# ON THE FOURIER TRANSFORM AND THE EXCHANGE PROPERTY

DRAGU ATANASIU AND PIOTR MIKUSIŃSKI

*Received 19 June 2005*

A simplified construction of tempered Boehmians is presented. The new construction shows that considering delta sequences and convergence arguments is not essential.

## 1. Introduction

Since Boehmians were introduced, extensions of the Fourier transform to spaces of Boehmians attracted a lot of attention (see [2, 3, 4, 5, 6, 7, 8, 9]). In some cases, the range of the extended Fourier transform is a space of functions. In other constructions, the range is a space of distributions or a space of Boehmians.

In this paper, we would like to consider the space of tempered Boehmians presented in [8]. In this case, the range of the Fourier transform is the space of all distributions $\mathcal{D}'$. This work is motivated by [1].

First we recall briefly the construction of the space of tempered Boehmians. A continuous function $f : \mathbb{R}^N \to \mathbb{C}$ is called *slowly increasing* if there is a polynomial $p$ on $\mathbb{R}^N$ such that $|f(x)| \le p(x)$ for all $x \in \mathbb{R}^N$. The space of slowly increasing functions will be denoted by $\mathcal{W}(\mathbb{R}^N)$ or simply $\mathcal{W}$.

An infinitely differentiable function $f : \mathbb{R}^N \to \mathbb{C}$ is called *rapidly decreasing* if

$$\sup_{|\alpha|\le m} \sup_{x\in\mathbb{R}^N} \left(1+x_1^2+\cdots+x_N^2\right)^m \left|D^\alpha f(x)\right| < \infty \tag{1.1}$$

for every nonnegative integer $m$, where $x = (x_1,\dots,x_N)$, $\alpha = (\alpha_1,\dots,\alpha_N)$, $\alpha_n$'s are nonnegative integers, $|\alpha| = \alpha_1 + \cdots + \alpha_N$, and

$$D^\alpha = \frac{\partial^{|\alpha|}}{\partial x^\alpha} = \frac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1}\cdots\partial x_N^{\alpha_N}}. \tag{1.2}$$

The space of rapidly decreasing functions is denoted by $\mathcal{S}(\mathbb{R}^N)$ or simply $\mathcal{S}$.

If $f \in \mathcal{W}$ and $\varphi \in \mathcal{S}$, then the convolution

$$f * \varphi(x) = \int_{\mathbb{R}^N} f(y)\varphi(x-y)\,dy \tag{1.3}$$

is well defined and $f * \varphi \in \mathcal{W}$.


International Journal of Mathematics and Mathematical Sciences 2005:16 (2005) 2579–2584
DOI: 10.1155/IJMMS.2005.2579

A sequence $(\varphi_n) \in \mathcal{S}^{\mathbb{N}}$ is called a *delta sequence* if it satisfies the following conditions:

(a) $\int_{\mathbb{R}^N} \varphi_n(x)dx = 1$ for all $n \in \mathbb{N}$,

(b) $\int_{\mathbb{R}^N} |\varphi_n(x)|dx \leq C$ for some constant $C$ and all $n \in \mathbb{N}$,

(c) $\lim_{n\to\infty} \int_{\|x\|\geq\varepsilon} \|x\|^k |\varphi_n(x)|dx = 0$ for every $k \in \mathbb{N}$ and $\varepsilon > 0$.

The space of *tempered Boehmians* $\mathscr{B}$ is defined as the space of equivalence classes of pairs of sequences $(f_n, \varphi_n)$, where $f_n \in \mathscr{W}$ and $(\varphi_n)$ is a delta sequence as defined above, satisfying

$$f_m * \varphi_n = f_n * \varphi_m \quad \forall m, n \in \mathbb{N}, \tag{1.4}$$

with respect to the equivalence relation defined by

$$(f_n, \varphi_n) \sim (g_n, \gamma_n) \quad \text{if } f_m * \gamma_n = g_n * \varphi_m,\ \forall m, n \in \mathbb{N}. \tag{1.5}$$

It is shown in [8] that the Fourier transform can be defined for tempered Boehmians and that the range is exactly the space of all distributions $\mathscr{D}'$. Thus, the space of tempered Boehmians can be identified with the space of ultradistributions $\mathscr{Z}'$ (see, e.g., [10]). In the construction, the particular choice of delta sequences and the fact that the Fourier transform of a delta sequence converges to 1 uniformly on compact subsets of $\mathbb{R}^N$ seem to be essential. In this paper, we show that this is not the case. In fact, we give an equivalent construction where convergence plays no role. This approach indicates that the results of [8] follow from a more general principle.

