Internat. J. Math. & Math. Sci.
VOL. 21 NO. 2 (1998) 403-408

ON NEW STRENGTHENED HARDY-HILBERT'S INEQUALITY

BICHENG YANG

Department of Mathematics
Guangdong Education College
Guangzhou, Guangdong 510303, P.R. CHINA

and

LOKENATH DEBNATH

Department of Mathematics
University of Central Florida
Orlando, Florida 32816, U.S.A.

(Received February 27, 1997 and in revised form September 8, 1997)

ABSTRACT. In this paper, a new inequality for the weight coefficient $\omega(q,n)$ in the form

$$\omega(q,n):=\sum_{m=1}^{\infty}\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}<\frac{\pi}{\sin(\pi/p)}-\frac{1}{2n^{1/p}+n^{-1/q}}\left(q>1,\frac{1}{p}+\frac{1}{q}=1,\,n\in N\right)$$

is proved. This is followed by a strengthened version of the Hardy-Hilbert inequality.

KEY WORDS AND PHRASES: Hardy-Hilbert's inequality, weight coefficient, Holder's inequality.
1991 AMS SUBJECT CLASSIFICATION CODES: 26D15.

1. INTRODUCTION

If $a_n \geq 0$, $0<\sum\limits_{n=1}^{\infty} n^2 a_n^2<\infty$, then the Karlson's inequality is

$$\left(\sum_{n=1}^{\infty}a_n\right)^4<\pi^2\sum_{n=1}^{\infty}a_n^2\sum_{n=1}^{\infty}n^2a_n^2, \tag{1.1}$$

where the constant π^2 cannot be made smaller. However, it can be strengthened (see Mikhlin [1], p. 7) as

$$\left(\sum_{n=1}^{\infty}a_n\right)^4<\pi^2\sum_{n=1}^{\infty}a_n^2\sum_{n=1}^{\infty}\left(n-\frac{1}{2}\right)^2a_n^2. \tag{1.2}$$

In recent years, considerable attention has been given to develop some types of strengthened inequality (see [2]-[10]) by estimating the weight coefficient $\omega(q,n)$ as

$$\omega(q,n)=\sum_{m=1}^{\infty}\frac{1}{(m+n)}\left(\frac{n}{m}\right)^{1/q}\left(q>1,p^{-1}+q^{-1}=1,n\in N\right). \tag{1.3}$$

Some improvement of Hardy-Hilbert's inequality (see Hardy et al. [11]) has been made in the form

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_mb_n}{m+n}<\frac{\pi}{\sin(\pi/p)}\left\{\sum_{n=1}^{\infty}a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}b_n^q\right\}^{1/q}. \tag{1 4}$$

In their recent work, Xu and Gau [2] considered the following weight coefficient (1.3) and proved the following inequality

$$\omega(q,n) < \frac{\pi}{\sin(\pi/p)} - \frac{\eta_p}{n^{1/p}+n^{-1/q}}, \quad \eta_p = p-1. \tag{1.5}$$

Then a strengthened Hardy-Hilbert's inequality

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_m b_n}{m+n} < \left\{\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{p-1}{n^{1/p}+n^{-1/q}}\right]a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{q-1}{n^{1/q}+n^{-1/p}}\right]b_n^q\right\}^{1/q}. \tag{1.6}$$

was proved. The key is to estimate the corresponding weight coefficient effectively. Hsu and Wang [3] proved the following inequality

$$\omega(2,n) < \pi - \frac{\theta}{\sqrt{n}}, \quad \theta = \frac{3}{\sqrt{2}} - 1 = 1.12132^{+} \quad (n \in N). \tag{1.7}$$

Then they gave a new strengthened Hilbert's inequality which is the same as (1.6) with $p = 2$. Since θ in (1.7) is not the best possible, Gau [5] obtained the best possible value of $\theta = \pi - \sum_{k=1}^{\infty}\frac{1}{(1+k)}\left(\frac{1}{\sqrt{k}}\right) = 1.2811^{+}$. Subsequently, Gau [6] considered the general case and proved a new inequality for the weight coefficient $\omega(q,n)$ as

$$\omega(q,n) < \frac{\pi}{\sin(\pi/p)} - \frac{\theta_p}{n^{1/p}} \quad \left(q > 1, \frac{1}{p} + \frac{1}{q} = 1, n \in N\right), \tag{1.8}$$

where $\theta_p = (p-1)$. Recently, Gau [7] replaced $(p-1)$ by $\theta_p = \theta_p(n) > 0$ in (1.8). But the problem is that $\theta_p(n)$ depends on both p and q. Simultaneously, Yang [8] found that $\theta_p = \theta = 0.341295^{+}$, but the constant $\theta_p = \theta$ is not the best possible value. Finally, Yang and Gau [9] found the best possible value for $\theta_p = \theta = 1 - C = 0.42278433^{+}$, where C is a Euler constant. They also proved the following new Hardy-Hilbert's inequality

