

SOME RESULTS ON $[n, m]$ -PARACOMPACT AND $[n, m]$ -COMPACT SPACES

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(Received February 9, 1988 and in revised form March 20, 1996)

ABSTRACT. Let n and m be infinite cardinals with $n \leq m$ and n be a regular cardinal. We prove certain implications of $[n, m]$ -strongly paracompact, $[n, m]$ -paracompact and $[n, m]$ -metacompact spaces. Let X be $[n, \infty]$ -compact and Y be a $[n, m]$ -paracompact (resp. $[n, \infty]$ -paracompact), P_n -space (resp. wP_n -space). If $m = \sum_{k < n} m^k$ we prove that $X \times Y$ is $[n, m]$ -paracompact (resp. $[n, \infty]$ -paracompact).

KEY WORDS AND PHRASES: Strongly paracompact and metacompact spaces.

1990 AMS SUBJECT CLASSIFICATION CODES: Primary 54D20.

1. INTRODUCTION

Throughout this paper m and n will denote infinite cardinals with $n \leq m$ and n will be a regular cardinal. A space X is called $[n, m]$ -compact (see Alexandroff [1]) if every open cover α of X with $|\alpha| \leq m$ has a subcover of cardinality $< n$. For a set A , we denote by $|A|$, the cardinality of A . A family α of subsets of X is a *locally- n (point- n)* family (Mansfield [2]) if for every $x \in X$, there is an open neighborhood of x in X which meets $< n$ members of α (resp. x belongs to $< n$ members of α). An *open refinement* of a cover α of a space X is an open cover β such that each member of β is contained in some member of α . A space X is $[n, m]$ -paracompact (resp. $[n, m]$ -metacompact) if every open cover α of X with $|\alpha| \leq m$ has a locally- n (resp. point- n) open refinement. X is $[n, m]$ -strongly paracompact if every open cover of X with $|\alpha| \leq m$, has an open refinement β such that for each $B \in \beta$,

$$|\{C \in \beta : C \cap B \neq \emptyset\}| < n.$$

Originally, Singal and Singal introduced the concept of (m, k) -paracompactness in [3]. Our notation is slightly different than theirs. However, we note that a space X is (m, k) -paracompact, as defined in [3], if and only if X is $[k^+, m]$ -paracompact. A space X is $[n, \infty]$ -compact (resp. $[n, \infty]$ -paracompact, $[n, \infty]$ -metacompact, $[n, \infty]$ -strongly paracompact) if X is $[n, m]$ -compact (resp. $[n, m]$ -paracompact, $[n, m]$ -metacompact, $[n, m]$ -strongly paracompact for each cardinal $m \geq n$). A space X is a P_n -space [4] if for every family α of open subsets of X with $|\alpha| < n$, $\bigcap \alpha$ is open in X . We observe that the class of P_{ω_0} -spaces is the class of all topological spaces, where ω_0 denotes the first infinite cardinal number. Also we observe that if P is any of "compact", "paracompact", or "metacompact", then the class of $[\omega_0, \infty] - P$ spaces is the same as the class of P spaces in the ordinary sense.

Morita [5] studied m -paracompact spaces. A space X is m -paracompact if and only if X is $[\omega_0, m]$ -paracompact. Morita proved that if Y is an m -paracompact space and X is a compact space, then $X \times Y$ is m -paracompact. In case $m = \sum_{k < n} m^k$, we generalize Morita's result by showing that if X is an $[n, \infty]$ -compact space and Y is $[n, m]$ -paracompact, P_n -space, then $X \times Y$ is $[n, m]$ -paracompact. We note that for $n = \omega_0$ this result implies Morita's result. A subset W of a topological space Y is called n -open (Hdeib [6]) if for each $y \in W$ there exists an open set V in Y such that $y \in V$ and $|V \setminus W| < n$. A subset F of Y is called n -closed if $Y \setminus F$ is n -open. A space Y is called a *weak P_n -space* [6] or *wP_n -space* if $\cap \alpha$ is n -open for every family α of open subsets of Y with $|\alpha| < n$. We prove that if X is a $[n, \infty]$ -compact space and Y is an $[n, \infty]$ -paracompact, wP_n -space, then $X \times Y$ is $[n, \infty]$ -paracompact. This result is a variation of our generalization of Morita's result.

