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# ON HOPF GALOIS HIRATA EXTENSIONS

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Let $H$ be a finite-dimensional Hopf algebra over a field $k$, $H^*$ the dual Hopf algebra of $H$, and $B$ a right $H^*$-Galois and Hirata separable extension of $B^H$. Then $B$ is characterized in terms of the commutator subring $V_B(B^H)$ of $B^H$ in $B$ and the smash product $V_B(B^H)\#H$. A sufficient condition is also given for $B$ to be an $H^*$-Galois Azumaya extension of $B^H$.

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**1. Introduction.** Let $H$ be a finite-dimensional Hopf algebra over a field $k$, $H^*$ the dual Hopf algebra of $H$, and $B$ a right $H^*$-Galois extension of $B^H$. In [3], the class of $H^*$-Galois Azumaya extensions was investigated and in [8], it was shown that $B$ is a Hirata separable extension of $B^H$ if and only if the commutator subring $V_B(B^H)$ of $B^H$ in $B$ is a left $H$-Galois extension of $C$, where $C$ is the center of $B$ (see [8, Lemma 2.1, Theorem 2.6]). The purpose of the present paper is to characterize a right $H^*$-Galois and Hirata separable extension $B$ of $B^H$ in terms of the commutator subring $V_B(B^H)$ and the smash product $V_B(B^H)\#H$. Let $B$ be a right $H^*$-Galois extension of $B^H$ such that $B^H = B^{H^*}$. Then the following statements are equivalent:

(1) $B$ is a Hirata separable extension of $B^H$,
(2) $V_B(B^H)$ is an Azumaya $C$-algebra and $V_B(V_B(B^H)) = B^H$,
(3) $V_B(B^H)$ is a right $H^*$-Galois extension of $C$ and a direct summand of $V_B(B^H)\#H$ as a $V_B(B^H)$-bimodule,
(4) $V_B(B^H)$ is a right $H^*$-Galois extension of $C$ and $V_B(B^H)\#H$ is a direct summand of a finite direct sum of $V_B(B^H)$ as a bimodule over $V_B(B^H)$.

Moreover, an equivalent condition is given for a right $H^*$-Galois and Hirata separable extension $B$ of $B^H$ to be an $H^*$-Galois Azumaya extension which was studied in [3, 7]. Also, let $B$ be a right $H^*$-Galois and Hirata separable extension of $B^H$ and $A$ a subalgebra of $B^H$ over $C$ such that $B^H$ is a projective Hirata separable extension of $A$ containing $A$ as a direct summand as an $A$-bimodule. Then $V_{B^H}(A)$ is a separable subalgebra of $B^H$ over $C$, and there exists an $H$-submodule algebra $D$ in $B$ which is separable over $C$ such that $D^H = V_{B^H}(A)$ and $D \cong V_{B^H}(A) \otimes_Z F$ as Azumaya $Z$-algebras, where $Z$ is the center of $D$ and $F$ is an Azumaya $Z$-algebra in $D$.

**2. Basic definitions and notations.** Throughout, $H$ denotes a finite-dimensional Hopf algebra over a field $k$ with comultiplication $\Delta$ and counit $\varepsilon$, $H^*$ the dual Hopf algebra of $H$, $B$ a left $H$-module algebra, $C$ the center of $B$, $B^H = \{b \in B \mid hb = \varepsilon(h)b$ for all $h \in H\}$ which is called the $H$-invariants of $B$, and $B\#H$ the smash product of $B$ with $H$, where $B\#H = B \otimes_k H$ such that for all $b\#h$ and $b'\#h'$ in $B\#H$, $(b\#h)(b'\#h') = \sum b(h_1b')\#h_2h'$, where $\Delta(h) = \sum h_1 \otimes h_2$. The ring $B$ is called a right $H^*$-Galois extension of $B^H$ if $B$ is a right $H^*$-comodule algebra with structure map $\rho : B \to B \otimes_k H^*$ such that $\beta : B \otimes_{B^H} B \to B \otimes_k H^*$ is a bijection, where $\beta(a \otimes b) = (a \otimes 1)\rho(b)$.

