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A NOTE ON A PAPER BY BRENNER

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We note that a result of Brenner (1962) follows from a theorem of Lerch (1896) which also extends it.

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Let m and n be relatively prime integers with $n \geq 2$. Let $\sim$ be the equivalence relation on the set $S = (\mathbb{Z}/n\mathbb{Z}) \setminus \{0\}$ given by $t_1 \sim t_2$ if and only if there exists an integer k such that $m^k t_1 = t_2$. Denote by N the number of equivalence classes. Brenner proved the following result [1].

Theorem 1. *If n is odd, then $(-1)^N$ equals the Jacobi symbol (m/n).*

The purpose of this note is to point out that the above result is a consequence of a theorem of Lerch [3] dating back to 1896, which, moreover, extends Theorem 1 to the case of even n.

Theorem 2 (Lerch). *For relatively prime integers m and n, with $n \geq 2$, the sign of the permutation π induced by multiplication by m on $(\mathbb{Z}/n\mathbb{Z}) \setminus \{0\}$ equals*
(a) *the Jacobi symbol (m/n) if n is odd;*
(b) *1 if n is even and not divisible by 4;*
(c) *$(-1)^{(m-1)/2}$ if n is divisible by 4.*

Observe that N is the number of cycles $\tau_1, \dots, \tau_N$ in the decomposition of π into a product of disjoint cycles (1-cycles need to be included). Now if l_i is the length of τ_i, then the sign of τ_i equals $(-1)^{l_i - 1}$, so, if n is odd, the sign of π equals

$$(-1)^{\sum_{i=1}^{N}(l_i - 1)} = (-1)^{n-1-N} = (-1)^N. \tag{1}$$

Thus Theorem 1 follows from Theorem 2, as does the following extension.

Corollary 3. *For n even $(-1)^N$ equals -1, if $n \equiv 2 (\text{mod}\, 4)$, and $(-1)^{(m+1)/2}$, if $n \equiv 0 (\text{mod}\, 4)$.*

Lerch's theorem, which generalizes a result of Zolotareff [4] on the Legendre symbol, considerably simplifies the theory of quadratic residues (see, e.g., [2]) and deserves to be more widely known.

References

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