

# SEMIDISCRETIZATION FOR A NONLOCAL PARABOLIC PROBLEM

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A time discretization technique by Euler forward scheme is proposed to deal with a nonlocal parabolic problem. Existence and uniqueness of the approximate solution are proved.

## 1. Introduction

In this work, we study the time discretization by Euler forward scheme of the nonlocal initial boundary value problem

$$\begin{aligned}\frac{\partial u}{\partial t} - \Delta u &= \lambda \frac{f(u)}{(\int_{\Omega} f(u) dx)^2} \quad \text{in } \Omega \times ]0; T[, \\ u &= 0 \quad \text{on } \partial\Omega \times ]0; T[, \\ u(0) &= u_0 \quad \text{in } \Omega,\end{aligned}\tag{1.1}$$

with  $\Omega \subset \mathbb{R}^d$  ( $d \geq 1$ ) a bounded regular domain and  $\lambda$  a positive parameter. The hypotheses we will assume on  $f$  are the same as in [6]. We recall first that (1.1) arises by reducing the following system of two equations modeling the thermistor problem:

$$\begin{aligned}u_t &= \nabla \cdot (k(u) \nabla u) + \sigma(u) |\nabla \varphi|^2, \\ \nabla (\sigma(u) \nabla \varphi) &= 0,\end{aligned}\tag{1.2}$$

where  $u$  represents the temperature generated by the electric current flowing through a conductor,  $\varphi$  the electric potential,  $\sigma(u)$  and  $k(u)$  are, respectively, the electric and thermal conductivities. For more description, we refer to [5, 6, 7, 8, 11] among others.

We recall also that the Euler forward method was used by several authors to treat semidiscretization of nonlinear parabolic problems, see [3, 4]. Concerning problem (1.1), results of existence and uniqueness of solutions are known under particular forms of  $f$ , we refer to [2] and the references therein. On the other hand, little is known about

the solutions to the discrete problem

$$\begin{aligned} U^n - \tau \Delta U^n &= U^{n-1} + \lambda \tau \frac{f(U^n)}{(\int_{\Omega} f(U^n) dx)^2} \quad \text{in } \Omega, \\ U^n &= 0 \quad \text{on } \partial\Omega, \\ U^0 &= u_0 \quad \text{in } \Omega. \end{aligned} \tag{1.3}$$

Whereas, semidiscretization has been involved for the equations of the thermistor problem in [1, 9]. Our aim here is to continue the study of problem (1.1) initiated in [6], where an a priori  $L^\infty$ -estimate is derived. In addition to habitual existence and uniqueness questions concerning the solutions of (1.3), we will prove some results of stability and proceed to error estimates analysis. In [1], the authors derived an  $L^2$  and  $H^1$ -norm error by requiring more regularity on the solution  $u$ , for instance  $u, u_t$  in  $H^2(\Omega) \cap W^{1,\infty}(\Omega)$ . Unfortunately, such smoothness is not always possible since the function  $f$  is nonlinear.

## 2. The semidiscrete problem

**2.1. Existence and uniqueness.** We consider the Euler scheme (1.3), with  $N\tau = T$ ,  $T > 0$  fixed, and  $1 \leq n \leq N$ , under the following hypotheses.

(H1)  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a locally Lipschitzian function.

(H2) There exist positive constants  $\sigma, c_1, c_2$ , and  $\alpha$  such that  $\alpha < 4/(d-2)$  and for all  $\xi \in \mathbb{R}$ ,

$$\sigma \leq f(\xi) \leq c_1 |\xi|^{\alpha+1} + c_2. \tag{2.1}$$

In the sequel, we will denote the norms in the spaces  $L^\infty(\Omega)$ ,  $L^k(\Omega)$  by  $|\cdot|_{L^\infty(\Omega)}$  and  $|\cdot|_k$ , respectively,  $(\cdot, \cdot)$  will denote the associated inner product in  $L^2(\Omega)$  or the duality product between  $H_0^1(\Omega)$  and its dual  $H^{-1}(\Omega)$ .

