

# 1974 CONJECTURE OF ANDREWS ON PARTITIONS

PADMAVATHAMMA and M. R. SALESTINA

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The case  $k = a$  of the 1974 conjecture of Andrews on two partition functions  $A_{\lambda,k,a}(n)$  and  $B_{\lambda,k,a}(n)$  was proved by the first author and Sudha (1993) and the case  $k = a + 1$  was established by the authors (2000). In this paper, we prove that the conjecture is false and give a revised conjecture for a particular case when  $\lambda$  is even.

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**1. Introduction.** Andrews [3] proved a general theorem from which the well-known Rogers-Ramanujan identities, Gordon's theorem [7], the Göllnitz-Gordon identities [6] and their generalization [1], Schur's theorem and its generalization [10] could be deduced. In 1969, Andrews [2] proved the following theorem.

**THEOREM 1.1** [2, Theorem 2]. *If  $\lambda$ ,  $k$ , and  $a$  are positive integers with  $\lambda/2 \leq a \leq k$ ,  $k \geq 2\lambda - 1$ , then for every positive integer,*

$$A_{\lambda,k,a}(n) = B_{\lambda,k,a}(n), \quad (1.1)$$

where  $A_{\lambda,k,a}(n)$  and  $B_{\lambda,k,a}(n)$  are defined as follows.

**DEFINITION 1.2.** For an even integer  $\lambda$ , let  $A_{\lambda,k,a}(n)$  denote the number of partitions of  $n$  into parts such that no part which is not equivalent to  $0 \pmod{\lambda + 1}$  may be repeated and no part is equivalent to  $0, \pm(a - \lambda/2)(\lambda + 1) \pmod{(2k - \lambda + 1)(\lambda + 1)}$ . For an odd integer  $\lambda$ , let  $A_{\lambda,k,a}(n)$  denote the number of partitions of  $n$  into parts such that no part which is not equivalent to  $0 \pmod{((\lambda + 1)/2)}$  may be repeated, no part is equivalent to  $\lambda + 1 \pmod{2\lambda + 2}$ , and no part is equivalent to  $0, \pm(2a - \lambda)((\lambda + 1)/2) \pmod{(2k - \lambda + 1)(\lambda + 1)}$ .

**DEFINITION 1.3.** Let  $B_{\lambda,k,a}(n)$  denote the number of partitions of  $n$  of the form  $b_1 + \dots + b_s$  with  $b_i \geq b_{i+1}$ , no part which is not equivalent to  $0 \pmod{\lambda + 1}$  is repeated,  $b_i - b_{i+k-1} \geq \lambda + 1$  with strict inequality if  $\lambda + 1/b_i, \sum_{i=j}^{\lambda-j+1} f_i \leq a - j$  for  $1 \leq j \leq (\lambda + 1)/2$ , and  $f_1 + \dots + f_{\lambda+1} \leq a - 1$ , where  $f_j$  is the number of appearances of  $j$  in the partition.

Since Schur's theorem [10] is the case  $\lambda = k = a = 2$ , it is not a particular case of Theorem 1.1 as  $k \geq 2\lambda - 1$  is not satisfied. Hence Andrews [2] conjectured that Theorem 1.1 may be still true if  $k \geq \lambda$ . In fact, he gave a proof of this result [4].

In the conclusion of [4], Andrews stated the following two conjectures.

**CONJECTURE 1.4.** For  $\lambda/2 < a \leq k < \lambda$ , let  $n^c = (k + \lambda - a + 1)(k + \lambda - a)/2 + (k - \lambda + 1)(\lambda + 1)$ . Then

$$\begin{aligned} B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) \quad \text{for } 0 \leq n < n^c, \\ B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) + 1 \quad \text{for } n = n^c. \end{aligned} \quad (1.2)$$

**CONJECTURE 1.5.** For all positive integers  $n$ ,  $A_{4,3,3}(n) = B_{4,3,3}^0(n)$ , where  $B_{4,3,3}^0(n)$  denotes the number of partitions of  $n$  enumerated by  $B_{4,3,3}(n)$  with the added restrictions:

$$\begin{aligned} f_{5j+2} + f_{5j+3} &\leq 1 \quad \text{for } j \geq 0, \\ f_{5j+4} + f_{5j+5} &\leq 1 \quad \text{for } j \geq 0, \\ f_{5j-1} + f_{5j} + f_{5j+5} + f_{5j+6} &\leq 3 \quad \text{for } j \geq 1. \end{aligned} \quad (1.3)$$

[Conjecture 1.5](#) is designed to show that when the condition  $k \geq \lambda$  is removed with some additional restrictions on the summands, some partition identities can be obtained in a few cases. In 1994, Andrews et al. [5] proved [Conjecture 1.5](#).

