

ON LOCALLY CONFORMAL KÄHLER SPACE FORMS

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ABSTRACT. An m -dimensional locally conformal Kähler manifold (l.c.K-manifold) is characterized as a Hermitian manifold admitting a global closed 1-form α_λ (called the Lee form) whose structure $(F_\mu^\lambda, g_{\mu\lambda})$ satisfies

$$\nabla_\nu F_{\mu\lambda} = -\beta_\mu g_{\nu\lambda} + \beta_\lambda g_{\nu\mu} - \alpha_\mu F_{\nu\lambda} + \alpha_\lambda F_{\nu\mu},$$

where ∇_λ denotes the covariant differentiation with respect to the Hermitian metric $g_{\mu\lambda}$, $\beta_\lambda = -F_\lambda^\epsilon \alpha_\epsilon$, $F_{\mu\lambda} = F_\mu^\epsilon g_{\epsilon\lambda}$ and the indices ν, μ, \dots, λ run over the range $1, 2, \dots, m$.

For l.c.K-manifolds, I. Vaisman [4] gave a typical example and T. Kashiwada ([1], [2], [3]) gave a lot of interesting properties about such manifolds.

In this paper, we shall study certain properties of l.c.K-space forms. In §2, we shall mainly get the necessary and sufficient condition that an l.c.K-space form is an Einstein one and the Riemannian curvature tensor with respect to $g_{\mu\lambda}$ will be expressed without the tensor field $P_{\mu\lambda}$. In §3, we shall get the necessary and sufficient condition that the length of the Lee form is constant and the sufficient condition that a compact l.c.K-space form becomes a complex space form. In the last §4, we shall prove that there does not exist a non-trivial recurrent l.c.K-space form.

KEY WORDS & PHRASES: *L.c.K-manifolds, Lee form, l.c.K-space forms, hybrid, recurrent l.c.K-space form.*

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1. INTRODUCTION.

This paper is directed to specialist readers with background in the area and appreciative of its relation of this area of study.

Let $M(F_\mu^\lambda, g_{\mu\lambda}, \alpha_\lambda)$ be an l.c.K-manifold. Then, by the definition, at any point of M there exists a neighborhood in which a conformal metric $g^* = e^{-2\rho} g$ is a Kähler one, i.e.,

$$\nabla_\nu^*(e^{-2\rho} F_{\mu\lambda}) = 0, \quad d\rho = \alpha,$$

where ∇_λ^* denotes the covariant differentiation with respect to g^* . Then we have

$$\nabla_{\nu} F_{\mu\lambda} = -\alpha_{\mu} F_{\nu\lambda} + \alpha_{\epsilon} F_{\epsilon\lambda} g_{\nu\mu} + \alpha_{\lambda} F_{\nu\mu} + \alpha_{\epsilon} F_{\mu\epsilon} g_{\nu\lambda}. \quad (1.1)$$

The following proposition was proved by T.Kashiwada [1]

PROPOSITION 1.1. A Hermitian manifold $M(F_{\mu}^{\lambda}, g_{\mu\lambda})$ is an l.c.K-manifold if and only if there exists a global closed 1-form α_{λ} satisfying (1.1).

In an l.c.K-manifold M , we define a tensor field $P_{\mu\lambda}$ as follows;

$$P_{\mu\lambda} = -\nabla_{\mu} \alpha_{\lambda} - \alpha_{\mu} \alpha_{\lambda} + \frac{1}{2} \|\alpha\|^2 g_{\mu\lambda}, \quad (1.2)$$

where $\|\alpha\|$ denotes the length of the Lee form α_{λ} with respect to $g_{\mu\lambda}$.

In an m -dimensional l.c.K-manifold M , we know the following formula;

$$R_{\mu\epsilon} F_{\lambda}^{\epsilon} + R_{\lambda\epsilon} F_{\mu}^{\epsilon} - (m-2)(P_{\mu\epsilon} F_{\lambda}^{\epsilon} + P_{\lambda\epsilon} F_{\mu}^{\epsilon}) = 0, \quad (1.3)$$

where $R_{\mu\lambda}$ denotes the Ricci tensor with respect to $g_{\mu\lambda}$ [1]. Thus we have

PROPOSITION 1.2. In an m -dimensional ($m \neq 2$) l.c.K-manifold M , the tensor field $P_{\mu\lambda}$ is hybrid, i.e.,

$$P_{\mu\epsilon} F_{\lambda}^{\epsilon} + P_{\lambda\epsilon} F_{\mu}^{\epsilon} = 0, \quad (1.4)$$

if and only if the Ricci tensor $R_{\mu\lambda}$ is hybrid.