In what follows, we will denote by $\mathcal{S}'$ the space of tempered distributions, that is, the space of continuous linear functionals on $\mathcal{S}$. If $f \in \mathcal{S}'$ and $\varphi \in \mathcal{S}$, then the convolution $f * \varphi$ is defined as $(f * \varphi)(x) = f(\varphi_x)$, where $\varphi_x(z) = \varphi(x - z)$. It can be shown that, if $f \in \mathcal{S}'$ and $\varphi \in \mathcal{S}$, then $f * \varphi \in \mathscr{W}$. The Fourier transform of a tempered distribution $f$, denoted by $\hat{f}$, is the functional on $\mathcal{S}$ defined by $\hat{f}(\varphi) = f(\hat{\varphi})$, where $\hat{\varphi}$ is the Fourier transform of $\varphi$.

## 2. The exchange property

For a family $\{\varphi_j\}_{j\in J} = \{\varphi_j\}_J$, where $J$ is an index set and $\varphi_j \in \mathcal{S}$ for all $j \in J$, we define

$$M\big(\{\varphi_j\}_J\big) = \{x \in \mathbb{R}^N : \hat{\varphi}_j(x) = 0\ \forall j \in J\}. \tag{2.1}$$

A family of pairs $\{(f_j, \varphi_j)\}_J$, where $f_j \in \mathcal{S}'$ and $\varphi_j \in \mathcal{S}$, is said to have the *exchange property* if

$$f_j * \varphi_k = f_k * \varphi_j \quad \forall j, k \in J. \tag{2.2}$$

Theorem 2.1. *If $\{(f_j, \varphi_j)\}_J$ has the exchange property and $\Omega = M(\{\varphi_j\}_J)^c$ (the complement of $M(\{\varphi_j\}_J)$ in $\mathbb{R}^N$), then there exists a unique $F \in \mathscr{D}'(\Omega)$ such that*

$$\hat{f}_j = F\hat{\varphi}_j \quad \forall j \in J. \tag{2.3}$$

*Proof.* For every $x \in \Omega$ there exists a $j \in J$ and $\varepsilon > 0$ such that $|\hat{\varphi}_j(x)| > \varepsilon$ in an open neighborhood of $x$. Then we can define $F = \hat{f}_j/\hat{\varphi}_j$ in that neighborhood. We need to show that this definition of $F$ is independent of $j$. Suppose, for some $\varepsilon > 0$, we have $|\widehat{\varphi}_j(x)| > \varepsilon$ for all $x \in U$ and $|\widehat{\varphi}_k(x)| > \varepsilon$ for all $x \in V$. Then, since $f_j * \varphi_k = f_k * \varphi_j$, we have $\widehat{f}_j\widehat{\varphi}_k = \widehat{f}_k\widehat{\varphi}_j$ and

$$\frac{\widehat{f}_j}{\widehat{\varphi}_j} = \frac{\widehat{f}_k}{\widehat{\varphi}_k} \tag{2.4}$$

on $U \cap V$. Clearly, $F$ is unique. □

We will denote by $\mathcal{A}$ the collection of all families of pairs $\{(f_j,\varphi_j)\}_J$, where $J$ is an index set, $f_j \in \mathcal{S}'$ and $\varphi_j \in \mathcal{S}$ for all $j \in J$, satisfying the exchange property and such that $M(\{\varphi_j\}_J) = \varnothing$.

Note that in the definition of $\mathcal{A}$ the index set is not fixed. If $f \in \mathcal{S}'$ is arbitrary and $\omega(x) = e^{-x\cdot x}$, then $\{(f,\omega)\} \in \mathcal{A}$. In this case the index set has only one element.

If $(\varphi_j)$ is a delta sequence, then obviously $M(\{\varphi_j\}_{\mathbb{N}}) = \varnothing$. However, it is possible that $M(\{\varphi_j\}_J) = \varnothing$ and $\{\varphi_j\}$ does not contain any subsequence which is a delta sequence. Consider, for example, a sequence $\{\varphi_j\}_{\mathbb{N}}$ such that $\{\widehat{\varphi}_j\}_{\mathbb{N}}$ is a partition of unity. More generally, let $\{U_j\}_J$ be an open covering of $\mathbb{R}^N$ and let $\{\varphi_j\}_J$ be such that $|\widehat{\varphi}_j(x)| > 0$ for $x \in U_j$. A family $\{\varphi_j\}_J$ such that $M(\{\varphi_j\}_J) = \varnothing$ will be called *total*.