For a subring $A$ of $B$ with the same identity 1, we denote the commutator subring of $A$ in $B$ by $V_B(A)$. We call $B$ a separable extension of $A$ if there exist $\{a_i, b_i$ in $B$, $i = 1, 2, \dots, m$, for some integer $m\}$ such that $\sum a_ib_i = 1$ and $\sum ba_i \otimes b_i = \sum a_i \otimes b_ib$ for all $b$ in $B$, where $\otimes$ is over $A$. An Azumaya algebra is a separable extension of its center. A ring $B$ is called a Hirata separable extension of $A$ if $B \otimes_A B$ is isomorphic to a direct summand of a finite direct sum of $B$ as a $B$-bimodule. A right $H^*$-Galois extension $B$ is called an $H^*$-Galois Azumaya extension if $B$ is separable over $B^H$ which is an Azumaya algebra over $C^H$. A right $H^*$-Galois extension $B$ of $B^H$ is called an $H^*$-Galois Hirata extension if $B$ is also a Hirata separable extension of $B^H$. Throughout, an $H^*$-Galois extension means a right $H^*$-Galois extension unless it is stated otherwise.

**3. The $H^*$-Galois Hirata extensions.** In this section, we will characterize an $H^*$-Galois Hirata extension $B$ of $B^H$ in terms of the commutator subring $V_B(B^H)$ of $B^H$ in $B$ and the smash product $V_B(B^H)\#H$. A relationship between an $H^*$-Galois Hirata extension and an $H^*$-Galois Azumaya extension is also given. We begin with some properties of an $H^*$-Galois Hirata extension $B$ of $B^H$. Throughout, we assume $B^H = B^{H^*}$.

**Lemma 3.1.** *If $A_1$ and $A_2$ are $H^*$-Galois extensions such that $A_1^H = A_2^H$ and $A_1 \subset A_2$, then $A_1 = A_2$.*

**Proof.** By [3, Theorem 5.1], there exist $\{x_i, y_i \in A_1 \mid i = 1, 2, \dots, n\}$ for some integer $n$ such that, for all $h \in H$, $\sum x_i(hy_i) = T(h)1_{A_1}$, where $T \in \int_{H^*}^r$, the set of right integrals in $H^*$. Let $t \in \int_H^l$, the set of left integrals in $H$, such that $T(t) = 1$, then $\{x_i, f_i = t(y_i -) \mid i = 1, 2, \dots, n\}$ is a dual basis of the finitely generated and projective right module $A_1$ over $A_1^H$. Since $A_1 \subset A_2$ such that $A_1^H = A_2^H$, $\{x_i, f_i \mid i = 1, 2, \dots, n\}$ is also a dual basis of the finitely generated and projective right module $A_2$ over $A_1^H$. This implies that $A_1 = A_2$. □

**Lemma 3.2.** *If $B$ is an $H^*$-Galois Hirata extension of $B^H$, then $B^H$ is a direct summand of $B$ as a $B^H$-bimodule.*

**Proof.** We use the argument as given in [2]. Since $B$ is an $H^*$-Galois and a Hirata separable extension of $B^H$, $V_B(B^H)$ is a left $H$-Galois extension of $C$ (see [8, Lemma 2.1, Theorem 2.6]). Hence, $V_B(B^H)$ is a finitely generated and projective module over $C$ (see [3, Theorem 2.2]). Let $\Omega = \mathrm{Hom}_C(V_B(B^H), V_B(B^H))$. Since $C$ is commutative, $V_B(B^H)$ is a progenerator of $C$. Thus, $B$ is a right $\Omega$-module such that $B \cong V_B(B^H) \otimes_C \mathrm{Hom}_\Omega(V_B(B^H), B) \cong V_B(B^H) \otimes_C B^{H^*}$ as $C$-algebras, where $f(1) \in B^{H^*}$ for each $f \in \mathrm{Hom}_\Omega(V_B(B^H), B)$ by the proof of [2, Lemma 2.8]. But $V_B(V_B(B^H)) = B^H$ (see [2, Lemma 2.5]), so $B \cong V_B(B^H) \otimes_C B^H$. This implies that $V_B(B^H)$ is an $H^*$-Galois extension of $C$ (see [2, Lemma 2.8]); and so $C$ is a direct summand of $V_B(B^H)$ as a $C$-bimodule (see [2, Corollaries 1.9 and 1.10]). Therefore, $B^H$ is a direct summand of $B$ as a $B^H$-bimodule. □

