

## ON LOCALLY CONFORMAL KÄHLER SPACE FORMS

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ABSTRACT. An  $m$ -dimensional locally conformal Kähler manifold (l.c.K-manifold) is characterized as a Hermitian manifold admitting a global closed 1-form  $\alpha_\lambda$  (called the Lee form) whose structure  $(F_\mu^\lambda, g_{\mu\lambda})$  satisfies

$$\nabla_\nu F_{\mu\lambda} = -\beta_\mu g_{\nu\lambda} + \beta_\lambda g_{\nu\mu} - \alpha_\mu F_{\nu\lambda} + \alpha_\lambda F_{\nu\mu},$$

where  $\nabla_\lambda$  denotes the covariant differentiation with respect to the Hermitian metric  $g_{\mu\lambda}$ ,  $\beta_\lambda = -F_\lambda^\epsilon \alpha_\epsilon$ ,  $F_{\mu\lambda} = F_\mu^\epsilon g_{\epsilon\lambda}$  and the indices  $\nu, \mu, \dots, \lambda$  run over the range  $1, 2, \dots, m$ .

For l.c.K-manifolds, I. Vaisman [4] gave a typical example and T. Kashiwada ([1], [2], [3]) gave a lot of interesting properties about such manifolds.

In this paper, we shall study certain properties of l.c.K-space forms. In §2, we shall mainly get the necessary and sufficient condition that an l.c.K-space form is an Einstein one and the Riemannian curvature tensor with respect to  $g_{\mu\lambda}$  will be expressed without the tensor field  $F_{\mu\lambda}$ . In §3, we shall get the necessary and sufficient condition that the length of the Lee form is constant and the sufficient condition that a compact l.c.K-space form becomes a complex space form. In the last §4, we shall prove that there does not exist a non-trivial recurrent l.c.K-space form.

KEY WORDS & PHRASES: *l.c.K-manifolds, Lee form, l.c.K-space forms, hybrid, recurrent l.c.K-space form.*

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### 1. INTRODUCTION.

This paper is directed to specialist readers with background in the area and appreciative of its relation of this area of study.

Let  $M(F_\mu^\lambda, g_{\mu\lambda}, \alpha_\lambda)$  be an l.c.K-manifold. Then, by the definition, at any point of  $M$  there exists a neighborhood in which a conformal metric  $g^* = e^{-2\rho} g$  is a Kähler one, i.e.,

$$\nabla_\nu^*(e^{-2\rho} F_{\mu\lambda}) = 0, \quad d\rho = \alpha,$$

where  $\nabla_\lambda^*$  denotes the covariant differentiation with respect to  $g^*$ . Then we have

$$\nabla_{\nu} F_{\mu\lambda} = -\alpha_{\mu} F_{\nu\lambda} + \alpha_{\varepsilon} F_{\varepsilon\lambda} g_{\nu\mu} + \alpha_{\lambda} F_{\nu\mu} + \alpha_{\varepsilon} F_{\mu\varepsilon} g_{\nu\lambda}. \quad (1.1)$$

The following proposition was proved by T.Kashiwada [1]

PROPOSITION 1.1. A Hermitian manifold  $M(F_{\mu}^{\lambda}, g_{\mu\lambda})$  is an l.c.K-manifold if and only if there exists a global closed 1-form  $\alpha_{\lambda}$  satisfying (1.1).

In an l.c.K-manifold  $M$ , we define a tensor field  $P_{\mu\lambda}$  as follows;

$$P_{\mu\lambda} = -\nabla_{\mu} \alpha_{\lambda} - \alpha_{\mu} \alpha_{\lambda} + \frac{1}{2} \|\alpha\|^2 g_{\mu\lambda}, \quad (1.2)$$

where  $\|\alpha\|$  denotes the length of the Lee form  $\alpha_{\lambda}$  with respect to  $g_{\mu\lambda}$ .