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_m b_n}{m+n} < \left\{\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{1-C}{n^{1/p}}\right]a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{1-C}{n^{1/q}}\right]b_n^q\right\}^{1/q}. \tag{1.9}$$

It is important to point out that (1.5) and (1.8) are different, and the constant η_p in (1.5) depends on p.

The main objective of this paper is to prove an improved version of (1.5) as

$$\omega(q,n) < \frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/p}+n^{-1/q}} \quad \left(q > 1, \frac{1}{p} + \frac{1}{q} = 1, n \in N\right), \tag{1.10}$$

and then prove a strengthened version of Hardy-Hilbert's inequality as follows:

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_m b_n}{m+n} < \left\{\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/p}+n^{-1/q}}\right]a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/q}+n^{-1/p}}\right]b_n^q\right\}^{1/q}. \tag{1.11}$$

For this, we need the following inequality (see Yang [8] Lemma 1): If

$$f(x) > 0, f^{(2r-1)}(x) < 0, f^{(2r)}(x) \geq 0, x \in [1,\infty)(r = 1,2), f^{(r)}(\infty) = 0(r = 0,1,2,3,4),$$

and $\int_1^{\infty} f(x)dx < \infty$, then

$$\sum_{m=1}^{\infty} f(m) \leq \int_1^{\infty} f(x)dx + \frac{1}{2}f(1) - \frac{1}{12}f'(1). \tag{1.12}$$

2. SOME LEMMAS

LEMMA 2.1. If $q > 1$, $p^{-1} + q^{-1} = 1$, $n \in N$, then

$$\omega(q,n) < \frac{\pi}{\sin(\pi/p)} - \frac{1}{n^{1/p}}[f_n(p) + g_n(p)], \tag{2.1}$$

where $\omega(q,n)$ is defined by (1.5), and

$$f_n(p) := p + \frac{1}{12p} + \frac{1}{(1+p)n} + \frac{1}{12pn^2} + \frac{1}{3(1+3p)n^3},$$
$$g_n(p) := \frac{-1}{12pn} - \frac{1}{2(1+2p)n^2} - \frac{7}{12} - \frac{1}{2n} + \frac{1}{12n^2} - \frac{7}{12n^3}.$$

PROOF. Let

$$f(x) = \frac{1}{(x+n)x^{1/q}}, \quad x \in [1,\infty)\,(q > 1, n \in N).$$

By (1.12), we obtain that

$$\sum_{m=1}^{\infty} \frac{1}{(m+n)m^{1/q}} \le \int_1^{\infty} \frac{1}{(x+n)x^{1/q}}\,dx + \left(\frac{7}{12} - \frac{1}{12p}\right)\frac{1}{1+n} + \frac{1}{12(1+n)^2}. \tag{2.2}$$

Since

$$\begin{aligned}
\int_0^{1/n} \frac{1}{(1+y)y^{1/q}}\,dy &= \int_0^{1/n} \sum_{\nu=0}^{\infty}(-1)^{\nu} y^{\nu-1/q}dy \\
&= \sum_{\nu=0}^{\infty}(-1)^{\nu} \int_0^{1/n} y^{\nu-1/q}dy = \frac{p}{n^{1/p}} \sum_{\nu=0}^{\infty} \frac{(-1)^{\nu}}{(1+\nu p)n^{\nu}} \\
&> \frac{p}{n^{1/p}} \sum_{\nu=0}^{3} \frac{(-1)^{\nu}}{(1+\nu p)n^{\nu}} = \frac{1}{n^{1/p}}\left[p + \sum_{\nu=1}^{3}\frac{(-1)^{\nu}}{\nu n^{\nu}} - \sum_{\nu=1}^{3}\frac{(-1)^{\nu}}{\nu(1+\nu p)n^{\nu}}\right].
\end{aligned}$$

