

ON FUZZY DOT SUBALGEBRAS OF BCH-ALGEBRAS

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ABSTRACT. We introduce the notion of fuzzy dot subalgebras in BCH-algebras, and study its various properties.

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1. Introduction. In [4], Hu and Li introduced the notion of BCH-algebras which are a generalization of BCK/BCI-algebras. In 1965, Zadeh [6] introduced the concept of fuzzy subsets. Since then several researchers have applied this notion to various mathematical disciplines. Jun [5] applied it to BCH-algebras, and he considered the fuzzification of ideals and filters in BCH-algebras. In this paper, we introduce the notion of a fuzzy dot subalgebra of a BCH-algebra as a generalization of a fuzzy subalgebra of a BCH-algebra, and then we investigate several basic properties related to fuzzy dot subalgebras.

2. Preliminaries. A BCH-algebra is an algebra $(X, *, 0)$ of type $(2, 0)$ satisfying the following conditions:

- (i) $x * x = 0$,
- (ii) $x * y = 0 = y * x$ implies $x = y$,
- (iii) $(x * y) * z = (x * z) * y$ for all $x, y, z \in X$.

In any BCH-algebra X , the following hold (see [2]):

- (P1) $x * 0 = x$,
- (P2) $x * 0 = 0$ implies $x = 0$,
- (P3) $0 * (x * y) = (0 * x) * (0 * y)$.

A BCH-algebra X is said to be *medial* if $x * (x * y) = y$ for all $x, y \in X$. A nonempty subset S of a BCH-algebra X is called a *subalgebra* of X if $x * y \in S$ whenever $x, y \in S$. A map f from a BCH-algebra X to a BCH-algebra Y is called a *homomorphism* if $f(x * y) = f(x) * f(y)$ for all $x, y \in X$.

We now review some fuzzy logic concepts. A fuzzy subset of a set X is a function $\mu : X \rightarrow [0, 1]$. For any fuzzy subsets μ and ν of a set X , we define

$$\begin{aligned}\mu \subseteq \nu &\iff \mu(x) \leq \nu(x) \quad \forall x \in X, \\ (\mu \cap \nu)(x) &= \min \{ \mu(x), \nu(x) \} \quad \forall x \in X.\end{aligned}\tag{2.1}$$

Let $f : X \rightarrow Y$ be a function from a set X to a set Y and let μ be a fuzzy subset of X .

The fuzzy subset ν of Y defined by

$$\nu(y) := \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \emptyset, \forall y \in Y, \\ 0 & \text{otherwise,} \end{cases} \quad (2.2)$$

is called the *image* of μ under f , denoted by $f[\mu]$. If ν is a fuzzy subset of Y , the fuzzy subset μ of X given by $\mu(x) = \nu(f(x))$ for all $x \in X$ is called the *preimage* of ν under f and is denoted by $f^{-1}[\nu]$.

A fuzzy relation μ on a set X is a fuzzy subset of $X \times X$, that is, a map $\mu : X \times X \rightarrow [0, 1]$. A fuzzy subset μ of a BCH-algebra X is called a *fuzzy subalgebra* of X if $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y \in X$.

3. Fuzzy product subalgebras. In what follows let X denote a BCH-algebra unless otherwise specified.

DEFINITION 3.1. A fuzzy subset μ of X is called a *fuzzy dot subalgebra* of X if $\mu(x * y) \geq \mu(x) \cdot \mu(y)$ for all $x, y \in X$.

EXAMPLE 3.2. Consider a BCH-algebra $X = \{0, a, b, c\}$ having the following Cayley table (see [1]):

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	c	0	c
c	c	0	0	0

Define a fuzzy set μ in X by $\mu(0) = 0.5$, $\mu(a) = 0.6$, $\mu(b) = 0.4$, $\mu(c) = 0.3$. It is easy to verify that μ is a fuzzy dot subalgebra of X .

Note that every fuzzy subalgebra is a fuzzy dot subalgebra, but the converse is not true. In fact, the fuzzy dot subalgebra μ in [Example 3.2](#) is not a fuzzy subalgebra since

$$\mu(a * a) = \mu(0) = 0.5 < 0.6 = \mu(a) = \min\{\mu(a), \mu(a)\}. \quad (3.1)$$

PROPOSITION 3.3. If μ is a fuzzy dot subalgebra of X , then

$$\mu(0) \geq (\mu(x))^2, \quad \mu(0^n * x) \geq (\mu(x))^{2n+1}, \quad (3.2)$$

for all $x \in X$ and $n \in \mathbb{N}$ where $0^n * x = 0 * (0 * (\dots (0 * x) \dots))$ in which 0 occurs n times.

