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ON WEAK SOLUTION OF A HYPERBOLIC DIFFERENTIAL INCLUSION WITH NONMONOTONE DISCONTINUOUS NONLINEAR TERM

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ABSTRACT. In this paper, a hyperbolic differential inclusion with nonmonotone discontinuous and nonlinear term, which the generalized velocity acts as its variable, is studied and the existence and decay of its weak solution are obtained.

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1. Introduction. In the present paper, we investigate the initial boundary value problem of the following degenerate multi-valued hyperbolic differential inclusion:

$$\begin{gathered}\ddot{u}(t)+B(u)(t)+\varphi(\dot{u})(t)\ni f(t),\quad \text{a.e. } t\in[0,T],\\ u(x,t)=0,\quad \text{a.e. } (x,t)\in\sum=\partial\Omega\times[0,T],\\ u(0)=u_0,\qquad \dot{u}(0)=u_1,\end{gathered}\tag{1.1}$$

where B is a linear and symmetric operator; φ is a discontinuous, nonmonotone, and nonlinear set-valued mapping.

Physical motivations for studying equation (1.1) come partly from problems of continuum mechanics, where nonmonotone, nonlinear, discontinuous, and multi-valued constitutive laws and boundary constraints lead to the above variational inequalities (differential inclusions). For example, when elastobody is constrainted by boundary friction, (1.1) denotes its control equation; if we study viscoelastical body and the unilateral problem of plate, (1.1) is also their control equation, etc. [10, 8, 5].

When φ is a nonmonotone multi-valued mapping, generally, for such nonmonotone and discontinuous multi-valued systems, usual monotonicity methods are not valid [1, 6]. When φ degenerates into a class of single-valued mappings and satisfies appropriate conditions, inequation (1.1) become an equation. Equation (1.1) and some of its evolution equations with which it is associated have been investigated and applied intensively [7, 3, 2, 9, 11].

In this paper, we investigate the existence and decay of the weak solutions of the hyperbolic in equation (1.1), with φ and B satisfying adequate conditions under zero boundary conditions.

2. Preliminaries. Let Ω be a bounded open set of R^n with regular boundary Γ. Let T denote a positive real number, $Q=\Omega\times[0,T]$. Suppose that $b\in L^\infty_{\text{loc}}(R)$. For every

$\rho > 0$, set

$$\underline{b}_p(\xi) = \operatorname*{ess\,inf}_{|\xi_1-\xi|<p} b(\xi_1), \qquad \overline{b}_p(\xi) = \operatorname*{ess\,sup}_{|\xi_1-\xi|<p} b(\xi_1), \tag{2.1}$$

and

$$\underline{b}(\xi) = \lim_{p\to 0^+} \underline{b}_p(\xi), \qquad \overline{b}(\xi) = \lim_{p\to 0^+} \overline{b}_p(\xi), \qquad \varphi(\xi) = [\underline{b}(\xi), \overline{b}(\xi)]. \tag{2.2}$$

Let $J(\xi) = \int_0^\xi b(t)\,dt$. Then $\partial^c J(\xi) \subseteq \varphi(\xi)$, where $\partial^c J(\xi)$ denotes the Clarke-subdifferential of J.

Remark. If $b(\xi_\pm)$ exists for every $\xi \in R$, then $\varphi(\xi) = \partial^c J(\xi)$. Furthermore, if J is convex, $\varphi(\xi)$ is maximal monotone. If b is continuous at ξ, then $\varphi(\xi)$ is single-valued at ξ ([3]).

Let $V = H_0^1(\Omega)$, $\langle\cdot,\cdot\rangle$ denote the dual pair between $V = H_0^1(\Omega)$ and $V' = H^{-1}(\Omega)$, and $(\cdot,\cdot)$ the inner product of $L^2(\Omega)$ which is compatible with the dual pair. Let $|x|_X$ denote the norm of an element x of a Banach space X.

