

MARCINKIEWICZ-TYPE STRONG LAW OF LARGE NUMBERS FOR DOUBLE ARRAYS OF PAIRWISE INDEPENDENT RANDOM VARIABLES

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ABSTRACT. Let $\{X_{ij}\}$ be a double sequence of pairwise independent random variables. If $P\{|X_{mn}| \geq t\} \leq P\{|X| \geq t\}$ for all nonnegative real numbers t and $E|X|^p(\log^+|X|)^3 < \infty$, for $1 < p < 2$, then we prove that

$$\frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{(mn)^{1/p}} \rightarrow 0 \quad \text{a.s. as } m \vee n \rightarrow \infty. \quad (0.1)$$

Under the weak condition of $E|X|^p \log^+|X| < \infty$, it converges to 0 in L^1 . And the results can be generalized to an r -dimensional array of random variables under the conditions $E|X|^p(\log^+|X|)^{r+1} < \infty$, $E|X|^p(\log^+|X|)^{r-1} < \infty$, respectively, thus, extending Choi and Sung's result [1] of the one-dimensional case.

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1. Introduction. Etemadi [3] extended the classical law of large numbers for i.i.d. random variables to the case where the random variables are pairwise i.i.d., i.e., if $\{X_n\}$ is a sequence of pairwise i.i.d. random variables with $E|X_1| < \infty$, then

$$\frac{\sum_{i=1}^n (X_i - EX_i)}{n} \rightarrow 0 \quad \text{a.s.} \quad (1.1)$$

In 1985, Choi and Sung [1] have shown that if $\{X_n\}$ are pairwise independent and are dominated in distribution by a random variable X with $E|X|^p(\log^+|X|)^2 < \infty$, $1 < p < 2$, then $\frac{\sum_{i=1}^n (X_i - EX_i)}{n^{1/p}} \rightarrow 0$ a.s. In addition, if $E|X|^p < \infty$, then it converges to 0 in L^1 .

For a double sequence $\{X_{ij}\}$ of pairwise i.i.d. random variables, also Etemadi [3] proved that if $E|X_{11}| \log^+|X_{11}| < \infty$, then

$$\frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{mn} \rightarrow 0 \quad \text{a.s. as } m \vee n \rightarrow \infty. \quad (1.2)$$

Now, we are interested in the extension of Choi and Sung's result of the one-dimensional case to a multi-dimensional array of pairwise independent random variables, which is established in the next section.

2. Main results. Let $\{X_{ij}\}$ be a double sequence of random variables and let $X'_{ij} = X_{ij}I\{|X_{ij}| \leq (ij)^{1/p}\}$, $X''_{ij} = X_{ij}I\{|X_{ij}| > (ij)^{1/p}\}$ for $1 < p < 2$. Throughout this paper,

c denotes an unimportant positive constant which is allowed to change and d_k the number of all divisors of integer k .

To prove the main theorem, we need the following lemmas.

LEMMA 2.1. *Let $\{X_{ij}\}$ be a double sequence of pairwise independent random variables. If $P\{|X_{mn}| \geq t\} \leq P\{|X| \geq t\}$ for all nonnegative real numbers t , then*

$$(a) \quad \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{E|X'_{ij}|^2}{(ij)^{2/p}} \leq cE|X|^p \log^+ |X|, \quad (2.1)$$

$$(b) \quad \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{E|X''_{ij}|}{(ij)^{1/p}} \leq cE|X|^p \log^+ |X| \quad \text{for } 1 < p < 2.$$

PROOF. The estimation of $E|X'_{ij}|^2$ is given by

$$\begin{aligned} E|X'_{ij}|^2 &\leq \int_0^{(ij)^{2/p}} P(|X_{ij}|^2 \geq t) dt \\ &\leq \int_0^{(ij)^{2/p}} P(|X|^2 \geq t) dt \\ &= \int_0^{(ij)^{2/p}} \{P(t \leq |X|^2 < (ij)^{2/p}) + P((ij)^{2/p} \leq |X|^2)\} dt \\ &= \int_0^{(ij)^{1/p}} x^2 dF(x) + (ij)^{2/p} P((ij)^{2/p} \leq |X|^2), \end{aligned} \quad (2.2)$$