From now on in this paper, we assume that the tensor field $P_{\mu\lambda}$ is hybrid.

REMARK. In an m -dimensional ($m \neq 2$) Einstein l.c.K-manifold, the tensor field $P_{\mu\lambda}$ is hybrid, identically.

An l.c.K-manifold M is called an l.c.K-space form if the holomorphic sectional curvature of the section $\{X, FX\}$ at each point of M has the constant value. Let $M(H)$ be an l.c.K-space form with constant holomorphic sectional curvature H . Then the Riemannian curvature tensor $R_{\omega\nu\mu\lambda}$ with respect to $g_{\mu\lambda}$ can be written as

$$\begin{aligned} 4R_{\omega\nu\mu\lambda} = & H(g_{\omega\lambda}g_{\nu\mu} - g_{\omega\mu}g_{\nu\lambda} + F_{\omega\lambda}F_{\nu\mu} - F_{\omega\mu}F_{\nu\lambda} - 2F_{\omega\nu}F_{\mu\lambda}) + 3(P_{\omega\lambda}g_{\nu\mu} - P_{\omega\mu}g_{\nu\lambda} \\ & + g_{\omega\lambda}P_{\nu\mu} - g_{\omega\mu}P_{\nu\lambda}) - \{\tilde{P}_{\omega\lambda}F_{\nu\mu} - \tilde{P}_{\omega\mu}F_{\nu\lambda} + F_{\omega\lambda}\tilde{P}_{\nu\mu} - F_{\omega\mu}\tilde{P}_{\nu\lambda} - 2(\tilde{P}_{\omega\nu}F_{\mu\lambda} \\ & + F_{\omega\nu}\tilde{P}_{\mu\lambda})\}, \end{aligned} \quad (1.5)$$

where $\tilde{P}_{\mu\lambda} = P_{\mu}^{\epsilon} F_{\epsilon\lambda}$ [1].

2. L.C.K-SPACE FORMS.

In this section, we shall consider the necessary and sufficient condition that an l.c.K-space form becomes an Einstein one. Next, we shall get an expression of the Riemannian curvature $R_{\omega\nu\mu\lambda}$ that does not include the tensor field $P_{\mu\lambda}$.

Let $M(H)$ be an m -dimensional l.c.K-space form with constant holomorphic sectional curvature H . Then we have (1.5). Transvecting (1.5) with $g^{\omega\lambda}$, we have from the straightforward calculation

$$4R_{\mu\lambda} = \{(m+2)H + 3P\}g_{\mu\lambda} + 3(m-4)P_{\mu\lambda}, \quad (2.1)$$

where $P = P_{\mu\lambda} g^{\mu\lambda}$ and it can be written as

$$P = -\nabla_{\epsilon} \alpha^{\epsilon} + \frac{1}{2}(m-2)\|\alpha\|^2. \quad (2.2)$$

Thus we have

PROPOSITION 2.1. A 4-dimensional l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is an Einstein one and then the scalar field P is constant.

We have from (2.2) and the Green's theorem [5]

PROPOSITION 2.2. A compact m -dimensional l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid has a non-negative P .

Next, we shall prove the following ;

THEOREM 2.3. An m -dimensional ($m \neq 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is an Einstein one if and only if the tensor field $P_{\mu\lambda}$ is proportional to $g_{\mu\lambda}$.

PROOF. If the tensor field $P_{\mu\lambda}$ is proportional to $g_{\mu\lambda}$, then the tensor field $P_{\mu\lambda}$ can be written as

$$P_{\mu\lambda} = \frac{P}{m} g_{\mu\lambda}. \quad (2.3)$$

Thus we have from (2.1) and (2.3)

$$R_{\mu\lambda} = \{(m+2)H + \frac{6(m-2)}{m}P\}g_{\mu\lambda}.$$

The inverse is trivial, so we omit its proof.

COROLLARY 2.4. An m -dimensional ($m \neq 4$) Einstein l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is a complex space form if $P = 0$.