**THEOREM 2.1.** *Let (H1)-(H2) be satisfied. Then, for each  $n$ , there exists a unique solution  $U^n$  of (1.3) in  $H_0^1(\Omega) \cap L^\infty(\Omega)$  provided that  $\tau$  is small enough.*

*Proof.* For simplicity, we write  $U = U^n$ ,  $h(x) = U^{n-1}$ . Then (1.3) becomes

$$\begin{aligned} U - \tau \Delta U &= h(x) + \lambda \frac{f(U)}{(\int_{\Omega} f(U) dx)^2} \quad \text{in } \Omega, \\ U &= 0 \quad \text{on } \partial\Omega. \end{aligned} \tag{2.2}$$

*Existence.* Define the map  $S(\mu, \cdot)$  by  $U = S(\mu, v)$ ,  $\mu \in [0, 1]$  if and only if

$$\begin{aligned} U - \tau \Delta U &= \mu g(x, v) \quad \text{in } \Omega, \\ U &= 0 \quad \text{on } \partial\Omega, \\ U^0 &= \mu u_0, \end{aligned} \tag{2.3}$$

where  $g(x, v) = h(x) + \lambda(f(v)/(\int_{\Omega} f(v)dx)^2)$ . For a fixed  $v \in H_0^1(\Omega)$ , (2.3) has a unique solution  $U \in H_0^1(\Omega)$ . Then, for each  $\mu \in [0, 1]$ , the operator  $S(\mu, \cdot)$  is well defined. Moreover,  $S(\mu, \cdot)$  is compact from  $H_0^1(\Omega)$  into it self. Indeed, using (H2), we have the estimate

$$|U|_2^2 + \tau |\nabla U|_2^2 \leq c_3. \quad (2.4)$$

We can easily see that  $\mu \rightarrow S(\mu, v)$  is continuous and that  $S(0, v) = U$ , for any  $v$ , if and only if  $U = 0$ . From Leray-Schauder fixed point theorem, there exists therefore a fixed point  $U$  of  $S(\mu, \cdot)$ .  $\square$

Now, we derive an a priori estimate.

LEMMA 2.2. *If  $u_0 \in L^\infty(\Omega)$ , then for all  $n \in \{1, \dots, N\}$ ,  $U^n \in L^\infty(\Omega)$ .*

*Proof.* The proof is similar to the one used by De Th  lin in [10] concerning a very different problem and we will give here only a sketch. Suppose that  $d \geq 2$  and define

$$\delta = \begin{cases} \frac{2d}{d-2} & \text{if } 2 < d, \\ 2(\alpha+2) & \text{if } d = 2. \end{cases} \quad (2.5)$$

For each  $k \in \mathbb{N}^*$ , we consider the number

$$q_k = \left\{ \left( \frac{\delta}{2} \right)^{k-1} (\delta - \gamma) - (2 - \gamma) \right\} \frac{\delta}{\delta - 2}, \quad k \geq 2, \quad (2.6)$$

$$q_1 = \delta,$$

we have

$$q_{k+1} = (q_k + 2 - \gamma) \frac{\delta}{2} \quad \text{with } \gamma = \alpha + 2, \quad \forall k \in \mathbb{N}^*. \quad (2.7)$$

$\square$

LEMMA 2.3. *For all  $k \in \mathbb{N}^*$ ,  $U^n \in L^{q_k}(\Omega)$ , and moreover*

$$|U^n|_\infty = \overline{\lim} |U^n|_{q_k} < +\infty. \quad (2.8)$$

*Proof.* We prove by recurrence that  $U \in L^{q_k}$ . The property is true for  $k = 1$ , since  $H_0^1(\Omega) \subset L^\delta(\Omega)$ . We show now that  $U \in L^{q_{k+1}}$ . Let  $m \in \mathbb{N}$ ,  $1 \leq m \leq k$ . Multiplying (2.2) by  $|U|^{q_m - \gamma} U$ , using (H2), and Young's inequality, we get

$$(q_m - \gamma + 1) \int_{\Omega} |\nabla U|^2 |U|^{q_m - \gamma} dx \leq c_4 |U|_{q_m}^{q_m} + c_5. \quad (2.9)$$

