

WEAKLY COMPATIBLE MAPS IN 2-NON-ARCHIMEDEAN MENGER PM-SPACES

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The aim of this paper is to introduce the concept of weakly compatible maps in 2-non-Archimedean Menger probabilistic metric (PM) spaces and to prove a theorem for these mappings without appeal to continuity. We also provide an application.

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1. Introduction. In 1999, Chugh and Sumitra [2] introduced the concept of 2-N.A. Menger PM-space as follows.

DEFINITION 1.1. Let X be any nonempty set and L the set of all left continuous distribution functions. An ordered pair (X, F) is said to be a 2-non-Archimedean probabilistic metric space (briefly 2-N.A. PM-space) if F is a mapping from $X \times X \times X$ into L satisfying the following conditions (where the value of F at $x, y, z \in X \times X \times X$ is represented by $F_{x,y,z}$ or $F(x, y, z)$ for all $x, y, z \in X$):

- (i) $F_{x,y,z}(t) = 1$ for all $t > 0$ if and only if at least two of the three points are equal,
- (ii) $F_{x,y,z} = F_{x,z,y} = F_{z,y,x}$,
- (iii) $F_{x,y,z}(0) = 0$,
- (iv) if $F_{x,y,s}(t_1) = F_{x,s,z}(t_2) = F_{s,y,z}(t_3) = 1$, then $F_{x,y,z}(\max\{t_1, t_2, t_3\}) = 1$.

DEFINITION 1.2. A t -norm is a function $\Delta : [0, 1] \times [0, 1] \times [0, 1] \rightarrow [0, 1]$ which is associative, commutative, nondecreasing in each coordinate and $\Delta(a, 1, 1) = a$ for every $a \in [0, 1]$.

DEFINITION 1.3. A 2-N.A. Menger PM-space is an order triplet (X, F, Δ) where Δ is a t -norm and (X, F) is 2-N.A. PM-space satisfying the following condition:

- (v) $F_{x,y,z}(\max\{t_1, t_2, t_3\}) \geq \Delta(F_{x,y,s}(t_1), F_{x,s,z}(t_2), F_{s,y,z}(t_3))$ for all $x, y, z, s \in X$ and $t_1, t_2, t_3 \geq 0$.

DEFINITION 1.4. Let (X, F, Δ) be a 2-N.A. Menger PM-space and Δ a continuous t -norm, then (X, F, Δ) is a Hausdorff in the topology induced by the family of neighbourhoods of x

$$\{U_x(\epsilon, \lambda, a_1, a_2, \dots, a_n), x, a_i \in X, \epsilon > 0, i = 1, 2, \dots, n, n \in \mathbb{Z}^+\}, \quad (1.1)$$

where \mathbb{Z}^+ is the set of all positive integers and

$$\begin{aligned} U_x(\epsilon, \lambda, a_1, a_2, \dots, a_n) &= \{y \in X; F_{x,y,a_i}(\epsilon) > 1 - \lambda, 1 \leq i \leq n\} \\ &= \bigcap_{i=1}^n \{y \in X; F_{x,y,a_i}(\epsilon) > 1 - \lambda, 1 \leq i \leq n\}. \end{aligned} \quad (1.2)$$

DEFINITION 1.5. A 2-N.A. Menger PM-space (X, F, Δ) is said to be of type $(C)_g$ if there exists a $g \in \Omega$ such that

$$g(F_{x,y,z}(t)) \leq g(F_{x,y,a}(t)) + g(F_{x,a,z}(t)) + g(F_{a,y,z}(t)) \quad (1.3)$$

for all $x, y, z, a \in X$ and $t \geq 0$, where $\Omega = \{g; g: [0, 1] \rightarrow [0, \infty)\}$ is continuous, strictly decreasing, $g(1) = 0$ and $g(0) < \infty$.

DEFINITION 1.6. A 2-N.A. Menger PM-space (X, F, Δ) is said to be of type $(D)_g$ if there exists a $g \in \Omega$ such that

$$g(\Delta(t_1, t_2, t_3)) \leq g(t_1) + g(t_2) + g(t_3) \quad \forall t_1, t_2, t_3 \in [0, 1]. \quad (1.4)$$

DEFINITION 1.7. Let (X, F, Δ) be a 2-N.A. Menger PM-space where Δ is a continuous t -norm and $A, S: X \rightarrow X$ be mappings. The mappings A and S are said to be weakly compatible if they commute at the coincidence point, that is, the mappings A and S are weakly compatible if and only if $Ax = Sx$ implies $ASx = SAx$.