PROOF. Since $x * x = 0$ for all $x \in X$, it follows that

$$\mu(0) = \mu(x * x) \geq \mu(x) \cdot \mu(x) = (\mu(x))^2 \quad (3.3)$$

for all $x \in X$. The proof of the second part is by induction on n . For $n = 1$, we have $\mu(0 * x) \geq \mu(0) \cdot \mu(x) \geq (\mu(x))^3$ for all $x \in X$. Assume that $\mu(0^k * x) \geq (\mu(x))^{2k+1}$ for

all $x \in X$. Then

$$\begin{aligned}\mu(0^{k+1} * x) &= \mu(0 * (0^k * x)) \geq \mu(0) \cdot \mu(0^k * x) \\ &\geq (\mu(x))^2 \cdot (\mu(x))^{2k+1} = (\mu(x))^{2(k+1)+1}.\end{aligned}\quad (3.4)$$

Hence $\mu(0^n * x) \geq (\mu(x))^{2n+1}$ for all $x \in X$ and $n \in \mathbb{N}$. \square

PROPOSITION 3.4. *Let μ be a fuzzy dot subalgebra of X . If there exists a sequence $\{x_n\}$ in X such that $\lim_{n \rightarrow \infty} (\mu(x_n))^2 = 1$, then $\mu(0) = 1$.*

PROOF. According to [Proposition 3.3](#), $\mu(0) \geq (\mu(x_n))^2$ for each $n \in \mathbb{N}$. Since $1 \geq \mu(0) \geq \lim_{n \rightarrow \infty} (\mu(x_n))^2 = 1$, it follows that $\mu(0) = 1$. \square

THEOREM 3.5. *If μ and ν are fuzzy dot subalgebras of X , then so is $\mu \cap \nu$.*

PROOF. Let $x, y \in X$, then

$$\begin{aligned}(\mu \cap \nu)(x * y) &= \min \{ \mu(x * y), \nu(x * y) \} \\ &\geq \min \{ \mu(x) \cdot \mu(y), \nu(x) \cdot \nu(y) \} \\ &\geq (\min \{ \mu(x), \nu(x) \}) \cdot (\min \{ \mu(y), \nu(y) \}) \\ &= ((\mu \cap \nu)(x)) \cdot ((\mu \cap \nu)(y)).\end{aligned}\quad (3.5)$$

Hence $\mu \cap \nu$ is a fuzzy dot subalgebra of X . \square

Note that a fuzzy subset μ of X is a fuzzy subalgebra of X if and only if a nonempty level subset

$$U(\mu; t) := \{x \in X \mid \mu(x) \geq t\} \quad (3.6)$$

is a subalgebra of X for every $t \in [0, 1]$. But, we know that if μ is a fuzzy dot subalgebra of X , then there exists $t \in [0, 1]$ such that

$$U(\mu; t) := \{x \in X \mid \mu(x) \geq t\} \quad (3.7)$$

is not a subalgebra of X . In fact, if μ is the fuzzy dot subalgebra of X in [Example 3.2](#), then

$$U(\mu; 0.4) = \{x \in X \mid \mu(x) \geq 0.4\} = \{0, a, b\} \quad (3.8)$$

is not a subalgebra of X since $b * a = c \notin U(\mu; 0.4)$.

THEOREM 3.6. *If μ is a fuzzy dot subalgebra of X , then*

$$U(\mu; 1) := \{x \in X \mid \mu(x) = 1\} \quad (3.9)$$

is either empty or is a subalgebra of X .

PROOF. If x and y belong to $U(\mu; 1)$, then $\mu(x * y) \geq \mu(x) \cdot \mu(y) = 1$. Hence $\mu(x * y) = 1$ which implies $x * y \in U(\mu; 1)$. Consequently, $U(\mu; 1)$ is a subalgebra of X . \square

THEOREM 3.7. *Let X be a medial BCH-algebra and let μ be a fuzzy subset of X such that*

$$\mu(0 * x) \geq \mu(x), \quad \mu(x * (0 * y)) \geq \mu(x) \cdot \mu(y), \quad (3.10)$$

for all $x, y \in X$. Then μ is a fuzzy dot subalgebra of X .