Consider the following initial boundary value problem of a hyperbolic variational inequation (inclusion):

$$\begin{gathered} \ddot{u}(t) + Bu(t) + g(t) = f(t), \quad \text{a.e. } t \in [0,T], \\ g(x,t) \in \varphi(\dot{u}(x,t)), \quad \text{a.e. } (x,t) \in Q_T = \Omega\times[0,T], \\ u(x,t) = 0, \quad \text{a.e. } (x,t) \in \sum = \partial\Omega\times[0,T], \\ u(0) = u_0, \qquad \dot{u}(0) = u_1, \end{gathered} \tag{2.3}$$

where f, u_0, and u_1 are given.

3. Existence of solution

Theorem 1. *Assume that $f \in L^2(0,T;L^2(\Omega)), u_0 \in H_0^1(\Omega), u_1 \in L^2(\Omega)$. If*

(1) $\exists c > 0, |b(\xi)| \le c(1+|\xi|)$, *a.e.* $\xi \in R$,

(2) $B : H_0^1(\Omega) \to H^{-1}(\Omega)$ *is linear, continuous, symmetric, and semicoercive, i.e.,* $\exists c_1 > 0$, $c_2 > 0$, $c_3 \ge 0$,

$$\begin{gathered} |Bv|_{H^{-1}(\Omega)} \le c_1|v|_{H_0^1(\Omega)}, \\ \langle Bu,v\rangle = \langle Bv,u\rangle \quad \forall u,v \in H_0^1(\Omega), \\ \langle Bv,v\rangle + c_3|v|^2_{L^2(\Omega)} \ge c_2|v|^2_{H_0^1(\Omega)} \quad \forall v \in H_0^1(\Omega), \end{gathered} \tag{3.1}$$

then there exists a function u, defined in $\Omega\times[0,T]$, such that

$$\begin{aligned} &u \in L^\infty(0,T;H_0^1(\Omega)) \cap C([0,T];L^2(\Omega)), \\ &\dot{u} \in L^\infty(0,T;L^2(\Omega)) \cap C([0,T];H^{-1}(\Omega)), \\ &\ddot{u} \in L^2(0,T;H^{-1}(\Omega)), \end{aligned} \tag{3.2}$$

and

$$\begin{gathered} \ddot{u}(t) + Bu(t) + g(t) = f(t) \quad \text{in } L^2(0,T;H^{-1}(\Omega)), \\ g(t) \in \varphi(\dot{u}(x,t)), \quad \text{a.e. } (x,t) \in \Omega\times[0,T], \\ u(0) = u_0, \qquad \dot{u}(0) = u_1. \end{gathered} \tag{3.3}$$

Proof. Let $\{e_n\}_{n=1}^{\infty}$ be a subset of $V = H_0^1(\Omega)$ satisfying $\overline{\operatorname{span}\{e_n\}} = V$, $(e_i, e_j) = \delta_{ij}$. Moreover, let $x_n = \sum_1^n \omega_i^1 e_i \to u_0$ strongly in V, $y_n = \sum_1^n \omega_i^2 e_i \to u_1$ strongly in $L^2(\Omega)$.

Consider the following regularized equation of inequation (1.1)

$$\ddot{\xi}^n = N^n + h, \qquad \xi^n|_{t=0} = \omega^{1n}, \qquad \dot{\xi}^n|_{t=0} = \omega^{2n}, \tag{3.4}$$

where

$$\begin{gathered}\xi^n = \{\xi_i^n\}_{1\times n}, \qquad \omega^{1n} = \{\omega_i^1\}_{1\times n}, \qquad \omega^{2n} = \{\omega_i^2\}_{1\times n}, \qquad h = \{\langle f, e_i\rangle\}_{1\times n}, \\ N^n = \{N_i^n\}_{1\times n}, \qquad N_i^n = -\Big\langle B\Big(\sum_1^n \xi_j^n e_j\Big), e_i\Big\rangle - \Big\langle b\Big(\sum_1^n \dot{\xi}_j^n e_j\Big), e_i\Big\rangle,\end{gathered} \tag{3.5}$$

where "$\cdot$" denotes time derivate.

