(j)} (c_i^{(j)} - g_j(c_i^{(j)})) = 1$. Therefore, $\sum_{i=1}^{m_j} b_i^{(j)} c_i^{(j)} = 1 + \sum_{i=1}^{m_j} b_i^{(j)} g_j(c_i^{(j)})$. Let $b_{m_j+1}^{(j)} = -\sum_{i=1}^{m_j} b_i^{(j)} g_j(c_i^{(j)})$ and $c_{m_j+1}^{(j)} = 1$. Then $\sum_{i=1}^{m_j+1} b_i^{(j)} c_i^{(j)} = 1$ and $\sum_{i=1}^{m_j+1} b_i^{(j)} g_j(c_i^{(j)}) = 0$. Let $b_{i_2, i_3, \dots, i_n} = b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)}$ and $c_{i_2, i_3, \dots, i_n} = c_{i_2}^{(2)} c_{i_3}^{(3)} \cdots c_{i_n}^{(n)}$ for $i_j = 1, 2, \dots, m_j+1$ and $j = 2, 3, \dots, n$. Then

$$\begin{aligned}
& \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2, i_3, \dots, i_n} c_{i_2, i_3, \dots, i_n} = \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)} c_{i_2}^{(2)} c_{i_3}^{(3)} \cdots c_{i_n}^{(n)} \\
& = \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} c_{i_2}^{(2)} b_{i_3}^{(3)} c_{i_3}^{(3)} \cdots b_{i_n}^{(n)} c_{i_n}^{(n)} \\
& = \sum_{i_2=1}^{m_2+1} b_{i_2}^{(2)} c_{i_2}^{(2)} \sum_{i_3=1}^{m_3+1} b_{i_3}^{(3)} c_{i_3}^{(3)} \cdots \sum_{i_n=1}^{m_n+1} b_{i_n}^{(n)} c_{i_n}^{(n)} = 1
\end{aligned} \tag{3.1}$$

and for each $j \neq 1$

$$\begin{aligned}
& \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2, i_3, \dots, i_n} g_j(c_{i_2, i_3, \dots, i_n}) \\
&= \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)} g_j(c_{i_2}^{(2)} c_{i_3}^{(3)} \cdots c_{i_n}^{(n)}) \\
&= \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} b_{i_3}^{(3)} \cdots b_{i_n}^{(n)} g_j(c_{i_2}^{(2)}) g_j(c_{i_3}^{(3)}) \cdots g_j(c_{i_n}^{(n)}) \quad (3.2) \\
&= \sum_{i_2=1}^{m_2+1} \sum_{i_3=1}^{m_3+1} \cdots \sum_{i_n=1}^{m_n+1} b_{i_2}^{(2)} g_j(c_{i_2}^{(2)}) b_{i_3}^{(3)} g_j(c_{i_3}^{(3)}) \cdots b_{i_n}^{(n)} g_j(c_{i_n}^{(n)}) \\
&= \sum_{i_2=1}^{m_2+1} b_{i_2}^{(2)} g_j(c_{i_2}^{(2)}) \sum_{i_3=1}^{m_3+1} b_{i_3}^{(3)} g_j(c_{i_3}^{(3)}) \cdots \sum_{i_n=1}^{m_n+1} b_{i_n}^{(n)} g_j(c_{i_n}^{(n)}) = 0.
\end{aligned}$$

Thus, $\{b_{i_2, i_3, \dots, i_n} \in B; c_{i_2, i_3, \dots, i_n} \in C, i_j = 1, 2, \dots, m_j + 1 \text{ and } j = 2, 3, \dots, n\}$ is a Galois system for B . This complete the proof of (1).

(2) By (1), B is a Galois extension of B^G with a Galois system $\{b_i \in B, c_i \in C, i = 1, 2, \dots, m\}$ for some integer m . Let $f_i : B \rightarrow B^G$ given by $f_i(b) = \sum_{j=1}^n g_j(c_i b)$ for all $b \in B, i = 1, 2, \dots, m$. Then it is easy to check that f_i is a homomorphism as B^G -bimodule and $b = \sum_{i=1}^m b_i c_i b = \sum_{j=1}^n \sum_{i=1}^m b_i g_j(c_i) g_j(b) = \sum_{i=1}^m b_i \sum_{j=1}^n g_j(c_i b) = \sum_{i=1}^m b_i f_i(b)$ for all $b \in B$. Hence $\{b_i; f_i, i = 1, 2, \dots, m\}$ is a dual bases for B as B^G -bimodule, and so B is finitely generated and projective as B^G -bimodule. Therefore, B is a direct summand of a finite direct sum of B^G as a B^G -bimodule. Thus B is centrally projective over B^G .