In an  $m$ -dimensional l.c.K-manifold  $M$ , we know the following formula;

$$R_{\mu\varepsilon} F_{\lambda}^{\varepsilon} + R_{\lambda\varepsilon} F_{\mu}^{\varepsilon} - (m-2)(P_{\mu\varepsilon} F_{\lambda}^{\varepsilon} + P_{\lambda\varepsilon} F_{\mu}^{\varepsilon}) = 0, \quad (1.3)$$

where  $R_{\mu\lambda}$  denotes the Ricci tensor with respect to  $g_{\mu\lambda}$  [1]. Thus we have

PROPOSITION 1.2. In an  $m$ -dimensional ( $m \neq 2$ ) l.c.K-manifold  $M$ , the tensor field  $P_{\mu\lambda}$  is hybrid, i.e.,

$$P_{\mu\varepsilon} F_{\lambda}^{\varepsilon} + P_{\lambda\varepsilon} F_{\mu}^{\varepsilon} = 0, \quad (1.4)$$

if and only if the Ricci tensor  $R_{\mu\lambda}$  is hybrid.

From now on in this paper, we assume that the tensor field  $P_{\mu\lambda}$  is hybrid.

REMARK. In an  $m$ -dimensional ( $m \neq 2$ ) Einstein l.c.K-manifold, the tensor field  $P_{\mu\lambda}$  is hybrid, identically.

An l.c.K-manifold  $M$  is called an l.c.K-space form if the holomorphic sectional curvature of the section  $\{X, FX\}$  at each point of  $M$  has the constant value. Let  $M(H)$  be an l.c.K-space form with constant holomorphic sectional curvature  $H$ . Then the Riemannian curvature tensor  $R_{\omega\nu\mu\lambda}$  with respect to  $g_{\mu\lambda}$  can be written as

$$\begin{aligned} 4R_{\omega\nu\mu\lambda} = & H(g_{\omega\lambda}g_{\nu\mu} - g_{\omega\mu}g_{\nu\lambda} + F_{\omega\lambda}F_{\nu\mu} - F_{\omega\mu}F_{\nu\lambda} - 2F_{\omega\nu}F_{\mu\lambda}) + 3(P_{\omega\lambda}g_{\nu\mu} - P_{\omega\mu}g_{\nu\lambda} \\ & + g_{\omega\lambda}P_{\nu\mu} - g_{\omega\mu}P_{\nu\lambda}) - \{\tilde{P}_{\omega\lambda}F_{\nu\mu} - \tilde{P}_{\omega\mu}F_{\nu\lambda} + F_{\omega\lambda}\tilde{P}_{\nu\mu} - F_{\omega\mu}\tilde{P}_{\nu\lambda} - 2(\tilde{P}_{\omega\nu}F_{\mu\lambda} \\ & + F_{\omega\nu}\tilde{P}_{\mu\lambda})\}, \end{aligned} \quad (1.5)$$

where  $\tilde{P}_{\mu\lambda} = P_{\mu}^{\varepsilon} F_{\varepsilon\lambda}$  [1].

## 2. L.C.K-SPACE FORMS.

In this section, we shall consider the necessary and sufficient condition that an l.c.K-space form becomes an Einstein one. Next, we shall get an expression of the Riemannian curvature  $R_{\omega\nu\mu\lambda}$  that does not include the tensor field  $P_{\mu\lambda}$ .

Let  $M(H)$  be an  $m$ -dimensional l.c.K-space form with constant holomorphic sectional curvature  $H$ . Then we have (1.5). Transvecting (1.5) with  $g^{\omega\lambda}$ , we have from the straightforward calculation

$$4R_{\mu\lambda} = \{(m+2)H + 3P\}g_{\mu\lambda} + 3(m-4)P_{\mu\lambda}, \quad (2.1)$$

where  $P = P_{\mu\lambda} g^{\mu\lambda}$  and it can be written as

$$P = -\nabla_{\varepsilon} \alpha^{\varepsilon} + \frac{1}{2}(m-2)\|\alpha\|^2. \quad (2.2)$$

Thus we have

PROPOSITION 2.1. A 4-dimensional l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid is an Einstein one and then the scalar field  $P$  is constant.