Equation (3.4) is a set of second-order ordinary differential equation and its local solution ξ^n exists on $I_n = [0, T_n]$, $0 < T_n \le T$.

Set $u_n(t) = \sum_1^n \xi_j^n e_j$ $(t \in I_n)$. Equation (3.4) is equivalent to

$$\langle \ddot{u}_n, e_i\rangle = -\langle Bu_n, e_i\rangle - \langle b(\dot{u}_n), e_i\rangle + \langle f, e_i\rangle, \quad i = 1, 2, \dots, n. \tag{3.6}$$

Multiplying equation (3.6) by $\dot{\xi}_i^n$, summing from $i = 1$ to $i = n$, and integrating over $[0, t]$ $(t \le I_n)$, we get

$$\begin{aligned} |\dot{u}_n(t)|_{L^2(\Omega)}^2 + \langle Bu_n(t), u_n(t)\rangle + 2\int_0^t \langle b(\dot{u}_n), \dot{u}_n\rangle d\tau & \\ = 2\int_0^t \langle f, \dot{u}_n\rangle d\tau + (y_n, y_n) + \langle Bx_n, x_n\rangle, & \\ |b(\dot{u}_n)|_{L^2(0,t;L^2(\Omega))}^2 = \int_0^t |b(\dot{u}_n)|_{L^2(\Omega)}^2 d\tau \le c\int_0^t \int_\Omega (1 + |\dot{u}_n|)^2 dx\, d\tau & \\ \le \frac{c}{2}\int_0^t \Big(|\Omega| + |\dot{u}_n(t)|_{L^2(\Omega)}^2\Big) d\tau & \\ \le c_4 + \frac{c}{2}\int_0^t |\dot{u}_n(\tau)|_{L^2(\Omega)}^2 d\tau, & \end{aligned} \tag{3.7}$$

where $|\Omega|$ denotes the Lebesgue measure of domain Ω.

$$\begin{aligned} \int_0^t \langle b(\dot{u}_n), \dot{u}_n\rangle d\tau &\le |b(\dot{u}_n)|_{L^2(0,t;L^2(\Omega))} \cdot |\dot{u}_n|_{L^2(0,t;L^2(\Omega))} \\ &\le \frac{1}{2}\Big(|b(\dot{u}_n)|_{L^2(0,t;L^2(\Omega))}^2 + |\dot{u}_n|_{L^2(0,t;L^2(\Omega))}^2\Big) \\ &\le \frac{1}{2}\Big\{c_4 + \Big(\frac{c}{2} + 1\Big)\int_0^t |\dot{u}_n(\tau)|_{L^2(\Omega)}^2 d\tau\Big\}, \end{aligned} \tag{3.8}$$

$$\begin{aligned} \int_0^t \langle f, \dot{u}_n\rangle d\tau &\le |f|_{L^2(0,T;L^2(\Omega))} \cdot |\dot{u}_n|_{L^2(0,t;L^2(\Omega))} \\ &\le \frac{1}{2}\Big(|f|_{L^2(0,T;L^2(\Omega))}^2 + |\dot{u}_n|_{L^2(0,t;L^2(\Omega))}^2\Big). \end{aligned} \tag{3.9}$$

From (3.7), $\exists c_5 > 0$ such that

$$\begin{aligned} &|\dot{u}_n(t)|_{L^2(\Omega)}^2 + c_2|u_n(t)|_{H_0^1(\Omega)}^2 \\ &\quad \le c_5 + c_3|u_n(t)|_{L^2(\Omega)}^2 + \frac{1}{2}\Big(\frac{c}{2} + 1\Big)\int_0^t |u_n(\tau)|_{L^2(\Omega)}^2 d\tau. \end{aligned} \tag{3.10}$$

