

A GENERALIZATION OF MULHOLLAND'S INEQUALITY

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By introducing three parameters r , s , and λ , we give a generalization of Mulholland's inequality with a best constant factor involving the β function. As its applications, we also consider its equivalent form and some particular results.

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If $p > 1$, $1/p + 1/q = 1$, and $\{a_n\}$ and $\{b_n\}$ are nonnegative sequences of real numbers such that $0 < \sum_{n=2}^{\infty} (1/n) a_n^p < \infty$ and $0 < \sum_{n=1}^{\infty} (1/n) b_n^q < \infty$, then the Mulholland's inequality is (cf. [4])

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m n \ln m n} < \frac{\pi}{\sin(\pi/p)} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} a_n^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} b_n^q \right\}^{1/q}. \quad (1)$$

Inequality (1) is similar to the well-known Hardy-Hilbert's inequality as (cf. [3])

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{m + n} < \frac{\pi}{\sin(\pi/p)} \left\{ \sum_{n=1}^{\infty} a_n^p \right\}^{1/p} \left\{ \sum_{n=1}^{\infty} b_n^q \right\}^{1/q}. \quad (2)$$

In this paper, we introduce three parameters r , s , and λ to generalize Mulholland's inequality and then derive several equivalent forms of our generalized results with special cases.

THEOREM 1. *If $p > 1$, $1/p + 1/q = 1$, $\{a_n\}$ and $\{b_n\}$ are nonnegative sequences of real numbers, $2 - \min\{p, q\} < \lambda \leq 2$ and $r, s \in \mathbb{R}$, such that $0 < \sum_{n=2}^{\infty} ((\ln n)^{1-\lambda}/n)(n^{1-r} a_n)^p < \infty$ and $0 < \sum_{n=2}^{\infty} ((\ln n)^{1-\lambda}/n)(n^{1-s} b_n)^q < \infty$, then*

$$\begin{aligned} & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s (\ln m n)^{\lambda}} \\ & < B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p \right\}^{1/p} \\ & \quad \times \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-s} b_n)^q \right\}^{1/q}, \end{aligned} \quad (3)$$

where the constant factor $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ is the best possible. In particular,

(i) for $r = 1/q$ and $s = 1/p$,

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^{1/q} n^{1/p} (\ln mn)^{\lambda}} < B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \left\{ \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} a_n^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q \right\}^{1/q}; \quad (4)$$

(ii) for $\lambda = 1$,

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s \ln mn} < \frac{\pi}{\sin(\pi/p)} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} (n^{1-r} a_n)^p \right\}^{1/q} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} (n^{1-s} b_n)^q \right\}^{1/q}; \quad (5)$$

(iii) for $r = s = 0$,

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{(\ln mn)^{\lambda}} < B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \times \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n a_n)^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n b_n)^q \right\}^{1/q}. \quad (6)$$

PROOF. By Hölder's inequality, we find

$$\begin{aligned} & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s (\ln mn)^{\lambda}} \\ &= \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \left[\frac{a_m}{(\ln mn)^{\lambda/p}} \left(\frac{\ln m}{\ln n} \right)^{(2-\lambda)/pq} \left(\frac{m^{1/q-r}}{n^{1/p}} \right) \right] \\ & \quad \times \left[\frac{b_n}{(\ln mn)^{\lambda/q}} \left(\frac{\ln n}{\ln m} \right)^{(2-\lambda)/pq} \left(\frac{n^{1/p-s}}{m^{1/q}} \right) \right] \\ &\leq \left\{ \sum_{m=2}^{\infty} \sum_{n=2}^{\infty} \frac{a_m^p}{(\ln mn)^{\lambda}} \left(\frac{\ln m}{\ln n} \right)^{(2-\lambda)/q} \left(\frac{m^{p(1-r)-1}}{n} \right) \right\}^{1/p} \\ & \quad \times \left\{ \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{b_n^q}{(\ln mn)^{\lambda}} \left(\frac{\ln n}{\ln m} \right)^{(2-\lambda)/p} \left(\frac{n^{q(1-s)-1}}{m} \right) \right\}^{1/q} \\ &= \left\{ \sum_{m=2}^{\infty} \omega_{\lambda}(q, m) m^{p(1-r)-1} a_m^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \omega_{\lambda}(p, n) n^{q(1-s)-1} b_n^q \right\}^{1/q}, \end{aligned} \quad (7)$$