The first author and Sudha [9] have proved the case  $k = a$  of [Conjecture 1.4](#) while the authors in [8] have established the case  $k = a + 1$  of [Conjecture 1.4](#). The objective of the present paper is to prove that [Conjecture 1.4](#) is false if  $n$  exceeds  $(2k - a - \lambda/2 + 1)(\lambda + 1)$  for even  $\lambda$  and  $k \geq a + 2$ . For odd  $\lambda$ , we have verified and checked that [Conjecture 1.4](#) is false when  $\lambda = 11$ ,  $k = 9$ , and  $a = 6$ . We also give the following revised conjecture for a particular case when  $\lambda$  is even.

**REVISED CONJECTURE 1.6.** Let  $\lambda$  be even,  $a - \lambda/2 = 1$ ,  $\theta = k - a$ ,  $\theta(\theta - 1)/2 < [a - \lambda/2](\lambda + 1)$ , and  $0 \leq \theta \leq \lambda/2 - 3$ . Then

$$\begin{aligned} B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) \quad \text{for } n < \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1), \\ B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) + B_{\lambda,k,a}(x), \\ \text{where } n &= \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + x, \quad 0 \leq x \leq \frac{\theta(\theta - 1)}{2}. \end{aligned} \quad (1.4)$$

These results support (i) Andrews' contention that  $k \geq \lambda$  is essential for the truth of [Theorem 1.1](#) and (ii) his belief that [Theorem 1.1](#) was the best possible one, but his conjecture about first counterexamples when  $k \geq \lambda$  is false.

**2. Preliminaries.** Let  $P_{B_{\lambda,k,a}}(n)$  and  $P_{A_{\lambda,k,a}}(n)$  denote the sets of partitions enumerated by  $B_{\lambda,k,a}(n)$  and  $A_{\lambda,k,a}(n)$ , respectively. Let  $P'_A(n)$  (resp.,  $P'_B(n)$ ) denote the set of partitions enumerated by  $A_{\lambda,k,a}(n)$  (resp.,  $B_{\lambda,k,a}(n)$ ) but not by  $B_{\lambda,k,a}(n)$  (resp.,  $A_{\lambda,k,a}(n)$ ).

$\pi \in P'_A(n)$  implies that it violates one of the conditions on  $f$ 's or  $b$ 's. Let  $S_j$  ( $j = 1, 2, \dots, \lambda/2$ ) denote the condition  $\sum_{i=j}^{\lambda-j+1} f_i \leq a - j$ , let  $S$  denote the condition  $\sum_{i=1}^{\lambda+1} f_i \leq a - 1$ , and let  $S^*$  be the condition on  $b$ 's.

Let  $(2k - a - \lambda/2 + 1)(\lambda + 1) \leq n < (2k - a - \lambda/2 + 1)(\lambda + 1) + \theta(\theta - 1)/2$ , where  $\theta(\theta - 1)/2 < (a - \lambda/2)(\lambda + 1)$  and  $\theta = k - a$ . Then

$$P'_B(n) = Q^1 \cup \dots \cup Q^{a-1} \cup R(n), \quad (2.1)$$

where for  $1 \leq i \leq a-1$ ,

$$\begin{aligned} Q^i &= \left\{ \pi \in P'_B(n) : \left(a - \frac{\lambda}{2}\right)(\lambda+1) \text{ appears } i \text{ times} \right\}, \\ R(n) &= \left\{ \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) + \pi : \pi \text{ is a partition of} \right. \\ &\quad \left. n - \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda+1) \text{ into parts with } C \right\}. \end{aligned} \quad (2.2)$$

Here  $C$  stands for “subjected to the conditions in the definition of  $B$ .” Clearly,  $\#R(n) = B_{\lambda,k,a}[n - (2k - a - \lambda/2 + 1)(\lambda + 1)]$ .