It is well known (Dungundji [7]) that if a space X is locally compact and Hausdorff, then X is paracompact if and only if X is a disjoint topological sum of σ -compact spaces. We prove that if $n > \omega_0$, then a locally $[n, \infty]$ -compact, regular space X is $[n, \infty]$ -paracompact if and only if X is a disjoint topological sum of $[n, \infty]$ -compact spaces. A space X is, by definition, locally $[n, \infty]$ -compact if for each point $x \in X$ and an open neighborhood G of x , there exists an $[n, \infty]$ -compact neighborhood H of x such that $H \subseteq G$.

In this paper we also prove certain implications concerning $[n, m]$ -paracompact, metacompact, strongly paracompact spaces.

For a space X , the *density* $d(X)$ of X is defined as the smallest cardinal number that is the cardinal number of a dense subset of X . For terminology not defined here see Engelking [8].

2. $[n, m]$ -PARACOMPACT SPACES

It is clear that each $[n, m]$ -strongly paracompact space is $[n, m]$ -paracompact which in turn is $[n, m]$ -metacompact. However, in general, the converses of these implications do not hold.

The following two theorems are interesting in this respect.

THEOREM 2.1. Let γ be an open cover of a space X such that $|\gamma| \leq m$ and $d(A) < n$ for each $A \in \gamma$. Then X is $[n, m]$ -strongly paracompact if and only if X is $[n, m]$ -metacompact.

PROOF. We only need to prove "if" part. Let X be $[n, m]$ -metacompact. Let α be an open cover of X with $|\alpha| \leq m$. Let $\beta = \{A \cap W : A \in \alpha \text{ and } W \in \gamma\}$. Then $|\beta| \leq m$, β is an open refinement of α and $d(B) < n$ for each $B \in \beta$. Since X is $[n, m]$ -metacompact, then there exists a point- n open refinement λ of β . Each $L \in \lambda$ is contained in some $B_L \in \beta$. Since L is open and $d(B_L) < n$, then $d(L) < n$. Let $L \in \lambda$ and D be a dense set in L such that $|D| < n$. Let $\Delta = \{A \in \lambda : A \cap L \neq \emptyset\}$. Since D is dense in L , then $A \in \Delta$ if and only if $A \cap D \neq \emptyset$. Thus $\Delta = \{A \in \lambda : A \cap D \neq \emptyset\}$. For $d \in D$ let us set $\Delta_d = \{K \in \lambda : d \in K\}$. Then $|\Delta_d| < n$ since λ is point- n . Hence

$$|\Delta| \leq \sum_{d \in D} |\Delta_d| < n.$$

Since $|D| < n$ and n is a regular cardinal, it follows that X is $[n, m]$ -strongly paracompact.

COROLLARY 2.1 (Traylor [9]). Let X be a regular space with an open cover γ such that $d(G) \leq \omega_0$ for all $G \in \gamma$. Then X is strongly paracompact if and only if X is metalindelöf.

PROOF. The proof follows from Theorem 2.1 and Theorem 3, page 229 in [8].

THEOREM 2.3. Let X be a locally $[n, \infty]$ -compact space. Then X is $[n, \infty]$ -paracompact if and only if X is $[n, \infty]$ -strongly paracompact.