Putting $x = ny$, we find that

$$\begin{aligned}
\int_1^{\infty} \frac{1}{(x+n)x^{1/q}}\,dx &= \frac{1}{n^{1/q}} \int_{1/n}^{\infty} \frac{1}{(1+y)y^{1/q}}\,dy \\
&= \frac{1}{n^{1/q}}\left[\int_0^{\infty} \frac{1}{(1+y)y^{1/q}}\,dy - \int_0^{1/n} \frac{1}{(1+y)y^{1/q}}\,dy\right] \\
&= \frac{1}{n^{1/q}}\left[\frac{\pi}{\sin(\pi/p)} - \frac{p}{n^{1/p}} \sum_{\nu=0}^{\infty} \frac{(-1)^{\nu}}{(1+\nu p)n^{\nu}}\right] \\
&< \frac{1}{n^{1/q}} \frac{\pi}{\sin(\pi/p)} - \frac{1}{n}\left[p + \sum_{\nu=1}^{3}\frac{(-1)^{\nu}}{\nu n^{\nu}} - \sum_{\nu=1}^{3}\frac{(-1)^{\nu}}{\nu(1+\nu p)n^{\nu}}\right],
\end{aligned}$$

we then find that

$$\frac{1}{1+n} = \frac{1}{n}\left(1 + \frac{1}{n}\right)^{-1} < \frac{1}{n}\left(1 - \frac{1}{n} + \frac{1}{n^2}\right),$$

and

$$\frac{1}{(1+n)^2} = \frac{1}{n^2}\left(1 + \frac{1}{n}\right)^{-2} < \frac{1}{n^2}\left(1 - \frac{2}{n} + \frac{3}{n^2}\right).$$

Substituting the above results in (2.2), by (1.5), we have (2.1). This proves the lemma.

LEMMA 2.2. If $p > 1$, $n \in N$, then

$$f_n(p) + g_n(p) > \frac{1}{2} - \frac{1}{12n} - \frac{1}{2n^3}. \tag{2.3}$$

PROOF. Since

$$\begin{aligned} f_n'(p) &= 1 - \frac{1+n^2}{12n^2p^2} - \frac{1}{(1+p)^2 n} - \frac{1}{(1+3p)^2 n^3} \\ &> 1 - \frac{1+n^2}{12n^2} - \frac{1}{(1+1)^2 n} - \frac{1}{(1+3)^2 n^3} \\ &= \frac{11}{12} - \frac{1}{12n^2} - \frac{1}{4n} - \frac{1}{16n^3} > 0, \end{aligned}$$

and

$$g_n'(p) = \frac{1}{12p^2 n} + \frac{1}{(1+2p)^2 n^2} > 0,$$

then $f_n(p) + g_n(p)$ is strictly increasing for $p \in (1, \infty)$, and

$$f_n(p) + g_n(p) > \lim_{p \to 1}(f_n(p) + g_n(p)) = \frac{1}{2} - \frac{1}{12n} - \frac{1}{2n^3}.$$

Thus the lemma is proved.

LEMMA 2.3. If $q > 1$, $p^{-1} + q^{-1} = 1$, $n \in N$, then inequality (1.10) is valid. So is the following inequality:

$$\omega(p, n) < \frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/q} + n^{-1/p}}. \tag{2.4}$$

PROOF. Since for $n \geq 3$,

$$\left(\frac{1}{2} - \frac{1}{12n} - \frac{1}{2n^3}\right)\left(1 + \frac{1}{2n}\right) = \frac{1}{2} + \frac{1}{n}\left(\frac{1}{6} - \frac{1}{24n} - \frac{1}{2n^2} - \frac{1}{4n^3}\right) > \frac{1}{2},$$

then

$$\frac{1}{2} - \frac{1}{12n} - \frac{1}{2n^3} > \frac{1}{2 + n^{-1}} \quad (n \geq 3).$$

By (2.1) and (2.3), we have

$$\begin{aligned} \omega(q, n) &< \frac{\pi}{\sin(\pi/p)} - \frac{1}{n^{1/p}}\left(\frac{1}{2} - \frac{1}{12n} - \frac{1}{2n^3}\right) \\ &< \frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/p} + n^{-1/q}} \quad (n \geq 3). \end{aligned}$$