We note that

$$u_n(t) = u_n(0) + \int_0^t \dot{u}_n \, d\tau,$$
$$|u_n(t)|_{L^2(\Omega)} \le |u_n(0)|_{L^2(\Omega)} + \int_0^t |\dot{u}_n|_{L^2(\Omega)} \, d\tau. \tag{3.11}$$

By Hölder inequality, $\exists c_6, c_7 > 0$ such that

$$|u_n(t)|^2_{L^2(\Omega)} \le c_6 + c_7 \int_0^t |\dot{u}_n(\tau)|^2_{L^2(\Omega)} d\tau. \tag{3.12}$$

From (3.10) and (3.12), we obtain: $\exists c_8, c_9 > 0$ such that

$$|\dot{u}_n(t)|^2_{L^2(\Omega)} + c_2 |u_n(t)|^2_{H_0^1(\Omega)} \le c_8 + c_9 \int_0^t |\dot{u}_n(\tau)|^2_{L^2(\Omega)} d\tau, \quad (t \in I_n). \tag{3.13}$$

Hence,

$$|\dot{u}_n(t)|^2_{L^2(\Omega)} \le c_8 + c_9 \int_0^t |\dot{u}_n(\tau)|^2_{L^2(\Omega)} d\tau, \quad (t \in I_n). \tag{3.14}$$

By Gronwall's inequality, we have

$$|\dot{u}_n(t)|^2_{L^2(\Omega)} \le c_8 \exp(c_9 t), \quad (t \in I_n). \tag{3.15}$$

Therefore, from (3.12), (3.15), and (3.16), there exists $c_{10} > 0$ such that

$$|\dot{u}_n(t)|_{L^2(\Omega)} \le C_{10}, \qquad |u_n(t)|_{L^2(\Omega)} \le C_{10}, \qquad |u_n(t)|_{H_0^1(\Omega)} \le C_{10}, \quad (t \in I_n), \tag{3.16}$$

where $c_4, c_5, c_6, c_7, c_8, c_9, c_{10}$ are positive constants independent of n and T_n. The estimate (3.16) implies that we can prolongate the solution of equation (3.4) to the interval $[0,T]$, i.e., $I_n = [0,T]$ $(\forall n)$.

From (3.6), we see that, for every $\eta \in \operatorname{span}\{e_1, e_2, \dots, e_n\}$,

$$\begin{aligned} |\langle \ddot{u}_n, \eta \rangle| \le{}& |f(t)|_{L^2(\Omega)} \cdot |\eta|_{L^2(\Omega)} + |b(\dot{u}_n)|_{L^2(\Omega)} \cdot |\eta|_{L^2(\Omega)} \\ &+ |B| \cdot |u_n|_{H_0^1(\Omega)} \cdot |\eta|_{H_0^1(\Omega)}, \end{aligned} \tag{3.17}$$

where $|B|$ is the norm of the linear continuous operator B.

$$\begin{aligned} |\ddot{u}_n(t)|_{H^{-1}(\Omega)} &= \sup_{|\eta|_V = 1} |\langle \ddot{u}_n(t), \eta \rangle| = \sup_{\substack{\eta \in \operatorname{span}\{e_1, \dots, e_n\} \\ |\eta|_V = 1}} |\langle \ddot{u}_n(t), \eta \rangle| \\ &\le c_{11} \Big(|f(t)|_{L^2(\Omega)} + |b(\dot{u}_n)|_{L^2(\Omega)} \Big) + |B| \cdot |u_n(t)|_{H_0^1(\Omega)}, \end{aligned} \tag{3.18}$$

where c_{11} is the imbedding constant which $H_0^1(\Omega)$ imbeds in $L^2(\Omega)$.

$$|b(\dot{u}_n)(t)|^2_{L^2(\Omega)} \le c \int_\Omega (1 + |\dot{u}_n(t)|)^2 dx \le \frac{c}{2} \Big(|\Omega| + |\dot{u}_n(t)|^2_{L^2(\Omega)} \Big). \tag{3.19}$$

This shows that $\{b(\dot{u}_n)\}$ is also a bounded subset of $L^\infty(0,T;L^2(\Omega))$. Hence, (3.18) implies that $\{\ddot{u}_n\}$ is a bounded subset of $L^2(0,T;H^{-1}(\Omega))$.

