

PROPERTIES OF SOME $*$ -DENSE-IN-ITSELF SUBSETS

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\mathcal{I} -open sets were introduced and studied by Janković and Hamlett (1990) to generalize the well-known Banach category theorem. Quasi- \mathcal{I} -openness was introduced and studied by Abd El-Monsef et al. (2000). These are $*$ -dense-in-itself sets of the ideal spaces. In this note, properties of these sets are further investigated and characterizations of these sets are given. Also, their relation with \mathcal{I} -dense sets and \mathcal{I} -locally closed sets is discussed. Characterizations of completely codense ideals are given in terms of semi-preopen sets.

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1. Introduction and preliminaries. The subject of ideals in topological spaces has been studied by Kuratowski [12] and Vaidyanathaswamy [20]. An *ideal* \mathcal{I} on a topological space (X, τ) is a collection of subsets of X which satisfies that (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a topological space (X, τ) with an ideal \mathcal{I} on X and if $\wp(X)$ is the set of all subsets of X , a set operator $(\cdot)^* : \wp(X) \rightarrow \wp(X)$, called a *local function* [12] of A with respect to \mathcal{I} and τ , is defined as follows: for $A \subset X$, $A^*(\mathcal{I}, \tau) = \{x \in X \mid U \cap A \notin \mathcal{I} \text{ for every } U \in \tau(x)\}$, where $\tau(x) = \{U \in \tau \mid x \in U\}$. We will make use of the basic facts concerning the local functions [10, Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator $\text{cl}^*(\cdot)$ for a topology $\tau^*(\mathcal{I}, \tau)$, called the $*$ -topology, finer than τ , is defined by $\text{cl}^*(A) = A \cup A^*(\mathcal{I}, \tau)$ [19]. When there is no chance for confusion, we will simply write A^* for $A^*(\mathcal{I}, \tau)$ and τ^* or $\tau^*(\mathcal{I})$ for $\tau^*(\mathcal{I}, \tau)$. If \mathcal{I} is an ideal on X , then (X, τ, \mathcal{I}) is called an ideal space. By a space, we always mean a topological space (X, τ) with no separation properties assumed. If $A \subset X$, $\text{cl}(A)$ and $\text{int}(A)$ will denote the closure and interior of A in (X, τ) , respectively, and $\text{cl}^*(A)$ and $\text{int}^*(A)$ will denote the closure and interior of A in (X, τ^*) , respectively. A subset A of a space (X, τ) is *semiopen* [13] if there exists an open set G such that $G \subset A \subset \text{cl}(G)$ or, equivalently, $A \subset \text{cl}(\text{int}(A))$. The complement of a semiopen set is semiclosed. The smallest semiclosed set containing A is called the *semiclosure* of A and is denoted by $\text{scl}(A)$. Also, $\text{scl}(A) = A \cup \text{int}(\text{cl}(A))$ [4, Theorem 1.5(a)]. The largest semiopen set contained in A is called the *semi-interior* of A and is denoted by $\text{sint}(A)$. A subset A of a space (X, τ) is an α -set [15] if $A \subset \text{int}(\text{cl}(\text{int}(A)))$. The family of all α -sets in (X, τ) is denoted by τ^α . τ^α is a topology on X which is finer than τ . The complement of an α -set is called an α -closed set. The closure and interior of A in (X, τ^α) are denoted by $\text{cl}_\alpha(A)$ and $\text{int}_\alpha(A)$, respectively. If \mathcal{N} is the ideal of all nowhere dense subsets in (X, τ) , then $\tau^*(\mathcal{N}, \tau) = \tau^\alpha$ and $\text{cl}_\alpha(A) = A \cup A^*(\mathcal{N})$ [10]. An open subset A of a space (X, τ) is said to be *regular open*

if $A = \text{int}(\text{cl}(A))$. The complement of a regular open set is *regular closed*. A subset A of a space (X, τ) is said to be *preopen* [14] if $A \subset \text{int}(\text{cl}(A))$. The family of all preopen sets is denoted by $\text{PO}(X, \tau)$ or simply $\text{PO}(X)$. The largest preopen set contained in A is called the *preinterior* of A and is denoted by $\text{pint}(A)$ and $\text{pint}(A) = A \cap \text{int}(\text{cl}(A))$ [4]. A is preopen if and only if there is a regular open set G such that $A \subset G$ and $\text{cl}(A) = \text{cl}(G)$ [7, Proposition 2.1]. A subset A of a space (X, τ) is *semi-preopen* [4] if there exists a preopen set G such that $G \subset A \subset \text{cl}(G)$. The family of all semi-preopen sets in (X, τ) is denoted by $\text{SPO}(X, \tau)$ or simply $\text{SPO}(X)$. The complement of a semi-preopen set is called *semi-preclosed*. The largest semi-preopen set contained in A is called the *semi-preinterior* of A and is denoted by $\text{spint}(A)$. Also, $\text{spint}(A) = A \cap \text{cl}(\text{int}(\text{cl}(A)))$ for every A of X [4]. Given a space (X, τ) and ideals \mathcal{J} and \mathcal{I} on X , the *extension* of \mathcal{J} via \mathcal{I} [11], denoted by $\mathcal{J} * \mathcal{I}$, is the ideal given by $\mathcal{J} * \mathcal{I} = \{A \subset X \mid A^*(\mathcal{J}) \in \mathcal{I}\}$. In particular, $\mathcal{J} * \mathcal{N} = \{A \subset X \mid \text{int}(A^*(\mathcal{J})) = \emptyset\}$ is an ideal containing both \mathcal{J} and \mathcal{N} and $\mathcal{J} * \mathcal{N}$ is usually denoted by $\tilde{\mathcal{J}}$. The following lemmas will be useful in the sequel.