Taking $\theta_p = 1 - C$, by (1.8) (see Yang and Gau [9]), we find that

$$\omega(q, 1) < \frac{\pi}{\sin(\pi/p)} - \frac{1 - C}{1} < \frac{\pi}{\sin(\pi/p)} - \frac{1}{2 \times 1 + 1}. \tag{2.5}$$

Since $C < 3/5 = 0.6$, then we have

$$\frac{1}{2 \times 2^{1/p} + 2^{-1/q}} < \frac{1 - C}{2^{1/p}},$$

and

$$\omega(q, 2) < \frac{\pi}{\sin(\pi/p)} - \frac{1 - C}{2^{1/p}} < \frac{\pi}{\sin(\pi/p)} - \frac{1}{2 \times 2^{1/p} + 2^{-1/q}}. \tag{2.6}$$

It follows that for $n = 1, 2$, (1.10) also holds. Then (1.10) is valid for any $n \in N$. Interchanging p, q in (1.10), since $\frac{\pi}{\sin(\pi/p)} = \frac{\pi}{\sin(\pi/q)}$, we have (2.4). The lemma is proved.

3. MAIN RESULTS

THEOREM 3.1. If $p > 1,\ p^{-1} + q^{-1} = 1,\ a_n \geq 0,\ b_n \geq 0,$ and $0 < \sum_{n=1}^{\infty} a_n^p < \infty,$ $0 < \sum_{n=1}^{\infty} b_n^q < \infty,$ then inequality (1.11) is valid. We also have

$$\sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{a_n}{m+n}\right)^p < \left[\frac{\pi}{\sin(\pi/p)}\right]^{p-1}\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/p}+n^{-1/q}}\right]a_n^p. \tag{3.1}$$

When $p = q = 2,$ this inequality reduces to the form

$$\sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{a_n}{m+n}\right)^2 < \pi\sum_{n=1}^{\infty}\left[\pi - \frac{1}{2\sqrt{n}+\sqrt{n^{-1}}}\right]a_n^2. \tag{3.2}$$

PROOF. By Holder's inequality, we have

$$\begin{aligned}
\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{a_m b_n}{m+n} &= \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{1}{(m+n)^{1/p}}\left(\frac{m}{n}\right)^{1/pq}a_m\right]\left[\frac{1}{(m+n)^{1/q}}\left(\frac{n}{m}\right)^{1/pq}b_n\right]\\
&\leq \left\{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m+n}\left(\frac{m}{n}\right)^{1/q}a_m^p\right\}^{1/p}\left\{\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/p}b_n^q\right\}^{1/q}\\
&= \left\{\sum_{n=1}^{\infty}\left[\sum_{m=1}^{\infty}\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}\right]a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}\left[\sum_{m=1}^{\infty}\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/p}\right]b_n^q\right\}^{1/q}\\
&= \left\{\sum_{n=1}^{\infty}\omega(q,n)a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}\omega(p,n)b_n^q\right\}^{1/q}.
\end{aligned}$$

Hence, by (1.10) and (2.4), inequality (1.11) holds.

Since by (2.4), $\omega(p,n) < \frac{\pi}{\sin(\pi/p)}$, then by Holder's inequality, we obtain

$$\begin{aligned}
\sum_{n=1}^{\infty}\frac{a_n}{m+n} &= \sum_{n=1}^{\infty}\left[\frac{a_n}{(m+n)^{1/p}}\left(\frac{n}{m}\right)^{1/pq}\right]\left[\frac{1}{(m+n)^{1/q}}\left(\frac{m}{n}\right)^{1/pq}\right]\\
&\leq \left\{\sum_{n=1}^{\infty}\left[\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}\right]a_n^p\right\}^{1/p}\left\{\sum_{n=1}^{\infty}\frac{1}{m+n}\left(\frac{m}{n}\right)^{1/p}\right\}^{1/q}\\
&= \left\{\sum_{n=1}^{\infty}\left[\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}\right]a_n^p\right\}^{1/p}\{\omega(p,n)\}^{1/q},\\
&< \left\{\sum_{n=1}^{\infty}\left[\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}\right]a_n^p\right\}^{1/p}\left\{\frac{\pi}{\sin(\pi/p)}\right\}^{1/q}.
\end{aligned}$$