We have from (2.2) and the Green's theorem [5]

PROPOSITION 2.2. A compact  $m$ -dimensional l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid has a non-negative  $P$ .

Next, we shall prove the following ;

THEOREM 2.3. An  $m$ -dimensional ( $m \neq 4$ ) l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid is an Einstein one if and only if the tensor field  $P_{\mu\lambda}$  is proportional to  $g_{\mu\lambda}$ .

PROOF. If the tensor field  $P_{\mu\lambda}$  is proportional to  $g_{\mu\lambda}$ , then the tensor field  $P_{\mu\lambda}$  can be written as

$$P_{\mu\lambda} = \frac{P}{m} g_{\mu\lambda}. \quad (2.3)$$

Thus we have from (2.1) and (2.3)

$$R_{\mu\lambda} = \{(m+2)H + \frac{6(m-2)}{m}P\}g_{\mu\lambda}.$$

The inverse is trivial, so we omit its proof.

COROLLARY 2.4. An  $m$ -dimensional ( $m \neq 4$ ) Einstein l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid is a complex space form if  $P = 0$ .

Transvecting (2.1) with  $g^{\mu\lambda}$ , we have

$$4R = m(m+2)H + 6(m-2)P, \quad (2.4)$$

where  $R$  denotes the scalar curvature with respect to  $g_{\mu\lambda}$ . By virtue of (2.1) and (2.4), we can easily see that

$$3P_{\mu\lambda} = \frac{4}{m-4}R_{\mu\lambda} - \frac{(m-4)(m+2)H + 4R}{2(m-2)(m-4)}g_{\mu\lambda}, \quad (2.5)$$

$$\tilde{P}_{\mu\lambda} = \frac{4}{3(m-4)}\tilde{R}_{\mu\lambda} - \frac{(m-4)(m+2)H + 4R}{6(m-2)(m-4)}g_{\mu\lambda}, \quad (2.6)$$

where  $\tilde{R}_{\mu\lambda} = R_{\mu}^{\epsilon} F_{\epsilon\lambda}$ . Substituting (2.5) and (2.6) into (1.5), we obtain

$$\begin{aligned} R_{\omega\nu\mu\lambda} = & -\frac{(m-4)H + R}{(m-2)(m-4)}(g_{\omega\lambda}g_{\nu\mu} - g_{\omega\mu}g_{\nu\lambda}) + \frac{(m-4)(m-1)H + R}{3(m-2)(m-4)}(F_{\omega\lambda}F_{\nu\mu} \\ & - F_{\omega\mu}F_{\nu\lambda} - 2F_{\omega\nu}F_{\mu\lambda}) + \frac{1}{(m-4)}(R_{\omega\lambda}g_{\nu\mu} - R_{\omega\mu}g_{\nu\lambda} + g_{\omega\lambda}R_{\nu\mu} - g_{\omega\mu}R_{\nu\lambda}) \\ & + \frac{1}{3(m-4)}\{\tilde{R}_{\omega\lambda}F_{\nu\mu} - \tilde{R}_{\omega\mu}F_{\nu\lambda} + F_{\omega\lambda}\tilde{R}_{\nu\mu} - F_{\omega\mu}\tilde{R}_{\nu\lambda} - 2(\tilde{R}_{\omega\nu}F_{\mu\lambda} + F_{\omega\nu}\tilde{R}_{\mu\lambda})\}. \end{aligned} \quad (2.7)$$

Thus we have

PROPOSITION 2.5. In an  $m$ -dimensional ( $m \neq 2, 4$ ) l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid, the Riemannian curvature tensor  $R_{\omega\nu\mu\lambda}$  can be written as (2.7) without  $P_{\mu\lambda}$ .