From the method explained in [8, 9], it follows that the partitions violating  $S_1, \dots, S_{\lambda/2}$  will be mapped onto  $Q^1 \cup \dots \cup Q^{a-1}$ . If  $a - \lambda/2 = 1$ , then  $S$  reduces to  $S_1$ . As such, any contribution to  $R(n)$  can come only from those partitions of  $P'_A$  which violate  $S^*$  but do not violate any of  $S_1, \dots, S_{\lambda/2}$ . For the counterexample in Section 3, we enumerate separately the partitions counted by  $R(n)$ . If there are no partitions of  $n$  violating only  $S^*$ , then for such  $n$ , we have that  $P'_A(n)$  is the union of the partitions violating  $S_1, \dots, S_{\lambda/2}$  and  $Q^{a-1}$  is the set containing  $a-1$  times  $\lambda+1$ . This set is identified with the first stage of  $S_1$  where all the parts from  $1, \dots, \lambda$  appear.  $Q^{a-2}$  will be the union of the two sets, one containing  $a-2$  times  $\lambda+1$  and the other containing  $a-2$  times  $\lambda+1$  plus a part between 1 and  $\lambda$ . These two sets are, respectively, identified with the first stage of  $S_2$  where all the parts from  $2, \dots, \lambda-1$  appear, and the second stage of  $S_1$  in which all the parts except one part from  $1, \dots, \lambda$  appear and so on.

**3. Counterexample.** Let  $\lambda = 12, k = 11, a = 7, \theta = 4, a - \lambda/2 = 1, \theta(\theta-1)/2 = 6 < (a - \lambda/2)(\lambda+1) = 13$ , and  $n^c = 136$ . In this case,

$$\begin{aligned} S_{\lambda/2} &= S_6 : f_7 + f_6 \leq 1, & S_5 &: f_8 + f_7 + f_6 + f_5 \leq 2, & S_4 &: f_9 + \dots + f_4 \leq 3, \\ S_3 &: f_{10} + \dots + f_3 \leq 4, & S_2 &: f_{11} + \dots + f_2 \leq 5, & S_1 &: f_{12} + \dots + f_1 \leq 6, \\ & & S &: f_{13} + \dots + f_1 \leq 6; \end{aligned} \quad (3.1)$$

$$P'_B(n) = Q^1 \cup \dots \cup Q^6 \cup R(n),$$

where  $Q^i = \{\pi \in P'_B(n) : 13 \text{ appears } i \text{ times}\}$ ,  $1 \leq i \leq 6$ , and  $R(n) = \{130 + \pi : \pi \text{ is a partition of } n - 130 \text{ into parts with } C\}$ . Here  $\#R(n) = B_{12,11,7}(x)$ , where  $x = n - 130$ . We now prove

$$B_{12,11,7}(n) = A_{12,11,7}(n), \quad n < 130, \quad (3.2)$$

$$B_{12,11,7}(n) = A_{12,11,7}(n) + B_{12,11,7}(x), \quad n = 130 + x, \quad 0 \leq x < 6, \quad (3.3)$$

$$B_{12,11,7}(136) = A_{12,11,7}(136) + B_{12,11,7}(6) - 1 = A_{12,11,7}(136) + 3, \quad (3.4)$$

since  $B(6) = 4$  as  $6, 5+1, 4+2$ , and  $3+2+1$  are the only relevant partitions of 6 enumerated by  $B$ .

**Proof of (3.2), (3.3), and (3.4).** Equation (3.2) follows from [8]. We now prove that for  $1 \leq n < 136$ , there are no partitions of  $n$  violating only  $S^*$  and that

$$18 + 17 + 16 + 15 + 14 + 12 + 11 + 10 + 9 + 8 + 6 \quad (3.5)$$

is the only partition of 136 violating only  $S^*$ .

In [8, 9] we have shown that for  $n < 130$ , if a partition violates  $S^*$ , then it violates either  $S$  or  $S_1$ . However, for  $130 \leq n \leq 136$ , we now investigate such partitions.