PROOF. We only need to prove "only if" part. Let X be $[n, \infty]$ -paracompact. Let α be an open cover of X . Since X is locally $[n, \infty]$ -compact then there exists a cover σ of X such that

- (i) σ refines α
- (ii) $\beta = \{\text{int } H : H \in \sigma\}$ is a cover of X ,

(iii) if $H \in \sigma$, then H is $[n, \infty]$ -compact

Since X is $[n, \infty]$ -paracompact, then β has a locally- n open refinement γ . Now, let $G \in \gamma$ and

$$\Delta = \{L \in \gamma : G \cap L \neq \emptyset\}.$$

Since γ refines β , then $G \subseteq \text{int } H \subseteq H$ for some $H \in \sigma$. For each $x \in H$, there is an open set W_x containing x such that W_x meets $< n$ members of γ . We have

$$H = \bigcup \{W_x \cap H : x \in H\}.$$

Since H is $[n, \infty]$ -compact, then there exists a subset T of H such that $|T| < n$ and

$$H = \bigcup \{W_x \cap H : x \in T\}.$$

For $x \in T$. Let us set

$$\Delta_x = \{L \in \gamma : W_x \cap L \neq \emptyset\}.$$

We see that

$$\Delta \subseteq \{\Delta_x : x \in T\}.$$

Hence

$$|\Delta| \leq \sum_{x \in T} |\Delta_x| < n.$$

Since $|T| < n$, $|\Delta_x| < n$ for each $x \in T$ and n is a regular cardinal.

COROLLARY 2.4. Let X be a regular, locally Lindelöf space. Then X is strongly paracompact if and only if X is paralindelöf

PROOF. The proof follows from Theorem 2.3 and Theorem 3, page 229 in [8].

It is well known in [7] that if X is a locally compact Hausdorff space, then X is paracompact if and only if X is a disjoint topological sum of σ -compact spaces. It is natural to ask when X is a locally $[n, \infty]$ -compact, $[n, \infty]$ -paracompact space, whether X is a disjoint topological sum of σ - $[n, \infty]$ -compact spaces. The result above is the answer to the case when $n = \omega_0$ and X is Hausdorff. So we are only interested in the case when $n > \omega_0$. The following theorem provides the answer to this question

THEOREM 2.5. Let $n > \omega_0$ and X be a locally $[n, \infty]$ -compact regular space. Then X is $[n, \infty]$ -paracompact if and only if X is a disjoint topological sum of $[n, \infty]$ -compact spaces

PROOF. It is obvious that if X is a disjoint topological sum of $[n, \infty]$ -compact spaces, then X is $[n, \infty]$ -paracompact. Thus let us assume that X is $[n, \infty]$ -paracompact. Let

$$\alpha = \{U : U \subseteq X \text{ and } U \text{ is } [n, \infty]\text{-compact}\}.$$

Then $\beta = \{\text{int } U : U \in \alpha\}$ is an open cover of X since X is locally $[n, \infty]$ -compact. Since X is regular, then there is an open cover γ of X such that $\bar{\gamma} = \{c\ell G : G \in \gamma\}$ refines β . Since X is a locally $[n, \infty]$ -compact, $[n, \infty]$ -paracompact space, then by Theorem 2.3, X is $[n, \infty]$ -strongly paracompact. Hence there exists an open refinement σ of γ such that for each $L \in \sigma$ the set $\Delta_L = \{H \in \sigma : L \cap H \neq \emptyset\}$ has cardinality n . For a positive integer t , a chain of length t in σ is a sequence L_1, \dots, L_t in σ such that $L_i \cap L_{i+1} \neq \emptyset$ for $1 \leq i \leq t-1$. If $t = 1$ we simply require $L_1 \neq \emptyset$. For $x, y \in X$ we define $x \sim y$ if there is a chain L_1, \dots, L_t in σ such that $x \in L_1$ and $y \in L_t$. Clearly " \sim " is an equivalence relation since σ is an open cover of X . Let R be an equivalence class and $a \in R$. If $y \in R$, then there is a chain L_1, \dots, L_t in σ such that $a \in L_1$ and $y \in L_t$. Clearly each point in L_t is equivalent to a with respect to " \sim ", hence $L_t \subseteq R$. So R is open. Let $z \in c\ell R$. There exists $L \in \sigma$ such that $z \in L$. Since $z \in c\ell R$, then $L \cap R \neq \emptyset$. Thus if $w \in L \cap R$, then $z \sim w$, i.e., $z \in R$. This shows that R is also closed. Let $a \in L$ and $L \in \sigma$. We know that $L \subseteq R$. For a positive integer t , let