Therefore, there exists a subsequence of $\{u_n\}$ (still denoted by itself) and a function u such that $u \in L^\infty(0,T;H_0^1(\Omega)), \dot{u} \in L^\infty(0,T;L^2(\Omega)), \ddot{u} \in L^2(0,T;H^{-1}(\Omega))$ satisfying

$$\begin{aligned} u_n &\longrightarrow u \quad \text{weakly-star in } L^\infty(0,T;H_0^1(\Omega)), \\ \dot{u}_n &\longrightarrow \dot{u} \quad \text{weakly-star in } L^\infty(0,T;L^2(\Omega)), \\ \ddot{u}_n &\longrightarrow \ddot{u} \quad \text{weakly in } L^2(0,T;H^{-1}(\Omega)), \\ b(\dot{u}_n) &\longrightarrow g \quad \text{weakly-star in } L^\infty(0,T;L^2(\Omega)). \end{aligned} \tag{3.20}$$

Furthermore, $\dot{u}_n(t,x) \to \dot{u}(t,x)$, a.e. $(t,x) \in [0,T]\times\Omega$.

It is well known that the space $W(V)$, defined by $W(V) = \{u \in L^2(0,T;V), \dot{u} \in L^2(0,T;V')\}$ with the norm $|u|_W = |u|_{L^2(0,T;V)} + |\dot{u}|_{L^2(0,T;V')}$, is continuously imbedded in $C([0,T];\, L^2(\Omega))$. It is obvious that $u \in C(0,T;L^2(\Omega))$, $\dot{u} \in C(0,T;H^{-1}(\Omega))$. Hence, $u(0)$, $\dot{u}(0)$ make sense.

For $\lambda \in L^2(0,T)$, by (3.6), we have

$$\begin{aligned} \int_0^T \langle \ddot{u}_n, \lambda e_i\rangle dt = &-\int_0^T \langle B(u_n), \lambda e_i\rangle dt - \int_0^T \langle b(\dot{u}_n), \lambda e_i\rangle dt \\ &+ \int_0^T \langle f(t), \lambda e_i\rangle dt, \quad i = 1,2,\dots,n. \end{aligned} \tag{3.21}$$

For every given positive integer i, let $n \to \infty$ in (3.21). Then, it follows that

$$\begin{aligned} \int_0^T \langle \ddot{u}, \lambda e_i\rangle dt = &-\int_0^T \langle B(u), \lambda e_i\rangle dt - \int_0^T \langle g, \lambda e_i\rangle dt \\ &+ \int_0^T \langle f(t), \lambda e_i\rangle dt, \quad i = 1,2,\dots. \end{aligned} \tag{3.22}$$

Therefore,

$$\ddot{u}(t) + B(u) + g(t) = f(t) \quad \text{in } L^2(0,T;H^{-1}(\Omega)). \tag{3.23}$$

In the following, we show that

$$g(x,t) \in \varphi(\dot{u}(x,t)), \quad \text{a.e. } (x,t) \in Q_T = \Omega\times[0,T]. \tag{3.24}$$

Since $\dot{u}_n(x,t) \to \dot{u}(x,t)$, a.e. $(x,t) \in Q_T$, by Eropob's theorem [12], for every $\delta > 0$, there exists a subset $Q_\delta \subseteq Q_T = \Omega\times[0,T], |Q_\delta| \le \delta$,

$$\dot{u}_n(x,t) \longrightarrow \dot{u}(x,t) \quad \text{uniformly in } Q_T/Q_\delta. \tag{3.25}$$