LEMMA 1.1. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$. If $A \subset A^*$, then*

- (a) $A^* = \text{cl}(A) = \text{cl}^*(A)$,
- (b) $A^*(\tilde{\mathcal{J}}) = A^*(\mathcal{N})$.

PROOF. Clearly, for every subset A of X , $\text{cl}^*(A) \subset \text{cl}(A)$. Let $x \notin \text{cl}^*(A)$. Then there exists a τ^* -open set G containing x such that $G \cap A = \emptyset$. There exists $V \in \tau$ and $I \in \mathcal{J}$ such that $x \in V - I \subset G$. $G \cap A = \emptyset \Rightarrow (V - I) \cap A = \emptyset \Rightarrow (V \cap A) - I = \emptyset \Rightarrow ((V \cap A) - I)^* = \emptyset \Rightarrow (V \cap A)^* - I^* = \emptyset \Rightarrow (V \cap A)^* = \emptyset \Rightarrow V \cap A^* = \emptyset \Rightarrow V \cap A = \emptyset$. Since V is an open set containing x , $x \notin \text{cl}(A)$ and so $\text{cl}(A) \subset \text{cl}^*(A)$. Hence $\text{cl}(A) = \text{cl}^*(A)$. Since $A \subset A^* \subset \text{cl}(A)$, $\text{cl}(A) = A^*$. This proves (a).

(b) By [11, Theorem 3.2], $A^*(\tilde{\mathcal{J}}) = \text{cl}(\text{int}(A^*))$ and so by (a), $A^*(\tilde{\mathcal{J}}) = \text{cl}(\text{int}(\text{cl}(A))) = A^*(\mathcal{N})$. \square

LEMMA 1.2. *Let (X, τ) be a space and $A \subset X$. If A is semiopen, then $\text{cl}(A) = \text{cl}_\alpha(A)$ and if A is semiclosed, then $\text{int}(A) = \text{int}_\alpha(A)$ [18, Lemma 2.1].*

LEMMA 1.3. *If (X, τ, \mathcal{J}) is an ideal space, then the following are equivalent.*

- (a) *For every $A \in \tau$, $A \subset A^*$.*
- (b) *For every $A \in \text{SO}(X, \tau)$, $A \subset A^*$.*

PROOF. Since $\tau \subset \text{SO}(X, \tau)$, it is enough to prove that (a) \Rightarrow (b). Suppose $A \in \text{SO}(X, \tau)$. Then there exists an open set H such that $H \subset A \subset \text{cl}(H)$. Since H is open, $H \subset H^*$ and so, by Lemma 1.1, $A \subset \text{cl}(H) = H^* \subset A^*$. Hence $A \subset A^*$. \square

2. Completely codense ideal. An ideal \mathcal{J} on a space (X, τ) is said to be *codense* [6] if $\tau \cap \mathcal{J} = \{\emptyset\}$ or, equivalently, $X = X^*$ [10]. \mathcal{J} is said to be *completely codense* [6] if $\text{PO}(X) \cap \mathcal{J} = \{\emptyset\}$ or, equivalently, $\mathcal{J} \subset \mathcal{N}$ [6, Theorem 4.13]. Every completely codense ideal is codense. The converse implication is not true, since in \mathbb{R} , the set of all real numbers with the usual topology, the ideal \mathcal{C} of all countable subsets is codense but not completely codense [6]. The following theorem characterizes completely codense ideals.

THEOREM 2.1. *Let (X, τ, \mathcal{J}) be an ideal space. Then the following are equivalent.*

- (a) \mathcal{J} is completely codense.
- (b) $\text{SPO}(X) \cap \mathcal{J} = \{\phi\}$.
- (c) $A \subset A^*$ for every $A \in \text{SPO}(X)$.
- (d) $\text{spint}(A) = \phi$ for every $A \in \mathcal{J}$.

PROOF. (a) \Rightarrow (b). Suppose $A \in \text{SPO}(X) \cap \mathcal{J}$. $A \in \mathcal{J} \Rightarrow A \in \mathcal{N}$ and so $\text{int}(\text{cl}(A)) = \phi$. $A \in \text{SPO}(X) \Rightarrow A \subset \text{cl}(\text{int}(\text{cl}(A))) \Rightarrow A = \phi$. Therefore, $\text{SPO}(X) \cap \mathcal{J} = \{\phi\}$.

(b) \Rightarrow (c). Suppose $A \in \text{SPO}(X)$ and $x \notin A^*$. Then there exists an open set G containing x such that $G \cap A \in \mathcal{J}$. Since $A \in \text{SPO}(X)$, $G \cap A \in \text{SPO}(X)$, by [4, Theorem 2.7] and so by hypothesis, $G \cap A = \phi$ which implies that $x \notin A$.

(c) \Rightarrow (d). Let $A \in \mathcal{J}$ such that $\text{spint}(A) \neq \phi$. Then there exists a nonempty semi-preopen set G such that $G \subset A$ and so $G^* \subset A^* = \phi$. Since $G \subset G^*$, $G = \phi$ which is a contradiction. Therefore, $\text{spint}(A) = \phi$.