(3) By (1), B is a Galois extension of B^G with Galois group G . Hence $B * G \cong \text{Hom}_{B^G}(B, B)$ [2, Theorem 1]. By (2), B is centrally projective over B^G . Thus, $B * G (\cong \text{Hom}_{B^G}(B, B))$ is H -separable over B [6, Proposition 11].

(4) We first claim that $V_{B*G}(C) = B$. Clearly, $B \subset V_{B*G}(C)$. Let $\sum_{j=1}^n b_j g_j$ in $V_{B*G}(C)$ for some $b_j \in B$. Then $c(\sum_{j=1}^n b_j g_j) = (\sum_{j=1}^n b_j g_j)c$ for each $c \in C$, so $cb_j = b_j g_j(c)$, that is, $b_j(c - g_j(c)) = 0$ for each $g_j \in G$ and $c \in C$. Since $B = BJ_j^{(C)}$ for each $g_j \neq 1$, there exist $b_i^{(j)} \in B$ and $c_i^{(j)} \in C, i = 1, 2, \dots, m$ such that $\sum_{i=1}^m b_i^{(j)}(c_i^{(j)} - g_j(c_i^{(j)})) = 1$. Hence $b_j = \sum_{i=1}^m b_i^{(j)}(c_i^{(j)} - g_j(c_i^{(j)}))b_j = \sum_{i=1}^m b_i^{(j)}b_j(c_i^{(j)} - g_j(c_i^{(j)})) = 0$ for each $g_j \neq 1$. This implies that $\sum_{j=1}^n b_j g_j = b_1 \in B$. Hence $V_{B*G}(C) \subseteq B$, and so $V_{B*G}(C) = B$. Therefore, $V_{B*G}(B) \subset V_{B*G}(C) = B$. Thus $V_{B*G}(B) = V_B(B) = C$. \square

We now show some characterizations of a center Galois extension B .

THEOREM 3.2. *The following statements are equivalent.*

- (1) B is a center Galois extension of B^G .
- (2) $B = BJ_j^{(C)}$ for each $g_j \neq 1$ in G .
- (3) B is a Galois extension of B^G with a Galois system $\{b_i \in B, c_i \in C, i = 1, 2, \dots, m\}$ for some integer m .
- (4) B is a Galois central extension of B^G .
- (5) $B^G C = B^G C J_j^{(C)}$ for each $g_j \neq 1$ in G .

PROOF. (1) \Rightarrow (2). By hypothesis, C is a Galois extension of C^G with Galois group $G|_C \cong G$. Hence $C = CJ_j^{(C)}$ for each $g_j \neq 1$ in G [3, Proposition 1.2, page 80]. Thus, $B = BJ_j^{(C)}$ for each $g_j \neq 1$ in G .

(2) \Rightarrow (1). Since $B = BJ_j^{(C)}$ for each $g_j \neq 1$ in G , $B*G$ is H -separable over B by Lemma 3.1(3) and $V_{B*G}(B) = C$ by Lemma 3.1(4). Thus C is a Galois extension of C^G with Galois group $G|_C \cong G$ by [1, Proposition 4].

(1) \Rightarrow (3). This is Lemma 3.1(1).

(3) \Rightarrow (1). Since B is Galois extension of B^G with a Galois system $\{b_i \in B, c_i \in C, i = 1, 2, \dots, m\}$ for some integer m , we have $\sum_{i=1}^m b_i g_j(c_i) = \delta_{1,g}$. Hence $\sum_{i=1}^m b_i(c_i - g_j(c_i)) = 1$ for each $g_j \neq 1$ in G . So for every $b \in B$, $b = \sum_{i=1}^m b b_i(c_i - g_j(c_i)) \in BJ_j^{(C)}$. Therefore, $B = BJ_i^{(C)}$ for each $g_i \neq 1$ in G . Thus, B is a center Galois extension of B^G by (2) \Rightarrow (1).

(1) \Rightarrow (4). Since C is a Galois algebra with Galois group $G|_C \cong G$, B and $B^G C$ are Galois extensions of B^G with Galois group $G|_{B^G C} \cong G$. Noting that $B^G C \subset B$, we have $B = B^G C$, that is, B is a central extension of B^G . But B is a Galois extension of B^G , so B is a Galois central extension of B^G .