where $F(x)$ is the distribution of X . If we use the fact that $\sum_{k=i+1}^{\infty} d_k/k^{2/p} = O(\log i/(i+1)^{2/p-1})$, we obtain

$$\begin{aligned} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{1}{(ij)^{2/p}} \int_0^{(ij)^{1/p}} x^2 dF(x) &\leq c \sum_{k=1}^{\infty} \frac{d_k}{k^{2/p}} \int_0^{k^{1/p}} x^2 dF(x) \\ &\leq c \sum_{i=0}^{\infty} \left(\sum_{k=i+1}^{\infty} \frac{d_k}{k^{2/p}} \right) \int_{i^{1/p}}^{(i+1)^{1/p}} x^2 dF(x) \\ &\leq c \sum_{i=0}^{\infty} \frac{\log i}{(i+1)^{2/p-1}} \int_{i^{1/p}}^{(i+1)^{1/p}} x^2 dF(x) \\ &\leq cE|X|^p \log^+ |X| < \infty. \end{aligned} \quad (2.3)$$

And

$$\begin{aligned} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P((ij)^{2/p} \leq |X|^2) &= \sum_{k=1}^{\infty} d_k P(k \leq |X|^p) \\ &= \sum_{k=1}^{\infty} \left(\sum_{j=1}^k d_j \right) P(k \leq |X|^p < k+1) \\ &= c \sum_{k=1}^{\infty} k \log k P(k \leq |X|^p < k+1) \\ &\leq cE|X|^p \log^+ |X| < \infty, \end{aligned} \quad (2.4)$$

where we use the fact that $\sum_{k=1}^n d_k = O(n \log n)$. It follows that

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{E|X'_{ij}|^2}{(ij)^{2/p}} < \infty, \quad \text{which proves (a).} \quad (2.5)$$

By the fact that $\sum_{k=1}^n d_k/k^{1/p} = O(n^{1-(1/p)} \log n)$, we can obtain (b) by the same method. \square

The following lemma is a two parameter analog of [5, Lem. 3.6.1a].

LEMMA 2.2. *Let $\{X_{ij}\}$ be a double sequence of pairwise independent random variables with $EX_{ij} = 0$, and let $S_{mn} = \sum_{i=1}^m \sum_{j=1}^n X_{ij}$. Then*

$$E \left(\max_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} |S_{ij}| \right)^2 \leq c(\log m)^2 (\log n)^2 \sum_{k=1}^m \sum_{l=1}^n E|X_{kl}|^2. \quad (2.6)$$

PROOF. For $m = 1$ and $n = 1$, the inequality is trivial. If $m > 1$, let s be an integer such that $2^{s-1} < m \leq 2^s$. And if $n > 1$, let t be an integer such that $2^{t-1} < n \leq 2^t$. We can assume that $m, n > 1$. We assign X_{ij} to the point (i, j) of integer in $(0, 2^s] \times (0, 2^t]$ (if $m < i \leq 2^s$ or $n < j \leq 2^t$, set $X_{ij} = 0$). Divide the interval $(0, 2^s]$ into $(0, 2^{s-1}]$ and $(2^{s-1}, 2^s]$, each of these two intervals into two halves, and so on. Then the elements of the i th partition are of length 2^{s-i} , $i = 0, \dots, s$. Also, divide the interval $(0, 2^t]$ in the same way. Then we obtain the (i, j) th partition P_{ij} of $(0, 2^s] \times (0, 2^t]$ by the i th partition of $(0, 2^s]$ and the j th partition of $(0, 2^t]$. Every rectangle $(0, i] \times (0, j]$ is the sum of at most $(s+1)(t+1)$ disjoint subrectangles each of which belongs to a different partition. We can write $S_{ij} = \sum_{k=0}^s \sum_{l=0}^t Y_{kl;ij}$, where $Y_{kl;ij}$ is the sum of all r.v.'s belonging to the rectangle $(a, b] \times (c, d]$, $b - a = 2^k$ and $d - c = 2^l$, which may or may not be a summand of $(0, i] \times (0, j]$ so that some $Y_{kl;ij}$ may vanish. Let $T_{ij} = \sum_{k=1}^{2^i} \sum_{l=1}^{2^j} |Y_{kl}|^2$, where Y_{kl} is the sum of all r.v.'s which belong to the (k, l) -element of P_{ij} . If we put $T = \sum_{i=0}^s \sum_{j=0}^t T_{ij}$, by the elementary Schwarz inequality, we obtain

$$|S_{ij}|^2 \leq (s+1)(t+1) \sum_{k=0}^s \sum_{l=0}^t |Y_{kl;ij}|^2 \leq (s+1)(t+1)T. \quad (2.7)$$