$$\mu_t = \{H \in \gamma : \text{there is a chain } L_1, \dots, L_t \text{ in } \sigma \text{ such that } L = L_1 \text{ and } L_t = H\}.$$

Clearly $\mu_1 = \{L\}$. Thus $|\mu_1| < n$. Assume that $|\mu_t| < n$. If $K \in \mu_{t+1}$, then there is a chain $L_1, L_2, \dots, L_t, L_{t+1}$ in σ such that $L = L_1$ and $K = L_{t+1}$. Then $L_t \in \mu_t$. Thus

$$\mu_{t+1} \subseteq \bigcup \{\Delta_H : H \in \mu_t\}.$$

Hence

$$|\mu_{t+1}| \leq \sum_{H \in \mu_t} |\Delta_H| < n,$$

since $|\mu_t| < n$ and n is a regular cardinal. This inductive argument shows that $|\mu_t| < n$ for all $t \geq 1$. We show that $R = \bigcup \{R_t : t \geq 1\}$ where $R_t = \bigcup \{c\ell H : H \in \mu_t\}$. If $H \in \mu_t$, then by the definition of " \sim " we get $H \subseteq R$. Since R is closed, then $c\ell H \subseteq R$. So $R \supseteq \bigcup_t R_t$. Conversely let $y \in R$. Then there is a chain L_1, \dots, L_t in σ such that $a \in L_1$ and $y \in L_t$. Since $a \in L_1 \cap L$, then L, L_1, \dots, L_t is a chain in σ . Thus $L_t \in \mu_{t+1}$; and consequently $y \in \bigcup_t R_t$. This proves the result.

Now, if $H \in \sigma$, then $H \subseteq c\ell E \subseteq U$ for some $G \in \gamma$ and $U \in \alpha$. Thus $c\ell G$ and consequently $c\ell H$ is $[n, \infty]$ -compact. Since $|\mu_t| < n$ when t is a positive integer, then R_t is also $[n, \infty]$ -compact. Since $n > \omega_0$, then $R = \bigcup_t R_t$ is also $[n, \infty]$ -compact. This proves the theorem since X is the disjoint topological sum of the equivalence classes of " \sim ".

3. PRODUCT THEOREMS

In this section we prove theorems concerning $[n, m]$ -paracompact of a product space $X \times Y$. Our first theorem is a generalization of a result by Morita [5] which states that if X is a compact space and Y is an m -paracompact space, then $X \times Y$ is an m -paracompact space.

THEOREM 3.1. Let the cardinal m satisfy $m = \Sigma\{m^k : k \text{ is a cardinal and } k < n\}$. Let X be an $[n, \infty]$ -compact space and Y be an $[n, m]$ -paracompact P_n -space. Then $X \times Y$ is $[n, m]$ -paracompact.

PROOF. Let α be an open cover of $X \times Y$ with $|\alpha| \leq m$. For each subset β of α with $|\beta| < n$, let $W_\beta = \{y \in Y : X \times \{y\} \subseteq \bigcup \beta\}$. Let $\beta \subseteq \alpha$ and $|\beta| < n$. Then W_β is open in Y . For let $y \in W_\beta$. Then $X \times \{y\}$ is contained in $G = \bigcup \beta$. For each $x \in X$, there exists a basic open set $B_x \times C_x$ in $X \times Y$ such that $(x, y) \in B_x \times C_x \subseteq G$. Now $\{B_x : x \in X\}$ is an open cover of X . Thus there is a subcover $\{B_x : x \in S\}$ where $|S| < n$. $C = \bigcap \{C_x : x \in S\}$ is open in Y , since Y is a P_n -space and $y \in C$. Moreover, $X \times C \subseteq \bigcup \{B_x \times C : x \in S\} \subseteq G$. It follows that $y \in C \subseteq W_\beta$. So W_β is open. Let us set