### 3. COMPACT L.C.K-SPACE FORMS.

In this section, we shall mainly deal with compact l.c.K-space form.

Let  $M(H)$  be an  $m$ -dimensional l.c.K-space form with constant holomorphic sectional curvature  $H$ . If we assume that the scalar curvature  $R$  is constant, then by virtue of (2.4) all of the scalar fields  $R, H$  and  $P$  are constant. Under this assumption, differentiating (2.1) covariantly, we get

$$4\nabla_{\omega} R_{\nu\mu} = 3(m-4)\nabla_{\omega} P_{\nu\mu}. \quad (3.1)$$

Substituting (1.2) into the above equation, we have

$$4\nabla_{\omega} R_{\nu\mu} = 3(m-4)\{-\nabla_{\omega}\nabla_{\nu}\alpha_{\mu} - (\nabla_{\omega}\alpha_{\nu})\alpha_{\mu} - \alpha_{\nu}\nabla_{\omega}\alpha_{\mu} + \frac{1}{2}(\nabla_{\omega}\|\alpha\|^2)g_{\nu\mu}\}. \quad (3.2)$$

By virtue of the Ricci identity [5] and the assumption  $\nabla_{\mu}\alpha_{\lambda} = \nabla_{\lambda}\alpha_{\mu}$ , the equation (3.2) implies

$$4(\nabla_{\omega}^R R_{\nu\mu} - \nabla_{\nu}^R R_{\omega\mu}) = 3(m-4)\{R_{\omega\nu\mu}^{\epsilon} \alpha_{\epsilon} + \alpha_{\omega}(\nabla_{\nu} \alpha_{\mu}) - \alpha_{\nu}(\nabla_{\omega} \alpha_{\mu}) + \frac{1}{2}(\nabla_{\omega} \|\alpha\|^2 g_{\nu\mu} - \nabla_{\nu} \|\alpha\|^2 g_{\omega\mu})\}.$$

Transvecting the above equation with  $g^{\nu\mu}$  and taking account of the formula  $2\nabla_{\epsilon} R_{\lambda}^{\epsilon} = \nabla_{\lambda} R$  [5], we obtain

$$R_{\omega}^{\epsilon} \alpha_{\epsilon} + (\nabla_{\epsilon} \alpha^{\epsilon})_{\alpha_{\omega}} + \frac{1}{2}(m-2)\nabla_{\omega} \|\alpha\|^2 = 0. \tag{3.3}$$

Substituting (2.1) into (3.3), we obtain

$$\{(m+2)H + 3\|\alpha\|^2 + \nabla_{\epsilon} \alpha^{\epsilon}\}_{\alpha_{\omega}} + \frac{m-4}{2}\nabla_{\omega} \|\alpha\|^2 = 0. \tag{3.4}$$

Thus we have

**THEOREM 3.1.** In an  $m$ -dimensional ( $m \neq 2,4$ ) l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid and the scalar curvature  $R$  is constant, the length  $\|\alpha\|$  of the Lee form  $\alpha_{\lambda}$  is non-zero constant if and only if

$$(m+2)H + 3\|\alpha\|^2 + \nabla_{\epsilon} \alpha^{\epsilon} = 0. \tag{3.5}$$

By virtue of (3.5) and the Green's theorem, we have

**COROLLARY 3.2.** In a compact  $m$ -dimensional ( $m \neq 2,4$ ) l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid and the scalar curvature  $R$  is constant, if the length  $\|\alpha\|$  of the Lee form  $\alpha_{\lambda}$  is non-zero constant, then there exists the following relation between the holomorphic sectional curvature  $H$  and the length  $\|\alpha\|$  of the Lee form  $\alpha_{\lambda}$ ;

$$(m+2)H + 3\|\alpha\|^2 = 0. \tag{3.6}$$

**COROLLARY 3.3.** There does not exist a compact  $m$ -dimensional ( $m \neq 2,4$ ) l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid and the holomorphic sectional curvature  $H$  is positive if the length  $\|\alpha\|$  of the Lee form  $\alpha_{\lambda}$  and the scalar curvature  $R$  are constant. Especially, if  $H = 0$ , then the manifold  $M$  must be locally Euclidean, that is, the Riemannian curvature tensor  $R_{\omega\nu\mu\lambda}$  is identically zero.