Since $ET_{ij} \leq \sum_{k=1}^m \sum_{l=1}^n E|X_{kl}|^2$, $ET \leq (s+1)(t+1) \sum_{k=1}^m \sum_{l=1}^n E|X_{kl}|^2$. It follows that

$$\begin{aligned} E \left(\max_{\substack{1 \leq i \leq m \\ 1 \leq j \leq n}} |S_{ij}|^2 \right) &\leq (s+1)^2 (t+1)^2 \sum_{k=1}^m \sum_{l=1}^n E|X_{kl}|^2 \\ &\leq c(\log m)^2 (\log n)^2 \sum_{k=1}^m \sum_{l=1}^n E|X_{kl}|^2. \end{aligned} \quad (2.8)$$

\square

THEOREM 2.3. *Let $\{X_{ij}\}$ be a double sequence of pairwise independent random variables. If $P\{|X_{mn}| \geq t\} \leq P\{|X| \geq t\}$ for all nonnegative real numbers t and $E|X|^p (\log^+ |X|)^3 < \infty$, for $1 < p < 2$, then*

$$\lim_{m \vee n \rightarrow \infty} \frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{(mn)^{1/p}} = 0 \quad a.s. \quad (2.9)$$

PROOF. We denote by $S_{mn} = \sum_{i=1}^m \sum_{j=1}^n X_{ij}$, $S'_{mn} = \sum_{i=1}^m \sum_{j=1}^n X'_{ij}$. Then we obtain the inequalities

$$\begin{aligned}
\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} P\{X_{ij} \neq X'_{ij}\} &= \sum_{k=1}^{\infty} d_k P\{|X_{11}| > k^{1/p}\} \\
&\leq \sum_{k=1}^{\infty} d_k P\{|X| > k^{1/p}\} \\
&= \sum_{i=1}^{\infty} \left(\sum_{k=1}^i d_k \right) \int_{i^{1/p}}^{(i+1)^{1/p}} dF(x) \\
&\leq c \sum_{i=1}^{\infty} i \log i \int_{i^{1/p}}^{(i+1)^{1/p}} dF(x) \\
&\leq c E|X|^p \log^+ |X| < \infty,
\end{aligned} \tag{2.10}$$

Hence, by the Borel-Cantelli lemma,

$$\frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - X'_{ij})}{(mn)^{1/p}} \rightarrow 0 \quad \text{a.s.} \tag{2.11}$$

Now, we use Chebyshev's inequality and Lemma 2.1 to obtain

$$\begin{aligned}
\sum_{k=1}^{\infty} \sum_{l=1}^{\infty} P \left\{ \left| \frac{S'_{2^k 2^l} - ES'_{2^k 2^l}}{(2^k 2^l)^{1/p}} \right| > \epsilon \right\} &\leq c \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \frac{\text{Var} S'_{2^k 2^l}}{(2^k 2^l)^{2/p}} \\
&= c \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \frac{1}{(2^k 2^l)^{2/p}} \sum_{i=1}^{2^k} \sum_{j=1}^{2^l} \text{Var} X'_{ij} \\
&\leq c \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{E X'_{ij}^2}{(ij)^{2/p}} \\
&\leq c E|X|^p \log^+ |X|^p < \infty,
\end{aligned} \tag{2.12}$$

which follows easily by summation by parts. It follows that

$$\frac{S'_{2^k 2^l} - ES'_{2^k 2^l}}{(2^k 2^l)^{1/p}} \rightarrow 0 \quad \text{a.s.} \tag{2.13}$$

And let

$$\begin{aligned}
T_{kl} &= \max_{\substack{2^k \leq m < 2^{k+1} \\ 2^l \leq n < 2^{l+1}}} \left| \frac{S'_{2^k 2^l}}{(2^k 2^l)^{1/p}} - \frac{S^*_{mn}}{(mn)^{1/p}} \right| \\
&\leq \frac{|S'_{2^k 2^l}|}{(2^k 2^l)^{1/p}} + \max_{\substack{2^k \leq m < 2^{k+1} \\ 2^l \leq n < 2^{l+1}}} \frac{|S^*_{mn}|}{(mn)^{1/p}},
\end{aligned} \tag{2.14}$$

where $S^*_{mn} = S'_{mn} - ES'_{mn}$.

