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ON A GENERALIZATION OF HAUSDORFF SPACE

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ABSTRACT. Here, a new separation axiom as a generalization of that of Hausdorff is introduced. Its simple consequences and relations with some other known separation axioms are studied. That a non-indiscrete topological group satisfies this axiom is shown.

KEY WORDS AND PHRASES. *Separation Axiom, Hausdorff space.*

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1. INTRODUCTION. Five well known separation axioms are introduced and these significances are studies in literature [1,2,3,4]. In addition to this, other separation axioms are formulated and their consequences with interrelations were discussed by several investigators. In this connection the papers of C. E. Aull [5] and A. Wilansky [6] are informative and of much interest.

Here a new separation axiom, which may be taken as a generalization of the Hausdorff axiom is stated and then its relations with T_0, T_1, T_2 separation axioms and also with other separation axioms KC, US [6]. After that simple consequences of the above axioms are studied. Finally non-indiscrete topological groups always imply as H-separation axiom.

DEFINITION. Let (X,T) be a topological space. In a non singletone space, for every $x \in X$ there is a $y \in X$ such that $x \in G$, $y \in H$ and $G \cap H = \phi$ for some $G,H \in T$. Then the space is called H-space and also every singletone space is H-space.

REMARK 1. It is clear that every Hausdorff space is H-space. But converse is not necessarily true by the following example.

EXAMPLE 1. Consider $X = \{1,2,3,4\}$ and $T = \{\phi, X,\{1,2\},\{3,4\}\}$ there (X,T) is a H-space but (X,T) is not a Hausdorff space. The space is also non-T_0 space.

REMARK 2. Example 1 and the following example show that a H-space and T-space are independent of each other.

EXAMPLE 2. Consider $X = \{1,2,3,4\}$ and $T = \{\phi, X,\{1\},\{1,2\},\{1,2,3\}\}$, then (X,T) is T_0-space but it is not a H-space.

REMARK 3. The following example shows that in the property of being H-space is non-hereditary property.

EXAMPLE 3. Consider $X = \{1,2,3,4,5\}$ and $T = \{\phi, X, \{1,2,3\}, \{4,5\}\}$ then (X,T) is H-space. Now consider the sub-space $\{1,2,3\}$ which is not a H-space.

REMARK 4. A T_0-space which is also H-space is not necessarily a T_1-space (by the following example).

EXAMPLE 4. Consider $X = \{1,2,3,4\}$ and $T = \{\phi, X, \{1\}, \{1,2\}, \{3\}, \{3,4\}, \{1,3\}, \{1,3,4\}, \{1,2,3\}$. Now it is clearly a T_0-space and also a H-space. But (X,T) is not a T_1-space.

REMARK 5. Example 1 and the following example shows that a H-space and a T_1-space are independent of each other.

EXAMPLE 5. Consider R is the set of all real numbers with cofinite topology. It is clear that the space is T_1 but it is not H-space.

REMARK 6. A T_1-space which is also H-space is not necessarily a T_2-space (by the following example).

EXAMPLE 6. Let us consider $X = \{1,2,3,4,\ldots\}$ and the topology T is cofintite topology. Now let $X^* = \{0,1,2,3,\ldots\}$ and $T^* = \{G, G \cup \{0\} : G \in T\}$.

Then clearly (X^*,T^*) is a topological space and it is clear that the space is T_1-space as well as H-space. But the space is not a T_2-space.

DEFINITION [6]. A topological space is called KC-space if every compact set is closed.

REMARK 7. Example 1 and the following example shows that a H-space and a KC-space are independent of each other.

EXAMPLE 7. Let us consider R^+ be the set of all positive real numbers with co-countable topology. It is clear that the space is KC-space. But it is not a H-space.

REMARK 8. A KC-space which is also H-space is not necessarily a T_2-space (by the following example).

EXAMPLE 8. Consider R^+ be the set of all positive real numbers with co-countable topology T. Now let $\bar{R}$ be the set of all non-negative real numbers and $\bar{T} = \{G, G \cup \{0\}: G \in T\}$. Then clearly $(\bar{R},\bar{T})$ is a topological space and it is clear that the space is KC-space as well as H-space. But the space is not a T_2-space.

DEFINITION [6]. A topological space is called a US-space if every convergent sequence has exactly one limit to which it converges.

REMARK 9. (a) Remark 5 and Remark 7 shows that US-space and H-space are independent of each other. Since $T_2 \Rightarrow$ KC $\Rightarrow$ US $\Rightarrow T_1$ (forom [6]).