(d) \Rightarrow (a). Let $A \in \text{PO}(X) \cap \mathcal{J}$. Then $A \in \text{PO}(X) \Rightarrow A \subset \text{int}(\text{cl}(A)) \subset \text{cl}(\text{int}(\text{cl}(A)))$. $A \in \mathcal{J} \Rightarrow \text{spint}(A) = \phi \Rightarrow A \cap \text{cl}(\text{int}(\text{cl}(A))) = \phi \Rightarrow A = \phi$. \square

COROLLARY 2.2. *Let (X, τ, \mathcal{J}) be an ideal space with a completely codense ideal \mathcal{J} .*

(a) *If $A \in \text{SPO}(X)$, then $A^*(\mathcal{J})$ is regular closed, $A^*(\mathcal{J}) = A^*(\mathcal{N})$, and $\text{cl}(A) = \text{cl}^*(A) = \text{cl}_\alpha(A)$.*

(b) *If A is semi-preclosed, then $\text{int}(A) = \text{int}^*(A) = \text{int}_\alpha(A)$.*

PROOF. (a) If $A \in \text{SPO}(X)$, by Theorem 2.1(c), $A \subset A^* \subset \text{cl}(A)$ and so $A^* = \text{cl}(A)$ which implies that A^* is regular closed, since the closure of a semi-preopen set is regular closed [4, Theorem 2.4]. Therefore, $A^* = \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(\text{cl}(A))) = A^*(\mathcal{N})$. $\text{cl}(A) = \text{cl}^*(A)$ by Theorem 2.1(c) and Lemma 1.1. Also, $\text{cl}^*(A) = A \cup A^*(\mathcal{J}) = A \cup A^*(\mathcal{N}) = \text{cl}_\alpha(A)$. This proves (a).

(b) The proof follows from (a). \square

3. \mathcal{J} -open sets. A subset A of an ideal space (X, τ, \mathcal{J}) is τ^* -closed [10] (resp., *-dense in itself [9], *-perfect [9]) if $A^* \subset A$ (resp., $A \subset A^*$, $A = A^*$). Clearly, A is *-perfect if and only if A is τ^* -closed and *-dense in itself. The following Theorem 3.1 is useful in the sequel.

THEOREM 3.1. *Let (X, τ, \mathcal{J}) be an ideal space and let U and A be subsets of X such that $A \subset U \subset A^*$. Then U is *-dense in itself, and U^* and A^* are *-perfect.*

PROOF. $A \subset U \subset A^*$ implies that $U^* = A^*$ and so U is *-dense in itself. Since $(A^*)^* \subset A^*$, $A \subset A^*$ implies that A^* is *-perfect and so U^* is *-perfect. \square

A subset A of an ideal space (X, τ, \mathcal{J}) is \mathcal{J} -locally closed, [5] if $A = G \cap V$, where G is open and V is *-perfect. Clearly, every *-perfect set is \mathcal{J} -locally closed. The following theorem gives a characterization of \mathcal{J} -locally closed sets.

THEOREM 3.2. *Let (X, τ, \mathcal{J}) be an ideal space. A subset A of X is \mathcal{J} -locally closed if and only if $A = G \cap A^*$ for some open set G .*

PROOF. Suppose A is \mathcal{J} -locally closed. Then $A = G \cap V$ where G is open and V is $*$ -perfect. Now $A^* = (G \cap V)^* \supset G \cap V^* = G \cap V = A$. Also, $A \subset V$ implies that $A^* \subset V^* = V$. Therefore, $G \cap A^* = G \cap (A^* \cap V) = (G \cap V) \cap A^* = A \cap A^* = A$. Conversely, if $A = G \cap A^*$ where G is open, then $A \subset A^*$ and so by [Theorem 3.1](#), A^* is $*$ -perfect and so A is \mathcal{J} -locally closed. \square

The following corollary follows from [10, Theorems 2.1 and 2.2 and Theorem 6.1(d)].

COROLLARY 3.3. *Let (X, τ, \mathcal{J}) be an ideal space.*

- (a) *Every \mathcal{J} -locally closed set is $*$ -dense in itself.*
- (b) *Every open, $*$ -dense-in-itself subset of X is \mathcal{J} -locally closed.*
- (c) *If \mathcal{J} is codense, then every open set is \mathcal{J} -locally closed.*

A subset A of an ideal space (X, τ, \mathcal{J}) is \mathcal{J} -open [11] if $A \subset \text{int}(A^*)$. The family of all \mathcal{J} -open sets is denoted by $\text{IO}(X, \tau, \mathcal{J})$, $\text{IO}(X, \tau)$, or $\text{IO}(X)$. The complement of an \mathcal{J} -open set is said to be \mathcal{J} -closed. The largest \mathcal{J} -open set contained in A is called the \mathcal{J} -interior of A and is denoted by $\text{Int}(A)$ and $\text{Int}(A) = A \cap \text{int}(A^*)$ [11, Theorem 4.1(3)]. The following theorem gives some properties of \mathcal{J} -open sets.

THEOREM 3.4. *If A is an \mathcal{J} -open subset of an ideal space (X, τ, \mathcal{J}) , then*

- (a) *A is $*$ -dense in itself,*
- (b) *$A^* = \text{cl}(A) = \text{cl}^*(A)$ and $\text{cl}(A)$ and A^* are regular closed,*
- (c) *A^* is $*$ -perfect and \mathcal{J} -locally closed,*
- (d) *$\text{int}(A^*)$ is $*$ -dense in itself and \mathcal{J} -locally closed,*
- (e) *$\text{cl}(\text{int}(A^*)) = A^*(\tilde{\mathcal{J}})$ is $*$ -dense in itself,*
- (f) *$A^* = (\text{int}(A^*))^* = (\text{cl}(\text{int}(A^*)))^* = (A^*(\tilde{\mathcal{J}}))^*(\mathcal{J})$,*
- (g) *$(\text{int}(A^*))^*$ and $(\text{cl}(\text{int}(A^*)))^*$ are \mathcal{J} -locally closed,*
- (h) *$\text{int}(A^*)$ is \mathcal{J} -open.*