The following proposition was proved by T.Kashiwada [1];

**PROPOSITION 3.4.** In a compact  $m$ -dimensional ( $m \neq 2$ ) l.c.K-manifold  $M$ , if

$$\tilde{H}_{\epsilon}^{\epsilon} - R \geq 0 \tag{3.7}$$

holds good, then the manifold  $M$  is a Kähler manifold, where  $\tilde{H}_{\mu\lambda} = \frac{1}{2}R_{\mu}^{\epsilon} \delta_{\nu}^{\delta} F^{\delta\gamma} F_{\epsilon\lambda}$ . The inequality  $\geq$  in this case is naturally reduced to =.

Now, let  $M(H)$  be a compact  $m$ -dimensional ( $m \neq 2,4$ ) l.c.K-space form. Then transvecting (2.5) with  $F^{\omega\nu} F^{\mu\lambda}$ , we get

$$\frac{1}{2}R_{\omega\nu\mu\lambda} F^{\omega\nu} F^{\mu\lambda} = \frac{-m(m+2)H + R}{3}. \tag{3.8}$$

By virtue of (2.4) and (3.8), we obtain

$$H_{\epsilon}^{\epsilon} - R = \frac{m(m+2)H - 4R}{3}. \tag{3.9}$$

Thus we have from PROPOSITION 3.4 and (3.9)

**THEOREM 3.5.** In a compact  $m$ -dimensional ( $m \neq 2,4$ ) l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid, if the inequality  $m(m+2)H \geq 4R$  holds good, then the manifold  $M$  is a complex space form.

4. RECURRENT L.C.K-SPACE FORMS.

A Riemannian manifold  $M$  is said to be recurrent if the Riemannian curvature tensor

$R_{\omega\nu\mu\lambda}$  satisfies

$$\nabla_{\kappa} R_{\omega\nu\mu\lambda} = \theta_{\kappa} R_{\omega\nu\mu\lambda} \quad (4.1)$$

for a certain non-zero vector field  $\theta_{\kappa}$ . For a recurrent Riemannian manifold, it is trivial that

$$\nabla_{\nu} R_{\mu\lambda} = \theta_{\nu} R_{\mu\lambda}, \quad \nabla_{\lambda} R = \theta_{\lambda} R. \quad (4.2)$$

Now, let  $M(H)$  be an  $m$ -dimensional ( $m \neq 2, 4$ ) recurrent l.c.K-space form which the tensor field  $P_{\mu\lambda}$  is hybrid. Then we have (2.7) and (4.1). Differentiating (2.7) covariantly and taking account of (4.1) and (4.2), we have