Lemma 2.2. *If $\{\varphi_j\}_J$ and $\{\gamma_k\}_K$ are total, then $\{\varphi_j * \gamma_k\}_{J\times K}$ is total.*

Theorem 2.3. *$\{(f_j,\varphi_j)\}_J \in \mathcal{A}$ if and only if there exists a unique $F \in \mathcal{D}'(\mathbb{R}^N)$ such that $\widehat{f}_j = \widehat{\varphi}_j F$ for all $j \in J$.*

*Proof.* We only need to show that existence of such an $F \in \mathcal{D}'(\mathbb{R}^N)$ implies the exchange property. Indeed, for any $j,k \in J$ we have

$$\widehat{f}_j\widehat{\varphi}_k = F\widehat{\varphi}_j\widehat{\varphi}_k = F\widehat{\varphi}_k\widehat{\varphi}_j = \widehat{f}_k\widehat{\varphi}_j. \tag{2.5}$$

□

*Definition 2.4.* If $\{(f_j,\varphi_j)\}_J \in \mathcal{A}$, then the unique $F \in \mathcal{D}'(\mathbb{R}^N)$ such that $\widehat{f}_j = \widehat{\varphi}_j F$ for all $j \in J$ will be denoted by $\mathcal{F}(\{(f_j,\varphi_j)\}_J)$ and called the *Fourier transform* of $\{(f_j,\varphi_j)\}_J$.

Theorem 2.5. *For every $F \in \mathcal{D}'(\mathbb{R}^N)$ there exists $\{(f_j,\varphi_j)\}_J \in \mathcal{A}$ such that $F = \mathcal{F}(\{(f_j,\varphi_j)\}_J)$.*

*Proof.* Let $\{\varphi_j\}_{\mathbb{N}}$ be a total sequence such that $\widehat{\varphi}_j \in \mathcal{D}(\mathbb{R}^N)$ for all $j \in \mathbb{N}$, where $\mathcal{D}(\mathbb{R}^N)$ denotes the space of smooth functions with compact support. Then, for every $j \in \mathbb{N}$, there is an $f_j \in \mathcal{S}'$ such that $\widehat{f}_j = \widehat{\varphi}_j F$. Clearly, $\{(f_j,\varphi_j)\}_{\mathbb{N}} \in \mathcal{A}$ and $F = \mathcal{F}(\{(f_j,\varphi_j)\}_{\mathbb{N}})$. □

Let $\{(f_j,\varphi_j)\}_J, \{(g_k,\gamma_k)\}_K \in \mathcal{A}$. If $f_j * \gamma_k = g_k * \varphi_j$ for all $j \in J$ and $k \in K$, then we write $\{(f_j,\varphi_j)\}_J \sim \{(g_k,\gamma_k)\}_K$. This relation is clearly symmetric and reflexive. We will show that it is also transitive.

Let $\{(f_j,\varphi_j)\}_J, \{(g_k,\gamma_k)\}_K, \{(h_l,\psi_l)\}_L \in \mathcal{A}$. If $\{(f_j,\varphi_j)\}_J \sim \{(g_k,\gamma_k)\}_K$ and $\{(g_k,\gamma_k)\}_K \sim \{(h_l,\psi_l)\}_L$, then

$$f_j * \gamma_k = g_k * \varphi_j, \qquad g_k * \psi_l = h_l * \gamma_k \tag{2.6}$$

for all $j \in J$, $k \in K$, $l \in L$. Hence

$$f_j * \gamma_k * \psi_l = g_k * \varphi_j * \psi_l, \qquad g_k * \psi_l * \varphi_j = h_l * \gamma_k * \varphi_j \tag{2.7}$$

for all $j \in J$, $k \in K$, $l \in L$. Since $*$ is commutative, we have

$$f_j * \psi_l * \gamma_k = h_l * \varphi_j * \gamma_k \tag{2.8}$$

for all $j \in J$, $k \in K$, $l \in L$. Now fix $j \in J$ and $l \in L$. Since $M(\{\gamma_k\}_K) = \varnothing$ and (2.8) holds for every $k \in K$, we conclude that $f_j * \psi_l = h_l * \varphi_j$ for all $j \in J$ and $l \in L$, which means that $\{(f_j, \varphi_j)\}_J \sim \{(h_l, \psi_l)\}_L$.