On the other hand, we have

$$|U|_{q_{m+1}}^{q_{m+1} + 2 - \gamma} \leq c_6 \left( 1 + \frac{q_m - \gamma}{2} \right)^2 \int_{\Omega} |\nabla U|^2 |U|^{q_m - \gamma} dx. \quad (2.10)$$

Therefore, we obtain

$$|U|_{q_{m+1}}^{q_m+2-\gamma} \leq (c_7 + c_8 |U|_{q_m}^{q_m}) (q_m + 2 - \gamma). \quad (2.11)$$

Thus,

$$(|U|_{q_{k+1}}^{q_{k+1}})^{2/\delta} \leq (c_7 + c_8 |U|_{q_k}^{q_k}) (q_k + 2 - \gamma). \quad (2.12)$$

The rest of the proof follows the same lines as in [10, pages 383-384].

*Uniqueness.* Consider  $U$  and  $V$  two different solutions of (2.2) and define  $w = U - V$ . Then, we have

$$\begin{aligned} w - \tau \Delta w &= \frac{\lambda \tau}{(\int_{\Omega} f(U) dx)^2} (f(U) - f(V)) \\ &+ \lambda \tau \frac{(\int_{\Omega} f(U) - f(V) dx) (\int_{\Omega} f(V) + f(U) dx)}{(\int_{\Omega} f(U) dx)^2 (\int_{\Omega} f(V) dx)^2} f(V). \end{aligned} \quad (2.13)$$

Multiplying (2.13) by  $w$ , integrating on  $\Omega$ , and using the  $L^\infty$ -estimate obtained in Lemma 2.2, we get

$$|w|_2^2 + \tau |\nabla w|_2^2 \leq c_9 \tau |w|_2^2. \quad (2.14)$$

Therefore,  $w = 0$  if  $\tau \leq 1/c_9$ . □

We address now the question of stability.

### 3. Stability

**THEOREM 3.1.** *Assume (H1)-(H2) hold. Then, there exists  $c(T, u_0) > 0$  depending on data but not on  $N$  such that for any  $n \in \{1, \dots, N\}$ ,*

- (a)  $|U^n|_{L^\infty(\Omega)} \leq c(T, u_0)$ ;
- (b)  $|U^n|_2^2 + \tau \sum_{k=1}^n |\nabla U^k|_2^2 \leq c(T, u_0)$ ;
- (c)  $\sum_{k=1}^n |U^k - U^{k-1}|_2^2 \leq c(T, u_0)$ .

*Proof.* (i) Multiplying (1.3) by  $|U^k|^m U^k$  for some integer  $m \geq 1$ , using Lemma 2.2, and Hölder's inequality, we obtain after simplification

$$|U^k|_{m+2} \leq |U^{k-1}|_{m+2} + c_{10} \tau. \quad (3.1)$$

By induction and taking the limit in the resulting inequality as  $m \rightarrow +\infty$ , we get

$$|U^k|_{L^\infty(\Omega)} \leq c(T, u_0). \quad (3.2)$$

(ii) Multiplying the first equation of (1.3) by  $U^k$  and using the hypotheses on  $f$ , one easily has

$$(U^k - U^{k-1}, U^k) + \tau |\nabla U^k|_2^2 \leq c_{11} \tau |U^k|_1. \quad (3.3)$$