REMARK 1.8. (1) If 2-N.A. PM-space (X, F, Δ) is of type $(D)_g$, then (X, F, Δ) is of type $(C)_g$.

(2) If (X, F, Δ) is a 2-N.A. PM-space and $\Delta \geq \Delta_m$, where $\Delta_m(r, s, t) = \max\{r + s + t - 1, 0, 0\}$, then (X, F, Δ) is of type $(D)_g$ for $g \in \Omega$ defined by $g(t) = 1 - t$.

Throughout this paper, let (X, F, Δ) be a complete 2-N.A. Menger PM-space of type $(D)_g$ with a continuous strictly increasing t -norm Δ .

Let $\phi: [0, \infty) \rightarrow [0, \infty)$ be a function satisfying the condition (Φ) :

(Φ) ϕ is upper semi-continuous from right and $\phi(t) < t$ for all $t > 0$.

LEMMA 1.9 (see [1]). *If a function $\phi: [0, \infty) \rightarrow [0, \infty)$ satisfies the condition (Φ) , then*

- (1) *for all $t \geq 0$, $\lim_{n \rightarrow \infty} \phi^n(t) = 0$ where $\phi^n(t)$ is the n th iteration of $\phi(t)$;*
- (2) *if $\{t_n\}$ is a nondecreasing sequence of real numbers and $t_{n+1} \leq \phi(t_n)$, $n = 1, 2, \dots$, then $\lim_{n \rightarrow \infty} t_n = 0$. In particular, if $t \leq \phi(t)$ for all $t \geq 0$, then $t = 0$.*

LEMMA 1.10 (see [1]). *Let $\{y_n\}$ be a sequence in X such that $\lim_{n \rightarrow \infty} F_{y_n, y_{n+1}, a}(t) = 1$ for all $t > 0$. If the sequence $\{y_n\}$ is not Cauchy sequence in X , then there exist $\epsilon_0 > 0$, $t_0 > 0$, and two sequences $\{m_i\}$ and $\{n_i\}$ of positive integers such that*

- (i) $m_i > n_i + 1$ and $n_i \rightarrow \infty$ as $i \rightarrow \infty$,
- (ii) $F_{y_{m_i}, y_{n_i}, a}(t_0) < 1 - \epsilon_0$ and $F_{y_{m_i-1}, y_{n_i}, a}(t_0) > 1 - \epsilon_0$, $i = 1, 2, \dots$

Chugh and Sumitra [2] proved the following theorem.

THEOREM 1.11. *Let $A, B, S, T: X \rightarrow X$ be mappings satisfying the following conditions:*

- (i) $A(X) \subset T(X)$ and $B(X) \subset S(X)$;
- (ii) *the pairs A, S and B, T are weak compatible of type (A) ;*
- (iii) *S and T are continuous;*

(iv) for all $a \in X$ and $t > 0$,

$$g(F_{Ax,By,a}(t)) \leq \phi \left(\max \left\{ g(F_{Sx,Ty,a}(t)), g(F_{Sx,Ax,a}(t)), g(F_{Ty,By,a}(t)), \frac{1}{2}(g(F_{Sx,By,a}(t)) + g(F_{Ty,Ax,a}(t))) \right\} \right), \quad (1.5)$$

where a function $\phi : [0, \infty) \rightarrow [0, \infty)$ satisfies the condition (Φ) .

Then A, B, S , and T have a unique common fixed points in X .

Now we prove the following theorem.

THEOREM 1.12. *Let $A, B, S, T : X \rightarrow X$ be mappings satisfying*

$$A(X) \subset T(X), \quad B(X) \subset S(X), \quad (1.6)$$

the pairs A, S and B, T are weakly compatible, (1.7)

$$g(F_{Ax,By,a}(t)) \leq \phi \left(\max \left\{ g(F_{Sx,Ty,a}(t)), g(F_{Sx,Ax,a}(t)), g(F_{Ty,By,a}(t)), \frac{1}{2}(g(F_{Sx,By,a}(t)) + g(F_{Ty,Ax,a}(t))) \right\} \right) \quad (1.8)$$

for all $t > 0, a \in X$ where a function $\phi : [0, \infty) \rightarrow (0, \infty)$ satisfies the condition (Φ) . Then A, B, S , and T have a unique common fixed point in X .