That is, for every $\varepsilon > 0$, there exists a positive integer $\overline{N}$, when $n \ge \overline{N}$,

$$|\dot{u}_n(x,t) - \dot{u}(x,t)| \le \varepsilon \quad \forall (x,t) \in Q_T/Q_\delta. \tag{3.26}$$

It is obvious that

$$\underline{b}_\varepsilon(\dot{u}(x,t)) \le b(\dot{u}_n(x,t)) \le \overline{b}_\varepsilon(\dot{u}(x,t)) \quad \forall (x,t) \in Q_T/Q_\delta. \tag{3.27}$$

For every $\mu \in L^1(0,T;L^2(\Omega))$, $\mu \ge 0$

$$\begin{aligned} \int_{Q_T\setminus Q_\delta} g(x,t)\mu(x,t)\,dx\,dt &= \lim_{n\to\infty}\int_{Q_T\setminus Q_\delta} b(\dot{u}_n(x,t))\mu(x,t)\,dx\,dt \\ &\le \int_{Q_T\setminus Q_\delta} \overline{b}_\varepsilon(\dot{u}(x,t))\mu(x,t)\,dx\,dt, \end{aligned} \tag{3.28}$$

$$\int_{Q_T\setminus Q_\delta} g(x,t)\mu(x,t)\,dx\,dt \le \limsup_{\varepsilon\to 0^+}\int_{Q_T\setminus Q_\delta}\overline{b}_\varepsilon(\dot{u}(x,t))\mu(x,t)\,dx\,dt$$
$$\le \int_{Q_T\setminus Q_\delta}\overline{b}(\dot{u}(x,t))\mu(x,t)\,dx\,dt. \tag{3.29}$$

Analogously, we can obtain

$$\int_{Q_T\setminus Q_\delta} g(x,t)\mu(x,t)\,dx\,dt \ge \int_{Q_T\setminus Q_\delta}\underline{b}(\dot{u}(x,t))\mu(x,t)\,dx\,dt. \tag{3.30}$$

Hence,

$$g(x,t)\in\varphi(\dot{u}(x,t)),\quad \text{a.e. } (x,t)\in Q_T/Q_\delta. \tag{3.31}$$

Letting $\delta\longrightarrow 0^+$, we get

$$g(x,t)\in\varphi(\dot{u}(x,t)),\quad \text{a.e. } (x,t)\in Q_T=\Omega\times[0,T]. \tag{3.32}$$

Let $\lambda\in C^1[0,T],\lambda(T)=0$. Integrating by parts the left-hand sides of equations (3.21) and (3.22) gives

$$\begin{aligned}-\langle\dot{u}_n(0),\lambda(0)e_i\rangle-\int_0^T\langle\dot{u}_n,\dot{\lambda}e_i\rangle dt &= \text{ the right of (3.21)},\\ -\langle\dot{u}(0),\lambda(0)e_i\rangle-\int_0^T\langle\dot{u},\dot{\lambda}e_i\rangle dt &= \text{ the right of (3.22)}.\end{aligned} \tag{3.33}$$

Making a comparison between the two equations of (3.33), we get

$$\lim_{n\to\infty}\langle\dot{u}_n(0)-\dot{u}(0),e_i\rangle=0,\quad i=1,2,\dots. \tag{3.34}$$

Therefore,

$$\dot{u}_n(0)\longrightarrow\dot{u}(0)\text{ weakly in } H^{-1}(\Omega). \tag{3.35}$$

The uniqueness of the limit implies that $\dot{u}(0)=u_1$ (in $H^{-1}(\Omega)$).