By using Lemma 2.2, we obtain, for any $\epsilon > 0$,

$$\begin{aligned}
\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} P \left[\max_{\substack{2^k \leq m \leq 2^{k+1} \\ 2^l \leq n \leq 2^{l+1}}} \frac{|S_{mn}^*|}{(mn)^{1/p}} \geq \frac{\epsilon}{2} \right] &\leq c \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{1}{(2^k 2^l)^{2/p}} E \left(\max_{\substack{2^k \leq m \leq 2^{k+1} \\ 2^l \leq n \leq 2^{l+1}}} |S_{mn}^*| \right)^2 \\
&\leq c \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(k+1)^2 (l+1)^2}{(2^{k+1} 2^{l+1})^{2/p}} \sum_{i=1}^{2^{k+1}} \sum_{j=1}^{2^{l+1}} E |X'_{ij}|^2 \quad (2.15) \\
&\leq c \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{(\log_2 ij)^2}{(ij)^{2/p}} E |X'_{ij}|^2,
\end{aligned}$$

where the last inequality follows easily by summation by parts. But

$$\begin{aligned}
\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{(\log_2 ij)^2}{(ij)^{2/p}} E |X'_{ij}|^2 &\leq \sum_{k=1}^{\infty} \frac{d_k (\log_2 k)^2}{k^{2/p}} \int_0^{k^{1/p}} x^2 dF(x) \\
&\leq \sum_{i=0}^{\infty} \left(\sum_{k=i+1}^{\infty} \frac{d_k (\log_2 k)^2}{k^{2/p}} \right) \int_{i^{1/p}}^{(i+1)^{1/p}} x^2 dF(x) \quad (2.16) \\
&\leq c \sum_{i=0}^{\infty} i^{1-(2/p)} (\log i)^3 \int_{i^{1/p}}^{(i+1)^{1/p}} x^2 dF(x) \\
&\leq c E |X|^p (\log^+ |X|^p)^3 < \infty,
\end{aligned}$$

where we use $\sum_{k=1}^{\infty} \frac{d_k (\log_2 k)^2}{k^{2/p}} = O\left(\frac{(\log i)^3}{i^{(2/p)-1}}\right)$ which follows by summation by parts. Hence, (2.13), (2.15), and (2.16) give us

$$\frac{S'_{mn} - ES'_{mn}}{(mn)^{1/p}} \rightarrow 0 \quad \text{a.s.} \quad (2.17)$$

Combining (2.11) and (2.17), we get

$$\frac{S_{mn} - ES_{mn}}{(mn)^{1/p}} \rightarrow 0 \quad \text{a.s.} \quad (2.18)$$

Since

$$\frac{S_{mn} - ES_{mn}}{(mn)^{1/p}} = \frac{S_{mn} - ES'_{mn}}{(mn)^{1/p}} - \frac{\sum_{i=1}^m \sum_{j=1}^n E |X''_{ij}|}{(mn)^{1/p}}, \quad (2.19)$$

it remains to prove that the second term of the right-hand side converges to 0 a.s. By Lemma 2.1(b), we obtain

$$\begin{aligned}
\sum_{k=1}^{\infty} \sum_{l=1}^{\infty} \frac{\sum_{i=1}^{2^k} \sum_{j=1}^{2^l} E |X''_{ij}|}{(2^k 2^l)^{1/p}} &\leq c \sum_{i,j=1}^{\infty} \frac{E |X''_{ij}|}{(ij)^{1/p}} \\
&\leq c E |X|^p \log^+ |X| < \infty,
\end{aligned} \quad (2.20)$$

from which, it follows that

$$\lim_{k \vee l \rightarrow \infty} \frac{\sum_{i=1}^{2^k} \sum_{j=1}^{2^l} E |X''_{ij}|}{(2^k 2^l)^{1/p}} = 0. \quad (2.21)$$