$$\Lambda = \{W_\beta : \beta \subseteq \alpha \text{ and } |\beta| < n\}.$$

Let $y \in Y$. For each $x \in X$, there exists $A_x \in \alpha$ such that $(x, y) \in A_x$. There is a basic open set $D_x \times E_x$ in $X \times Y$ such that $(x, y) \in D_x \times E_x \subseteq A_x$. Now, $\{D_x : x \in X\}$ is an open cover of X . Thus it has a subcover $\{D_x : x \in T\}$ such that $|T| < n$.

Let $\beta = \{A_x : x \in T\}$. Then $|\beta| < n$ and $X \times \{y\} \subseteq \bigcup_{x \in T} D_x \times \{y\} \subseteq \bigcup \beta$. Thus $y \in W_\beta$. This shows that Λ is an open cover of Y . Further notice that

$$|\Lambda| \leq \sum_{k < n} m^k = m.$$

Thus there exists a locally- n open refinement μ of Λ since Y is $[n, m]$ -paracompact. For each $M \in \mu$ we pick $\beta_M \subseteq \alpha$ such that $|\beta_M| < n$ and $M \subseteq W_{\beta_M}$. For $A \in \beta_M$ we define $G(M, A) = (X \times M) \cap A$. Let $\rho = \{G(M, A) : M \in \mu, A \in \beta_M\}$. If $(x, y) \in X \times Y$, then $y \in M \subseteq W_{\beta_M}$ for some $M \in \mu$. Since $y \in W_{\beta_M}$, then $X \times \{y\} \subseteq \bigcup \beta_M$. Thus $(x, y) \in A$ for some $A \in \beta_M$. Hence $(x, y) \in G(M, A)$.

This shows that ρ is an open cover of $X \times Y$. Clearly ρ refines α . Let $(x, y) \in X \times Y$. There exists an open set N in Y such that $y \in N$ and N meets $< n$ members of μ . Let $\mu' = \{M \in \mu : N \cap M \neq \emptyset\}$. Thus we have $|\mu'| < n$. If $M \notin \mu'$, then $(X \times N) \cap G(M, A) = \emptyset$ for all $A \in \beta_M$. Thus the open neighborhood $X \times N$ of (x, y) can only meet those $G(M, A)$ with $M \in \mu'$ and $A \in \beta_M$. The cardinality of such $G(M, A)$'s is at most $\sum_{M \in \mu'} |\beta_M|$ which is less than n since $|\mu'| < n$, $|\beta_M| < n$ for each $M \in \mu'$ and n is a regular cardinal. Hence ρ is a locally- n family.

In Theorem 3.1 if we assume the stronger condition that Y is $[n, \infty]$ -paracompact then we can show that $X \times Y$ is $[n, \infty]$ -paracompact if we only assume that y is a wP_n -space. Before we prove this result we first prove two theorems which are interesting in their own rights.

Let A and B be topological spaces and $f : A \rightarrow B$ be a function. f is called n -closed if for every closed subset F of A , $f(F)$ is an n -closed subset of B .

THEOREM 3.2. Let X be an $[n, \infty]$ -compact space and Y be a wP_n -space. Then the projection mapping $P : X \times Y \rightarrow Y$ is an n -closed map.

