

1974 CONJECTURE OF ANDREWS ON PARTITIONS

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The case $k = a$ of the 1974 conjecture of Andrews on two partition functions $A_{\lambda,k,a}(n)$ and $B_{\lambda,k,a}(n)$ was proved by the first author and Sudha (1993) and the case $k = a + 1$ was established by the authors (2000). In this paper, we prove that the conjecture is false and give a revised conjecture for a particular case when λ is even.

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1. Introduction. Andrews [3] proved a general theorem from which the well-known Rogers-Ramanujan identities, Gordon's theorem [7], the Göllnitz-Gordon identities [6] and their generalization [1], Schur's theorem and its generalization [10] could be deduced. In 1969, Andrews [2] proved the following theorem.

THEOREM 1.1 [2, Theorem 2]. *If λ , k , and a are positive integers with $\lambda/2 \leq a \leq k$, $k \geq 2\lambda - 1$, then for every positive integer,*

$$A_{\lambda,k,a}(n) = B_{\lambda,k,a}(n), \quad (1.1)$$

where $A_{\lambda,k,a}(n)$ and $B_{\lambda,k,a}(n)$ are defined as follows.

DEFINITION 1.2. For an even integer λ , let $A_{\lambda,k,a}(n)$ denote the number of partitions of n into parts such that no part which is not equivalent to $0(\bmod \lambda + 1)$ may be repeated and no part is equivalent to $0, \pm(a - \lambda/2)(\lambda + 1) \bmod [(2k - \lambda + 1)(\lambda + 1)]$. For an odd integer λ , let $A_{\lambda,k,a}(n)$ denote the number of partitions of n into parts such that no part which is not equivalent to $0(\bmod ((\lambda + 1)/2))$ may be repeated, no part is equivalent to $\lambda + 1(\bmod 2\lambda + 2)$, and no part is equivalent to $0, \pm(2a - \lambda)((\lambda + 1)/2) \bmod [(2k - \lambda + 1)(\lambda + 1)]$.

DEFINITION 1.3. Let $B_{\lambda,k,a}(n)$ denote the number of partitions of n of the form $b_1 + \dots + b_s$ with $b_i \geq b_{i+1}$, no part which is not equivalent to $0(\bmod \lambda + 1)$ is repeated, $b_i - b_{i+k-1} \geq \lambda + 1$ with strict inequality if $\lambda + 1/b_i$, $\sum_{i=j}^{\lambda-j+1} f_i \leq a - j$ for $1 \leq j \leq (\lambda + 1)/2$, and $f_1 + \dots + f_{\lambda+1} \leq a - 1$, where f_j is the number of appearances of j in the partition.

Since Schur's theorem [10] is the case $\lambda = k = a = 2$, it is not a particular case of **Theorem 1.1** as $k \geq 2\lambda - 1$ is not satisfied. Hence Andrews [2] conjectured that **Theorem 1.1** may be still true if $k \geq \lambda$. In fact, he gave a proof of this result [4].

In the conclusion of [4], Andrews stated the following two conjectures.

CONJECTURE 1.4. For $\lambda/2 < a \leq k < \lambda$, let $n^c = (k + \lambda - a + 1)(k + \lambda - a)/2 + (k - \lambda + 1)(\lambda + 1)$. Then

$$\begin{aligned} B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) \quad \text{for } 0 \leq n < n^c, \\ B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) + 1 \quad \text{for } n = n^c. \end{aligned} \quad (1.2)$$

CONJECTURE 1.5. For all positive integers n , $A_{4,3,3}(n) = B_{4,3,3}^0(n)$, where $B_{4,3,3}^0(n)$ denotes the number of partitions of n enumerated by $B_{4,3,3}(n)$ with the added restrictions:

$$\begin{aligned} f_{5j+2} + f_{5j+3} &\leq 1 \quad \text{for } j \geq 0, \\ f_{5j+4} + f_{5j+6} &\leq 1 \quad \text{for } j \geq 0, \\ f_{5j-1} + f_{5j} + f_{5j+5} + f_{5j+6} &\leq 3 \quad \text{for } j \geq 1. \end{aligned} \quad (1.3)$$

Conjecture 1.5 is designed to show that when the condition $k \geq \lambda$ is removed with some additional restrictions on the summands, some partition identities can be obtained in a few cases. In 1994, Andrews et al. [5] proved **Conjecture 1.5**.