PROOF. By (1.6) since $A(X) \subset T(X)$, for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$. Since $B(X) \subset S(X)$, for this x_1 , we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$ and so on, inductively, we can define a sequence $\{y_n\}$ in X such that

$$y_{2n} = Ax_{2n} = Tx_{2n+1}, \quad y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}, \quad \text{for } n = 0, 1, 2, \dots \quad (1.9)$$

□

First we prove the following lemma.

LEMMA 1.13. *Let $A, B, S, T : X \rightarrow X$ be mappings satisfying conditions (1.6) and (1.8), then the sequence $\{y_n\}$ defined by (1.9), such that $\lim_{n \rightarrow \infty} g(F_{y_n, y_{n+1}, a}(t)) = 0$ for all $t > 0, a \in X$, is a Cauchy sequence in X .*

PROOF. Since $g \in \Omega$, it follows that $\lim_{n \rightarrow \infty} (F_{y_n, y_{n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$ if and only if $\lim_{n \rightarrow \infty} g(F_{y_n, y_{n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$. By Lemma 1.10, if $\{y_n\}$ is not a Cauchy sequence in X , there exist $\epsilon_0 > 0, t_0 > 0$, and two sequences $\{m_i\}, \{n_i\}$ of positive integers such that

(A) $m_i > n_i + 1$ and $n_i \rightarrow \infty$ as $i \rightarrow \infty$,

(B) $g(F_{y_{m_i}, y_{n_i}, a}(t_0)) > g(1 - \epsilon_0)$ and $g(F_{y_{m_i-1}, y_{n_i}, a}(t_0)) \leq g(1 - \epsilon_0)$, $i = 1, 2, \dots$

Thus we have

$$\begin{aligned} g(1 - \epsilon_0) &< g(F_{y_{m_i}, y_{n_i}, a}(t_0)) \leq g(F_{y_{m_i}, y_{n_i}, y_{m_i-1}}(t_0)) \\ &\quad + g(F_{y_{m_i}, y_{m_i-1}, a}(t_0)) + g(F_{y_{m_i-1}, y_{n_i}, a}(t_0)) \\ &\leq g(F_{y_{m_i}, y_{n_i}, y_{m_i-1}}(t_0)) + g(F_{y_{m_i}, y_{m_i-1}, a}(t_0)) + g(1 - \epsilon_0). \end{aligned} \quad (1.10)$$

Letting $i \rightarrow \infty$ in (1.10), we have

$$\lim_{n \rightarrow \infty} g(F_{y_{m_i}, y_{n_i}, a}(t_0)) = g(1 - \epsilon_0). \quad (1.11)$$

On the other hand, we have

$$\begin{aligned} g(1 - \epsilon_0) &< g(F_{y_{m_i}, y_{n_i}, a}(t_0)) \leq g(F_{y_{m_i}, y_{n_i}, y_{n_i+1}}(t_0)) \\ &\quad + g(F_{y_{m_i}, y_{n_i+1}, a}(t_0)) + g(F_{y_{n_i+1}, y_{n_i}, a}(t_0)). \end{aligned} \quad (1.12)$$

Now, consider $g(F_{y_{m_i}, y_{n_i+1}, a}(t_0))$ in (1.12), without loss of generality, assume that both n_i and m_i are even.