(b) From the above example 8 it is clear that a US-space which is also H-space is not necessarily a T_2-space.

RESULT 1. Let (X,T_1) and (Y,T_2) be two topological spaces. If a non-constant function $f: X \to Y$ is continuous and Y is T_2-space. Then X is a H-space.

PROOF. Since $f: X \to Y$ is a non-constant function, so for every $x \in X$ there is a $y \in X$ such that $f(x) \neq f(y)$. Since $f(x), f(y) \in Y$ and Y is T_2-space . Hence there are $U,V \in T_2$ such that $f(x) \in U$, $f(y) \in V$ and $U \cap V = \phi$. Then $f^{-1}(U)$ and $f^{-1}(V)$ are mutually disjoint non-empty open in X [since f is continuous].

$x \in f^{-1}(U)$ and $y \in f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(U) = \phi$. Hence X is H-space.

RESULT 2. Let (X,T_1) and (Y,T_2) be two topological spaces. If (X,T_1) is a H-space. Then the product space $X \times Y$ is also a H-space.

PROOF. Let (x,y) be any point in $X \times Y$. Since X is a H-space, then there is a $x_1 \in X$ such that $x \in V_1$, $x_1 \in V_2$ and $V_1 \cap V_2 = \phi$ for some $V_1, V_2 \in T_1$. And if $y \in U \in T_2$, then $(x,y) \in V_1 \times U$. $(x_1,y) \in V_2 \times U$ and $(V_1 \times U) \cap (V_2 \times U) = \phi$ (since $V_1 \cap V_2 = \phi$). Hence $X \times Y$ is a H-space.

RESULT 3. Let (X,T) be a non-indiscrete topological group. Then (X,T) is H-space.

PROOF. Let $x \in X$ and V be a non-empty proper open set in X. Case I: Let $x \in V$, since V be a non-empty proper open set in X, so there is a $y \in X$ such that $y \bar{\in} V$. Let $A = x^{-1}V$. Then A is a open neighborhood of e(identity). Let $B = A \cap A^{-1}$. Then B is a open neighborhood of e and $B = B^{-1}$. Let $U = yB$. Then U is a open neighborhood of y. We claim that $x \bar{\in} U$. For suppose $x \in U$. Then $x \in yB$ so $x = yb$ for some $b \in B$. Then $x^{-1} = b^{-1}y^{-1}$. But $b^{-1} \in B^{-1} = B$. So $x^{-1} \in B^{-1}y^{-1} = By^{-1}$. Now $B \subset A$, then $x^{-1} \in By^{-1} \subset Ay^{-1} = x^{-1}Vy^{-1}$. Then $e \in Vy^{-1}$. So $y \in V$ - a contradiction. So $x \bar{\in} U$. Hence we get, for every $x \in X$, there is $y \in X$ such that $x \in V$, $y \in U$ and $x \bar{\in} U$, $y \bar{\in} V$ for some $V,U \in T$. Let V' be the complement of V, so V' is closed and $x \bar{\in} V'$, $y \in V'$. Since every topological group is regular, so there are $U_1, V_1 \in T$ such that $x \in V_1$, $V' \subset U_1$ and $V_1 \cap U_1 = \phi$. Then $x \in V_1$ and $y \in U_1$ such that $V_1 \cap U_1 = \phi$ for some $V_1, U_1 \in T$. Hence (X,T) is H-space.

Case II: If $x \bar{\in} V$ then $x \in V'$ (complement of V). Since V is open in X so V' is closed in X. Since V is non-empty so there is a $y \in V$, so $y \bar{\in} V'$. Since every topological group is regular space. So there are $V_1, V_2 \in T$ such that $y \in V_1$ and $V' \subset V_2$ such that $V_1 \cap V_2 = \phi$. Hence $x \in V_2$, $y \in V_1$ and $V_1 \cap V_2 = \phi$. Hence it is H-space.

REFERENCES

1. KELLEY, J. L., General Topology, D. Van Nostrand Co. Inc., 1955.
2. ALEKSANOROV, P. S., Combinatorial Topology, Graylock Press, Rochester, N. Y., 1956.
3. PONTRJAGIN, L. S., Topologische Gruppen, B. G. Teubner Verlagsesellschaft, Leipzig, 1957.
4. DUTTA, M., DEBNATH, L., and MUKHERJEE, T. K. Elements of General Topology, World Press, 1964.
5. AULL, C. E., Separation of B_1-compact set, Math. Ann., 158(1965), 197-202.
6. WILANSKY, A., Between T_1 and T_2, Amer. Math. Monthly, 74(1967), 261-264.

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