Using the elementary identity  $2a(a-b) = a^2 - b^2 + (a-b)^2$  and summing from  $k = 1$  to  $n$ , we obtain

$$|U^n|_2^2 + \sum_{k=1}^n |U^k - U^{k-1}|_2^2 + \tau \sum_{k=1}^n |\nabla U^k|_2^2 \leq |u_0|_2^2 + \tau c_{12} \sum_{k=1}^n |U^k|_1. \quad (3.4)$$

Then, the inequalities (b)-(c) hold by using the uniform bound of  $U^n$  in  $L^\infty$  which is established in part (a).  $\square$

#### 4. Error estimates for solutions

We will adopt the following notations concerning the time discretization for problem (1.1). We denote the time step  $\tau = T/N$ ,  $t^n = n\tau$ , and  $I_n = (t^n, t^{n+1})$  for  $n = 1, \dots, N$ . If  $z$  is a continuous function (resp., summable), defined in  $(0, T)$  with values in  $H^{-1}(\Omega)$  or  $L^2(\Omega)$  or  $H_0^1(\Omega)$ , we define  $z^n = z(t^n, \cdot)$ ,  $\bar{z}^n = (1/\tau) \int_{I_n} z(t, \cdot) dt$ ,  $\bar{z}^0 = z^0 = z(0, \cdot)$ ; the error  $e_n = u(t) - U^n$  for all  $t \in I_n$  and the local errors  $e_u^n$  and  $e^n$  defined by  $e_u^n = \bar{u}^n(t) - U^n$ ,  $e^n = u^n - U^n$ .

We have the following theorem.

**THEOREM 4.1.** *Let (H1)-(H2) hold. Then, the following error bounds are satisfied:*

- (1)  $\|e_n\|_{L^\infty(0, T, H^{-1}(\Omega))}^2 + \int_0^T |e_n|^2 dt \leq c_{13} \tau$ ,
- (2)  $\|e^m\|_{H^{-1}(\Omega)} \leq c_{14} \tau^{1/2}$ ,
- (3)  $|\nabla \int_0^T e_n(t) dt|_2 \leq c_{15} \tau^{1/4}$ .

*Proof.* For the proof, we consider the following variational formulation of discrete problem (1.3):

$$(U^n - U^{n-1}, \varphi) + \tau(\nabla U^n, \nabla \varphi) = \frac{\lambda \tau}{(\int_\Omega f(U^n) dx)^2} (f(U^n), \varphi), \quad \forall \varphi \in H_0^1(\Omega). \quad (4.1)$$

Integrating the continuous problem (1.1) over  $I_n$ , we get

$$(u^n - u^{n-1}, \varphi) + \tau(\nabla \bar{u}^n, \nabla \varphi) = \lambda \tau \frac{(\overline{f(u^n)}, \varphi)}{(\int_\Omega f(u^n) dx)^2}, \quad \forall \varphi \in H_0^1(\Omega). \quad (4.2)$$

Subtracting (4.2) from (4.1) and adding from  $n = 1$  to  $m$  with  $m \leq N$ , we obtain

$$\begin{aligned} & \sum_{n=1}^m (e^n - e^{n-1}, \varphi) + \tau \sum_{n=1}^m (\nabla e_u^n, \nabla \varphi) \\ & \leq c_{16} \tau \left| \sum_{n=1}^m (\overline{f(u)}^n - f(U^n), \varphi) \right| + c_{17} \tau \left| \sum_{n=1}^m (f(U^n), \varphi) \right|. \end{aligned} \quad (4.3)$$