Then by (1.8), we have

$$\begin{aligned} g(F_{y_{m_i}, y_{n_i+1}, a}(t_0)) &= g(F_{Axm_i, Bxn_i+1, a}(t_0)) \\ &\leq \phi \left(\max \left\{ g(F_{Sxm_i, Txn_i+1, a}(t_0)), \right. \right. \\ &\quad g(F_{Sxm_i, Axm_i, a}(t_0)), g(F_{Txn_i+1, Bxn_i+1, a}(t_0)), \\ &\quad \left. \left. \frac{1}{2}(g(F_{Sxm_i, Bxn_i+1, a}(t_0)) + g(F_{Txn_i+1, Axm_i+1, a}(t_0))) \right\} \right) \\ &= \phi \left(\max \left\{ g(F_{y_{m_i-1}, y_{n_i}, a}(t_0)), \right. \right. \\ &\quad g(F_{y_{m_i-1}, y_{m_i}, a}(t_0)), g(F_{y_{n_i}, y_{n_i+1}, a}(t_0)), \\ &\quad \left. \left. \frac{1}{2}(g(F_{y_{m_i-1}, y_{n_i+1}, a}(t_0)) + g(F_{y_{n_i}, y_{m_i}, a}(t_0))) \right\} \right). \end{aligned} \quad (1.13)$$

By (1.11), (1.12), and (1.13), letting $i \rightarrow \infty$ in (1.13), we have

$$g(1 - \epsilon_0) \leq \phi(\max \{g(1 - \epsilon_0), 0, 0, g(1 - \epsilon_0)\}) = \phi(g(1 - \epsilon_0)) < g(1 - \epsilon_0) \quad (1.14)$$

which is a contradiction. Therefore, $\{\gamma_n\}$ is a Cauchy sequence in X . \square

Now, we are ready to prove our main theorem.

If we prove $\lim_{n \rightarrow \infty} g(F_{y_n, y_{n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$, then by Lemma 1.13, the sequence $\{\gamma_n\}$ defined by (1.9) is a Cauchy sequence in X . First we prove that $\lim_{n \rightarrow \infty} g(F_{y_n, y_{n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$. In fact, by (1.8) and (1.9), we have

$$\begin{aligned} g(F_{y_{2n}, y_{2n+1}, a}(t)) &= g(F_{Ax_{2n}, Bx_{2n+1}, a}(t)) \\ &\leq \phi \left(\max \left\{ g(F_{Sx_{2n}, Tx_{2n+1}, a}(t)), \right. \right. \\ &\quad g(F_{Sx_{2n}, Ax_{2n}, a}(t)), g(F_{Tx_{2n+1}, Bx_{2n+1}, a}(t)), \\ &\quad \left. \left. \frac{1}{2}(g(F_{Sx_{2n}, Bx_{2n+1}, a}(t)) + g(F_{Tx_{2n+1}, Ax_{2n}, a}(t))) \right\} \right) \\ &= \phi \left(\max \left\{ g(F_{y_{2n-1}, y_{2n}, a}(t)), g(F_{y_{2n-1}, y_{2n}, a}(t)), \right. \right. \\ &\quad g(F_{y_{2n}, y_{2n+1}, a}(t)), \frac{1}{2}(g(F_{y_{2n-1}, y_{2n+1}, a}(t)) + g(1)) \left. \right\} \right) \\ &\leq \phi \left(\max \left\{ g(F_{y_{2n-1}, y_{2n}, a}(t)), g(F_{y_{2n}, y_{2n+1}, a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2}(g(F_{y_{2n-1}, y_{2n}, a}(t)) + g(F_{y_{2n}, y_{2n+1}, a}(t))) \right\} \right). \end{aligned} \quad (1.15)$$

If $\mathcal{G}(F_{y_{2n-1}, y_{2n}, a}(t)) \leq \mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t))$ for all $t > 0$, then by (1.8),

$$\mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t)) \leq \phi(\mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t))) \quad (1.16)$$

and thus, by Lemma 1.9, $\mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$. Similarly, we have $\mathcal{G}(F_{y_{2n+1}, y_{2n+2}, a}(t)) = 0$, thus we have $\lim_{n \rightarrow \infty} \mathcal{G}(F_{y_n, y_{n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$. On the other hand, if $\mathcal{G}(F_{y_{2n-1}, y_{2n}, a}(t)) \geq \mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t))$, then by (1.8), we have

$$\mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t)) \leq \phi(\mathcal{G}(F_{y_{2n-1}, y_{2n}, a}(t))) \quad \forall a \in X, t > 0. \quad (1.17)$$