Transvecting (2.1) with $g^{\mu\lambda}$, we have

$$4R = m(m+2)H + 6(m-2)P, \quad (2.4)$$

where R denotes the scalar curvature with respect to $g_{\mu\lambda}$. By virtue of (2.1) and (2.4), we can easily see that

$$3P_{\mu\lambda} = \frac{4}{m-4}R_{\mu\lambda} - \frac{(m-4)(m+2)H + 4R}{2(m-2)(m-4)}g_{\mu\lambda}, \quad (2.5)$$

$$\tilde{P}_{\mu\lambda} = \frac{4}{3(m-4)}\tilde{R}_{\mu\lambda} - \frac{(m-4)(m+2)H + 4R}{6(m-2)(m-4)}g_{\mu\lambda}, \quad (2.6)$$

where $\tilde{R}_{\mu\lambda} = R_{\mu}^{\epsilon} F_{\epsilon\lambda}$. Substituting (2.5) and (2.6) into (1.5), we obtain

$$\begin{aligned} R_{\omega\nu\mu\lambda} = & -\frac{(m-4)H + R}{(m-2)(m-4)}(g_{\omega\lambda}g_{\nu\mu} - g_{\omega\mu}g_{\nu\lambda}) + \frac{(m-4)(m-1)H + R}{3(m-2)(m-4)}(F_{\omega\lambda}F_{\nu\mu} \\ & - F_{\omega\mu}F_{\nu\lambda} - 2F_{\omega\nu}F_{\mu\lambda}) + \frac{1}{(m-4)}(R_{\omega\lambda}g_{\nu\mu} - R_{\omega\mu}g_{\nu\lambda} + g_{\omega\lambda}R_{\nu\mu} - g_{\omega\mu}R_{\nu\lambda}) \\ & + \frac{1}{3(m-4)}\{\tilde{R}_{\omega\lambda}F_{\nu\mu} - \tilde{R}_{\omega\mu}F_{\nu\lambda} + F_{\omega\lambda}\tilde{R}_{\nu\mu} - F_{\omega\mu}\tilde{R}_{\nu\lambda} - 2(\tilde{R}_{\omega\nu}F_{\mu\lambda} + F_{\omega\nu}\tilde{R}_{\mu\lambda})\}. \end{aligned} \quad (2.7)$$

Thus we have

PROPOSITION 2.5. In an m -dimensional ($m \neq 2, 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid, the Riemannian curvature tensor $R_{\omega\nu\mu\lambda}$ can be written as (2.7) without $P_{\mu\lambda}$.

3. COMPACT L.C.K-SPACE FORMS.

In this section, we shall mainly deal with compact l.c.K-space form.

Let $M(H)$ be an m -dimensional l.c.K-space form with constant holomorphic sectional curvature H . If we assume that the scalar curvature R is constant, then by virtue of (2.4) all of the scalar fields R, H and P are constant. Under this assumption, differentiating (2.1) covariantly, we get

$$4\nabla_{\omega}R_{\nu\mu} = 3(m-4)\nabla_{\omega}P_{\nu\mu}. \quad (3.1)$$

Substituting (1.2) into the above equation, we have

$$4\nabla_{\omega}R_{\nu\mu} = 3(m-4)\{-\nabla_{\omega}\nabla_{\nu}\alpha_{\mu} - (\nabla_{\omega}\alpha_{\nu})\alpha_{\mu} - \alpha_{\nu}\nabla_{\omega}\alpha_{\mu} + \frac{1}{2}(\nabla_{\omega}\|\alpha\|^2)g_{\nu\mu}\}. \quad (3.2)$$

By virtue of the Ricci identity [5] and the assumption $\nabla_{\mu}\alpha_{\lambda} = \nabla_{\lambda}\alpha_{\mu}$, the equation (3.2) implies

$$4(\nabla_{\omega}^R R_{\nu\mu} - \nabla_{\nu}^R R_{\omega\mu}) = 3(m-4)\{R_{\omega\nu\mu}^{\epsilon} \alpha_{\epsilon} + \alpha_{\omega}(\nabla_{\nu}^R \alpha_{\mu}) - \alpha_{\nu}(\nabla_{\omega}^R \alpha_{\mu})\} \\ + \frac{1}{2}(\nabla_{\omega} \|\alpha\|^2 g_{\nu\mu} - \nabla_{\nu} \|\alpha\|^2 g_{\omega\mu}).$$