PROOF. (a) follows from the definition. (b) follows from (a), [Lemma 1.1](#), and the fact that every \mathcal{J} -open set is preopen [1] and the closure of a preopen set is regular closed [7, Proposition 2.1(ii)]. (c) follows from [Theorem 3.1](#) and from the fact that every $*$ -perfect set is \mathcal{J} -locally closed. (d) follows from [Theorem 3.1](#) and [Corollary 3.3\(b\)](#). (e) $\text{cl}(\text{int}(A^*)) = A^*(\tilde{\mathcal{J}})$ by [11, Theorem 3.2] and since $A \subset \text{int}(A^*) \subset \text{cl}(\text{int}(A^*)) \subset A^*$, by [Theorem 3.1](#), $\text{cl}(\text{int}(A^*))$ is $*$ -dense in itself. (f) From the inequality in the proof of (e), we have $A^* = (\text{int}(A^*))^* = (\text{cl}(\text{int}(A^*)))^*$. Each is equal to $(A^*(\tilde{\mathcal{J}}))^*(\mathcal{J})$ by (e). (g) and (h) follow from (c) and (f), respectively. \square

THEOREM 3.5. *Let (X, τ, \mathcal{J}) be an ideal space. If A is \mathcal{J} -open and V is semiopen, then*

- (a) *$V \cap A$ is $*$ -dense in itself,*
- (b) *$(V \cap A)^*$ is $*$ -perfect and \mathcal{J} -locally closed,*
- (c) *$\text{cl}(V) \cap A$ is $*$ -dense in itself,*
- (d) *$(\text{cl}(V) \cap A)^*$ is $*$ -perfect and \mathcal{J} -locally closed.*

PROOF. Since $V \cap A \subset \text{cl}(V) \cap A \subset (V \cap A)^*$ by [1, Theorem 2.10], $V \cap A$ is $*$ -dense in itself and by [Theorem 3.1](#), $\text{cl}(V) \cap A$ is $*$ -dense in itself and so by [Theorem 3.1](#), $(V \cap A)^*$ and $(\text{cl}(V) \cap A)^*$ are $*$ -perfect and so are \mathcal{J} -locally closed. \square

The following theorem shows that (X, τ) and (X, τ^α) have the same collection of \mathcal{J} -open sets.

THEOREM 3.6. *If (X, τ, \mathcal{J}) is an ideal space, then $\text{IO}(X, \tau, \mathcal{J}) = \text{IO}(X, \tau^\alpha, \mathcal{J})$.*

PROOF. $A \in \text{IO}(X, \tau)$ if and only if $A \subset \text{int}(A^*)$ if and only if $A \subset \text{int}_\alpha(A^*)$, by Lemma 1.2 if and only if $A \in \text{IO}(X, \tau^\alpha)$. \square

COROLLARY 3.7. *If (X, τ, \mathcal{J}) is an ideal space where \mathcal{J} is completely codense, then $\text{IO}(X, \tau) = \text{IO}(X, \tau^*) = \text{IO}(X, \tau^\alpha)$.*

PROOF. Follows from Corollary 2.2(b). \square

The following theorem and corollary are generalizations of [1, Theorem 2.6(iii) and Corollary 2.1(ii)], respectively.

THEOREM 3.8. *Let (X, τ, \mathcal{J}) be an ideal space. If $A \in \text{IO}(X)$ and $B \in \tau^\alpha$, then $A \cap B \in \text{IO}(X)$.*

PROOF. $A \in \text{IO}(X, \tau) \Rightarrow A \in \text{IO}(X, \tau^\alpha)$ and so by [1, Theorem 2.6(ii)], $A \cap B \in \text{IO}(X, \tau^\alpha)$ which implies that $A \cap B \in \text{IO}(X, \tau)$. \square

COROLLARY 3.9. *Let (X, τ, \mathcal{J}) be an ideal space. If A is \mathcal{J} -closed and B is α -closed, then $A \cup B$ is \mathcal{J} -closed.*

Every \mathcal{J} -open set is preopen but the converse need not be true [1, Example 2.3]. The following theorem characterizes \mathcal{J} -open sets in terms of preopen sets.

THEOREM 3.10. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$. Then the following are equivalent.*

- (a) A is \mathcal{J} -open.
- (b) $A \subset A^*$ and $\text{scl}(A) = \text{int}(\text{cl}(A))$.
- (c) $A \subset A^*$ and A is preopen.

PROOF. $A \in \text{IO}(X)$ if and only if $A \subset A^*$ and $A \subset \text{int}(A^*)$ if and only if $A \subset A^*$ and $A \subset \text{int}(\text{cl}(A))$, since $\text{cl}(A) = A^*$ if and only if $A \subset A^*$ and $A \cup \text{int}(\text{cl}(A)) = \text{int}(\text{cl}(A))$ if and only if $A \subset A^*$ and $\text{scl}(A) = \text{int}(\text{cl}(A))$. Therefore, (a) and (b) are equivalent. It is clear that (a) and (c) are equivalent. \square

COROLLARY 3.11. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$.*

- (a) *If A is semiclosed and \mathcal{J} -open, then A is regular open.*
- (b) *If A is semiopen and \mathcal{J} -closed, then A is regular closed.*
- (c) *If A is \mathcal{J} -open, then $\text{sint}(\text{scl}(A)) = \text{int}(\text{scl}(A)) = \text{int}(\text{cl}(A))$.*

For subsets of any ideal space (X, τ, \mathcal{J}) , openness and \mathcal{J} -openness are independent concepts [1, Examples 2.1 and 2.2]. The following Theorem 3.12 shows that the two concepts coincide for $*$ -perfect sets. Corollary 3.13 follows from the fact that every τ^* -closed, \mathcal{J} -open set is $*$ -perfect.