Similarly, $\mathcal{G}(F_{y_{2n+1}, y_{2n+2}, a}(t)) \leq \phi(\mathcal{G}(F_{y_{2n}, y_{2n+1}, a}(t)))$ for all $a \in X$ and $t > 0$. Thus we have $\mathcal{G}(F_{y_n, y_{n+1}, a}(t)) \leq \phi(\mathcal{G}(F_{y_{n-1}, y_n, a}(t)))$ for all $a \in X$ and $t > 0$ and $n = 1, 2, 3, \dots$, therefore by Lemma 1.9, $\lim_{n \rightarrow \infty} \mathcal{G}(F_{y_n, y_{n+1}, a}(t)) = 0$ for all $a \in X$ and $t > 0$, which implies that $\{y_n\}$ is a Cauchy sequence in X by Lemma 1.13. Since (X, F, Δ) is complete, the sequence $\{y_n\}$ converges to a point $z \in X$ and so the subsequences $\{Ax_{2n}\}$, $\{Bx_{2n+1}\}$, $\{Sx_{2n}\}$, $\{Tx_{2n+1}\}$ of $\{y_n\}$ also converge to the limit z . Since $B(X) \subset S(X)$, there exists a point $u \in X$ such that $z = Su$.

Now

$$\mathcal{G}(F_{Au, z, a}(t)) \leq \mathcal{G}(F_{Au, Bx_{2n+1}, Z}(t)) + \mathcal{G}(F_{Bx_{2n+1}, z, a}(t)) + \mathcal{G}(F_{Au, Bx_{2n+1}, a}(t)). \quad (1.18)$$

From (1.8), we have

$$\begin{aligned} \mathcal{G}(F_{Au, Bx_{2n+1}, a}(t)) &\leq \phi \left(\max \left\{ \mathcal{G}(F_{Su, Tx_{2n+1}, a}(t)), \mathcal{G}(F_{Su, Au, a}(t)), \mathcal{G}(F_{Tx_{2n+1}, Bx_{2n+1}, a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2} (\mathcal{G}(F_{Su, Bx_{2n+1}, a}(t)) + \mathcal{G}(F_{Tx_{2n+1}, Au, a}(t))) \right\} \right). \end{aligned} \quad (1.19)$$

From (1.18) and (1.19), letting $n \rightarrow \infty$, we have

$$\begin{aligned} \mathcal{G}(F_{Au, z, a}(t)) &\leq \phi \left(\max \left\{ \mathcal{G}(F_{Su, z, a}(t)), \mathcal{G}(F_{Su, Au, a}(t)), \mathcal{G}(F_{z, z, a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2} (\mathcal{G}(F_{Su, z, a}(t)) + \mathcal{G}(F_{z, Au, a}(t))) \right\} \right) \\ &= \phi(\mathcal{G}(F_{z, Au, a}(t))), \quad \forall a \in X, t > 0, \end{aligned} \quad (1.20)$$

which means $z = Au = Su$. Since $A(X) \subset T(X)$, there exists a point $v \in X$ such that $z = Tv$. Then, again using (1.8), we have

$$\begin{aligned} \mathcal{G}(F_{z, Bv, a}(t)) &= \mathcal{G}(F_{Au, Bv, a}(t)) \\ &\leq \phi \left(\max \left\{ \mathcal{G}(F_{Su, Tv, a}(t)), \mathcal{G}(F_{Su, Au, a}(t)), \mathcal{G}(F_{Tv, Bv, a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2} (\mathcal{G}(F_{Su, Bv, a}(t)) + \mathcal{G}(F_{Tv, Au, a}(t))) \right\} \right) \\ &= \phi(\mathcal{G}(Fz, Bv, a(t))), \quad \forall a \in X, t > 0, \end{aligned} \quad (1.21)$$

which implies that $Bv = z = Tv$.

Since pairs of maps A and S are weakly compatible, then $ASu = SAu$, that is, $Az = Sz$. Now we show that z is a fixed point of A . If $Az \neq z$, then by (1.8),

$$\begin{aligned} g(F_{Az,z,a}(t)) &= g(F_{Az,Bv,a}(t)) \\ &\leq \phi \left(\max \left\{ g(F_{Sz,Tv,a}(t)), g(F_{Sz,Az,a}(t)), g(F_{Tv,Bv,a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2}(g(F_{Sz,Bv,a}(t)) + g(F_{Tv,Az,a}(t))) \right\} \right) \\ &= \phi(\max \{g(F_{Az,z,a}(t))\}), \quad \text{implies } Az = z. \end{aligned} \quad (1.22)$$