PROOF. Let F be closed in $X \times Y$ and y be in $U = Y \setminus P(F)$. Then $(x, y) \notin F$ for each $x \in X$. Hence there are open sets U_x in X and V_x in Y , for each $x \in X$, such that $(x, y) \in U_x \times V_x$ and $F \cap (U_x \times V_x) = \emptyset$. $\alpha = \{U_x : x \in X\}$ is an open cover of X . Since X is $[n, \infty]$ -compact, then there exists a subset T of X such that $|T| < n$ and $\beta = \{U_x : x \in T\}$ covers X . $W = \bigcap \{V_x : x \in T\}$ is n -open in Y since Y is a wP_n -space and $y \in W$. Hence there exists an open set V in Y such that $y \in V$ and $|V \setminus W| < n$. Now, we have $X \times W \cap F = \emptyset$. Hence $W \subseteq U$. Thus $|V \setminus U| < n$. It follows that U is n -open. Thus P is n -closed.

THEOREM 3.3. Let $f : Z \rightarrow Y$ be a continuous, n -closed mapping such that $f^{-1}(y)$ is $[n, \infty]$ -compact for such $y \in Y$. If Y is $[n, \infty]$ -paracompact (resp. $[n, \infty]$ -compact) then Z is also $[n, \infty]$ -paracompact (resp. $[n, \infty]$ -compact).

PROOF. We will only prove the case when Y is $[n, \infty]$ -paracompact. The $[n, \infty]$ -compact case can be proved similarly.

Let α be an open cover of X . For each $y \in Y$ let α_y be a subcollection of α such that $|\alpha_y| < n$ and $f^{-1}(y) \subseteq \bigcup \alpha_y$. Such a subcollection exists since $f^{-1}(y)$ is $[n, \infty]$ -compact. For $y \in Y$, let $G_y = \bigcup \alpha_y$, and $W_y = Y \setminus f(X \setminus G_y)$. Then $y \in W_y$ and W_y is n -open since f is an n -closed map. Thus for each $y \in Y$, there is an open set V_y in Y such that $y \in V_y$ and $|V_y \setminus W_y| < n$. $\gamma = \{V_y : y \in Y\}$ is an open cover of Y and Y is $[n, \infty]$ -paracompact. Hence there exists a locally- n open refinement $\{T_i : i \in I\}$ of γ . For each $i \in I$, pick $y_i \in Y$ such that $T_i \subseteq V_{y_i}$. For $y \in Y$ let

$$\beta_y = \alpha_y \cup (\bigcup \{\alpha_t : t \in V_{y_i} \setminus W_{y_i}\}).$$

Then

$$|\beta_y| \leq |\alpha_y| + \sum \{|\alpha_t| : t \in V_{y_i} \setminus W_{y_i}\} < n,$$

since n is a regular cardinal. Moreover $f^{-1}(T_i) \subseteq \bigcup \beta_{y_i}$ since $T_i \subseteq V_{y_i}$. Let

$$\sigma = \{H \cap f^{-1}(T_i) : H \in \beta_{y_i}, i \in I\}.$$

Then clearly σ is an open refinement of α . Let $x \in X$ and $y = f(x)$. There is an open set N in Y and a subset J of I such that $|J| < n$, $y \in N$ and $N \cap T_i = \emptyset$ for all $i \in I \setminus J$. Let $M = f^{-1}(N)$ and $\Lambda = \{H \cap f^{-1}(T_i) : H \in \beta_{y_i}, i \in J\}$. Then $x \in M$ and $|\Lambda| \leq \sum_{i \in J} |\beta_{y_i}| < n$ since n is a regular cardinal. Moreover, if $L \in \sigma \setminus \Lambda$, then $L \cap M = \emptyset$. Hence σ is a locally- n family.

As a corollary of Theorem 3.2 and Theorem 3.3 we obtain the following variation of Theorem 3.1

THEOREM 3.4. Let X be an $[n, \infty]$ -compact space and Y be an $[n, \infty]$ -paracompact (resp $[n, \infty]$ -compact) wP_n -space, then $X \times Y$ is $[n, \infty]$ -paracompact (resp $[n, \infty]$ -compact)

ACKNOWLEDGMENT. This work was partially supported by Yarmouk University

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