Transvecting the above equation with $g^{\nu\mu}$ and taking account of the formula $2\nabla_{\epsilon}^R R_{\lambda}^{\epsilon} = \nabla_{\lambda}^R R$ [5], we obtain

$$R_{\omega}^{\epsilon} \alpha_{\epsilon} + (\nabla_{\epsilon}^R \alpha)_{\omega} + \frac{1}{2}(m-2)\nabla_{\omega} \|\alpha\|^2 = 0. \quad (3.3)$$

Substituting (2.1) into (3.3), we obtain

$$\{(m+2)H + 3\|\alpha\|^2 + \nabla_{\epsilon}^R \alpha_{\omega}\} + \frac{m-4}{2}\nabla_{\omega} \|\alpha\|^2 = 0. \quad (3.4)$$

Thus we have

THEOREM 3.1. In an m -dimensional ($m \neq 2, 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid and the scalar curvature R is constant, the length $\|\alpha\|$ of the Lee form α_{λ} is non-zero constant if and only if

$$(m+2)H + 3\|\alpha\|^2 + \nabla_{\epsilon}^R \alpha^{\epsilon} = 0. \quad (3.5)$$

By virtue of (3.5) and the Green's theorem, we have

COROLLARY 3.2. In a compact m -dimensional ($m \neq 2, 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid and the scalar curvature R is constant, if the length $\|\alpha\|$ of the Lee form α_{λ} is non-zero constant, then there exists the following relation between the holomorphic sectional curvature H and the length $\|\alpha\|$ of the Lee form α_{λ} ;

$$(m+2)H + 3\|\alpha\|^2 = 0. \quad (3.6)$$

COROLLARY 3.3. There does not exist a compact m -dimensional ($m \neq 2, 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid and the holomorphic sectional curvature H is positive if the length $\|\alpha\|$ of the Lee form α_{λ} and the scalar curvature R are constant. Especially, if $H = 0$, then the manifold M must be locally Euclidean, that is, the Riemannian curvature tensor $R_{\omega\nu\mu\lambda}$ is identically zero.

The following proposition was proved by T.Kashiwada [1];

PROPOSITION 3.4. In a compact m -dimensional ($m \neq 2$) l.c.K-manifold M , if

$$\tilde{H}_{\epsilon}^{\epsilon} - R \geq 0 \quad (3.7)$$

holds good, then the manifold M is a Kähler manifold, where $\tilde{H}_{\mu\lambda} = \frac{1}{2}R_{\mu}^{\epsilon} \delta_{\epsilon}^{\lambda} F^{\delta\gamma} F_{\epsilon\lambda}^{\gamma}$. The inequality \geq in this case is naturally reduced to $=$.

Now, let $M(H)$ be a compact m -dimensional ($m \neq 2, 4$) l.c.K-space form. Then transvecting (2.5) with $F^{\omega\nu} F^{\mu\lambda}$, we get

$$\frac{1}{2}R_{\omega\nu\mu\lambda} F^{\omega\nu} F^{\mu\lambda} = \frac{-m(m+2)H + R}{3}. \quad (3.8)$$

By virtue of (2.4) and (3.8), we obtain

$$H_{\epsilon}^{\epsilon} - R = \frac{m(m+2)H - 4R}{3}. \quad (3.9)$$

Thus we have from PROPOSITION 3.4 and (3.9)

THEOREM 3.5. In a compact m -dimensional ($m \neq 2, 4$) l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid, if the inequality $m(m+2)H \geq 4R$ holds good, then the manifold M is a complex space form.

4. RECURRENT L.C.K-SPACE FORMS.

A Riemannian manifold M is said to be recurrent if the Riemannian curvature tensor

$R_{\omega\nu\mu\lambda}$ satisfies

$$\nabla_{\kappa} R_{\omega\nu\mu\lambda} = \theta_{\kappa} R_{\omega\nu\mu\lambda} \quad (4.1)$$

for a certain non-zero vector field θ_{κ} . For a recurrent Riemannian manifold, it is trivial that

$$\nabla_{\nu} R_{\mu\lambda} = \theta_{\nu} R_{\mu\lambda}, \quad \nabla_{\lambda} R = \theta_{\lambda} R. \quad (4.2)$$