THEOREM 3.12. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$.*

- (a) *If A is $*$ -dense in itself, then $\text{lint}(A^*) = \text{int}(A^*)$.*

(b) If A is $*$ -perfect, then $\text{Iint}(A) = \text{int}(A)$ and so, for $*$ -perfect sets, the concepts open and \mathcal{J} -open coincide.

PROOF. Since A is $*$ -dense in itself, A^* is $*$ -perfect, by [Theorem 3.1](#). Now $\text{Iint}(A^*) = A^* \cap \text{int}((A^*)^*) = A^* \cap \text{int}(A^*) = \text{int}(A^*)$. This proves (a). (b) follows from (a). \square

COROLLARY 3.13. Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$. If A is τ^* -closed and \mathcal{J} -open, then A is open.

In [17, Remark 4], it was stated that \mathcal{J} is codense if and only if $\tau \subset \text{IO}(X)$. The following [Theorem 3.14\(a\)](#) follows from the above result. [Theorem 3.14\(b\)](#) follows from [Theorem 3.6](#) and the fact that $\text{SO}(X) \cap \mathcal{J} = \{\phi\}$ if and only if $\tau \cap \mathcal{J} = \{\phi\}$. [Theorem 3.15](#) is a characterization of completely codense ideals.

THEOREM 3.14. Let (X, τ, \mathcal{J}) be an ideal space.

- (a) If $\text{SO}(X) \subset \text{IO}(X)$, then \mathcal{J} is codense.
- (b) \mathcal{J} is codense if and only if $\tau^\alpha \subset \text{IO}(X)$.

THEOREM 3.15. Let (X, τ, \mathcal{J}) be an ideal space. Then \mathcal{J} is completely codense if and only if $\text{PO}(X) = \text{IO}(X)$.

PROOF. Suppose \mathcal{J} is completely codense and $G \in \text{PO}(X)$. Then $G \subset G^*$, by [Theorem 2.1\(c\)](#) and so $\text{cl}(G) = G^*$. $G \in \text{PO}(X)$ implies $G \subset \text{int}(\text{cl}(G)) = \text{int}(G^*)$ and so $G \in \text{IO}(X)$. Therefore, $\text{PO}(X) \subset \text{IO}(X)$. Clearly, $\text{IO}(X) \subset \text{PO}(X)$. Conversely, if $G \in \text{SPO}(X)$, then there exists $V \in \text{PO}(X)$ such that $V \subset G \subset \text{cl}(V)$ and by hypothesis, $V \subset V^*$ and so by [Lemma 1.1](#), $\text{cl}(V) = V^*$. Hence by [Theorem 3.1](#), G is $*$ -dense in itself and so by [Theorem 2.1](#), \mathcal{J} is completely codense. \square

In the following [Theorem 3.16](#), we show that if A is \mathcal{J} -open, then $\text{sint}(A^*)$ is regular closed.

THEOREM 3.16. Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$.

- (a) For every subset A of X , $\text{cl}(\text{Iint}(A)) = \text{cl}(\text{int}(A^*)) = \text{sint}(A^*)$.
- (b) If A is \mathcal{J} -open, then $A^* = \text{cl}(A) = \text{cl}(\text{int}(A^*)) = \text{sint}(A^*)$ and so $\text{sint}(A^*)$ is regular closed.

PROOF. If A is a subset of X , then $\text{sint}(A^*) = A^* \cap \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(A^*))$. To prove the other equality, since $\text{Iint}(A) = A \cap \text{int}(A^*)$, $\text{cl}(\text{Iint}(A)) = \text{cl}(A \cap \text{int}(A^*)) \supset \text{cl}(A) \cap \text{int}(A^*) = \text{int}(A^*)$ and so $\text{cl}(\text{Iint}(A)) \supset \text{cl}(\text{int}(A^*))$. To prove the reverse direction, note that $\text{Iint}(A) \subset \text{int}(A^*)$ and so $\text{cl}(\text{Iint}(A)) \subset \text{cl}(\text{int}(A^*))$. This completes the proof of (a). (b) follows from (a) and [Theorem 3.4\(b\)](#). \square

A subset A of an ideal space (X, τ, \mathcal{J}) is \mathcal{J} -dense [6] if $A^* = X$. Clearly, every \mathcal{J} -dense set is dense but the converse is not true. If G is any proper dense subset of an ideal space (X, τ, \mathcal{J}) where \mathcal{J} is the maximal ideal $\wp(X)$, then G is not \mathcal{J} -dense. In particular, if \mathcal{J} is not codense, then X is not \mathcal{J} -dense and hence no subset of X is \mathcal{J} -dense [6]. Therefore, the existence of an \mathcal{J} -dense set implies that the ideal is codense. The following theorem characterizes \mathcal{J} -open sets in terms of \mathcal{J} -dense sets.

THEOREM 3.17. *Let (X, τ, \mathcal{J}) be an ideal space with a codense ideal \mathcal{J} and $A \subset X$. Then the following are equivalent.*

- (a) *A is \mathcal{J} -open.*
- (b) *There is a regular open set G such that $A \subset G$ and $A^* = G^*$.*
- (c) *$A = G \cap D$ where G is regular open and D is \mathcal{J} -dense.*
- (d) *$A = G \cap D$ where G is open and D is \mathcal{J} -dense.*

PROOF. (a) \Rightarrow (b). That A is \mathcal{J} -open implies $A \subset \text{int}(A^*) \subset A^*$. Let $G = \text{int}(A^*)$. Then $A \subset G$ and $\text{int}(\text{cl}(G)) = \text{int}(\text{cl}(\text{int}(A^*))) = \text{int}(\text{cl}(\text{int}(\text{cl}(A)))) = \text{int}(\text{cl}(A)) = \text{int}(A^*) = G$ and so G is regular open. $G^* = (\text{int}(A^*))^* = A^*$, by Theorem 3.4(f).