(4) \Rightarrow (1). By hypothesis, $B = B^G C$ is a Galois extension of B^G . Hence there exists a Galois system $\{a_i; b_i \in B, i = 1, 2, \dots, m\}$ for some integer m such that $\sum_{i=1}^m a_i g_j(b_i) = \delta_{1,j}$. But $B = B^G C$, so $a_i = \sum_{k=1}^{n_{a_i}} b_k^{(a_i)} c_k^{(a_i)}$ and $b_i = \sum_{l=1}^{n_{b_i}} b_l^{(b_i)} c_l^{(b_i)}$ for some $a_k^{(a_i)}$, $b_l^{(b_i)}$ in B^G and $c_k^{(a_i)}$, $c_l^{(b_i)}$ in C , $k = 1, 2, \dots, n_{a_i}$, $l = 1, 2, \dots, n_{b_i}$, $i = 1, 2, \dots, m$. Therefore,

$$\begin{aligned} \delta_{1,j} &= \sum_{i=1}^m a_i g_j(b_i) = \sum_{i=1}^m \sum_{k=1}^{n_{a_i}} b_k^{(a_i)} c_k^{(a_i)} g_j \left(\sum_{l=1}^{n_{b_i}} b_l^{(b_i)} c_l^{(b_i)} \right) \\ &= \sum_{i=1}^m \sum_{k=1}^{n_{a_i}} b_k^{(a_i)} c_k^{(a_i)} \sum_{l=1}^{n_{b_i}} b_l^{(b_i)} g_j(c_l^{(b_i)}) = \sum_{i=1}^m \sum_{k=1}^{n_{a_i}} \sum_{l=1}^{n_{b_i}} (b_k^{(a_i)} c_k^{(a_i)} b_l^{(b_i)}) g_j(c_l^{(b_i)}). \end{aligned} \quad (3.3)$$

This shows that $\{b_{k,l}^{(a_i, b_i)} = b_k^{(a_i)} c_k^{(a_i)} b_l^{(b_i)} \in B; c_{k,l}^{(a_i, b_i)} = c_l^{(b_i)} \in C, k = 1, 2, \dots, n_{a_i}, l = 1, 2, \dots, n_{b_i}, i = 1, 2, \dots, m\}$ is a Galois system for B . Thus, B is a center Galois extension of B^G by (3) \Rightarrow (1).

(1) \Rightarrow (5). Since B is a center Galois extension of B^G , $B = BJ_j^{(C)}$ for each $g_j \neq 1$ in G by (1) \Rightarrow (2) and $B = B^G C$ by (1) \Rightarrow (4). Thus, $B^G C = B^G CJ_j^{(C)}$ for each $g_j \neq 1$ in G .

(5) \Rightarrow (1). Since $B^G C = B^G CJ_j^{(C)}$ for each $g_j \neq 1$ in G , $B = BJ_j^{(C)}$ for each $g_j \neq 1$ in G . Thus, B is a center Galois extension of B^G by (2) \Rightarrow (1). \square

The characterization of a commutative Galois extension C in terms of the ideals generated by $\{c - g(c) \mid c \in C\}$ for $g \neq 1$ in G is an immediate consequence of Theorem 3.2.

COROLLARY 3.3. *A commutative ring C is a Galois extension of C^G if and only if $C = CJ_j^{(C)}$, the ideal generated by $\{c - g_j(c) \mid c \in C\}$ is C for each $g_j \neq 1$ in G .*

PROOF. Let $B = C$ in Theorem 3.2. Then, the corollary is an immediate consequence of Theorem 3.2(2). \square

By Theorem 3.2, we derive several characterizations of a Galois central extension B .

COROLLARY 3.4. *If B is a central extension of B^G (that is, $B = B^G C$), then the following statements are equivalent.*

- (1) B is a Galois extension of B^G .
- (2) B is a center Galois extension of B^G .
- (3) $B * G$ is H -separable over B .
- (4) $B = CJ_j^{(B)}$ for each $g_j \neq 1$ in G .
- (5) $B = BJ_j^{(B)}$ for each $g_j \neq 1$ in G .

PROOF. (1) \Leftrightarrow (2). This is given by (1) \Leftrightarrow (4) in Theorem 3.2.

(2) \Rightarrow (3). This is Lemma 3.1(3).

(3) \Rightarrow (1). Since $B * G$ is H -separable over B , B is a Galois extension of B^G [1, Proposition 2].

Since $B = B^G C$ by hypothesis, it is easy to see that $J_j^{(B)} = B^G J_j^{(C)}$ for each g_j in G . Thus, $B = CJ_j^{(B)}$, $B = BJ_j^{(B)}$, and $B = BJ_j^{(C)}$ are equivalent. This implies that (2) \Leftrightarrow (4) \Leftrightarrow (5) by Theorem 3.2(2). \square

We call a ring B the DeMeyer-Kanzaki Galois extension of B^G if B is an Azumaya C -algebra and B is a center Galois extension of B^G (for more about the DeMeyer-Kanzaki Galois extensions, see [2]). Clearly, the class of center Galois extensions is broader than the class of the DeMeyer-Kanzaki Galois extensions. We conclude the present paper with two examples. (1) The DeMeyer-Kanzaki Galois extension of B^G and (2) a center Galois extension of B^G , but not the DeMeyer-Kanzaki Galois extension of B^G .