$$\begin{aligned} & \frac{H}{m-2} \theta_{\kappa} (g_{\omega\lambda} g_{\nu\mu} - g_{\omega\mu} g_{\nu\lambda}) - \frac{(m-1)H}{3(m-2)} \theta_{\kappa} (F_{\omega\lambda} F_{\nu\mu} - F_{\omega\mu} F_{\nu\lambda} - 2F_{\omega\nu} F_{\mu\lambda}) \\ & + \frac{(m-4)(m-1)H + R}{3(m-2)(m-4)} \{ (g_{\kappa\mu} F_{\nu\lambda} - g_{\kappa\lambda} F_{\nu\mu} + 2g_{\kappa\nu} F_{\mu\lambda}) \beta_{\omega} + (g_{\kappa\lambda} F_{\omega\mu} - g_{\kappa\mu} F_{\omega\lambda} \\ & - 2g_{\kappa\omega} F_{\mu\lambda}) \beta_{\nu} + (g_{\kappa\nu} F_{\omega\lambda} - g_{\kappa\omega} F_{\nu\lambda} + 2g_{\kappa\lambda} F_{\omega\nu}) \beta_{\mu} + (g_{\kappa\omega} F_{\nu\mu} - g_{\kappa\nu} F_{\omega\mu} - 2g_{\kappa\mu} F_{\omega\nu}) \beta_{\lambda} \\ & + (F_{\kappa\mu} F_{\nu\lambda} - F_{\kappa\lambda} F_{\nu\mu} + 2F_{\kappa\nu} F_{\mu\lambda}) \alpha_{\omega} + (F_{\kappa\lambda} F_{\omega\mu} - F_{\kappa\mu} F_{\omega\lambda} - 2F_{\kappa\omega} F_{\mu\lambda}) \alpha_{\nu} \\ & + (F_{\kappa\nu} F_{\omega\lambda} - F_{\kappa\omega} F_{\nu\lambda} + 2F_{\kappa\lambda} F_{\kappa\nu}) \alpha_{\mu} + (F_{\kappa\omega} F_{\nu\mu} - F_{\kappa\nu} F_{\omega\mu} - 2F_{\omega\nu} F_{\kappa\mu}) \alpha_{\lambda} \} \\ & - \frac{1}{3(m-4)} [ \{ R_{\omega}^{\epsilon} g_{\kappa\mu} F_{\nu\lambda} - R_{\omega}^{\epsilon} g_{\kappa\lambda} F_{\nu\mu} - R_{\nu}^{\epsilon} g_{\kappa\mu} F_{\omega\lambda} + R_{\nu}^{\epsilon} g_{\kappa\lambda} F_{\omega\mu} + 2(R_{\omega}^{\epsilon} g_{\kappa\nu} F_{\mu\lambda} \\ & + R_{\mu}^{\epsilon} g_{\kappa\lambda} F_{\omega\mu}) \} \beta_{\epsilon} + \{ R_{\omega}^{\epsilon} F_{\kappa\mu} F_{\nu\lambda} - R_{\omega}^{\epsilon} F_{\kappa\lambda} F_{\nu\mu} - R_{\nu}^{\epsilon} F_{\kappa\mu} F_{\omega\lambda} + R_{\nu}^{\epsilon} F_{\kappa\lambda} F_{\omega\mu} \\ & + 2(R_{\omega}^{\epsilon} F_{\kappa\nu} F_{\mu\lambda} + R_{\mu}^{\epsilon} F_{\kappa\lambda} F_{\omega\nu}) \} \alpha_{\epsilon} + (g_{\kappa\mu} \tilde{R}_{\nu\lambda} - g_{\kappa\lambda} \tilde{R}_{\nu\mu} + 2g_{\kappa\nu} \tilde{R}_{\mu\lambda}) \beta_{\omega} + \{ g_{\kappa\lambda} \tilde{R}_{\omega\mu} \\ & - g_{\kappa\mu} \tilde{R}_{\omega\lambda} - 2(F_{\mu\lambda} R_{\omega\kappa} + g_{\kappa\omega} \tilde{R}_{\mu\lambda}) \} \beta_{\nu} + (g_{\kappa\nu} \tilde{R}_{\omega\lambda} - F_{\nu\lambda} R_{\omega\kappa} + F_{\omega\lambda} R_{\kappa\nu} - g_{\kappa\omega} \tilde{R}_{\nu\lambda} \\ & + 2g_{\kappa\lambda} \tilde{R}_{\omega\nu}) \beta_{\mu} + \{ F_{\nu\mu} R_{\omega\kappa} - g_{\kappa\nu} \tilde{R}_{\omega\mu} + g_{\kappa\omega} \tilde{R}_{\nu\mu} - F_{\omega\mu} R_{\kappa\nu} - 2(g_{\omega\nu} \tilde{R}_{\kappa\mu} + F_{\omega\nu} R_{\kappa\mu}) \} \beta_{\lambda} \\ & + (F_{\kappa\mu} \tilde{R}_{\nu\lambda} - F_{\kappa\lambda} \tilde{R}_{\nu\mu} + 2F_{\kappa\nu} \tilde{R}_{\mu\lambda}) \alpha_{\omega} + \{ F_{\kappa\lambda} \tilde{R}_{\omega\mu} - F_{\kappa\mu} \tilde{R}_{\omega\lambda} - 2(F_{\mu\lambda} \tilde{R}_{\kappa\omega} + F_{\omega\nu} \tilde{R}_{\mu\lambda}) \} \alpha_{\nu} \\ & + (F_{\kappa\nu} \tilde{R}_{\omega\lambda} - F_{\nu\lambda} \tilde{R}_{\kappa\omega} + F_{\omega\lambda} \tilde{R}_{\kappa\nu} - F_{\kappa\omega} \tilde{R}_{\nu\lambda} + 2F_{\kappa\lambda} \tilde{R}_{\omega\nu}) \alpha_{\mu} + \{ F_{\nu\mu} \tilde{R}_{\kappa\omega} - F_{\kappa\nu} \tilde{R}_{\omega\mu} \\ & + F_{\kappa\omega} \tilde{R}_{\nu\mu} - F_{\omega\mu} \tilde{R}_{\kappa\nu} - 2(F_{\kappa\mu} \tilde{R}_{\omega\nu} + F_{\omega\nu} \tilde{R}_{\kappa\mu}) \} \alpha_{\lambda} ] = 0. \end{aligned} \quad (4.3)$$