Let  $(-\Delta)^{-1}$  be the green operator satisfying

$$(\nabla(-\Delta)^{-1}\psi, \nabla\varphi) = (\psi, \varphi)_{H^{-1}(\Omega), H_0^1(\Omega)} \quad (4.4)$$

for all  $\psi \in H^{-1}(\Omega)$ ,  $\varphi \in H_0^1(\Omega)$ . Choosing  $\varphi = (-\Delta)^{-1}(e^n)$  as test function, we then obtain

$$I_1 + I_2 \leq I_3 + I_4, \quad (4.5)$$

where

$$\begin{aligned} I_1 &= \sum_{n=1}^m (e^n - e^{n-1}, (-\Delta)^{-1}(e^n)), \\ I_2 &= \tau \sum_{n=1}^m (e_u^n, e^n), \\ I_3 &\leq c_{16}\tau \left| \sum_{n=1}^m (\overline{f(u)}^n - f(U^n), (-\Delta)^{-1}(e^n)) \right|, \\ I_4 &= c_{17}\tau \left| \sum_{n=1}^m (f(U^n), (-\Delta)^{-1}(e^n)) \right|. \end{aligned} \quad (4.6)$$

With the aid of the elementary identity  $2a(a-b) = a^2 - b^2 + (a-b)^2$  and the property of  $(-\Delta)^{-1}$ ,  $I_1$  reduces after straightforward calculations to

$$I_1 = \frac{1}{2} \|e^m\|_{H^{-1}(\Omega)}^2 + \frac{1}{2} \sum_{n=1}^m \|e^n - e^{n-1}\|_{H^{-1}(\Omega)}^2. \quad (4.7)$$

On the other hand,

$$\begin{aligned} I_2 &= \tau \sum_{n=1}^m (e_u^n, e^n) \\ &= \sum_{n=1}^m \int_{I_n} (u(t) - U^n, u(t) - U^n) dt + \sum_{n=1}^m \int_{I_n} (u(t) - U^n, u^n - u(t)) dt \\ &= I_{21} + I_{22}, \end{aligned} \quad (4.8)$$

where

$$\begin{aligned} I_{21} &= \sum_{n=1}^m \int_{I_n} (u(t) - U^n, u(t) - U^n) dt = \sum_{n=1}^m \int_{I_n} |e_n|^2 dt, \\ I_{22} &= \sum_{n=1}^m \int_{I_n} (u(t), u^n - u(t)) dt - \sum_{n=1}^m \int_{I_n} (U^n, u^n - u(t)) dt \\ &= I_{22}^1 + I_{22}^2. \end{aligned} \quad (4.9)$$

We now estimate  $I_{22}^1$ . Using the boundedness of  $\partial u/\partial s$  (see [6]), we have

$$\begin{aligned}
 |I_{22}^1| &= \left| \sum_{n=1}^m \int_{I_n} \left( u(t), \int_t^{t^n} \frac{\partial u}{\partial s} ds \right) dt \right| \\
 &\leq \sum_{n=1}^m \int_{I_n} \left( \int_t^{t^n} \left\| \frac{\partial u}{\partial s} \right\|_{H^{-1}(\Omega)} ds \right) \|u(t)\|_{H_0^1(\Omega)} dt \\
 &\leq \tau \left\| \frac{\partial u}{\partial s} \right\|_{L^2(0, t^m, H^{-1}(\Omega))} \|u\|_{L^2(0, t^m, H_0^1(\Omega))} \\
 &\leq c_{18} \tau.
 \end{aligned} \tag{4.10}$$

In the same manner, we have

$$\begin{aligned}
 |I_{22}^2| &\leq \tau \left\| \frac{\partial u}{\partial s} \right\|_{L^2(0, t^m, H^{-1}(\Omega))} \left( \tau \sum_{n=1}^m \|U^n\|_{H_0^1(\Omega)}^2 \right)^{1/2} \\
 &\leq c_{18} \tau.
 \end{aligned} \tag{4.11}$$