By (1.10), we find

$$\begin{aligned}
\sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{a_n}{m+n}\right)^p &< \left[\frac{\pi}{\sin(\pi/p)}\right]^{p/q}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}a_n^p\\
&= \left[\frac{\pi}{\sin(\pi/p)}\right]^{p-1}\sum_{n=1}^{\infty}\left[\sum_{m=1}^{\infty}\frac{1}{m+n}\left(\frac{n}{m}\right)^{1/q}\right]a_n^p\\
&= \left[\frac{\pi}{\sin(\pi/p)}\right]^{p-1}\sum_{n=1}^{\infty}\omega(q,n)a_n^p\\
&< \left[\frac{\pi}{\sin(\pi/p)}\right]^{p-1}\sum_{n=1}^{\infty}\left[\frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/p}+n^{-1/q}}\right]a_n^p.
\end{aligned}$$

This proves result (3.1). Thus the proof of Theorem 3.1 is complete.

4. CONCLUDING REMARKS

(a) Inequality (1.11) is a definite improvement over (1.6).

(b) Since, for $n \geq 3$, $C > \left(\frac{n+1}{2n+1}\right)$, then

$$\frac{\pi}{\sin(\pi/p)} - \frac{1}{2n^{1/p} + n^{-1/q}} < \frac{\pi}{\sin(\pi/p)} - \frac{(1-C)}{n^{1/p}}, \quad (n \geq 3). \tag{4.1}$$

In view of (2.5), (2.6) and (3.3), it follows that (1.9) and (1.11) represent two distinct versions of strengthened inequalities. But they are not comparable.

(c) Inequality (3.1) reduces to

$$\sum_{m=1}^{\infty}\left(\sum_{n=1}^{\infty}\frac{a_n}{m+n}\right)^p < \left[\frac{\pi}{\sin(\pi/p)}\right]^p \sum_{n=1}^{\infty} a_n^p, \tag{4.2}$$

This is an equivalent form of Hardy-Hilbert's inequality (1.4) (see Hardy et al. [11], Chapter 9).

REFERENCES

[1] MIKHLIN, S.G., *Constants in Some Inequalities of Analysis*, John Wiley & Sons, New York, 1986.

[2] XU, L.C. and GAU, Y.K., Note on Hardy-Riesz's extension of Hilbert's inequality, *Chinese Quarterly Journal of Mathematics*, **6**, 1 (1991), 75-77.

[3] HSU, L.C. and WANG, Y.J., A refinement of Hilbert's double series theorem, *J. Math. Res. Exp.* **11**, 1 (1991), 143-144.

[4] ZHAO, D.J., On a refinement of Hilbert's double series theorem, *Math. Practice and Theory*, 1 (1993), 85-90.

[5] GAU, M.Z., A note on Hilbert double series theorem, *Hunan Annals of Mathematics*, **12**, 1-2 (1992), 142-147.

[6] GAU, M.Z., An improvement of Hardy-Riesz's extension of the Hilbert inequality, *J. Math. Res. Exp.* **14**, 2 (1994), 255-259.

[7] GAU, M.Z., A note on the Hardy-Hilbert inequality, *J. Math. Ana. Appl.* **204** (1996), 346-351.

[8] YANG, B.C., A refinement on the general Hilbert's double series theorem, *J. Math. Study*, **29**, 2 (1996), 64-70.

[9] YANG, B.C. and GAU, M.Z., On a best value of Hardy-Hilbert's inequality, *Advances in Math.*, **26**, 2 (1997), 159-164.

[10] YANG, B.C. and DEBNATH, L., Some inequalities involving the constant e, and an application to Carleman's inequality, *J. Math. Anal. and Appl.*, to appear (1997).

[11] HARDY, G.H., LITTLEWOOD, J.E. and POLYA, G., *Inequalities*, Cambridge University Press, Cambridge, 1952.

Journal of Applied Mathematics and Decision Sciences

Special Issue on Intelligent Computational Methods for Financial Engineering

Call for Papers

As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods**: artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning
- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management
- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

Authors should follow the Journal of Applied Mathematics and Decision Sciences manuscript format described at the journal site http://www.hindawi.com/journals/jamds/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/, according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

Lean Yu, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; yulean@amss.ac.cn

Shouyang Wang, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China; sywang@amss.ac.cn

K. K. Lai, Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskklai@cityu.edu.hk