If a partition violates  $S^*$ , then there exist a partition

$$n = b_1 + \cdots + b_i + \cdots + b_{i+10} + \cdots + b_s \quad (3.6)$$

and an integer  $i$  with  $b_i - b_{i+10} < 13$ . We get the following possibilities.

**CASE 1.** If  $b_{i+10} \geq 13$ , then the number being partitioned is greater than or equal to

$$\begin{aligned} (12 + x_{11}) + \cdots + (12 + x_1) + \cdots, \\ 11(12 + 1), \quad \text{where } x_{11} - x_1 < 13. \end{aligned} \quad (3.7)$$

If (3.7) contains the part 13 more than 6 times, then it violates  $S$ . Let  $x \leq 6$  be the number of 13's and let  $y$  denote the number of terms greater than 13 in (3.7) so that  $x + y = 11$ . Then (3.7) becomes

$$13x + (12 + 2) + \cdots + (12 + 11 - x) = 11(13) + \frac{(11 - x)(11 - x - 1)}{2}. \quad (3.8)$$

Let  $n^c$  denote the  $n$  in the conjecture. If  $k = a + \theta$ , then

$$\begin{aligned} n^c &= \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + \frac{\theta(\theta - 1)}{2} \\ &= k(\lambda + 1) + \left(k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + \frac{(k - a)(k - a - 1)}{2} \\ &< k(\lambda + 1) + \left(k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + \frac{(k - x)(k - x - 1)}{2} \\ &< k(\lambda + 1) + \frac{(k - x)(k - x - 1)}{2} \quad \text{since } k - a - \frac{\lambda}{2} + 1 < 0. \end{aligned} \quad (3.9)$$

In this case, we have that  $n^c < 11(13) + (11 - x)(11 - x - 1)/2$ .

**CASE 2.** Let  $b_{i+10} < 13$  and  $b_i < 13$ . Then (3.6) violates  $S_1$ .

**CASE 3.** Let  $b_{i+10} < 13$  and  $b_i \geq 13$ . Let  $\beta$  denote the number of parts among  $1, 2, \dots, 13$ . If  $\beta \geq 7$ , then (3.6) violates  $S$  or  $S_1$ . Hence,  $1 \leq \beta \leq 6$ . Let  $\alpha$  denote the number of parts 13 so that  $5 \leq \alpha \leq 10$  and  $\alpha + \beta = 11$ . Then the number being partitioned is

$$(12 + x_\alpha) + \cdots + (12 + x_1) + y_1 + \cdots + y_\beta. \quad (3.10)$$

Since  $(12 + x_\alpha) - y_\beta < 13$ , we have  $x_\alpha = y_\beta$ . Now,  $x_1 \geq 2, x_2 \geq 3, \dots, x_\alpha \geq \alpha + 1$ . Thus,  $y_\beta \geq \alpha + 1, \dots, y_1 \geq \alpha + \beta = 11$ . Hence, (3.10) is greater than or equal to

$$(12 + \alpha + 1) + \cdots + (12 + 2) + (\alpha + \beta) + \cdots + (\alpha + 1) \quad (3.11)$$

and equals

$$\frac{13\alpha + (\alpha + \beta)(\alpha + \beta + 1)}{2}. \quad (3.12)$$

Let  $\beta = 1, 2, 3, 4, 5$ . Then (3.10) is, respectively, 196, 183, 170, 157, and 144, all of which are greater than  $136 = n^c$ .

Now let  $\beta = 6$ . Since we have to choose 6 parts from  $1, 2, \dots, 13$  and 5 parts greater than 13 for a partition violating  $S^*$  (and not violating any of  $S, S_1, \dots, S_6$ ), it is clear that the minimum part should be 6. Let  $S_1^* = \{6, 7\}$  and  $S_2^* = \{8, 9, 10, 11, 12, 13\}$ . Since  $f_6 + f_7 \leq 1$ , we can choose either 6 or 7 from  $S_1^*$  and the other five must be from  $S_2^*$ . Also there are 5 parts greater than 13. In this case, the minimum value of  $n$  will be