PROOF. Since X is medial, we have $0 * (0 * y) = y$ for all $y \in X$. Hence

$$\mu(x * y) = \mu(x * (0 * (0 * y))) \geq \mu(x) \cdot \mu(0 * y) \geq \mu(x) \cdot \mu(y) \quad (3.11)$$

for all $x, y \in X$. Therefore μ is a fuzzy dot subalgebra of X . \square

THEOREM 3.8. *Let $g : X \rightarrow Y$ be a homomorphism of BCH-algebras. If ν is a fuzzy dot subalgebra of Y , then the preimage $g^{-1}[\nu]$ of ν under g is a fuzzy dot subalgebra of X .*

PROOF. For any $x_1, x_2 \in X$, we have

$$\begin{aligned} g^{-1}[\nu](x_1 * x_2) &= \nu(g(x_1 * x_2)) = \nu(g(x_1) * g(x_2)) \\ &\geq \nu(g(x_1)) \cdot \nu(g(x_2)) = g^{-1}[\nu](x_1) \cdot g^{-1}[\nu](x_2). \end{aligned} \quad (3.12)$$

Thus $g^{-1}[\nu]$ is a fuzzy dot subalgebra of X . \square

THEOREM 3.9. *Let $f : X \rightarrow Y$ be an onto homomorphism of BCH-algebras. If μ is a fuzzy dot subalgebra of X , then the image $f[\mu]$ of μ under f is a fuzzy dot subalgebra of Y .*

PROOF. For any $y_1, y_2 \in Y$, let $A_1 = f^{-1}(y_1)$, $A_2 = f^{-1}(y_2)$, and $A_{12} = f^{-1}(y_1 * y_2)$. Consider the set

$$A_1 * A_2 := \{x \in X \mid x = a_1 * a_2 \text{ for some } a_1 \in A_1, a_2 \in A_2\}. \quad (3.13)$$

If $x \in A_1 * A_2$, then $x = x_1 * x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$ so that

$$f(x) = f(x_1 * x_2) = f(x_1) * f(x_2) = y_1 * y_2, \quad (3.14)$$

that is, $x \in f^{-1}(y_1 * y_2) = A_{12}$. Hence $A_1 * A_2 \subseteq A_{12}$. It follows that

$$\begin{aligned} f[\mu](y_1 * y_2) &= \sup_{x \in f^{-1}(y_1 * y_2)} \mu(x) = \sup_{x \in A_{12}} \mu(x) \\ &\geq \sup_{x \in A_1 * A_2} \mu(x) \geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1 * x_2) \\ &\geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1) \cdot \mu(x_2). \end{aligned} \quad (3.15)$$

Since $\cdot : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous, for every $\varepsilon > 0$ there exists $\delta > 0$ such that if $\hat{x}_1 \geq \sup_{x_1 \in A_1} \mu(x_1) - \delta$ and $\hat{x}_2 \geq \sup_{x_2 \in A_2} \mu(x_2) - \delta$, then $\hat{x}_1 \cdot \hat{x}_2 \geq \sup_{x_1 \in A_1} \mu(x_1) \cdot \sup_{x_2 \in A_2} \mu(x_2) - \varepsilon$. Choose $a_1 \in A_1$ and $a_2 \in A_2$ such that $\mu(a_1) \geq$

$\sup_{x_1 \in A_1} \mu(x_1) - \delta$ and $\mu(a_2) \geq \sup_{x_2 \in A_2} \mu(x_2) - \delta$. Then

$$\mu(a_1) \cdot \mu(a_2) \geq \sup_{x_1 \in A_1} \mu(x_1) \cdot \sup_{x_2 \in A_2} \mu(x_2) - \varepsilon. \quad (3.16)$$

Consequently,

$$\begin{aligned} f[\mu](y_1 * y_2) &\geq \sup_{x_1 \in A_1, x_2 \in A_2} \mu(x_1) \cdot \mu(x_2) \\ &\geq \sup_{x_1 \in A_1} \mu(x_1) \cdot \sup_{x_2 \in A_2} \mu(x_2) \\ &= f[\mu](y_1) \cdot f[\mu](y_2), \end{aligned} \quad (3.17)$$

and hence $f[\mu]$ is a fuzzy dot subalgebra of Y . \square

DEFINITION 3.10. Let σ be a fuzzy subset of X . The *strongest fuzzy σ -relation* on X is the fuzzy subset μ_σ of $X \times X$ given by $\mu_\sigma(x, y) = \sigma(x) \cdot \sigma(y)$ for all $x, y \in X$.

THEOREM 3.11. Let μ_σ be the strongest fuzzy σ -relation on X , where σ is a fuzzy subset of X . If σ is a fuzzy dot subalgebra of X , then μ_σ is a fuzzy dot subalgebra of $X \times X$.