Note that

$$\{(f_j, \varphi_j)\}_J \sim \{(f_j * \psi_k,\ \varphi_j * \psi_k)\}_{J \times K} \tag{2.9}$$

for any total family $\{\psi_k\}_K$.

Theorem 2.6. *Let $\{(f_j, \varphi_j)\}_J$, $\{(g_k, \gamma_k)\}_K \in \mathcal{A}$. Then*

$$\{(f_j, \varphi_j)\}_J \sim \{(g_k, \gamma_k)\}_K \quad \textit{iff } \mathcal{F}\Big(\{(f_j, \varphi_j)\}_J\Big) = \mathcal{F}\Big(\{(g_k, \gamma_k)\}_K\Big). \tag{2.10}$$

*Proof.* Let $F = \mathcal{F}(\{(f_j, \varphi_j)\}_J)$ and $G = \mathcal{F}(\{(g_k, \gamma_k)\}_K)$.

If $\{(f_j, \varphi_j)\}_J \sim \{(g_k, \gamma_k)\}_K$, then

$$F\hat{\varphi}_j\hat{\gamma}_k = \hat{f}_j\hat{\gamma}_k = \hat{g}_k\hat{\varphi}_j = G\hat{\gamma}_k\hat{\varphi}_j \tag{2.11}$$

for all $j \in J$ and $k \in K$. Hence $F = G$, by Lemma 2.2.

Now assume $F = G$. Then

$$\hat{f}_j\hat{\gamma}_k = F\hat{\varphi}_j\hat{\gamma}_k = G\hat{\gamma}_k\hat{\varphi}_j = \hat{g}_k\hat{\varphi}_j \tag{2.12}$$

for all $j \in J$ and $k \in K$. Hence $\{(f_j, \varphi_j)\}_J \sim \{(g_k, \gamma_k)\}_K$. □

Now we define $\mathcal{B} = \mathcal{A}/\sim$, the space of equivalence classes. In view of Theorems 2.5 and 2.6, the Fourier transform is a bijection from $\mathcal{B}$ to $\mathcal{D}'$. Consequently, $\mathcal{B}$ can be identified with the space of ultradistributions $\mathcal{Z}'$. We will show that, with a properly defined convergence in $\mathcal{B}$, the spaces are isomorphic.

Note that the space $\mathcal{S}'$ can be identified with a subspace of $\mathcal{B}$ via $f \mapsto [\{(f * \omega, \omega)\}]$, where $\omega(x) = e^{-x \cdot x}$.

Theorem 2.7. *There exists a delta sequence $(\varphi_n)$ such that for every $T \in \mathcal{B}$, $T = [\{(f_n, \varphi_n)\}_{\mathbb{N}}]$ for some $f_n \in \mathcal{W}$.*

*Proof.* Let $(\psi_n)$ be a delta sequence such that $\hat{\psi}_n \in \mathcal{D}$. Then, for any $T \in \mathcal{B}$, we have $\hat{T}\hat{\psi}_n \in \mathcal{S}'$, since $\hat{T} \in \mathcal{D}'$. Consequently, $\hat{T}\hat{\psi}_n = \hat{g}_n$ for some $g_n \in \mathcal{S}'$. It is easy to check that $T = [\{(g_n * \psi_n, \psi_n * \psi_n)\}_{\mathbb{N}}]$. Since $f_n = g_n * \psi_n \in \mathcal{W}$ and $(\varphi_n) = (\psi_n * \psi_n)$ is a delta sequence, the proof is complete. Note that $(\varphi_n)$ does not depend on $T$. □

## 3. Algebraic properties and convergence

$\mathcal{B}$ becomes a vector space with the operations defined as follows:

$$\lambda\Big[\{(f_j,\varphi_j)\}_J\Big] = \Big[\{(\lambda f_j,\varphi_j)\}_J\Big], \quad \lambda \in \mathbb{C},$$
$$\Big[\{(f_j,\varphi_j)\}_J\Big] + \Big[\{(g_k,\psi_k)\}_K\Big] = \Big[\{(f_j * \psi_k + g_k * \varphi_j, \varphi_j * \psi_k)\}_{J\times K}\Big]. \tag{3.1}$$

If $[\{(f_j,\varphi_j)\}_J]$, $[\{(g_k,\psi_k)\}_K] \in \mathcal{B}$ and $g_k \in \mathcal{S}$ for all $k \in K$, then we can define

$$\Big[\{(f_j,\varphi_j)\}_J\Big] * \Big[\{(g_k,\psi_k)\}_K\Big] = \Big[\{(f_j * g_k, \varphi_j * \psi_k)\}_{J\times K}\Big]. \tag{3.2}$$

It is easy to check that these operations are well defined. Note that, in view of Theorem 2.7, the definition of addition can be simplified.