Let $\lambda\in C^2[0,T]$, $\lambda(T)=0$, $\dot{\lambda}(T)=0$. Analogously, integrating by parts the left-hand sides of equations (3.33), and making a comparison with the obtained results again gives: $u(0)=u_0$(in $L^2(\Omega)$). This completes the proof. □

Theorem 2. *Let $f\in L^2(0,T;L^2(\Omega))$, $u_0\in H_0^1(\Omega)$, $u_1\in L^2(\Omega)$. Assume that b satisfies (1)′ $b(\xi)\xi\ge-\delta$ for almost everywhere $\xi\in R$, and $\exists\overline{c}>0,|b(\xi)|\le\overline{c}(1+|\xi|^p)$, a.e. $\xi\in R$, if $n>2$, $0<p\le(2n)/(n-2)$; if $n\le 2$, $0\le p<\infty$, and condition (2) of Theorem 1 is valid. Then there exists a function v, defined in $\Omega\times[0,T]$, satisfying*

$$v\in L^\infty(0,T;H_0^1(\Omega)),\qquad \dot{v}\in L^\infty(0,T;L^2(\Omega)), \tag{3.36}$$

and

$$\begin{gathered}\ddot{v}+B(v)+\overline{g}(t)=f(t)\quad\textit{in } L^1(0,T;H^{-1}(\Omega)+L^1(\Omega)),\\ \overline{g}(x,t)\in\varphi(\dot{v}(x,t)),\quad \textit{a.e. } (x,t)\in Q_T=\Omega\times[0,T],\\ v(0)=u_0,\qquad \dot{v}(0)=u_1.\end{gathered} \tag{3.37}$$

Proof. Analogously to Theorem 1, we still may get (3.7), where $\{e_n\}_{n=1}^{\infty}$ is a basis of $H_0^1(\Omega)\cap L^\infty(\Omega)$ satisfying $(e_i,e_j)=\delta_{ij}$. Under assumption $(1)'$, $\int_0^t \langle b(\dot{u}_n),\dot{u}_n\rangle d\tau \ge -\delta$. From (3.7), we have

$$|\dot{u}_n(t)|_{L^2(\Omega)}^2 + c_2|u_n(t)|_{H_0^1(\Omega)}^2 \le c_4 + c_3|u_n(t)|_{L^2(\Omega)}^2 + 2\int_0^t \langle f,\dot{u}_n\rangle d\tau. \tag{3.38}$$

It is easy to see that equations (3.12), (3.13), (3.15), and (3.16) are still true and the solution of equation (3.4) may still be extended to the interval $[0,T]$.

By Sobolev Imbedding Theorem, we have, for a.e. $t\in[0,T]$, if $n>2$, then

$$H_0^1(\Omega)\subset L^{p^*}(\Omega)\subset L^p(\Omega), \qquad p^*=\frac{2n}{n-2}, \tag{3.39}$$

and if $n=2$, then

$$H_0^1(\Omega)\subset L^q(\Omega) \quad \forall 1\le q\le\infty, \tag{3.40}$$

so

$$|u_n(t)|_{L^p(\Omega)} \le c_{11}|u_n(t)|_{H_0^1(\Omega)} \le c_{11}c_9; \tag{3.41}$$

if $n=1$, then

$$H_0^1(\Omega)\subset C(\overline{\Omega}), \text{ and ditto, } |u_n(t)|_{C(\overline{\Omega})} = \max_{x\in\overline{\Omega}}|u_n(x,t)| \le c_{11}c_9, \tag{3.42}$$

where $\overline{\Omega}$ denotes the Closure of Ω and c_{11} is the imbedding constant which $H_0^1(\Omega)$ imbeds in $L^p(\Omega)$ or $C(\overline{\Omega})$. Everyway, we always have that $b(\dot{u}_n)\in L^\infty(0,T;L^1(\Omega))$ and $\{b(\dot{u}_n)\}$ is a bounded subset of $L^\infty(0,T;L^1(\Omega))$.