(b) \Rightarrow (c). Let G be a regular open set such that $A \subset G$ and $A^* = G^*$. Let $D = A \cup (X - G)$. Then $A = G \cap D$ where G is regular open. Now $D^* = (A \cup (X - G))^* = A^* \cup (X - G)^* = G^* \cup (X - G)^* = (G \cup (X - G))^* = X^* = X$, since \mathcal{J} is codense. Therefore, D is \mathcal{J} -dense which proves (c).

(c) \Rightarrow (d) is clear.

(d) \Rightarrow (a). Suppose $A = G \cap D$ where G is open and D is \mathcal{J} -dense. Now $G = G \cap X = G \cap D^* \subset (G \cap D)^*$ and so $G \subset \text{int}((G \cap D)^*) = \text{int}(A^*)$. Therefore, $A \subset G \subset \text{int}(A^*)$ which implies that A is \mathcal{J} -open. \square

The following theorem is a generalization of [1, Theorem 2.14(ii)].

THEOREM 3.18. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$. If A is \mathcal{J} -closed and α -open, then $A = \text{cl}(A) = \text{int}(\text{cl}(A)) = \text{cl}(\text{int}(A))$ and so A is both regular open and regular closed.*

PROOF. A is \mathcal{J} -closed $\Rightarrow X - A$ is \mathcal{J} -open $\Rightarrow X - A \subset \text{int}(X - A)^* \Rightarrow X - A \subset \text{int}(\text{cl}(X - A)) \Rightarrow X - A \subset X - \text{cl}(\text{int}(A)) \Rightarrow \text{cl}(\text{int}(A)) \subset A$. A is α -open $\Rightarrow A$ is semipreopen and preopen [16] $\Rightarrow \text{cl}(A) = \text{cl}(\text{int}(A))$ and $A \subset \text{int}(\text{cl}(A))$. Therefore, $\text{int}(\text{cl}(A)) \subset \text{cl}(A) = \text{cl}(\text{int}(A)) \subset A \subset \text{int}(\text{cl}(A))$ and so $A = \text{cl}(A) = \text{cl}(\text{int}(A)) = \text{int}(\text{cl}(A))$. \square

4. Quasi- \mathcal{J} -open sets. A subset A of an ideal space (X, τ, \mathcal{J}) is quasi- \mathcal{J} -open [2] if $A \subset \text{cl}(\text{int}(A^*))$. Every \mathcal{J} -open set is quasi- \mathcal{J} -open and every quasi- \mathcal{J} -open set is semipreopen but the converse implications need not be true [2, Examples 1 and 2]. Also, quasi- \mathcal{J} -openness and semipreopenness (resp., preopenness) are independent concepts [2, Examples 1 and 2]. The family of all quasi- \mathcal{J} -open sets is denoted by $Q\mathcal{J}O(X, \tau)$. The following theorem gives some of the properties of quasi- \mathcal{J} -open sets, the proof of which is similar to the proof of Theorem 3.4.

THEOREM 4.1. *Let (X, τ, \mathcal{J}) be an ideal space and A a quasi- \mathcal{J} -open subset of X . Then*

- (a) *A is $*$ -dense in itself,*
- (b) *$A^* = \text{cl}(A) = \text{cl}^*(A)$,*
- (c) *A^* is $*$ -perfect, regular closed, and \mathcal{J} -locally closed,*
- (d) *$\text{cl}(\text{int}(A^*)) = A^*(\tilde{\mathcal{J}})$ is $*$ -dense in itself,*
- (e) *$A^* = (\text{cl}(\text{int}(A^*)))^* = (A^*(\tilde{\mathcal{J}}))^*(\mathcal{J})$,*
- (f) *$(\text{cl}(\text{int}(A^*)))^*$ is $*$ -perfect and \mathcal{J} -locally closed.*

COROLLARY 4.2. *Let (X, τ, \mathcal{J}) be an ideal space. A subset A of X is quasi- \mathcal{J} -open if and only if $A \subset A^*(\tilde{\mathcal{J}})$ [2, Theorem 3].*

THEOREM 4.3. *Let (X, τ, \mathcal{J}) be an ideal space and let U and A be subsets of X such that $A \subset U \subset A^*$. Then U^* is $*$ -perfect, and if A is quasi- \mathcal{J} -open, then U is quasi- \mathcal{J} -open and so $\text{cl}(\text{int}(A^*))$ is quasi- \mathcal{J} -open.*

PROOF. By [Theorem 3.1](#), $U^* = A^*$ and U^* is $*$ -perfect. A is quasi- \mathcal{J} -open $\Rightarrow A \subset \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(U^*))$. Now $U \subset A^* \Rightarrow U \subset (\text{cl}(\text{int}(U^*)))^* \Rightarrow U \subset \text{cl}(\text{cl}(\text{int}(U^*))) = \text{cl}(\text{int}(U^*))$. Therefore, U is quasi- \mathcal{J} -open. Since $A \subset \text{cl}(\text{int}(A^*)) \subset A^*$, $\text{cl}(\text{int}(A^*))$ is quasi- \mathcal{J} -open. \square

Every quasi- \mathcal{J} -open set is semi-preopen but the converse is not true [2]. [2, Proposition 3(iii)] says that every semiopen set which is $*$ -dense in itself is quasi- \mathcal{J} -open. The following [Theorem 4.4](#) is a generalization of this result and shows that for $*$ -dense in itself, the concepts quasi- \mathcal{J} -open and semi-preopen are equivalent. [Theorem 4.5\(a\)](#) gives a characterization of codense ideals and [Theorem 4.5\(b\)](#) gives a characterization of completely codense ideals.