The first author and Sudha [9] have proved the case $k = a$ of **Conjecture 1.4** while the authors in [8] have established the case $k = a + 1$ of **Conjecture 1.4**. The objective of the present paper is to prove that **Conjecture 1.4** is false if n exceeds $(2k - a - \lambda/2 + 1)(\lambda + 1)$ for even λ and $k \geq a + 2$. For odd λ , we have verified and checked that **Conjecture 1.4** is false when $\lambda = 11$, $k = 9$, and $a = 6$. We also give the following revised conjecture for a particular case when λ is even.

REVISED CONJECTURE 1.6. Let λ be even, $a - \lambda/2 = 1$, $\theta = k - a$, $\theta(\theta - 1)/2 < [a - \lambda/2](\lambda + 1)$, and $0 \leq \theta \leq \lambda/2 - 3$. Then

$$\begin{aligned} B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) \quad \text{for } n < \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1), \\ B_{\lambda,k,a}(n) &= A_{\lambda,k,a}(n) + B_{\lambda,k,a}(x), \\ \text{where } n &= \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + x, \quad 0 \leq x \leq \frac{\theta(\theta - 1)}{2}. \end{aligned} \quad (1.4)$$

These results support (i) Andrews' contention that $k \geq \lambda$ is essential for the truth of **Theorem 1.1** and (ii) his belief that **Theorem 1.1** was the best possible one, but his conjecture about first counterexamples when $k \geq \lambda$ is false.

2. Preliminaries. Let $P_{B_{\lambda,k,a}}(n)$ and $P_{A_{\lambda,k,a}}(n)$ denote the sets of partitions enumerated by $B_{\lambda,k,a}(n)$ and $A_{\lambda,k,a}(n)$, respectively. Let $P'_A(n)$ (resp., $P'_B(n)$) denote the set of partitions enumerated by $A_{\lambda,k,a}(n)$ (resp., $B_{\lambda,k,a}(n)$) but not by $B_{\lambda,k,a}(n)$ (resp., $A_{\lambda,k,a}(n)$).

$\pi \in P'_A(n)$ implies that it violates one of the conditions on f' s or b' s. Let S_j ($j = 1, 2, \dots, \lambda/2$) denote the condition $\sum_{i=j}^{\lambda-j+1} f_i \leq a - j$, let S denote the condition $\sum_{i=1}^{\lambda+1} f_i \leq a - 1$, and let S^* be the condition on b' s.

Let $(2k - a - \lambda/2 + 1)(\lambda + 1) \leq n < (2k - a - \lambda/2 + 1)(\lambda + 1) + \theta(\theta - 1)/2$, where $\theta(\theta - 1)/2 < (a - \lambda/2)(\lambda + 1)$ and $\theta = k - a$. Then

$$P'_B(n) = Q^1 \cup \dots \cup Q^{a-1} \cup R(n), \quad (2.1)$$

where for $1 \leq i \leq a-1$,

$$\begin{aligned} Q^i &= \left\{ \pi \in P'_B(n) : \left(a - \frac{\lambda}{2} \right) (\lambda + 1) \text{ appears } i \text{ times} \right\}, \\ R(n) &= \left\{ \left(2k - a - \frac{\lambda}{2} + 1 \right) (\lambda + 1) + \pi : \pi \text{ is a partition of} \right. \\ &\quad \left. n - \left(2k - a - \frac{\lambda}{2} + 1 \right) (\lambda + 1) \text{ into parts with } C \right\}. \end{aligned} \quad (2.2)$$

Here C stands for “subjected to the conditions in the definition of B.” Clearly, $\#R(n) = B_{\lambda, k, a}[n - (2k - a - \lambda/2 + 1)(\lambda + 1)]$.