$$6 + 8 + 9 + 10 + 11 + 12 + 14 + 15 + 16 + 17 + 18 = 136. \quad (3.13)$$

Thus for all  $130 \leq n < 136$ , there are no partitions of  $n$  violating only  $S^*$ . It is easy to see that when  $n = 136$ ,

$$18 + \dots + 14 + 12 + \dots + 8 + 6 \quad (3.14)$$

is the only partition of 136 violating only  $S^*$ . Thus we find

$$P'_A(n) = \{\text{union of the partitions violating } S_1, \dots, S_6\} \quad \text{for } 1 \leq n < 136 \quad (3.15)$$

while

$$P'_A(136) = \{\text{union of the partitions violating } S_1, \dots, S_6\} + 1. \quad (3.16)$$

We now establish a bijection of  $Q^1 \cup \dots \cup Q^6$  onto  $P'_A(n)$  which is explained in Table 3.1. This also proves (3.3) and (3.4). Before writing the table, we observe that for a partition

$$\pi + 13 \times i + \alpha_1 + \dots + \alpha_j, \quad 1 \leq i \leq 6, \quad (3.17)$$

belonging to  $P'_B$ ,  $\pi$  is a partition of  $(n - 13 \times i - \alpha_1 - \dots - \alpha_j)$  into parts greater than 13 with  $C$ , where  $1 \leq \alpha_j < \dots < \alpha_1 \leq 12$ , and for a partition

$$\pi + \beta_1 + \dots + \beta_j \quad (3.18)$$

belonging to  $P'_A$ ,  $\pi$  is a partition of  $(n - \beta_1 - \dots - \beta_j)$  into parts greater than  $\beta_1$  such that 13 is not a part, where  $1 \leq \beta_j < \dots < \beta_1 \leq 12$ .

**REMARK 3.1.** In Table 3.1, some partitions in  $Q^2$  are not covered. They are

$$\begin{aligned} & \{\pi + 13 \times 2 + x_1 + x_2 + 1 : 2 \leq x_2 \leq 11, 3 \leq x_1 \leq 12, (x_1, x_2) \neq (7, 6)\} \\ & \cup \{\pi + 13 \times 2 + 12 + x_1 + x_2 : 3 \leq x_1 \leq 11, 2 \leq x_2 \leq 10, (x_1, x_2) \neq (7, 6)\}. \end{aligned} \quad (3.19)$$

Here we split  $13 \times 2$  into pairs  $(\alpha, \beta)$  and  $(\gamma, \delta)$  in the following order:

$$(7, 6) (8, 5) (9, 4) (10, 3) (11, 2) (12, 1). \quad (3.20)$$

TABLE 3.1

$P'_{B_{12,11,7}}(n)$	$P'_{A_{12,11,7}}(n)$
$Q^6 = \{\pi + 13 \times 6\}$	1st stage of $S_1 = \{\pi + 12 + \dots + 1\}$
$Q^5 = \{\pi + 13 \times 5\}$ $\cup \{\pi + 13 \times 5 + (13 - x_1) : 1 \leq (13 - x_1) \leq 12\}$	1st stage of $S_2 = \{\pi + 11 + \dots + 2\}$ 2nd stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 2 + 1 : 1 \leq x_1 \leq 12\}$
$Q^4 = \{\pi + 13 \times 4 + x : x = 0, 1, 2, 12\}$ $\cup \{\pi + 13 \times 4 + (13 - x_1) : 2 < (13 - x_1) \leq 11\}$ $\cup \{\pi + 13 \times 4 + (13 - x_1) + (13 - x_2) : 1 \leq (13 - x_2) < (13 - x_1) \leq 12\}$	1st stage of $S_3 = \{\pi + 10 + \dots + 3 + x : x = 0, 1, 2, 12\}$ 2nd stage of $S_2 = \{\pi + 11 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 2 : 2 \leq x_1 < 11\}$ 3rd stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_2 + 1) + (x_2 - 1) + \dots + 1 : 1 \leq x_2 < x_1 \leq 12, (x_i, x_j) \neq (7, 6)\}$
$Q^3 = \{\pi + 13 \times 3 + x : x = 0, 1, 2, 11, 12\}$ $\cup \{\pi + 13 \times 3 + (13 - x_1) : 3 \leq (13 - x_1) \leq 10\}$ $\cup \{\pi + 13 \times 3 + (13 - x_1) + (13 - x_2) : 2 \leq (13 - x_2) < (13 - x_1) \leq 11\}$	<b>Note 1.</b> If $(x_i, x_j) = (7, 6)$ , then it will be covered in the 3rd stage of $S_2$ . 1st stage of $S_4 = \{\pi + 9 + \dots + 4 + x : x = 0, 1, 2, 11, 12\}$ 2nd stage of $S_3 = \{\pi + 10 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 3 : 3 \leq x_1 \leq 10\}$ 3rd stage of $S_2 = \{\pi + 11 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_2 + 1) + (x_2 - 1) + \dots + 2 : 2 \leq x_2 < x_1 \leq 11, (x_i, x_j) \neq (7, 6)\}$
$\cup \{\pi + 13 \times 3 + x + y : (x, y) = \text{all possible pairs of } 1, 2, 11, 12 \text{ except } (11, 2)\}$ $\cup \{\pi + 13 \times 3 + (13 - x_1) + \dots + (13 - x_3) : 1 \leq (13 - x_3) < (13 - x_2) < (13 - x_1) \leq 12\}$	<b>Note 2.</b> If $(x_i, x_j) = (7, 6)$ , then it will be covered in the 3rd stage of $S_3$ . 4th stage of $S_4 = \{\pi + 9 + \dots + 4 + x + y : (x, y) = \text{all possible pairs of } 1, 2, 11, 12 \text{ except } (11, 2)\}$ 4th stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_3 + 1) + (x_3 - 1) + \dots + 1 : 1 \leq x_3 < x_2 < x_1 \leq 12, (x_i, x_j) \neq (7, 6)\}$
$Q^2 = \{\pi + 13 \times 2 + x : x = 0, 1, 2, 3, 10, 11, 12\}$ $\cup \{\pi + 13 \times 2 + (13 - x_1) : 4 \leq (13 - x_1) \leq 9\}$ $\cup \{\pi + 13 \times 2 + (13 - x_1) + (13 - x_2) : 3 \leq (13 - x_2) < (13 - x_1) \leq 10\}$	<b>Note 3.</b> If $(x_i, x_j) = (7, 6)$ , then it will be covered in the 4th stage of $S_2$ . 1st stage of $S_5 = \{\pi + 8 + 7 + 6 + 5 + x : x = 0, 1, 2, 3, 10, 11, 12\}$ 2nd stage of $S_4 = \{\pi + 9 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 4 : 4 \leq x_1 \leq 9\}$ 3rd stage of $S_3 = \{\pi + 10 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_2 + 1) + (x_2 - 1) + \dots + 3 : 3 \leq x_2 < x_1 \leq 10, (x_i, x_j) \neq (7, 6)\}$

TABLE 3.1. Continued.