THEOREM 4.4. *Let (X, τ, \mathcal{J}) be an ideal space. If A is semi-preopen and $*$ -dense in itself, then A is quasi- \mathcal{J} -open.*

PROOF. $A \subset A^* \Rightarrow \text{cl}(A) = A^*$, by [Lemma 1.1](#). A is semi-preopen $\Rightarrow A \subset \text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(\text{int}(A^*))$ and so A is quasi- \mathcal{J} -open. \square

THEOREM 4.5. *Let (X, τ, \mathcal{J}) be an ideal space. Then*

- (a) \mathcal{J} is codense if and only if $\text{SO}(X) \subset Q\mathcal{J}O(X)$,
- (b) \mathcal{J} is completely codense if and only if $\text{SPO}(X) = Q\mathcal{J}O(X)$.

PROOF. (a) Suppose \mathcal{J} is codense. Let $G \in \text{SO}(X)$. By [10, Theorem 6.1] and [Lemma 1.3](#), G is $*$ -dense in itself and so by [2, Proposition 3(iii)], $G \in Q\mathcal{J}O(X)$. Conversely, suppose that $\text{SO}(X) \subset Q\mathcal{J}O(X)$. If $G \in \text{SO}(X)$, then $G \in Q\mathcal{J}O(X)$ and so $G \subset G^*$. Therefore, \mathcal{J} is codense by [10, Theorem 6.1] and [Lemma 1.3](#).

(b) Suppose \mathcal{J} is completely codense and $G \in \text{SPO}(X)$. Then $G \subset G^*$, by [Theorem 2.1\(c\)](#) and so $\text{cl}(G) = G^*$. $G \in \text{SPO}(X) \Rightarrow G \subset \text{cl}(\text{int}(\text{cl}(G))) = \text{cl}(\text{int}(G^*))$ and so $G \in Q\mathcal{J}O(X)$. Therefore, $\text{SPO}(X) \subset Q\mathcal{J}O(X)$. Clearly, $Q\mathcal{J}O(X) \subset \text{SPO}(X)$. Conversely, if $G \in \text{SPO}(X)$, then $G \in Q\mathcal{J}O(X)$, by hypothesis, and so $G \subset G^*$, and so by [Theorem 2.1\(c\)](#), \mathcal{J} is completely codense. \square

In [2], it was established that the intersection of a quasi- \mathcal{J} -open set with an α -set is semi-preopen. The following theorem is a generalization of the above result.

THEOREM 4.6. *Let (X, τ, \mathcal{J}) be an ideal space. Then (a) $Q\mathcal{J}O(X, \tau) = Q\mathcal{J}O(X, \tau^\alpha)$ and (b) $A \in Q\mathcal{J}O(X, \tau)$ and $B \in \tau^\alpha$ implies $A \cap B \in Q\mathcal{J}O(X, \tau)$.*

PROOF. $A \in Q\mathcal{J}O(X, \tau)$ if and only if $A \subset \text{cl}(\text{int}(A^*))$ if and only if $A \subset \text{cl}_\alpha(\text{int}_\alpha(A^*))$ [3] if and only if $A \in Q\mathcal{J}O(X, \tau^\alpha)$ which proves (a). $A \in Q\mathcal{J}O(X, \tau)$ and $B \in \tau^\alpha \Rightarrow A \in Q\mathcal{J}O(X, \tau^\alpha)$ and $B \in \tau^\alpha \Rightarrow A \cap B \in Q\mathcal{J}O(X, \tau^\alpha)$; by [2, Proposition 2] implies $A \cap B \in Q\mathcal{J}O(X, \tau)$. \square

[2, Lemma 2] states that $W^*(\mathcal{N}) \subset W$ for every subset W of X in the ideal space (X, τ, \mathcal{N}) . That is, every subset of X is τ^* -closed and so τ^* is the discrete topology. This is not always the case. For example, if we consider \mathbb{R} with the usual topology τ and the ideal \mathcal{N} of nowhere dense subsets of \mathbb{R} , then $Q^* = \mathbb{R}$ and so Q is not τ^* -closed. Therefore, [2, Proposition 4] is no longer valid. Also, it was established that every τ^* -closed, quasi- \mathcal{J} -open set is semiopen [2, Proposition 3(iii)]. The following **Theorem 4.7(a)** is a generalization of the above result and also shows that the condition *preclosed* is not necessary in [2, Proposition 5(i)], and **Theorem 4.7(b)** shows that [2, Proposition 3(iii)] is also true if we replace the condition τ^* -closed by semiclosed.