By the proof of Lemma 3.2, $V_B(B^H)$ is an $H^*$-Galois extension of $C$.

**Corollary 3.3.** *If $B$ is an $H^*$-Galois Hirata extension of $B^H$, then $V_B(B^H)$ is an $H^*$-Galois extension of $C$.*

**Corollary 3.4.** *If $B$ is an $H^*$-Galois Hirata extension of $B^H$, then $B = B^H \cdot V_B(B^H)$ and the centers of $B$, $B^H$, and $V_B(B^H)$ are the same $C$.*

**Proof.** By Corollary 3.3, $V_B(B^H)$ is an $H^*$-Galois extension of $C$, so $B^H \cdot V_B(B^H)$ is also an $H^*$-Galois extension of $B^H$ $(= (B^H \cdot V_B(B^H))^H)$ with the same Galois system as $V_B(B^H)$ (see [3, Theorem 5.1]). Noting that $B^H \cdot V_B(B^H) \subset B$, we conclude that $B = B^H \cdot V_B(B^H)$ by Lemma 3.1. Moreover, $V_B(V_B(B^H)) = B^H$ (see [8, Lemma 2.5]), so the centers of $B^H$, $V_B(B^H)$, and $B$ are the same $C$. □

**Theorem 3.5.** *Let $B$ be an $H^*$-Galois extension of $B^H$. The following statements are equivalent:*

(1) *$B$ is a Hirata separable extension of $B^H$,*

(2) *$V_B(B^H)$ is an $H^*$-Galois extension of $C$ and a direct summand of $V_B(B^H)\#H$ as a $V_B(B^H)$-bimodule,*

(3) *$V_B(B^H)$ is an Azumaya $C$-algebra and $V_B(V_B(B^H)) = B^H$,*

(4) *$V_B(B^H)$ is an $H^*$-Galois extension of $C$ and $V_B(B^H)\#H$ is a direct summand of a finite direct sum of $V_B(B^H)$ as a bimodule over $V_B(B^H)$.*

**Proof.** (1)⇒(3). Since $B$ is an $H^*$-Galois and a Hirata separable extension of $B^H$, by Lemma 3.2, $B^H$ is a direct summand of $B$ as a $B^H$-bimodule. Thus, $V_B(V_B(B^H)) = B^H$ and $V_B(B^H)$ is a separable $C$-algebra (see [4, Propositions 1.3 and 1.4]). But the center of $V_B(B^H)$ is $C$ by Corollary 3.4, so $V_B(B^H)$ is an Azumaya $C$-algebra.

(3)⇒(1). Since $V_B(B^H)$ is an Azumaya $C$-algebra and $B$ is a bimodule over $V_B(B^H)$, $B \cong V_B(B^H) \otimes_C V_B(V_B(B^H)) = V_B(B^H) \otimes_C B^H$ as a bimodule over $V_B(B^H)$ (see [1, Corollary 3.6, page 54]). Noting that $B \cong V_B(B^H) \otimes_C B^H$ is also an isomorphism as $C$-algebras and that $V_B(B^H)$ is an Azumaya $C$-algebra, we conclude that $V_B(B^H) \otimes_C B^H$ is a Hirata separable extension of $B^H$; and so $B$ is a Hirata separable extension of $B^H$.