Next, we estimate the first term on the right-hand side of (4.5) by using Hölder's and Young's inequalities and (H1),

$$\begin{aligned}
 |I_3| &\leq \left| \sum_{n=1}^m \left( \int_{I_n} [f(u) - f(U^n)] dt, (-\Delta)^{-1}(e^n) \right) \right| \\
 &\leq c_{20} \tau^{1/2} \sum_{n=1}^m \left( \int_{I_n} |f(u) - f(U^n)|_2^2 dt \right)^{1/2} \|e^n\|_{H^{-1}(\Omega)} \\
 &\leq \eta \sum_{n=1}^m \left( \int_{I_n} |f(u) - f(U^n)|_2^2 dt \right) + \frac{c_{21}}{\eta} \tau \sum_{n=1}^m \|e^n\|_{H^{-1}(\Omega)}^2 \\
 &\leq c_{22} \eta \sum_{n=1}^m \left( \int_{I_n} |e_n|_2^2 dt \right) + \frac{c_{21}}{\eta} \tau \sum_{n=1}^m \|e^n\|_{H^{-1}(\Omega)}^2.
 \end{aligned} \tag{4.12}$$

Moreover, we have

$$|I_4| \leq c_{23} \tau + c_{24} \tau \sum_{n=1}^m \|e^n\|_{H^{-1}(\Omega)}^2. \tag{4.13}$$

Choosing suitable  $\eta$ , we conclude that

$$\begin{aligned}
 &\|e^m\|_{H^{-1}(\Omega)}^2 + \sum_{n=1}^m \|e^n - e^{n-1}\|_{H^{-1}(\Omega)}^2 + \sum_{n=1}^m \int_{I_n} |e_n|_2^2 dt \\
 &\leq c_{25} \tau + c_{26} \tau \sum_{n=1}^m \|e^n\|_{H^{-1}(\Omega)}^2.
 \end{aligned} \tag{4.14}$$

On the other hand, setting  $y^m = \sum_{n=1}^m \|e^n\|_{H^{-1}(\Omega)}^2$ , then from (4.14), we get

$$y^m - y^{m-1} \leq c_{25} \tau + c_{26} \tau y^m. \tag{4.15}$$

By applying the discrete Gronwall inequality, we deduce that  $y^m \leq c(T)$ . Therefore, we have

$$\|e^m\|_{H^{-1}(\Omega)} \leq c_{27}\tau^{1/2}. \quad (4.16)$$

On the other hand, we have

$$\sup_{t \in (0, t_m)} \|e_n(t)\|_{H^{-1}(\Omega)} - c_{27}\tau^{1/2} \leq \max_{1 \leq n \leq m} \|e_n(t^n)\|_{H^{-1}(\Omega)} = \max_{1 \leq n \leq m} \|e^n\|_{H^{-1}(\Omega)}. \quad (4.17)$$

Thus, we get

$$\|e_n\|_{L^\infty(0, T, H^{-1}(\Omega))} - c_{27}\tau^{1/2} \leq \max_{1 \leq n \leq m} \|e^n\|_{H^{-1}(\Omega)}. \quad (4.18)$$

From the last inequality, we obtain

$$\begin{aligned} \|e_n\|_{L^\infty(0, T, H^{-1}(\Omega))}^2 + \int_0^T \|e_n\|_2^2 dt &\leq c_{29}\tau, \\ \sum_{n=1}^m \|e^n - e^{n-1}\|_{H^{-1}(\Omega)}^2 &\leq c_{29}\tau. \end{aligned} \quad (4.19)$$

Choosing  $\varphi = \tau \sum_{n=1}^m (\bar{u}^n - U^n)$  in (4.3), we get

$$\begin{aligned} &\tau \int_{\Omega} (u^m - U^m) \left( \sum_{n=1}^m (\bar{u}^n - U^n) dx \right) + \tau^2 \left| \sum_{n=1}^m \nabla (\bar{u}^n - U^n) \right|_2^2 \\ &\leq c_{30}\tau^2 \left| \int_{\Omega} \sum_{n=1}^m (\overline{f(u)}^n - f(U^n)) \left( \sum_{n=1}^m (\bar{u}^n - U^n) \right) dx \right| \\ &\quad + c_{31}\tau^2 \left| \sum_{n=1}^m \left( f(U^n), \sum_{n=1}^m (\bar{u}^n - U^n) \right) \right|. \end{aligned} \quad (4.20)$$