PROOF. Assume that σ is a fuzzy dot subalgebra of X . For any $x_1, x_2, y_1, y_2 \in X$, we have

$$\begin{aligned} \mu_\sigma((x_1, y_1) * (x_2, y_2)) &= \mu_\sigma(x_1 * x_2, y_1 * y_2) \\ &= \sigma(x_1 * x_2) \cdot \sigma(y_1 * y_2) \\ &\geq (\sigma(x_1) \cdot \sigma(x_2)) \cdot (\sigma(y_1) \cdot \sigma(y_2)) \\ &= (\sigma(x_1) \cdot \sigma(y_1)) \cdot (\sigma(x_2) \cdot \sigma(y_2)) \\ &= \mu_\sigma(x_1, y_1) \cdot \mu_\sigma(x_2, y_2), \end{aligned} \quad (3.18)$$

and so μ_σ is a fuzzy dot subalgebra of $X \times X$. \square

DEFINITION 3.12. Let σ be a fuzzy subset of X . A fuzzy relation μ on X is called a *fuzzy σ -product relation* if $\mu(x, y) \geq \sigma(x) \cdot \sigma(y)$ for all $x, y \in X$.

DEFINITION 3.13. Let σ be a fuzzy subset of X . A fuzzy relation μ on X is called a *left fuzzy relation on σ* if $\mu(x, y) = \sigma(x)$ for all $x, y \in X$.

Similarly, we can define a right fuzzy relation on σ . Note that a left (resp., right) fuzzy relation on σ is a fuzzy σ -product relation.

THEOREM 3.14. Let μ be a left fuzzy relation on a fuzzy subset σ of X . If μ is a fuzzy dot subalgebra of $X \times X$, then σ is a fuzzy dot subalgebra of X .

PROOF. Assume that a left fuzzy relation μ on σ is a fuzzy dot subalgebra of $X \times X$. Then

$$\begin{aligned} \sigma(x_1 * x_2) &= \mu(x_1 * x_2, y_1 * y_2) = \mu((x_1, y_1) * (x_2, y_2)) \\ &\geq \mu(x_1, y_1) \cdot \mu(x_2, y_2) = \sigma(x_1) \cdot \sigma(x_2) \end{aligned} \quad (3.19)$$

for all $x_1, x_2, y_1, y_2 \in X$. Hence σ is a fuzzy dot subalgebra of X . \square

THEOREM 3.15. Let μ be a fuzzy relation on X satisfying the inequality $\mu(x, y) \leq \mu(x, 0)$ for all $x, y \in X$. Given $z \in X$, let σ_z be a fuzzy subset of X defined by $\sigma_z(x) = \mu(x, z)$ for all $x \in X$. If μ is a fuzzy dot subalgebra of $X \times X$, then σ_z is a fuzzy dot subalgebra of X for all $z \in X$.

PROOF. Let $z, x, y \in X$, then

$$\begin{aligned}\sigma_z(x * y) &= \mu(x * y, z) = \mu(x * y, z * 0) \\ &= \mu((x, z) * (y, 0)) \geq \mu(x, z) \cdot \mu(y, 0) \\ &\geq \mu(x, z) \cdot \mu(y, z) = \sigma_z(x) \cdot \sigma_z(y),\end{aligned}\tag{3.20}$$

completing the proof. \square

THEOREM 3.16. Let μ be a fuzzy relation on X and let σ_μ be a fuzzy subset of X given by $\sigma_\mu(x) = \inf_{y \in X} \mu(x, y) \cdot \mu(y, x)$ for all $x \in X$. If μ is a fuzzy dot subalgebra of $X \times X$ satisfying the equality $\mu(x, 0) = 1 = \mu(0, x)$ for all $x \in X$, then σ_μ is a fuzzy dot subalgebra of X .

PROOF. For any $x, y, z \in X$, we have

$$\begin{aligned}\mu(x * y, z) &= \mu(x * y, z * 0) = \mu((x, z) * (y, 0)) \\ &\geq \mu(x, z) \cdot \mu(y, 0) = \mu(x, z), \\ \mu(z, x * y) &= \mu(z * 0, x * y) = \mu((z, x) * (0, y)) \\ &\geq \mu(z, x) \cdot \mu(0, y) = \mu(z, x).\end{aligned}\tag{3.21}$$