EXAMPLE 3.5. Let \mathbb{C} be the field of complex numbers, that is, $\mathbb{C} = \mathbb{R} + \mathbb{R}\sqrt{-1}$ where \mathbb{R} is the field of real numbers, $B = \mathbb{C}[i, j, k]$ the quaternion algebra over \mathbb{C} , and $G = \{1, g \mid g(c_1 + c_i i + c_j j + c_k k) = g(c_1) + g(c_i)i + g(c_j)j + g(c_k)k \text{ for each } b = c_1 + c_i i + c_j j + c_k k \in \mathbb{C}[i, j, k] \text{ and } g(u + v\sqrt{-1}) = u - v\sqrt{-1} \text{ for each } c = u + v\sqrt{-1} \in \mathbb{C}\}$. Then

- (1) The center of B is \mathbb{C} .
- (2) B is an Azumaya C -algebra.
- (3) \mathbb{C} is a Galois extension of \mathbb{C}^G with Galois group $G|_{\mathbb{C}} \cong G$ and a Galois system $\{a_1 = 1/\sqrt{2}, a_2 = (1/\sqrt{2})\sqrt{-1}; b_1 = 1/\sqrt{2}, b_2 = -(1/\sqrt{2})\sqrt{-1}\}$.
- (4) B is the DeMeyer-Kanzaki Galois extension of B^G by (2) and (3).
- (5) $B^G = \mathbb{R}[i, j, k]$.
- (6) $B = B^G \mathbb{C}$, so B is a centeral extension of B^G .
- (7) $J_g^{(\mathbb{C})} = \mathbb{R}\sqrt{-1}$.
- (8) $B = BJ_g^{(C)}$ since $1 = -\sqrt{-1}\sqrt{-1} \in BJ_g^{(C)}$.
- (9) $J_g^{(B)} = \mathbb{R}\sqrt{-1} + \mathbb{R}\sqrt{-1}i + \mathbb{R}\sqrt{-1}j + \mathbb{R}\sqrt{-1}k$.
- (10) $B = \mathbb{C}J_g^{(B)}$.

EXAMPLE 3.6. By replacing in Example 3.5 the field of complex numbers \mathbb{C} with the ring $C = \mathbb{Z} \oplus \mathbb{Z}$ where \mathbb{Z} is the ring of integers, $g(a, b) = (b, a)$ for all $(a, b) \in C$, and $G = \{1, g \mid g(c_1 + c_i i + c_j j + c_k k) = g(c_1) + g(c_i)i + g(c_j)j + g(c_k)k \text{ for each } b = c_1 + c_i i + c_j j + c_k k \in B = C[i, j, k]\}$. Then

- (1) The center of B is C .
- (2) C is a Galois extension of C^G with Galois group $G|_C \cong G$ and a Galois system $\{a_1 = (1, 0), a_2 = (0, 1); b_1 = (1, 0), b_2 = (0, 1)\}$.

(3) B is not an Azumaya C -algebra (for $1/2 \notin C$), and so B is not the DeMeyer-Kanzaki Galois extension of B^G .

$$(4) C^G = \{(a, a) \mid a \in \mathbb{Z}\} \cong \mathbb{Z}.$$

$$(5) B^G = C^G[i, j, k].$$

$$(6) B = B^G C, \text{ so } B \text{ is a central extension of } B^G.$$

$$(7) J_g^{(C)} = \{(a, -a) \mid a \in \mathbb{Z}\} = \mathbb{Z}(1, -1).$$

$$(8) B = BJ_g^{(C)} \text{ since } 1 = (1, 1) = (1, -1)(1, -1) \in BJ_g^{(C)}.$$

$$(9) J_g^{(B)} = \mathbb{Z}(1, -1) + \mathbb{Z}(1, -1)i + \mathbb{Z}(1, -1)j + \mathbb{Z}(1, -1)k.$$

$$(10) B = CJ_g^{(B)}.$$

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SZETO: DEPARTMENT OF MATHEMATICS, BRADLEY UNIVERSITY, PEORIA, ILLINOIS 61625, USA
E-mail address: szeto@bradley.bradley.edu

XUE: DEPARTMENT OF MATHEMATICS, BRADLEY UNIVERSITY, PEORIA, ILLINOIS 61625, USA
E-mail address: lxue@bradley.bradley.edu

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Lead Guest Editor

Juan J. Nieto, Departamento de Análisis Matemático,
Facultad de Matemáticas, Universidad de Santiago de

Compostela, Santiago de Compostela 15782, Spain;
juanjose.nieto.roig@usc.es

Guest Editor

Donal O'Regan, Department of Mathematics, National
University of Ireland, Galway, Ireland;
donal.oregan@nuigalway.ie