(3)⇒(2). By the proof of (3)⇒(1), $B \cong V_B(B^H) \otimes_C B^H$ such that $V_B(B^H)$ is a finitely generated and projective module over $C$, so $V_B(B^H)$ is an $H^*$-Galois extension of $C$ (see [2, Lemma 2.8]). Moreover, since $V_B(B^H)$ is an Azumaya

$C$-algebra, $V_B(B^H)$ is a direct summand of $V_B(B^H)\otimes_C (V_B(B^H))^\circ$ as a $V_B(B^H)$-bimodule, where $(V_B(B^H))^\circ$ is the opposite algebra of $V_B(B^H)$. But $V_B(B^H)\otimes_C (V_B(B^H))^\circ \cong \mathrm{Hom}_C(V_B(B^H), V_B(B^H)) \cong V_B(B^H)\#H$ (see [3, Theorem 2.2]), so $V_B(B^H)$ is a direct summand of $V_B(B^H)\#H$ as a $V_B(B^H)$-bimodule.

(2)⇒(3). Since $V_B(B^H)$ is an $H^*$-Galois extension of $C$, $B^H\cdot V_B(B^H)$ is an $H^*$-Galois extension of $(B^H\cdot V_B(B^H))^H$. But $(B^H\cdot V_B(B^H))^H = B^H$, so $B^H\cdot V_B(B^H)$ and $B$ are $H^*$-Galois extensions of $B^H$ such that $B^H\cdot V_B(B^H)\subset B$. Hence, $B^H\cdot V_B(B^H) = B$ by Lemma 3.1. Thus, the centers of $B$ and $V_B(B^H)$ are the same $C$. Moreover, $V_B(B^H)$ is a direct summand of $V_B(B^H)\#H$ as a $V_B(B^H)$-bimodule by hypothesis, so it is a separable $C$-algebra (see [3, Theorem 2.3]). Thus, $V_B(B^H)$ is an Azumaya $C$-algebra. But then $B\cong V_B(B^H)\otimes_C V_B(V_B(B^H))$. On the other hand, by hypothesis, $V_B(B^H)$ is an $H^*$-Galois extension of $C$, so $B\cong V_B(B^H)\otimes_C B^H$ (see [2, Lemma 2.8]). Therefore, $V_B(V_B(B^H)) = B^H$.

(3)⇔(4). Since $V_B(B^H)$ is an $H^*$-Galois extension of $C$, it is a finitely generated and projective module over $C$ and $\mathrm{Hom}_C(V_B(B^H), V_B(B^H)) \cong V_B(B^H)\#H$ (see [3, Theorem 2.2]). But then $V_B(B^H)$ is a Hirata separable extension of $C$ if and only if $V_B(B^H)\#H$ is a direct summand of a finite direct sum of $V_B(B^H)$ as a bimodule over $V_B(B^H)$ (see [5, Corollary 3]). Thus, $V_B(B^H)$ is an Azumaya $C$-algebra if and only if $V_B(B^H)$ is an $H^*$-Galois extension of $C$ and $V_B(B^H)\#H$ is a direct summand of a finite direct sum of $V_B(B^H)$ as a bimodule over $V_B(B^H)$. □

By Theorem 3.5, we can obtain a relationship between the class of $H^*$-Galois Hirata extensions and the class of $H^*$-Galois Azumaya extensions which were studied in [3, 7].

**Corollary 3.6.** *Let $B$ be an $H^*$-Galois Azumaya extension of $B^H$. Then $B$ is an $H^*$-Galois Hirata extension of $B^H$ if and only if $C = C^H$.*

**Proof.** (⇒) Since $B$ is an $H^*$-Galois Hirata extension of $B^H$, $V_B(B^H)$ is an Azumaya algebra over $C$ and a left $H$-Galois extension of $C$ (see [8, Theorem 2.6]). Hence, $V_B(V_B(B^H)) = B^H$ (see [8, Lemma 2.5]). Thus, $C\subset B^H$; and so $C = C^H$.