It follows that

$$\begin{aligned}\mu(x * y, z) \cdot \mu(z, x * y) &\geq \mu(x, z) \cdot \mu(z, x) \\ &\geq (\mu(x, z) \cdot \mu(z, x)) \cdot (\mu(y, z) \cdot \mu(z, y))\end{aligned}\tag{3.22}$$

so that

$$\begin{aligned}\sigma_\mu(x * y) &= \inf_{z \in X} \mu(x * y, z) \cdot \mu(z, x * y) \\ &\geq \left(\inf_{z \in X} \mu(x, z) \cdot \mu(z, x) \right) \cdot \left(\inf_{z \in X} \mu(y, z) \cdot \mu(z, y) \right) \\ &= \sigma_\mu(x) \cdot \sigma_\mu(y).\end{aligned}\tag{3.23}$$

This completes the proof. \square

DEFINITION 3.17 (see Choudhury et al. [3]). A fuzzy map f from a set X to a set Y is an ordinary map from X to the set of all fuzzy subsets of Y satisfying the following conditions:

- (C1) for all $x \in X$, there exists $y_x \in X$ such that $(f(x))(y_x) = 1$,
- (C2) for all $x \in X$, $f(x)(y_1) = f(x)(y_2)$ implies $y_1 = y_2$.

One observes that a fuzzy map f from X to Y gives rise to a unique ordinary map $\mu_f : X \times X \rightarrow I$, given by $\mu_f(x, y) = f(x)(y)$. One also notes that a fuzzy map from X to Y gives a unique ordinary map $f_1 : X \rightarrow Y$ defined as $f_1(x) = y_x$.

DEFINITION 3.18. A fuzzy map f from a BCH-algebra X to a BCH-algebra Y is called a *fuzzy homomorphism* if

$$\mu_f(x_1 * x_2, y) = \sup_{y=y_1 * y_2} \mu_f(x_1, y_1) \cdot \mu_f(x_2, y_2) \quad (3.24)$$

for all $x_1, x_2 \in X$ and $y \in Y$.

One notes that if f is an ordinary map, then the above definition reduces to an ordinary homomorphism. One also observes that if a fuzzy map f is a fuzzy homomorphism, then the induced ordinary map f_1 is an ordinary homomorphism.

PROPOSITION 3.19. Let $f : X \rightarrow Y$ be a fuzzy homomorphism of BCH-algebras. Then

- (i) $\mu_f(x_1 * x_2, y_1 * y_2) \geq \mu_f(x_1, y_1) \cdot \mu_f(x_2, y_2)$ for all $x_1, x_2 \in X$ and $y_1, y_2 \in Y$.
- (ii) $\mu_f(0, 0) = 1$.
- (iii) $\mu_f(0 * x, 0 * y) \geq \mu_f(x, y)$ for all $x \in X$ and $y \in Y$.
- (iv) if Y is medial and $\mu_f(x, y) = t \neq 0$, then $\mu_f(0, y_x * y) = t$ for all $x \in X$ and $y \in Y$, where $y_x \in Y$ with $\mu_f(x, y_x) = 1$.

PROOF. (i) For every $x_1, x_2 \in X$ and $y_1, y_2 \in Y$, we have

$$\begin{aligned} \mu_f(x_1 * x_2, y_1 * y_2) &= \sup_{y_1 * y_2 = \tilde{y}_1 * \tilde{y}_2} \mu_f(x_1, \tilde{y}_1) \cdot \mu_f(x_2, \tilde{y}_2) \\ &\geq \mu_f(x_1, y_1) \cdot \mu_f(x_2, y_2). \end{aligned} \quad (3.25)$$

(ii) Let $x \in X$ and $y_x \in Y$ be such that $\mu_f(x, y_x) = 1$. Using (I) and (i), we get

$$\mu_f(0, 0) = \mu_f(x * x, y_x * y_x) \geq \mu_f(x, y_x) \cdot \mu_f(x, y_x) = 1 \quad (3.26)$$

and so $\mu_f(0, 0) = 1$.

(iii) The proof follows from (i) and (ii).

(iv) Assume that Y is medial and $\mu_f(x, y) = t \neq 0$ for all $x \in X$ and $y \in Y$, and let $y_x \in Y$ be such that $\mu_f(x, y_x) = 1$. Then

$$\begin{aligned} \mu_f(0, y_x * y) &= \mu_f(x * x, y_x * y) \geq \mu_f(x, y_x) \cdot \mu_f(x, y) \\ &= t = \mu_f(x, y) = \mu_f(x * 0, y_x * (y_x * y)) \\ &\geq \mu_f(x, y_x) \cdot \mu_f(0, y_x * y) = \mu_f(0, y_x * y), \end{aligned} \quad (3.27)$$

and hence $\mu_f(0, y_x * y) = t$. This completes the proof. \square

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