THEOREM 4.7. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$.*

(a) *If A is τ^* -closed and quasi- \mathcal{J} -open, then A is regular closed.*

(b) *If A is semiclosed and quasi- \mathcal{J} -open, then A is semiopen and $A^* = A^*(\mathcal{N})$.*

PROOF. (a) That A is τ^* -closed and quasi- \mathcal{J} -open implies $A = A^*$. Also, $A \in QJO(X) \Rightarrow A \subset \text{cl}(\text{int}(A^*)) \Rightarrow \text{int}(A^*) \subset A^* \subset \text{cl}(\text{int}(A^*)) \Rightarrow \text{cl}(\text{int}(A^*)) \subset A^* \subset \text{cl}(\text{int}(A^*))$. Therefore, $A = A^* = \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(A))$ and so A and A^* are regular closed. (b) A is semiclosed $\Rightarrow \text{int}(A) = \text{int}(\text{cl}(A))$ by [8, Proposition 1]. That A is quasi- \mathcal{J} -open implies $A \subset \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(\text{int}(A))$ and so A is semiopen. By **Theorem 4.1(b)**, $\text{cl}(A) = A^*$. Since $\text{int}(\text{cl}(A)) \subset A \subset \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(\text{cl}(A)))$, $\text{cl}(\text{int}(\text{cl}(A))) \subset \text{cl}(A) \subset \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(\text{cl}(A)))$ and so $A^* = \text{cl}(A) = \text{cl}(\text{int}(A^*)) = A^*(\mathcal{N})$. \square

The following theorem gives a characterization of quasi- \mathcal{J} -open sets.

THEOREM 4.8. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$. A is quasi- \mathcal{J} -open if and only if $A \subset A^*$ and $\text{cl}_\alpha(A) = \text{cl}(\text{int}(A^*))$.*

PROOF. Suppose $A \in QJO(X)$. Then $A \subset A^*$ and $\text{cl}(A) = A^*$. Also $A \subset \text{cl}(\text{int}(A^*)) \Rightarrow A \subset \text{cl}(\text{int}(\text{cl}(A))) \Rightarrow A \cup \text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(\text{int}(\text{cl}(A))) \Rightarrow \text{cl}_\alpha(A) = \text{cl}(\text{int}(A^*))$, since $\text{cl}_\alpha(A) = A \cup \text{cl}(\text{int}(\text{cl}(A)))$ [3]. Conversely, suppose the conditions hold. Then $\text{cl}_\alpha(A) = \text{cl}(\text{int}(\text{cl}(A)))$ and so $A \subset \text{cl}(\text{int}(\text{cl}(A))) = \text{cl}(\text{int}(A^*))$. Therefore, A is quasi- \mathcal{J} -open. \square

The quasi- \mathcal{J} -interior of a subset A in an ideal space (X, τ, \mathcal{J}) is the largest quasi- \mathcal{J} -open set contained in A and is denoted by $\text{qlint}(A)$. The following theorem deals with the properties of the quasi- \mathcal{J} -interior of subsets of ideal spaces. In [11], it was established that $\text{lint}(A) = \emptyset$ if and only if $A \in \tilde{\mathcal{J}}$. **Theorem 4.9(c)** is a partial generalization of this result.

THEOREM 4.9. *Let (X, τ, \mathcal{J}) be an ideal space and $A \subset X$. Then*

(a) *$\text{qlint}(A) = A \cap \text{cl}(\text{int}(A^*))$ for every subset A of X ,*

(b) *if A is α -closed, then $\text{qlint}(A) = \text{cl}(\text{int}(A^*))$ and the converse holds if $A \subset A^*$,*

(c) *$\text{qlint}(A) = \emptyset$ if and only if $A \in \tilde{\mathcal{J}}$.*

PROOF. (a) $A \cap \text{cl}(\text{int}(A^*)) \subset \text{cl}(\text{int}(A^*)) = \text{cl}(\text{int}(\text{int}(A^*))) = \text{cl}(\text{int}(A^* \cap (\text{int}A^*))) \subset \text{cl}(\text{int}((A \cap \text{int}(A^*))^*)) \subset \text{cl}(\text{int}((A \cap \text{cl}(\text{int}(A^*)))^*))$. Therefore, $A \cap \text{cl}(\text{int}(A^*))$ is a quasi- \mathcal{J} -open set contained in A and so $A \cap \text{cl}(\text{int}(A^*)) \subset \text{qlint}(A)$. Since $\text{qlint}(A)$ is

quasi- \mathcal{I} -open, $q\text{Int}(A) \subset \text{cl}(\text{int}(q\text{Int}(A))^*) \subset \text{cl}(\text{int}(A^*))$ and so $A \cap q\text{Int}(A) \subset A \cap \text{cl}(\text{int}(A^*))$ which implies that $q\text{Int}(A) \subset A \cap \text{cl}(\text{int}(A^*))$. Hence $q\text{Int}(A) = A \cap \text{cl}(\text{int}(A^*))$.

(b) A is α -closed $\Rightarrow \text{cl}(\text{int}(\text{cl}(A))) \subset A \Rightarrow \text{cl}(\text{int}(A^*)) \subset A \Rightarrow q\text{Int}(A) = \text{cl}(\text{int}(A^*))$. Conversely, if $A \subset A^*$, then $A^* = \text{cl}(A)$. $q\text{Int}(A) = \text{cl}(\text{int}(A^*)) \Rightarrow \text{cl}(\text{int}(A^*)) \subset A$ and so $\text{cl}(\text{int}(\text{cl}(A))) \subset A$ and so A is α -closed.

(c) $q\text{Int}(A) = \phi \Rightarrow A \cap \text{cl}(\text{int}(A^*)) = \phi \Rightarrow A \cap \text{int}(A^*) = \phi \Rightarrow \text{Int}(A) = \phi \Rightarrow A \in \tilde{\mathcal{J}}$. Conversely, $A \in \tilde{\mathcal{J}} \Rightarrow \text{int}(A^*) = \phi \Rightarrow \text{cl}(\text{int}(A^*)) = \phi \Rightarrow A \cap \text{cl}(\text{int}(A^*)) = \phi \Rightarrow q\text{Int}(A) = \phi$. \square

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