where the weight coefficient $\omega_\lambda(\gamma, n)$ is defined by

$$\omega_\lambda(\gamma, n) = \sum_{m=2}^{\infty} \frac{1}{m(\ln mn)^\lambda} \left(\frac{\ln n}{\ln m} \right)^{(2-\lambda)/\gamma} \quad (\gamma = p, q, n \in \mathbb{N} \setminus \{1\}). \quad (8)$$

For $0 \leq 2 - \gamma < \lambda \leq 2$ ($\gamma = p, q$), setting $u = \ln t / \ln n$ in (8), we have

$$\begin{aligned} \omega_\lambda(\gamma, n) &< \int_0^\infty \frac{1}{t(\ln nt)^\lambda} \left(\frac{\ln n}{\ln t} \right)^{(2-\lambda)/\gamma} dt \\ &= (\ln n)^{1-\lambda} \int_0^\infty \frac{1}{(1+u)^\lambda} u^{(\lambda-2)/\gamma} du. \end{aligned} \quad (9)$$

Since for the β function $B(p, q)$, we have (cf. [5])

$$B(p, q) = \int_0^\infty \frac{1}{(1+u)^{p+q}} u^{-1+p} du = B(q, p) \quad (p, q > 0), \quad (10)$$

then by (9) and (10), we obtain

$$\omega_\lambda(\gamma, n) < B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) (\ln n)^{1-\lambda} \quad (\gamma = p, q, n \in \mathbb{N} \setminus \{1\}). \quad (11)$$

In view of (7), (8), and (11), we have (3).

For $\varepsilon > 0$, such that $(2 - \lambda + \varepsilon)/p < 1$, setting

$$\tilde{a}_n = \frac{1}{n^{1-r}(\ln n)^{(2-\lambda+\varepsilon)/p}}, \quad \tilde{b}_n = \frac{1}{n^{1-s}(\ln n)^{(2-\lambda+\varepsilon)/q}} \quad (n \in \mathbb{N} \setminus \{1\}), \quad (12)$$

then we have

$$\begin{aligned} &\varepsilon \left\{ \sum_{n=2}^{\infty} n^{p(1-r)-1} (\ln n)^{1-\lambda} \tilde{a}_n^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} n^{q(1-s)-1} (\ln n)^{1-\lambda} \tilde{b}_n^q \right\}^{1/q} \\ &= \varepsilon \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1+\varepsilon}} < \varepsilon \left[\frac{1}{2(\ln 2)^2} + \frac{1}{3 \ln 3} + \sum_{n=4}^{\infty} \frac{1}{n(\ln n)^{(1+\varepsilon)}} \right] \\ &< \varepsilon \left[1 + \int_e^\infty \frac{1}{t(\ln t)^{1+\varepsilon}} dt \right] \\ &= \varepsilon + 1. \end{aligned} \quad (13)$$