But since

$$\begin{aligned} T'_{kl} &= \max_{\substack{2^k \leq m \leq 2^{k+1} \\ 2^l \leq n \leq 2^{l+1}}} \left| \frac{\sum_{i=1}^m \sum_{j=1}^n E |X''_{ij}|}{(mn)^{1/p}} - \frac{\sum_{i=1}^{2^k} \sum_{j=1}^{2^l} E |X''_{ij}|}{(2^k 2^l)^{1/p}} \right| \\ &\leq \frac{c}{(2^{k+1} 2^{l+1})^{1/p}} \sum_{i=1}^{2^{k+1}} \sum_{j=1}^{2^{l+1}} E |X''_{ij}|, \end{aligned} \quad (2.22)$$

T'_{kl} converges to 0 which implies that, by (2.21),

$$\frac{\sum_{i=1}^m \sum_{j=1}^n E |X''_{ij}|}{(mn)^{1/p}} \rightarrow 0. \quad (2.23)$$

This completes the proof. \square

COROLLARY 2.4. *Let $\{X_{ij}\}$ be a double sequence of pairwise i.i.d. random variables with $E|X_{11}|^p (\log^+ |X_{11}|)^3 < \infty$, for $1 < p < 2$. Then*

$$\lim_{m \vee n \rightarrow \infty} \frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{(mn)^{1/p}} = 0 \quad \text{a.s.} \quad (2.24)$$

REMARK. The generalization to r -dimensional arrays of random variables can be obtained easily under the condition $E|X|^p (\log^+ |X|)^{r+1} < \infty$.

THEOREM 2.5. *Let $\{X_{ij}\}$ be a double sequence of pairwise independent random variables. If $P\{|X_{ij}| \geq t\} \leq P\{|X| \geq t\}$ for all nonnegative real numbers t and $E|X|^p \log^+ |X| < \infty$, $1 < p < 2$, then*

$$\frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{(mn)^{1/p}} \rightarrow 0 \quad \text{in } L^1 \text{ as } m \vee n \rightarrow \infty. \quad (2.25)$$

PROOF. Since $\{X_{ij}\}$ are pairwise independent, $\{X'_{ij} - EX'_{ij}\}$ are orthogonal which implies that

$$E \left| \frac{\sum_{i=1}^m \sum_{j=1}^n (X'_{ij} - EX'_{ij})}{(mn)^{1/p}} \right|^2 \leq \frac{\sum_{i=1}^m \sum_{j=1}^n E |X'_{ij}|^2}{(mn)^{2/p}}. \quad (2.26)$$

Since

$$\begin{aligned} E \left| \frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{(mn)^{1/p}} \right| &\leq E \left| \frac{\sum_{i=1}^m \sum_{j=1}^n (X'_{ij} - EX'_{ij})}{(mn)^{1/p}} \right| \\ &\quad + 2 \frac{\sum_{i=1}^m \sum_{j=1}^n E |X''_{ij}|}{(mn)^{1/p}}, \end{aligned} \quad (2.27)$$

it suffices to show that $(\sum_{i=1}^m \sum_{j=1}^n E |X''_{ij}|^2)/(mn)^{2/p}$ converges to 0 as $m \vee n \rightarrow \infty$. But this can be shown by a method similar to that used in the proof of (2.23) in Theorem 2.3. \square

COROLLARY 2.6. *Let $\{X_{ij}\}$ be a double sequence of pairwise i.i.d. random variable with $E|X_{11}|^p \log^+ |X_{11}| < \infty$, for $1 < p < 2$. Then*

$$\frac{\sum_{i=1}^m \sum_{j=1}^n (X_{ij} - EX_{ij})}{(mn)^{1/p}} \rightarrow 0 \quad \text{in } L^1 \text{ as } m \vee n \rightarrow \infty. \quad (2.28)$$

REMARK. The generalization to r -dimensional arrays of random variables can be obtained under the condition $E|X|^p(\log^+|X|)^{r+1} < \infty$.

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