Now, let $M(H)$ be an m -dimensional ($m \neq 2, 4$) recurrent l.c.K-space form which the tensor field $P_{\mu\lambda}$ is hybrid. Then we have (2.7) and (4.1). Differentiating (2.7) covariantly and taking account of (4.1) and (4.2), we have

$$\begin{aligned} & \frac{H}{m-2} \theta_{\kappa} (g_{\omega\lambda} g_{\nu\mu} - g_{\omega\mu} g_{\nu\lambda}) - \frac{(m-1)H}{3(m-2)} \theta_{\kappa} (F_{\omega\lambda} F_{\nu\mu} - F_{\omega\mu} F_{\nu\lambda} - 2F_{\omega\nu} F_{\mu\lambda}) \\ & + \frac{(m-4)(m-1)H + R}{3(m-2)(m-4)} \{ (g_{\kappa\mu} F_{\nu\lambda} - g_{\kappa\lambda} F_{\nu\mu} + 2g_{\kappa\nu} F_{\mu\lambda}) \beta_{\omega} + (g_{\kappa\lambda} F_{\omega\mu} - g_{\kappa\mu} F_{\omega\lambda} \\ & - 2g_{\kappa\omega} F_{\mu\lambda}) \beta_{\nu} + (g_{\kappa\nu} F_{\omega\lambda} - g_{\kappa\omega} F_{\nu\lambda} + 2g_{\kappa\lambda} F_{\omega\nu}) \beta_{\mu} + (g_{\kappa\omega} F_{\nu\mu} - g_{\kappa\nu} F_{\omega\mu} - 2g_{\kappa\mu} F_{\omega\nu}) \beta_{\lambda} \\ & + (F_{\kappa\mu} F_{\nu\lambda} - F_{\kappa\lambda} F_{\nu\mu} + 2F_{\kappa\nu} F_{\mu\lambda}) \alpha_{\omega} + (F_{\kappa\lambda} F_{\omega\mu} - F_{\kappa\mu} F_{\omega\lambda} - 2F_{\kappa\omega} F_{\mu\lambda}) \alpha_{\nu} \\ & + (F_{\kappa\nu} F_{\omega\lambda} - F_{\kappa\omega} F_{\nu\lambda} + 2F_{\kappa\lambda} F_{\omega\nu}) \alpha_{\mu} + (F_{\kappa\omega} F_{\nu\mu} - F_{\kappa\nu} F_{\omega\mu} - 2F_{\omega\nu} F_{\kappa\mu}) \alpha_{\lambda} \} \\ & - \frac{1}{3(m-4)} [\{ R_{\omega}^{\epsilon} g_{\kappa\mu} F_{\nu\lambda} - R_{\omega}^{\epsilon} g_{\kappa\lambda} F_{\nu\mu} - R_{\nu}^{\epsilon} g_{\kappa\mu} F_{\omega\lambda} + R_{\nu}^{\epsilon} g_{\kappa\lambda} F_{\omega\mu} + 2(R_{\omega}^{\epsilon} g_{\kappa\nu} F_{\mu\lambda} \\ & + R_{\mu}^{\epsilon} g_{\kappa\lambda} F_{\omega\mu}) \} \beta_{\epsilon} + \{ R_{\omega}^{\epsilon} F_{\kappa\mu} F_{\nu\lambda} - R_{\omega}^{\epsilon} F_{\kappa\lambda} F_{\nu\mu} - R_{\nu}^{\epsilon} F_{\kappa\mu} F_{\omega\lambda} + R_{\nu}^{\epsilon} F_{\kappa\lambda} F_{\omega\mu} \\ & + 2(R_{\omega}^{\epsilon} F_{\kappa\nu} F_{\mu\lambda} + R_{\mu}^{\epsilon} F_{\kappa\lambda} F_{\omega\nu}) \} \alpha_{\epsilon} + (g_{\kappa\mu} \tilde{R}_{\nu\lambda} - g_{\kappa\lambda} \tilde{R}_{\nu\mu} + 2g_{\kappa\nu} \tilde{R}_{\mu\lambda}) \beta_{\omega} + \{ g_{\kappa\lambda} \tilde{R}_{\omega\mu} \\ & - g_{\kappa\mu} \tilde{R}_{\omega\lambda} - 2(F_{\mu\lambda} R_{\omega\kappa} + g_{\kappa\omega} \tilde{R}_{\mu\lambda}) \} \beta_{\nu} + (g_{\kappa\nu} \tilde{R}_{\omega\lambda} - F_{\nu\lambda} R_{\omega\kappa} + F_{\omega\lambda} R_{\kappa\nu} - g_{\kappa\omega} \tilde{R}_{\nu\lambda} \\ & + 2g_{\kappa\lambda} \tilde{R}_{\omega\nu}) \beta_{\mu} + \{ F_{\nu\mu} R_{\omega\kappa} - g_{\kappa\nu} \tilde{R}_{\omega\mu} + g_{\kappa\omega} \tilde{R}_{\nu\mu} - F_{\omega\mu} R_{\kappa\nu} - 2(g_{\omega\nu} \tilde{R}_{\kappa\mu} + F_{\omega\nu} R_{\kappa\mu}) \} \beta_{\lambda} \\ & + (F_{\kappa\mu} \tilde{R}_{\nu\lambda} - F_{\kappa\lambda} \tilde{R}_{\nu\mu} + 2F_{\kappa\nu} \tilde{R}_{\mu\lambda}) \alpha_{\omega} + \{ F_{\kappa\lambda} \tilde{R}_{\omega\mu} - F_{\kappa\mu} \tilde{R}_{\omega\lambda} - 2(F_{\mu\lambda} \tilde{R}_{\kappa\omega} + F_{\kappa\omega} \tilde{R}_{\mu\lambda}) \} \alpha_{\nu} \\ & + (F_{\kappa\nu} \tilde{R}_{\omega\lambda} - F_{\nu\lambda} \tilde{R}_{\kappa\omega} + F_{\omega\lambda} \tilde{R}_{\kappa\nu} - F_{\kappa\omega} \tilde{R}_{\nu\lambda} + 2F_{\kappa\lambda} \tilde{R}_{\omega\nu}) \alpha_{\mu} + \{ F_{\nu\mu} \tilde{R}_{\kappa\omega} - F_{\kappa\nu} \tilde{R}_{\omega\mu} \\ & + F_{\kappa\omega} \tilde{R}_{\nu\mu} - F_{\omega\mu} \tilde{R}_{\kappa\nu} - 2(F_{\mu\lambda} \tilde{R}_{\omega\nu} + F_{\omega\nu} \tilde{R}_{\kappa\mu}) \} \alpha_{\lambda}] = 0. \end{aligned} \quad (4.3)$$