Putting $u = \ln x / \ln y$ in the following, we find

$$\begin{aligned}
& \varepsilon \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{\tilde{a}_m \tilde{b}_n}{m^r n^s (\ln mn)^{\lambda}} \\
&= \varepsilon \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{1}{mn (\ln mn)^{\lambda}} \cdot \frac{1}{(\ln m)^{(2-\lambda+\varepsilon)/p}} \cdot \frac{1}{(\ln n)^{(2-\lambda+\varepsilon)/q}} \\
&> \varepsilon \int_e^{\infty} \left[\int_e^{\infty} \frac{1}{xy (\ln xy)^{\lambda}} \cdot \frac{1}{(\ln x)^{(2-\lambda+\varepsilon)/p}} \cdot \frac{1}{(\ln y)^{(2-\lambda+\varepsilon)/q}} dx \right] dy \\
&= \varepsilon \int_e^{\infty} \frac{1}{y (\ln y)^{1+\varepsilon}} \left[\int_{1/\ln y}^{\infty} \frac{1}{(1+u)^{\lambda}} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
&= \varepsilon \int_e^{\infty} \frac{1}{y (\ln y)^{1+\varepsilon}} \left[\int_0^{\infty} \frac{1}{(1+u)^{\lambda}} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \quad (14) \\
&\quad - \varepsilon \int_e^{\infty} \frac{1}{y (\ln y)^{1+\varepsilon}} \left[\int_0^{1/\ln y} \frac{1}{(1+u)^{\lambda}} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
&\geq B \left(\frac{p+\lambda-2}{p} - \frac{\varepsilon}{p}, \frac{q+\lambda-2}{q} + \frac{\varepsilon}{q} \right) \\
&\quad - \varepsilon \int_e^{\infty} \frac{1}{y \ln y} \left[\int_0^{1/\ln y} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
&= B \left(\frac{p+\lambda-2}{p} - \frac{\varepsilon}{p}, \frac{q+\lambda-2}{q} + \frac{\varepsilon}{q} \right) - \frac{\varepsilon}{[1 - (2-\lambda+\varepsilon)/p]^2}.
\end{aligned}$$

If the constant factor $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ in (3) is not the best possible, then there exists a positive number K (where $K < B((p+\lambda-2)/p, (q+\lambda-2)/q)$), such that (3) is valid if we change $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ to K . In particular, we have

$$\begin{aligned}
& \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{\tilde{a}_m \tilde{b}_n}{m^r n^s (\ln mn)^{\lambda}} \\
&< K \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} \tilde{a}_n)^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-s} \tilde{b}_n)^q \right\}^{1/q}. \quad (15)
\end{aligned}$$

In virtue of (13), (14), and (15), we have

$$B \left(\frac{p+\lambda-2}{p} - \frac{\varepsilon}{p}, \frac{q+\lambda-2}{q} + \frac{\varepsilon}{q} \right) - \frac{\varepsilon}{[1 - (2-\lambda+\varepsilon)/p]^2} < K(\varepsilon+1). \quad (16)$$

Setting $\varepsilon \rightarrow 0^+$, it follows that $B((p+\lambda-2)/p, (q+\lambda-2)/q) \leq K$, which contradicts the fact that $K < B((p+\lambda-2)/p, (q+\lambda-2)/q)$. Hence the constant factor $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ in (3) is the best possible. This proves the theorem. \square

REMARK 2. (i) For $\lambda = r = s = 1$, (3) changes to (1), it follows that (3) is a generalization of (1) with three parameters and (4), (5), and (6) are its some particular results with the best constant factors.

(ii) For $\lambda = 2$, (6) changes to

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{(\ln mn)^2} < \left\{ \sum_{n=2}^{\infty} \frac{1}{n \ln n} (na_n)^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{1}{n \ln n} (nb_n)^q \right\}^{1/q}, \quad (17)$$

which is a new inequality with a best constant factor 1.

(iii) For $p = q = 2$, (5) and (6) change, respectively, to

$$\begin{aligned} \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s \ln mn} &< \pi \left\{ \sum_{n=2}^{\infty} n^{1-2r} a_n^2 \sum_{n=2}^{\infty} n^{1-2s} b_n^2 \right\}^{1/2}; \\ \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{(\ln mn)^{\lambda}} &< B\left(\frac{\lambda}{2}, \frac{\lambda}{2}\right) \left\{ \sum_{n=2}^{\infty} n(\ln n)^{1-\lambda} a_n^2 \sum_{n=2}^{\infty} n(\ln n)^{1-\lambda} b_n^2 \right\}^{1/2}, \end{aligned} \quad (18)$$

which are new inequalities similar to (cf. [1, 2])

$$\begin{aligned} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{m^r n^s \ln e^{3/4} mn} &< \pi \left\{ \sum_{n=1}^{\infty} n^{1-2r} a_n^2 \sum_{n=1}^{\infty} n^{1-2s} b_n^2 \right\}^{1/2}; \\ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{(m+n)^{\lambda}} &< B\left(\frac{\lambda}{2}, \frac{\lambda}{2}\right) \left\{ \sum_{n=1}^{\infty} n^{1-\lambda} a_n^2 \sum_{n=1}^{\infty} n^{1-\lambda} b_n^2 \right\}^{1/2} \quad (0 < \lambda \leq 2). \end{aligned} \quad (19)$$