Similarly, pairs of maps B and T are weakly compatible, we have $Bz = Tz$. Therefore,

$$\begin{aligned} g(F_{Az,z,a}(t)) &= g(F_{Az,Bz,a}(t)) \\ &\leq \phi \left(\max \left\{ g(F_{Sz,Tz,a}(t)), g(F_{Sz,Az,a}(t)), g(F_{Tz,Bz,a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2}(g(F_{Sz,Bz,a}(t)) + g(F_{Tz,Az,a}(t))) \right\} \right) \\ &= \phi(\max \{g(F_{z,Tz,a}(t))\}). \end{aligned} \quad (1.23)$$

Thus we have $Bz = Tz = z$.

Therefore, $Az = Bz = Sz = Tz$ and z is a common fixed point of A, B, S , and T . The uniqueness follows from (1.8).

2. Application

THEOREM 2.1. *Let (X, F, Δ) be a complete 2-N.A. Menger PM-space and A, B, S , and T be the mappings from the product $X \times X$ to X such that*

$$\begin{aligned} A(X \times \{y\}) &\subseteq T(X \times \{y\}), \quad B(X \times \{y\}) \subseteq (X \times \{y\}), \\ g(F_{A(T(x,y),y),T(A(x,y),y),a}(t)) &\leq g(F_{A(x,y),T(x,y),a}(t)), \\ g(F_{B(S(x,y),y),S(B(x,y),y),a}(t)) &\leq g(F_{B(x,y),S(x,y),a}(t)) \end{aligned} \quad (2.1)$$

for all $a \in X$ and $t > 0$ and

$$\begin{aligned} g(F_{A(x,y),B(x',y'),a}(t)) &\leq \phi \left(\max \left\{ g(F_{S(x,y),T(x',y'),a}(t)), g(F_{S(x,y),A(x,y),a}(t)), g(F_{T(x',y'),B(x',y'),a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2}(g(F_{S(x,y),B(x',y'),a}(t)) + g(F_{T(x',y'),A(x,y),a}(t))) \right\} \right) \end{aligned} \quad (2.2)$$

for all $a \in X$, $t > 0$, and x, y, x', y' in X , then there exists only one point b in X such that

$$A(b,y) = S(b,y) = B(b,y) = T(b,y) \quad \forall y \text{ in } X. \quad (2.3)$$

PROOF. By (2.2),

$$\begin{aligned} g(F_{A(x,y),B(x',y')}(t)) &\leq \phi \left(\max \left\{ g(F_{S(x,y),T(x',y'),a}(t)), g(F_{S(x,y),A(x,y),a}(t)), g(F_{T(x',y'),B(x',y'),a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2}(g(F_{S(x,y),B(x',y'),a}(t)) + g(F_{T(x',y'),A(x,y),a}(t))) \right\} \right) \end{aligned} \quad (2.4)$$

for all $a \in X$ and $t > 0$; therefore by [Theorem 1.12](#), for each y in X , there exists only one $x(y)$ in X such that

$$A(x(y), y) = S(x(y), y) = B(x(y), y) = T(x(y), y) = x(y) \quad (2.5)$$

for every y, y' in X ,

$$\begin{aligned} & g(F_{x(y), x(y'), a}(t)) \\ &= g(F_{A(x(y), y), A(x(y'), y'), a}(t)) \\ &\leq \phi \left(\max \left\{ g(F_{A(x, y), A(x', y'), a}(t)), g(F_{A(x, y), A(x, y'), a}(t)), g(F_{T(x', y'), A(x', y'), a}(t)), \right. \right. \\ &\quad \left. \left. \frac{1}{2} (g(F_{A(x, y), A(x', y'), a}(t)) + g(F_{A(x', y'), A(x, y), a}(t))) \right\} \right) \\ &= g(F_{x(y), x(y'), a}(t)). \end{aligned} \quad (2.6)$$

This implies $x(y) = x(y')$ and hence $x(y)$ is some constant $b \in X$ so that

$$A(b, y) = b = T(b, y) = S(b, y) = B(b, y) \quad \forall y \text{ in } X. \quad (2.7)$$

□

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As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods:** artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning

- **Application fields:** asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects:** decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

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