*Definition 3.1.* Let $T_0, T_1, T_2, \ldots \in \mathcal{B}$. It is said that the sequence $(T_n)$ is convergent to $T_0$ and is written as $T_n \to T_0$ if there exists a total family $\{\varphi_j\}_J$ such that

(a) there exist tempered distributions $f_{j,n}$, where $j \in J$ and $n \in \mathbb{N}$, such that $T_n = [\{f_{j,n},\varphi_j\}_J]$ for all $n = 0,1,2,\ldots,$

(b) $f_{j,n} \to f_{j,0}$ in $\mathcal{S}'$ as $n \to \infty$ for every $j \in J$.

Theorem 3.2. *The Fourier transform is an isomorphism from $\mathcal{B}$ to $\mathcal{D}'$.*

*Proof.* Note that, since $T_n \to T_0$ in $\mathcal{B}$ if and only if $T_n - T_0 \to 0$, it suffices to prove continuity at 0.

Assume $T_n \to 0$ in $\mathcal{B}$. Then there exist tempered distributions $f_{j,n}$, where $j \in J$ and $n \in \mathbb{N}$, such that $T_n = [\{(f_{j,n},\varphi_j)\}_J]$ for all $n = 1,2,\ldots$ and $f_{j,n} \to 0$ in $\mathcal{S}'$ as $n \to \infty$ for every $j \in J$. If $\psi \in \mathcal{D}$, then there are $j_1,\ldots,j_k$ such that $\operatorname{supp}\psi \subset \bigcup_{m=1}^{k} \operatorname{supp}\widehat{\varphi}_{j_m}$. Then

$$\begin{aligned}\lim_{n\to\infty} \widehat{T}_n\psi &= \lim_{n\to\infty} \sum_{m=1}^{k} \big(\widehat{T}_n\widehat{\varphi}_{j_m}\big)\frac{\overline{\widehat{\varphi}_{j_m}}\psi}{\sum_{m=1}^{k}\big|\widehat{\varphi}_{j_m}\big|^2} \\ &= \sum_{m=1}^{k}\Big(\lim_{n\to\infty}\widehat{f}_{j_m,n}\Big)\frac{\overline{\widehat{\varphi}_{j_m}}\psi}{\sum_{m=1}^{k}\big|\widehat{\varphi}_{j_m}\big|^2} = 0,\end{aligned} \tag{3.3}$$

since $\lim_{n\to\infty}\widehat{f}_{j,n} = 0$ for every $j \in J$, by continuity of the Fourier transform in $\mathcal{S}'$. This proves continuity of $\mathcal{F} : \mathcal{B} \to \mathcal{D}'$, because $\lim_{n\to\infty}\widehat{T}_n\psi = 0$ in $\mathcal{S}'$ for every $\psi \in \mathcal{D}$ implies $\lim_{n\to\infty}\widehat{T}_n = 0$ in $\mathcal{D}'$.

Now assume $\lim_{n\to\infty}\widehat{T}_n = 0$ in $\mathcal{D}'$. By Theorem 2.7, there exists a delta sequence $(\varphi_j)$, $j \in \mathbb{N}$, such that for every $n \in \mathbb{N}$ we have $T_n = [\{(f_{j,n},\varphi_j)\}_{\mathbb{N}}]$ for some $f_{j,n} \in \mathcal{W}$. Let $(\psi_k)$, $k \in \mathbb{N}$, be a delta sequence such that $\widehat{\psi}_k \in \mathcal{D}$ for every $k \in \mathbb{N}$. Then $\lim_{n\to\infty}\widehat{T}_n\widehat{\varphi}_j\widehat{\psi}_k = 0$ in $\mathcal{S}'$ for every $j,k \in \mathbb{N}$. Since $\widehat{T}_n\widehat{\varphi}_j = f_{j,n}$ for every $j,k \in \mathbb{N}$, we have $\lim_{n\to\infty}\widehat{f}_{j,n}\widehat{\psi}_k = 0$ in $\mathcal{S}'$, which implies $\lim_{n\to\infty} f_{j,n} * \psi_k = 0$ in $\mathcal{S}'$. But

$$T_n = \Big[\{(f_{j,n},\varphi_j)\}_J\Big] = \Big[\{(f_{j,n} * \psi_k, \varphi_j * \psi_k)\}_{J\times K}\Big] \tag{3.4}$$

for all $n = 0,1,2,\ldots$, so we have $T_n \to 0$ in $\mathcal{B}$. □

**Acknowledgment**

The authors would like to thank Dennis Nemzer for helpful comments.