From the method explained in [8, 9], it follows that the partitions violating $S_1, \dots, S_{\lambda/2}$ will be mapped onto $Q^1 \cup \dots \cup Q^{a-1}$. If $a - \lambda/2 = 1$, then S reduces to S_1 . As such, any contribution to $R(n)$ can come only from those partitions of P'_A which violate S^* but do not violate any of $S_1, \dots, S_{\lambda/2}$. For the counterexample in Section 3, we enumerate separately the partitions counted by $R(n)$. If there are no partitions of n violating only S^* , then for such n , we have that $P'_A(n)$ is the union of the partitions violating $S_1, \dots, S_{\lambda/2}$ and Q^{a-1} is the set containing $a-1$ times $\lambda+1$. This set is identified with the first stage of S_1 where all the parts from $1, \dots, \lambda$ appear. Q^{a-2} will be the union of the two sets, one containing $a-2$ times $\lambda+1$ and the other containing $a-2$ times $\lambda+1$ plus a part between 1 and λ . These two sets are, respectively, identified with the first stage of S_2 where all the parts from $2, \dots, \lambda-1$ appear, and the second stage of S_1 in which all the parts except one part from $1, \dots, \lambda$ appear and so on.

3. Counterexample. Let $\lambda = 12$, $k = 11$, $a = 7$, $\theta = 4$, $a - \lambda/2 = 1$, $\theta(\theta-1)/2 = 6 < (a - \lambda/2)(\lambda + 1) = 13$, and $n^c = 136$. In this case,

$$\begin{aligned} S_{\lambda/2} &= S_6 : f_7 + f_6 \leq 1, & S_5 : f_8 + f_7 + f_6 + f_5 \leq 2, & S_4 : f_9 + \dots + f_4 \leq 3, \\ S_3 &: f_{10} + \dots + f_3 \leq 4, & S_2 : f_{11} + \dots + f_2 \leq 5, & S_1 : f_{12} + \dots + f_1 \leq 6, \\ S &: f_{13} + \dots + f_1 \leq 6; \\ P'_B(n) &= Q^1 \cup \dots \cup Q^6 \cup R(n), \end{aligned} \quad (3.1)$$

where $Q^i = \{\pi \in P'_B(n) : 13 \text{ appears } i \text{ times}\}$, $1 \leq i \leq 6$, and $R(n) = \{130 + \pi : \pi \text{ is a partition of } n - 130 \text{ into parts with } C\}$. Here $\#R(n) = B_{12, 11, 7}(x)$, where $x = n - 130$. We now prove

$$B_{12, 11, 7}(n) = A_{12, 11, 7}(n), \quad n < 130, \quad (3.2)$$

$$B_{12, 11, 7}(n) = A_{12, 11, 7}(n) + B_{12, 11, 7}(x), \quad n = 130 + x, \quad 0 \leq x < 6, \quad (3.3)$$

$$B_{12, 11, 7}(136) = A_{12, 11, 7}(136) + B_{12, 11, 7}(6) - 1 = A_{12, 11, 7}(136) + 3, \quad (3.4)$$

since $B(6) = 4$ as $6, 5+1, 4+2$, and $3+2+1$ are the only relevant partitions of 6 enumerated by B .

Proof of (3.2), (3.3), and (3.4). Equation (3.2) follows from [8]. We now prove that for $1 \leq n < 136$, there are no partitions of n violating only S^* and that

$$18 + 17 + 16 + 15 + 14 + 12 + 11 + 10 + 9 + 8 + 6 \quad (3.5)$$

is the only partition of 136 violating only S^* .