$P'_{B_{12,11,7}}(n)$	$P'_{A_{12,11,7}}(n)$
	<b>Note 4.</b> If $(x_i, x_j) = (7, 6)$ , then it will be covered in the 3rd stage of $S_4$ .
$\cup\{\pi + 13 \times 2 + x + y : (x, y)$ = all possible pairs of 1, 2, 3, 10 11, 12 except (10, 3)\}	4th stage of $S_5 = \{\pi + 8 + 7 + 6 + 5 +$ $x + y : (x, y) =$ all possible pairs of 1, 2, 3, 10, 11, 12 except (10, 3)\}
$\cup\{\pi + 13 \times 2 + (13 - x_1) + \cdots$ $+ (13 - x_3) : 2 \leq (13 - x_3)$ $< (13 - x_2) < (13 - x_1) \leq 11\}$	4th stage of $S_2 = \{\pi + 11 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + (x_3 + 1)$ $+ (x_3 - 1) + \cdots + 2 : 2 \leq x_3$ $< x_2 < x_1 \leq 11\}$ $(x_i, x_j) \neq (7, 6)\}$
	<b>Note 5.</b> If $(x_i, x_j) = (7, 6)$ , then it will be covered in the 4th stage of $S_3$ .
$\cup\{\pi + 13 \times 2 + (13 - x_1) + \cdots$ $+ (13 - x_4) : 1 \leq (13 - x_4)$ $< \cdots < (13 - x_1) \leq 12\}$	5th stage of $S_1 = \{\pi + 12 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + (x_4 + 1)$ $+ (x_4 - 1) + \cdots + 1 : 1 \leq x_4$ $< \cdots < x_1 \leq 12\}$ $(x_i, x_j) \neq (7, 6)\}$
	<b>Note 6.</b> If $(x_i, x_j) = (7, 6)$ , then it will be covered in the 5th stage of $S_2$ .
$Q^1 = \{\pi + 13 + x :$ $x = 0, 1, 2, 3, 4, 9, 10, 11, 12\}$	1st stage of $S_6 = \{\pi + 7 + 6 + x :$ $x = 0, 1, 2, 3, 4, 9, 10, 11, 12\}$
$\cup\{\pi + 13 + (13 - x_1) :$ $5 \leq (13 - x_1) \leq 8\}$	2nd stage of $S_5 = \{\pi + 8 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + 5$ $: 5 \leq x_1 \leq 8\}$
$\cup\{\pi + 13 + (13 - x_1) + (13 - x_2) :$ $4 \leq (13 - x_2) < (13 - x_1) \leq 9\}$	3rd stage of $S_4 = \{\pi + 9 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + (x_2 + 1)$ $+ (x_2 - 1) + \cdots + 4 : 4 \leq x_2$ $< x_1 \leq 9\}$
$\cup\{\pi + 13 + x + y : (x, y)$ = all possible pairs of 1, 2, 3, 4, 9, 10, 11, 12 except (9, 4)\}	4th stage of $S_5 = \{\pi + 7 + 6 + x + y : (x, y)$ = all possible pairs of 1, 2, 3, 4, 9, 10, 11, 12 except (9, 4)\}
$\cup\{\pi + 13 + (13 - x_1) + \cdots$ $+ (13 - x_3) : 3 \leq (13 - x_3)$ $< (13 - x_2) < (13 - x_1) \leq 10\}$	4th stage of $S_3 = \{\pi + 10 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + (x_3 + 1)$ $+ (x_3 - 1) + \cdots + 3 : 3 \leq x_3$ $< x_2 < x_1 \leq 10\}$
$\cup\{\pi + 13 + (13 - x_1) + \cdots + (13 - x_4) :$ $2 \leq (13 - x_4) < \cdots < (13 - x_1)$ $\leq 11\}$	5th stage of $S_2 = \{\pi + 11 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + (x_4 + 1)$ $+ (x_4 - 1) + \cdots + 2 : 2 \leq x_4$ $< \cdots < x_1 \leq 11\}$
$\cup\{\pi + 13 + (13 - x_1) + \cdots + (13 - x_5) :$ $1 \leq (13 - x_5) < \cdots < (13 - x_1)$ $\leq 12\}$	6th stage of $S_1 = \{\pi + 12 + \cdots + (x_1 + 1)$ $+ (x_1 - 1) + \cdots + (x_5 + 1)$ $+ (x_5 - 1) + \cdots + 1 : 1 \leq x_5$ $< \cdots < x_1 \leq 12\}$

We arrange  $\pi + \alpha + \beta + \gamma + \delta + x_1 + x_2 + \gamma$  ( $\gamma = 12$  or  $1$ ) in the decreasing order and associate it to the rearranged partition  $\pi^*$  which belongs to  $P'_A$ .

A similar procedure is adopted for some partitions in  $Q^1$  which are also not covered in Table 3.1. This completes the proof of (3.3) and (3.4).  $\square$

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Padmavathamma: Department of Studies in Mathematics, University of Mysore, Manasagan-  
gotri, Mysore 570 006, India

E-mail address: [padma\\_vathamma@yahoo.com](mailto:padma_vathamma@yahoo.com)

M. R. Salestina: Department of Studies in Mathematics, University of Mysore, Manasagan-  
gotri, Mysore 570 006, India

E-mail address: [salestina@rediffmail.com](mailto:salestina@rediffmail.com)



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