Thus,

$$\begin{aligned} \tau^2 \left| \sum_{n=1}^m \nabla (\bar{u}^n - U^n) \right|_2^2 &= \left| \nabla \int_0^{t^m} e_n dt \right|_2^2 \leq \tau \left| \int_{\Omega} (u^m - U^m) \left( \sum_{n=1}^m (\bar{u}^n - U^n) dx \right) \right| \\ &\quad + c_{30}\tau^2 \left| \int_{\Omega} \sum_{n=1}^m (\overline{f(u)}^n - f(U^n)) \left( \sum_{n=1}^m (\bar{u}^n - U^n) \right) dx \right| \\ &\quad + c_{31}\tau^2 \left| \sum_{n=1}^m \left( f(U^n), \sum_{n=1}^m (\bar{u}^n - U^n) \right) \right| \\ &\leq I + II + III. \end{aligned} \quad (4.21)$$



Clearly,

$$\begin{aligned} I &\leq \|e^m\|_{H^{-1}(\Omega)} \left( \sum_{n=1}^m \int_{I_n} \|u(t)\|_{H_0^1(\Omega)} dt + \tau \sum_{n=1}^m \|U^n\|_{H_0^1(\Omega)} \right) \\ &\leq c_{32} \|e^m\|_{H^{-1}(\Omega)} \\ &\leq c_{33} \tau^{1/2}. \end{aligned} \quad (4.22)$$

We also get

$$\begin{aligned} II &\leq \left( \int_{\Omega} \left( \sum_{n=1}^m \int_{I_n} (f(u) - f(U^n)) dt \right)^2 dx \right)^{1/2} \times \left( \int_{\Omega} \left( \sum_{n=1}^m \int_{I_n} (u(t) - U^n) dt \right)^2 dx \right)^{1/2} \\ &\leq T^2 \left( \sum_{n=1}^m \int_{I_n} |f(u) - f(U^n)|_2^2 dt \right)^{1/2} \times \left( \sum_{n=1}^m \int_{I_n} |u(t) - U^n|_2^2 dt \right)^{1/2} \\ &\leq T^2 \left( \sum_{n=1}^m \int_{I_n} |f(u) - f(U^n)|_2^2 dt \right)^{1/2} \times \left( 2\|u\|_{L^2(0,T,H_0^1(\Omega))}^2 + 2\tau \sum_{n=1}^m |U^n|_2^2 \right)^{1/2} \\ &\leq c_{34} \tau^{1/2}. \end{aligned} \quad (4.23)$$

The last inequality follows by using simultaneously the  $L^\infty$ -estimate of  $u(t)$  (see [6]),  $U^n$ , and the error bound given in (a). Arguing exactly as in the previous estimate, we get

$$III \leq T^2 \left( \sum_{n=1}^m \int_{I_n} |f(U^n)|_2^2 dt \right)^{1/2} \times \left( 2\|u\|_{L^2(0,T,H_0^1(\Omega))}^2 + 2\tau \sum_{n=1}^m |U^n|_2^2 \right)^{1/2}. \quad (4.24)$$

Using again the hypothesis (H1) and the estimates above, we obtain

$$III \leq c_{35} \tau^{1/2}. \quad (4.25)$$

Finally collecting these results, it follows that

$$\left| \nabla \int_0^T e_n dt \right|_2^2 \leq c_{36} \tau^{1/2}. \quad (4.26)$$

This completes the proof.  $\square$

**COROLLARY 4.2.** *Under hypotheses (H1)-(H2), problem (1.3) generates a continuous semi-group  $S_\tau$  defined by  $S_\tau U^{n-1} = U^n$ .*

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## Special Issue on Intelligent Computational Methods for Financial Engineering

### Call for Papers

As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

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