In [8, 9] we have shown that for $n < 130$, if a partition violates S^* , then it violates either S or S_1 . However, for $130 \leq n \leq 136$, we now investigate such partitions.

If a partition violates S^* , then there exist a partition

$$n = b_1 + \cdots + b_i + \cdots + b_{i+10} + \cdots + b_s \quad (3.6)$$

and an integer i with $b_i - b_{i+10} < 13$. We get the following possibilities.

CASE 1. If $b_{i+10} \geq 13$, then the number being partitioned is greater than or equal to

$$\begin{aligned} & (12 + x_{11}) + \cdots + (12 + x_1) + \cdots, \\ & 11(12 + 1), \quad \text{where } x_{11} - x_1 < 13. \end{aligned} \quad (3.7)$$

If (3.7) contains the part 13 more than 6 times, then it violates S . Let $x \leq 6$ be the number of 13's and let y denote the number of terms greater than 13 in (3.7) so that $x + y = 11$. Then (3.7) becomes

$$13x + (12 + 2) + \cdots + (12 + 11 - x) = 11(13) + \frac{(11 - x)(11 - x - 1)}{2}. \quad (3.8)$$

Let n^c denote the n in the conjecture. If $k = a + \theta$, then

$$\begin{aligned} n^c &= \left(2k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + \frac{\theta(\theta - 1)}{2} \\ &= k(\lambda + 1) + \left(k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + \frac{(k - a)(k - a - 1)}{2} \\ &< k(\lambda + 1) + \left(k - a - \frac{\lambda}{2} + 1\right)(\lambda + 1) + \frac{(k - x)(k - x - 1)}{2} \\ &< k(\lambda + 1) + \frac{(k - x)(k - x - 1)}{2} \quad \text{since } k - a - \frac{\lambda}{2} + 1 < 0. \end{aligned} \quad (3.9)$$

In this case, we have that $n^c < 11(13) + (11 - x)(11 - x - 1)/2$.

CASE 2. Let $b_{i+10} < 13$ and $b_i < 13$. Then (3.6) violates S_1 .

CASE 3. Let $b_{i+10} < 13$ and $b_i \geq 13$. Let β denote the number of parts among 1, 2, ..., 13. If $\beta \geq 7$, then (3.6) violates S or S_1 . Hence, $1 \leq \beta \leq 6$. Let α denote the number of parts 13 so that $5 \leq \alpha \leq 10$ and $\alpha + \beta = 11$. Then the number being partitioned is

$$(12 + x_\alpha) + \cdots + (12 + x_1) + y_1 + \cdots + y_\beta. \quad (3.10)$$

Since $(12 + x_\alpha) - y_\beta < 13$, we have $x_\alpha = y_\beta$. Now, $x_1 \geq 2, x_2 \geq 3, \dots, x_\alpha \geq \alpha + 1$. Thus, $y_\beta \geq \alpha + 1, \dots, y_1 \geq \alpha + \beta = 11$. Hence, (3.10) is greater than or equal to

$$(12 + \alpha + 1) + \cdots + (12 + 2) + (\alpha + \beta) + \cdots + (\alpha + 1) \quad (3.11)$$

and equals

$$\frac{13\alpha + (\alpha + \beta)(\alpha + \beta + 1)}{2}. \quad (3.12)$$

Let $\beta = 1, 2, 3, 4, 5$. Then (3.10) is, respectively, 196, 183, 170, 157, and 144, all of which are greater than $136 = n^c$.