(⇐) Since $B$ is an $H^*$-Galois Azumaya extension of $B^H$, $V_B(B^H)$ is separable over $C^H$ (see [3, Lemma 4.1]). Since $B$ is an $H^*$-Galois Azumaya extension of $B^H$ again, $V_B(B^H)$ is an $H^*$-Galois extension of $(V_B(B^H))^H$ (see [3, Lemma 4.1]), so both $B^H\cdot V_B(B^H)$ and $B$ are $H^*$-Galois extensions of $B^H$ such that $B^H\cdot V_B(B^H)\subset B$. Hence, $B^H\cdot V_B(B^H) = B$ by Lemma 3.1. This implies that the center of $V_B(B^H)$ is $C$. But by hypothesis, $C = C^H$, so $V_B(B^H)$ is an Azumaya $C$-algebra. Hence, $V_B(B^H)$ is a Hirata separable extension of $C$. But $B = B^H\cdot V_B(B^H)\cong B^H\otimes_C V_B(B^H)$ as Azumaya $C$-algebras, so $B$ is a Hirata separable extension of $B^H$. Thus, $B$ is an $H^*$-Galois Hirata extension of $B^H$. □

**Corollary 3.7.** *Let $B$ be an $H^*$-Galois Hirata extension of $B^H$. Then $B$ is an $H^*$-Galois Azumaya extension of $B^H$ if and only if $B$ is an Azumaya $C^H$-algebra.*

**Proof.** (⇒) Since $B$ is an $H^*$-Galois Azumaya extension of $B^H$, $B^H$ is an Azumaya $C^H$-algebra and $B$ is separable over $B^H$ (see [3, Theorem 3.4]). Hence, $B$ is separable over $C^H$ by the transitivity of separable extensions. But $B$ is an $H^*$-Galois Azumaya extension of $B^H$ and an $H^*$-Galois Hirata extension of $B^H$ by hypothesis, so $C = C^H$ by Corollary 3.6. This implies that $B$ is an Azumaya $C^H$-algebra.

(⇐) By hypothesis, $B$ is an Azumaya $C^H$-algebra. Hence, $C = C^H$. But $B$ is an $H^*$-Galois Hirata extension of $B^H$, so $V_B(B^H)$ is an Azumaya subalgebra of $B$ over $C$ by Theorem 3.5(3). Since $B$ is an $H^*$-Galois Hirata extension of $B^H$ again, $B$ is a Hirata separable extension of $B^H$ and a finitely generated and projective module over $B^H$. Thus, $V_B(V_B(B^H)) = B^H$ (see [8, Lemma 2.5]); and so $B^H$ $(= V_B(V_B(B^H)))$ is an Azumaya subalgebra of $B$ over $C^H$ by the commutator theorem for Azumaya algebras (see [1, Theorem 4.3, page 57]). This proves that $B$ is an $H^*$-Galois Azumaya extension of $B^H$. □

**4. Invariant subalgebras.** For an $H^*$-Galois Hirata extension $B$ as given in Theorem 3.5, let $A$ be a subalgebra of $B^H$ over $C$ such that $B^H$ is a projective Hirata separable extension of $A$ and contains $A$ as a direct summand as an $A$-bimodule. In this section, we show that $V_{B^H}(A)$ is the $H$-invariant subalgebra of a separable subalgebra $D$ in $B$ over $C$, that is, $D^H = V_{B^H}(A)$. We denote by $\mathcal{S}$ the set $\{A \mid A$ is a subalgebra of $B^H$ over $C$ such that $B^H$ is a projective Hirata separable extension of $A$ and contains $A$ as a direct summand as an $A$-bimodule$\}$.

**Lemma 4.1.** *Let $B$ be an $H^*$-Galois Hirata extension of $B^H$. For any $A \in \mathcal{S}$, $V_B(A)$ is an $H$-submodule algebra of $B$ and separable over $C$, and $(V_B(A))^H = V_{B^H}(A)$ which is a separable $C$-algebra.*