Transvecting (4.3) with  $F^{\omega\lambda}$ , we get

$$\begin{aligned} & \frac{(m+2)H}{3} \theta_{\kappa} F_{\nu\mu} = \frac{(m+2)\{(m-4)(m-1)H + R\}}{3(m-4)(m-2)} (g_{\kappa\nu} \beta_{\mu} - g_{\kappa\mu} \beta_{\nu} - F_{\kappa\mu} \alpha_{\nu} + F_{\kappa\nu} \alpha_{\mu}) \\ & \frac{1}{3(m-4)} [ \{ (m-1)R_{\nu}^{\epsilon} F_{\kappa\mu} - 5R_{\mu}^{\epsilon} F_{\kappa\nu} \} \alpha_{\epsilon} + \{ (m-1)R_{\nu}^{\epsilon} g_{\kappa\mu} - 5R_{\mu}^{\epsilon} g_{\kappa\nu} \} \beta_{\epsilon} \\ & + (RF_{\kappa\mu} + 5R_{\kappa\mu}) \alpha_{\nu} - \{ RF_{\kappa\nu} + (m-1)R_{\kappa\nu} \} \alpha_{\mu} + (Rg_{\kappa\mu} + 5R_{\kappa\mu}) \beta_{\nu} \\ & - \{ Rg_{\kappa\nu} + (m-1)R_{\kappa\nu} \} \beta_{\mu} ]. \end{aligned}$$

From this, we obtain

$$H\theta_{\kappa} = 0. \quad (4.4)$$

Thus we have

**THEOREM 4.1.** An  $m$ -dimensional ( $m \neq 2, 4$ ) recurrent l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid is trivial, that is, the manifold is locally symmetric or of zero holomorphic sectional curvature.

Let  $M(H)$  be a 4-dimensional recurrent l.c.K-space form. Then, by virtue of PROPOSITION 2.1, the manifold is Einstein. Thus we have from (2.1) and (4.2)

$$(2H + P)\theta_{\kappa} = 0. \quad (4.5)$$

Thus we have

THEOREM 4.2. A 4-dimensional recurrent l.c.K-space form  $M(H)$  which the tensor field  $P_{\mu\lambda}$  is hybrid is trivial or the manifold has a property  $2H + P = 0$ .

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