Now let $\beta = 6$. Since we have to choose 6 parts from $1, 2, \dots, 13$ and 5 parts greater than 13 for a partition violating S^* (and not violating any of S, S_1, \dots, S_6), it is clear that the minimum part should be 6. Let $S_1^* = \{6, 7\}$ and $S_2^* = \{8, 9, 10, 11, 12, 13\}$. Since $f_6 + f_7 \leq 1$, we can choose either 6 or 7 from S_1^* and the other five must be from S_2^* . Also there are 5 parts greater than 13. In this case, the minimum value of n will be

$$6 + 8 + 9 + 10 + 11 + 12 + 14 + 15 + 16 + 17 + 18 = 136. \quad (3.13)$$

Thus for all $130 \leq n < 136$, there are no partitions of n violating only S^* . It is easy to see that when $n = 136$,

$$18 + \dots + 14 + 12 + \dots + 8 + 6 \quad (3.14)$$

is the only partition of 136 violating only S^* . Thus we find

$$P'_A(n) = \{\text{union of the partitions violating } S_1, \dots, S_6\} \quad \text{for } 1 \leq n < 136 \quad (3.15)$$

while

$$P'_A(136) = \{\text{union of the partitions violating } S_1, \dots, S_6\} + 1. \quad (3.16)$$

We now establish a bijection of $Q^1 \cup \dots \cup Q^6$ onto $P'_A(n)$ which is explained in Table 3.1. This also proves (3.3) and (3.4). Before writing the table, we observe that for a partition

$$\pi + 13 \times i + \alpha_1 + \dots + \alpha_j, \quad 1 \leq i \leq 6, \quad (3.17)$$

belonging to P'_B , π is a partition of $(n - 13 \times i - \alpha_1 - \dots - \alpha_j)$ into parts greater than 13 with C , where $1 \leq \alpha_j < \dots < \alpha_1 \leq 12$, and for a partition

$$\pi + \beta_1 + \dots + \beta_j \quad (3.18)$$

belonging to P'_A , π is a partition of $(n - \beta_1 - \dots - \beta_j)$ into parts greater than β_1 such that 13 is not a part, where $1 \leq \beta_j < \dots < \beta_1 \leq 12$.

REMARK 3.1. In Table 3.1, some partitions in Q^2 are not covered. They are

$$\begin{aligned} & \{\pi + 13 \times 2 + x_1 + x_2 + 1 : 2 \leq x_2 \leq 11, 3 \leq x_1 \leq 12, (x_1, x_2) \neq (7, 6)\} \\ & \cup \{\pi + 13 \times 2 + 12 + x_1 + x_2 : 3 \leq x_1 \leq 11, 2 \leq x_2 \leq 10, (x_1, x_2) \neq (7, 6)\}. \end{aligned} \quad (3.19)$$

Here we split 13×2 into pairs (α, β) and (γ, δ) in the following order:

$$(7, 6) (8, 5) (9, 4) (10, 3) (11, 2) (12, 1). \quad (3.20)$$

TABLE 3.1

$P'_{B_{12,11,7}}(n)$	$P'_{A_{12,11,7}}(n)$
$Q^6 = \{\pi + 13 \times 6\}$	1st stage of $S_1 = \{\pi + 12 + \dots + 1\}$
$Q^5 = \{\pi + 13 \times 5\}$	1st stage of $S_2 = \{\pi + 11 + \dots + 2\}$
$\cup \{\pi + 13 \times 5 + (13 - x_1) : 1 \leq (13 - x_1) \leq 12\}$	2nd stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 2 + 1 : 1 \leq x_1 \leq 12\}$
$Q^4 = \{\pi + 13 \times 4 + x : x = 0, 1, 2, 12\}$	1st stage of $S_3 = \{\pi + 10 + \dots + 3 + x : x = 0, 1, 2, 12\}$
$\cup \{\pi + 13 \times 4 + (13 - x_1) : 2 < (13 - x_1) \leq 11\}$	2nd stage of $S_2 = \{\pi + 11 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 2 : 2 \leq x_1 < 11\}$
$\cup \{\pi + 13 \times 4 + (13 - x_1) + (13 - x_2) : 1 \leq (13 - x_2) < (13 - x_1) \leq 12\}$	3rd stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_2 + 1) + (x_2 - 1) + \dots + 1 : 1 \leq x_2 < x_1 \leq 12, (x_i, x_j) \neq (7, 6)\}$
$Q^3 = \{\pi + 13 \times 3 + x : x = 0, 1, 2, 11, 12\}$	Note 1. If $(x_i, x_j) = (7, 6)$, then it will be covered in the 3rd stage of S_2 .
$\cup \{\pi + 13 \times 3 + (13 - x_1) : 3 \leq (13 - x_1) \leq 10\}$	1st stage of $S_4 = \{\pi + 9 + \dots + 4 + x : x = 0, 1, 2, 11, 12\}$
$\cup \{\pi + 13 \times 3 + (13 - x_1) + (13 - x_2) : 2 \leq (13 - x_2) < (13 - x_1) \leq 11\}$	2nd stage of $S_3 = \{\pi + 10 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 3 : 3 \leq x_1 \leq 10\}$
$\cup \{\pi + 13 \times 3 + x + y : (x, y) = \text{all possible pairs of } 1, 2, 11, 12 \text{ except } (11, 2)\}$	3rd stage of $S_2 = \{\pi + 11 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_2 + 1) + (x_2 - 1) + \dots + 2 : 2 \leq x_2 < x_1 \leq 11, (x_i, x_j) \neq (7, 6)\}$
$\cup \{\pi + 13 \times 3 + (13 - x_1) + \dots + (13 - x_3) : 1 \leq (13 - x_3) < (13 - x_2) < (13 - x_1) \leq 12\}$	Note 2. If $(x_i, x_j) = (7, 6)$, then it will be covered in the 3rd stage of S_3 .
$Q^2 = \{\pi + 13 \times 2 + x : x = 0, 1, 2, 3, 10, 11, 12\}$	4th stage of $S_4 = \{\pi + 9 + \dots + 4 + x + y : (x, y) = \text{all possible pairs of } 1, 2, 11, 12 \text{ except } (11, 2)\}$
$\cup \{\pi + 13 \times 2 + (13 - x_1) : 4 \leq (13 - x_1) \leq 9\}$	4th stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_3 + 1) + (x_3 - 1) + \dots + 1 : 1 \leq x_3 < x_2 < x_1 \leq 12, (x_i, x_j) \neq (7, 6)\}$
$\cup \{\pi + 13 \times 2 + (13 - x_1) + (13 - x_2) : 3 \leq (13 - x_2) < (13 - x_1) \leq 10\}$	Note 3. If $(x_i, x_j) = (7, 6)$, then it will be covered in the 4th stage of S_2 .
	1st stage of $S_5 = \{\pi + 8 + 7 + 6 + 5 + x : x = 0, 1, 2, 3, 10, 11, 12\}$
	2nd stage of $S_4 = \{\pi + 9 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + 4 : 4 \leq x_1 \leq 9\}$
	3rd stage of $S_3 = \{\pi + 10 + \dots + (x_1 + 1) + (x_1 - 1) + \dots + (x_2 + 1) + (x_2 - 1) + \dots + 3 : 3 \leq x_2 < x_1 \leq 10, (x_i, x_j) \neq (7, 6)\}$

TABLE 3.1. Continued.