THEOREM 3. *If $p > 1$, $1/p + 1/q = 1$, $\{a_n\}$ is a nonnegative sequence of real numbers, $2 - \min\{p, q\} < \lambda \leq 2$ and $r \in \mathbb{R}$, such that*

$$0 < \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p < \infty, \quad (20)$$

then

$$\begin{aligned} \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^{\lambda}} \right]^p \\ < \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p, \end{aligned} \quad (21)$$

where the constant factor $[B((p+\lambda-2)/p, (q+\lambda-2)/q)]^p$ is the best possible. Equation (21) is equivalent to (3). In particular,

(i) for $r = 1/q$,

$$\begin{aligned} \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^{1/q} (\ln mn)^{\lambda}} \right]^p \\ < \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} a_n^p; \end{aligned} \quad (22)$$

(ii) for $\lambda = 1$,

$$\sum_{n=2}^{\infty} \frac{1}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r \ln mn} \right]^p < \left[\frac{\pi}{\sin(\pi/p)} \right]^p \sum_{n=2}^{\infty} \frac{1}{n} (n^{1-r} a_n)^p; \quad (23)$$

(iii) for $r = 0$,

$$\begin{aligned} & \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{(\ln mn)^{\lambda}} \right]^p \\ & < \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (na_n)^p. \end{aligned} \quad (24)$$

PROOF. Setting

$$b_n = (\ln n)^{(p-1)(\lambda-1)} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r n^{1/p} (\ln mn)^{\lambda}} \right]^{p-1} \quad (n = 2, 3, \dots), \quad (25)$$

then by (3), for $s = 1/p$, we have

$$\begin{aligned} 0 & < \left[\sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q \right]^p = \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^{\lambda}} \right]^p \right\}^p \\ & = \left[\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^{1/s} (\ln mn)^{\lambda}} \right]^p \\ & \leq \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p \left\{ \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q \right\}^{p-1}. \end{aligned} \quad (26)$$

Hence, we have

$$\begin{aligned} 0 & < \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q = \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^{\lambda}} \right]^p \\ & \leq \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p < \infty. \end{aligned} \quad (27)$$

In view of (3), neither (26) nor (27) keeps the form of equality. Hence, (21) is valid.

On the other hand, if (21) is valid, by Hölder's inequality, we have

$$\begin{aligned}
 & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s (\ln mn)^{\lambda}} \\
 &= \sum_{n=2}^{\infty} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r n^{1/p} (\ln mn)^{\lambda} (\ln n)^{(1-\lambda)/q}} \right] [(\ln n)^{(1-\lambda)/q} n^{1/p-s} b_n] \\
 &\leq \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^{\lambda}} \right]^p \right\}^{1/p} \\
 &\quad \times \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-s} b_n)^q \right\}^{1/q}. \tag{28}
 \end{aligned}$$

In view of (21), we have (3). Hence (21) is equivalent to (3). If the constant factor in (21) is not the best possible, then by (28), we can get a contradiction that the constant factor in (3) is not the best possible. This proves the theorem. \square

REMARK 4. For $r = 1$, (23) changes to

$$\sum_{n=2}^{\infty} \frac{1}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m \ln mn} \right]^p < \left[\frac{\pi}{\sin(\pi/p)} \right]^p \sum_{n=2}^{\infty} \frac{1}{n} a_n^p, \tag{29}$$

which is equivalent to (1). Both constant factors in (1) and (29) are the best possible.

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