Therefore, there exists a subsequence of $\{u_n\}$, still denoted by itself, and a function v, such that $v\in L^\infty(0,T;H_0^1(\Omega)), \dot{v}\in L^\infty(0,T;L^2(\Omega))$, satisfying

$$\begin{aligned} u_n &\longrightarrow v \quad \text{weakly-star in } L^\infty(0,T;H_0^1(\Omega)),\\ \dot{u}_n &\longrightarrow \dot{v} \quad \text{weakly-star in } L^\infty(0,T;L^2(\Omega)),\\ b(\dot{u}_n) &\longrightarrow \overline{g} \quad \text{weakly-star in } L^\infty(0,T;L^1(\Omega)). \end{aligned} \tag{3.43}$$

Since the dual of the space $H_0^1(\Omega)\cap L^\infty(\Omega)$ is the space $H^{-1}(\Omega)+L^1(\Omega)$, by (3.6), it is easy to obtain

$$\ddot{v}(t)+B(v)+\overline{g}(t)=f(t) \quad \text{in } L^1(0,T;H^{-1}(\Omega)+L^1(\Omega)). \tag{3.44}$$

The rest is analogous to that of Theorem 1.

This completes the proof. □

4. Decay of solution

Theorem 3. *Let $T=+\infty$, $f\equiv 0$. Suppose that $\langle Bw,w\rangle\ge 0, \forall w\in H_0^1(\Omega)$. If $(b(w),w)\ge \mu_0|w|_{L^2(\Omega)}^2$, then, under the conditions of Theorem 2, the solution in Theorem 2, obtained from the regularized equation (3.4), satisfies*

$$|\dot{u}(t)|_{L^2(\Omega)}^2 \le \mu_1\exp(-\mu_2 t), \quad \textit{a.e. } t\ge 0, \tag{4.1}$$

where μ_0, μ_1, and μ_2 are positive constants.

Proof. Let u_n be a solution of (3.4), i.e., u_n satisfies (3.6) and (3.7). Since $(b(w),w) \geq \mu_0|w|^2_{L^2(\Omega)}$, by (3.7), we have

$$|\dot{u}_n(t)|^2_{L^2(\Omega)} + \langle Bu_n(t), u_n(t)\rangle \leq c_{12} - 2\mu_0 \int_0^t |\dot{u}_n(\tau)|^2_{L^2(\Omega)} d\tau, \quad t \in [0,+\infty), \tag{4.2}$$

where c_{12} is a positive constant independent of n.

If $\langle Bw, w\rangle \geq 0, \forall w \in H_0^1(\Omega)$ and $\langle Bu_n(t), u_n(t)\rangle \geq 0$, then, by Gronwall inequality,

$$|\dot{u}_n(t)|^2_{L^2(\Omega)} \leq c_{12}\exp(-2\mu_0 t), \quad \text{a.e. } t \geq 0. \tag{4.3}$$

Since

$$|\dot{u}_n(t)|_{L^2(\Omega)} \leq c_9, \quad \dot{u}_n \longrightarrow \dot{u} \text{ weakly-star in } L^\infty(0,\infty;L^2(\Omega)), \tag{4.4}$$

it is easy to obtain that $\dot{u}_n(t) \to \dot{u}(t)$ weakly in $L^2(\Omega)$ for almost everywhere $t \geq 0$. But $L^2(\Omega)$ is a real Hilbert space, hence, $|\dot{u}(t)|_{L^2(\Omega)} \leq \underline{\lim}_{n\to\infty}|\dot{u}_n(t)|_{L^2(\Omega)}$, a.e. $t \geq 0$ (see [4]). Finally, we get

$$|\dot{u}(t)|^2_{L^2(\Omega)} \leq c_{12}\exp(-2\delta_0 t), \quad (\text{a.e. } t \geq 0). \tag{4.5}$$

□

Remark 1. If $Bu = -\Delta u$, $\varphi(u) = |u|^p u$ then (1.1) is the equation which was ever considered by J. L. Lions [6]. J. L. Lions ever obtained the existence and uniqueness. But at this case, the result of decay of solution is true since the conditions of Theorem 3 is satisfied.

Remark 2. When $Bu = -\Delta u$ and φ denotes the friction potential, equation (1.1) was considered by P. D. Panagiotopoulos under stronger conditions [8].

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