$P'_{B_{12,11,7}}(n)$	$P'_{A_{12,11,7}}(n)$
$\cup \{\pi + 13 \times 2 + x + y : (x, y)$ = all possible pairs of 1, 2, 3, 10 11, 12 except (10, 3)\}	Note 4. If $(x_i, x_j) = (7, 6)$, then it will be covered in the 3rd stage of S_4 . 4th stage of $S_5 = \{\pi + 8 + 7 + 6 + 5 +$ $x + y : (x, y) = \text{all possible}$ pairs of 1, 2, 3, 10, 11, 12 except (10, 3)\}
$\cup \{\pi + 13 \times 2 + (13 - x_1) + \dots$ $+ (13 - x_3) : 2 \leq (13 - x_3)$ $< (13 - x_2) < (13 - x_1) \leq 11\}$	4th stage of $S_2 = \{\pi + 11 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + (x_3 + 1)$ $+ (x_3 - 1) + \dots + 2 : 2 \leq x_3$ $< x_2 < x_1 \leq 11\}$ $(x_i, x_j) \neq (7, 6)\}$
$\cup \{\pi + 13 \times 2 + (13 - x_1) + \dots$ $+ (13 - x_4) : 1 \leq (13 - x_4)$ $< \dots < (13 - x_1) \leq 12\}$	Note 5. If $(x_i, x_j) = (7, 6)$, then it will be covered in the 4th stage of S_3 . 5th stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + (x_4 + 1)$ $+ (x_4 - 1) + \dots + 1 : 1 \leq x_4$ $< \dots < x_1 \leq 12\}$ $(x_i, x_j) \neq (7, 6)\}$
$Q^1 = \{\pi + 13 + x :$ $x = 0, 1, 2, 3, 4, 9, 10, 11, 12\}$ $\cup \{\pi + 13 + (13 - x_1) :$ $5 \leq (13 - x_1) \leq 8\}$	Note 6. If $(x_i, x_j) = (7, 6)$, then it will be covered in the 5th stage of S_2 . 1st stage of $S_6 = \{\pi + 7 + 6 + x :$ $x = 0, 1, 2, 3, 4, 9, 10, 11, 12\}$
$\cup \{\pi + 13 + (13 - x_1) + (13 - x_2) :$ $4 \leq (13 - x_2) < (13 - x_1) \leq 9\}$	2nd stage of $S_5 = \{\pi + 8 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + 5$ $: 5 \leq x_1 \leq 8\}$
$\cup \{\pi + 13 + x + y : (x, y)$ = all possible pairs of 1, 2, 3, 4, 9, 10, 11, 12 except (9, 4)\}	3rd stage of $S_4 = \{\pi + 9 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + (x_2 + 1)$ $+ (x_2 - 1) + \dots + 4 : 4 \leq x_2$ $< x_1 \leq 9\}$
$\cup \{\pi + 13 + (13 - x_1) + \dots$ $+ (13 - x_3) : 3 \leq (13 - x_3)$ $< (13 - x_2) < (13 - x_1) \leq 10\}$	4th stage of $S_5 = \{\pi + 7 + 6 + x + y : (x, y)$ = all possible pairs of 1, 2, 3, 4, 9, 10, 11, 12 except (9, 4)\}
$\cup \{\pi + 13 + (13 - x_1) + \dots + (13 - x_4) :$ $2 \leq (13 - x_4) < \dots < (13 - x_1)$ $\leq 11\}$	4th stage of $S_3 = \{\pi + 10 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + (x_3 + 1)$ $+ (x_3 - 1) + \dots + 3 : 3 \leq x_3$ $< x_2 < x_1 \leq 10\}$
$\cup \{\pi + 13 + (13 - x_1) + \dots + (13 - x_5) :$ $1 \leq (13 - x_5) < \dots < (13 - x_1)$ $\leq 12\}$	5th stage of $S_2 = \{\pi + 11 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + (x_4 + 1)$ $+ (x_4 - 1) + \dots + 2 : 2 \leq x_4$ $< \dots < x_1 \leq 11\}$
	6th stage of $S_1 = \{\pi + 12 + \dots + (x_1 + 1)$ $+ (x_1 - 1) + \dots + (x_5 + 1)$ $+ (x_5 - 1) + \dots + 1 : 1 \leq x_5$ $< \dots < x_1 \leq 12\}$

We arrange $\pi + \alpha + \beta + \gamma + \delta + x_1 + x_2 + \gamma$ ($\gamma = 12$ or 1) in the decreasing order and associate it to the rearranged partition π^* which belongs to P'_A .

A similar procedure is adopted for some partitions in Q^1 which are also not covered in Table 3.1. This completes the proof of (3.3) and (3.4). \square

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