Transvecting (4.3) with $F^{\omega\lambda}$, we get

$$\begin{aligned} & \frac{(m+2)H}{3} \theta_{\kappa} F_{\nu\mu} = \frac{(m+2)\{(m-4)(m-1)H + R\}}{3(m-4)(m-2)} (g_{\kappa\nu} \beta_{\mu} - g_{\kappa\mu} \beta_{\nu} - F_{\kappa\mu} \alpha_{\nu} + F_{\kappa\nu} \alpha_{\mu}) \\ & - \frac{1}{3(m-4)} [\{ (m-1)R_{\nu}^{\epsilon} F_{\kappa\mu} - 5R_{\mu}^{\epsilon} F_{\kappa\nu} \} \alpha_{\epsilon} + \{ (m-1)R_{\nu}^{\epsilon} g_{\kappa\mu} - 5R_{\mu}^{\epsilon} g_{\kappa\nu} \} \beta_{\epsilon} \\ & + (R F_{\kappa\mu} + 5R_{\kappa\mu}) \alpha_{\nu} - \{ R F_{\kappa\nu} + (m-1)R_{\kappa\nu} \} \alpha_{\mu} + (R g_{\kappa\mu} + 5R_{\kappa\mu}) \beta_{\nu} \\ & - \{ R g_{\kappa\nu} + (m-1)R_{\kappa\nu} \} \beta_{\mu}]. \end{aligned}$$

From this, we obtain

$$H\theta_{\kappa} = 0. \quad (4.4)$$

Thus we have

THEOREM 4.1. An m -dimensional ($m \neq 2, 4$) recurrent l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is trivial, that is, the manifold is locally symmetric or of zero holomorphic sectional curvature.

Let $M(H)$ be a 4-dimensional recurrent l.c.K-space form. Then, by virtue of PROPOSITION 2.1, the manifold is Einstein. Thus we have from (2.1) and (4.2)

$$(2H + P)\theta_{\kappa} = 0. \quad (4.5)$$

Thus we have

THEOREM 4.2. A 4-dimensional recurrent l.c.K-space form $M(H)$ which the tensor field $P_{\mu\lambda}$ is hybrid is trivial or the manifold has a property $2H + P = 0$.

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