**References**

[1] D. Atanasiu, *Fourier transform and the Boehme property*, to appear in Integral Transform. Spec. Funct.
[2] V. Karunakaran and N. V. Kalpakam, *Boehmians and Fourier transform*, Integral Transform. Spec. Funct. **9** (2000), no. 3, 197–216.
[3] V. Karunakaran and R. Roopkumar, *Ultra Boehmians and their Fourier transforms*, Fract. Calc. Appl. Anal. **5** (2002), no. 2, 181–194.
[4] V. Karunakaran and V. B. Thilaga, *Plancherel theorem for vector valued functions and Boehmians*, Rocky Mountain J. Math. **28** (1998), no. 4, 1321–1342.
[5] V. Karunakaran and T. Venugopal, *A new space for Fourier transform*, Integral Transform. Spec. Funct. **9** (2000), no. 2, 133–148.
[6] P. Mikusiński, *Fourier transform for integrable Boehmians*, Rocky Mountain J. Math. **17** (1987), no. 3, 577–582.
[7] ———, *The Fourier transform of tempered Boehmians*, Fourier Analysis (Orono, Me, 1992), Lecture Notes in Pure and Appl. Math., vol. 157, Dekker, New York, 1994, pp. 303–309.
[8] ———, *Tempered Boehmians and ultradistributions*, Proc. Amer. Math. Soc. **123** (1995), no. 3, 813–817.
[9] D. Nemzer, *The Boehmians as an F-space*, Ph.D. thesis, University of California, California, 1984.
[10] A. H. Zemanian, *Distribution Theory and Transform Analysis*, Dover, New York, 1987.

Dragu Atanasiu: School of Engineering, Borås University, Borås, SE 501 90, Sweden
*E-mail address*: dragu.atanasiu@hb.se

Piotr Mikusiński: Department of Mathematics, University of Central Florida, Orlando, FL 32816-1364, USA
*E-mail address*: piotrm@mail.ucf.edu

Mathematical Problems in Engineering

# Special Issue on Space Dynamics

## Call for Papers

Space dynamics is a very general title that can accommodate a long list of activities. This kind of research started with the study of the motion of the stars and the planets back to the origin of astronomy, and nowadays it has a large list of topics. It is possible to make a division in two main categories: astronomy and astrodynamics. By astronomy, we can relate topics that deal with the motion of the planets, natural satellites, comets, and so forth. Many important topics of research nowadays are related to those subjects. By astrodynamics, we mean topics related to spaceflight dynamics.

It means topics where a satellite, a rocket, or any kind of man-made object is travelling in space governed by the gravitational forces of celestial bodies and/or forces generated by propulsion systems that are available in those objects. Many topics are related to orbit determination, propagation, and orbital maneuvers related to those spacecrafts. Several other topics that are related to this subject are numerical methods, nonlinear dynamics, chaos, and control.

The main objective of this Special Issue is to publish topics that are under study in one of those lines. The idea is to get the most recent researches and published them in a very short time, so we can give a step in order to help scientists and engineers that work in this field to be aware of actual research. All the published papers have to be peer reviewed, but in a fast and accurate way so that the topics are not outdated by the large speed that the information flows nowadays.

Before submission authors should carefully read over the journal's Author Guidelines, which are located at http://www.hindawi.com/journals/mpe/guidelines.html. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

| | |
|---|---|
| Manuscript Due | July 1, 2009 |
| First Round of Reviews | October 1, 2009 |
| Publication Date | January 1, 2010 |

**Lead Guest Editor**

**Antonio F. Bertachini A. Prado,** Instituto Nacional de Pesquisas Espaciais (INPE), São José dos Campos, 12227-010 São Paulo, Brazil; prado@dem.inpe.br

**Guest Editors**

**Maria Cecilia Zanardi,** São Paulo State University (UNESP), Guaratinguetá, 12516-410 São Paulo, Brazil; cecilia@feg.unesp.br

**Tadashi Yokoyama,** Universidade Estadual Paulista (UNESP), Rio Claro, 13506-900 São Paulo, Brazil; tadashi@rc.unesp.br

**Silvia Maria Giuliatti Winter,** São Paulo State University (UNESP), Guaratinguetá, 12516-410 São Paulo, Brazil; silvia@feg.unesp.br