**Proof.** Since $A \in \mathcal{S}$, $B^H$ is a projective Hirata separable extension of $A$ and contains $A$ as a direct summand as an $A$-bimodule. But $B$ is an $H^*$-Galois Hirata extension of $B^H$, so $B$ is a projective Hirata separable extension of $B^H$. Hence, by the transitivity property of projective Hirata separable extensions, $B$ is a projective Hirata separable extension of $A$. Also $B^H$ is a direct summand of $B$ as a $B^H$-bimodule by Lemma 3.2, so $A$ is a direct summand of $B$ as an $A$-bimodule. Thus, $V_B(A)$ is a separable algebra over $C$ (see [6, Theorem 1]). Moreover, it is clear that $(V_B(A))^H = V_{B^H}(A)$, so $V_{B^H}(A)$ is a separable $C$-algebra (see Corollary 3.4 and [6, Theorem 1]). □

Next we want to show which separable subalgebra of $B^H$ over $C$ is an $H$-invariant subring of an $H$-submodule algebra in $B$. Let $\mathcal{T} = \{E \subset B \mid E$ is a separable $C$-subalgebra of $B^H$ and satisfies the double centralizer property in $B^H$ such that $V_{B^H}(E) \in \mathcal{S}\}$. Next we show that for any $E \in \mathcal{T}$, $E$ is the $H$-invariant subring of an $H$-submodule algebra $D$ in $B$ which is separable over $C$.

**Theorem 4.2.** *Let $E$ be in $\mathcal{T}$. Then there exists an $H$-submodule algebra $D$ in $B$ which is separable over $C$ such that $D^H = E$.*

**Proof.** Since $E$ is in $\mathcal{T}$, $V_{B^H}(E)$ is in $\mathcal{S}$ such that $V_{B^H}(V_{B^H}(E)) = E$. Now by Lemma 4.1, $V_B(V_{B^H}(E))$ is an $H$-submodule algebra of $B$ and separable over $C$ such that $(V_B(V_{B^H}(E)))^H = V_{B^H}(V_{B^H}(E))$. But $V_{B^H}(V_{B^H}(E)) = E$, so

$$\left(V_B\left(V_{B^H}(E)\right)\right)^H = E. \tag{4.1}$$

Let $D = V_B(V_{B^H}(E))$. Then $D$ satisfies the theorem. □

By Theorem 4.2, we obtain an expression for the separable $H$-submodule algebra $D$ for a given $E$ in $\mathcal{T}$.

**Corollary 4.3.** *By keeping the notations as given in Theorem 4.2, let $Z$ be the center of $E$. Then $D \cong E \otimes_Z V_D(E)$ as Azumaya $Z$-algebras.*

**Proof.** Since $E$ satisfies the double centralizer property in $B^H$, $V_{B^H}(V_{B^H}(E)) = E$. Hence, the centers of $E$ and $V_{B^H}(E)$ are the same $Z$. Similarly as given in the proof of Lemma 4.1, since $V_{B^H}(E)$ is in $\mathcal{S}$, $B$ $(= B^H \cdot V_B(B^H))$ is a projective Hirata separable extension of $V_{B^H}(E)$ and contains $V_{B^H}(E)$ as a direct summand as a $V_{B^H}(E)$-bimodule by the transitivity property of projective Hirata separable extensions and the direct summand conditions. Thus, $V_{B^H}(E)$ satisfies the double centralizer property in $B$, that is, $V_B(V_B(V_{B^H}(E))) = V_{B^H}(E)$. This implies that the centers of $V_{B^H}(E)$ and $V_B(V_{B^H}(E))$ are the same. Therefore, $D$ and $E$ have the same center $Z$. Noting that $D$ and $E$ are separable $C$-algebras by Theorem 4.2, we conclude that $E$ $(= D^H)$ is an Azumaya subalgebra of $D$ over $Z$; and so $D \cong E \otimes_Z V_D(E)$ as Azumaya $Z$-algebras (see [1, Theorem 4.3, page 57]). □

**Remark 4.4.** When $B$ is an $H^*$-Galois Azumaya extension of $B^H$, the correspondence $A \to V_B(A)$ as given in Lemma 4.1 recovers the one-to-one correspondence between the set of separable subalgebras of $B^H$ and the set of $H^*$-Galois extensions in $B$ containing $V_B(B^H)$ as given in [3].

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