

A GENERALIZATION OF MULHOLLAND'S INEQUALITY

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Received 6 March 2002

By introducing three parameters r , s , and λ , we give a generalization of Mulholland's inequality with a best constant factor involving the β function. As its applications, we also consider its equivalent form and some particular results.

2000 Mathematics Subject Classification: 26D15.

If $p > 1$, $1/p + 1/q = 1$, and $\{a_n\}$ and $\{b_n\}$ are nonnegative sequences of real numbers such that $0 < \sum_{n=2}^{\infty} (1/n) a_n^p < \infty$ and $0 < \sum_{n=1}^{\infty} (1/n) b_n^q < \infty$, then the Mulholland's inequality is (cf. [4])

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{mn \ln mn} < \frac{\pi}{\sin(\pi/p)} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} a_n^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} b_n^q \right\}^{1/q}. \quad (1)$$

Inequality (1) is similar to the well-known Hardy-Hilbert's inequality as (cf. [3])

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{m+n} < \frac{\pi}{\sin(\pi/p)} \left\{ \sum_{n=1}^{\infty} a_n^p \right\}^{1/p} \left\{ \sum_{n=1}^{\infty} b_n^q \right\}^{1/q}. \quad (2)$$

In this paper, we introduce three parameters r , s , and λ to generalize Mulholland's inequality and then derive several equivalent forms of our generalized results with special cases.

THEOREM 1. *If $p > 1$, $1/p + 1/q = 1$, $\{a_n\}$ and $\{b_n\}$ are nonnegative sequences of real numbers, $2 - \min\{p, q\} < \lambda \leq 2$ and $r, s \in \mathbb{R}$, such that $0 < \sum_{n=2}^{\infty} ((\ln n)^{1-\lambda}/n) (n^{1-r} a_n)^p < \infty$ and $0 < \sum_{n=2}^{\infty} ((\ln n)^{1-\lambda}/n) (n^{1-s} b_n)^q < \infty$, then*

$$\begin{aligned} & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s (\ln mn)^\lambda} \\ & < B \left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q} \right) \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p \right\}^{1/p} \\ & \quad \times \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-s} b_n)^q \right\}^{1/q}, \end{aligned} \quad (3)$$

where the constant factor $B((p + \lambda - 2)/p, (q + \lambda - 2)/q)$ is the best possible. In particular,

(i) for $r = 1/q$ and $s = 1/p$,

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^{1/q} n^{1/p} (\ln mn)^{\lambda}} < B\left(\frac{p + \lambda - 2}{p}, \frac{q + \lambda - 2}{q}\right) \left\{ \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} a_n^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q \right\}^{1/q}; \quad (4)$$

(ii) for $\lambda = 1$,

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s \ln mn} < \frac{\pi}{\sin(\pi/p)} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} (n^{1-r} a_n)^p \right\}^{1/q} \left\{ \sum_{n=2}^{\infty} \frac{1}{n} (n^{1-s} b_n)^q \right\}^{1/q}; \quad (5)$$

(iii) for $r = s = 0$,

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{(\ln mn)^{\lambda}} < B\left(\frac{p + \lambda - 2}{p}, \frac{q + \lambda - 2}{q}\right) \times \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n a_n)^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n b_n)^q \right\}^{1/q}. \quad (6)$$

PROOF. By Hölder's inequality, we find

$$\begin{aligned} & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s (\ln mn)^{\lambda}} \\ &= \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \left[\frac{a_m}{(\ln mn)^{\lambda/p}} \left(\frac{\ln m}{\ln n} \right)^{(2-\lambda)/pq} \left(\frac{m^{1/q-r}}{n^{1/p}} \right) \right] \\ & \quad \times \left[\frac{b_n}{(\ln mn)^{\lambda/q}} \left(\frac{\ln n}{\ln m} \right)^{(2-\lambda)/pq} \left(\frac{n^{1/p-s}}{m^{1/q}} \right) \right] \\ &\leq \left\{ \sum_{m=2}^{\infty} \sum_{n=2}^{\infty} \frac{a_m^p}{(\ln mn)^{\lambda}} \left(\frac{\ln m}{\ln n} \right)^{(2-\lambda)/q} \left(\frac{m^{p(1-r)-1}}{n} \right) \right\}^{1/p} \\ & \quad \times \left\{ \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{b_n^q}{(\ln mn)^{\lambda}} \left(\frac{\ln n}{\ln m} \right)^{(2-\lambda)/p} \left(\frac{n^{q(1-s)-1}}{m} \right) \right\}^{1/q} \\ &= \left\{ \sum_{m=2}^{\infty} \omega_{\lambda}(q, m) m^{p(1-r)-1} a_m^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \omega_{\lambda}(p, n) n^{q(1-s)-1} b_n^q \right\}^{1/q}, \end{aligned} \quad (7)$$

where the weight coefficient $\omega_\lambda(\gamma, n)$ is defined by

$$\omega_\lambda(\gamma, n) = \sum_{m=2}^{\infty} \frac{1}{m(\ln mn)^\lambda} \left(\frac{\ln n}{\ln m} \right)^{(2-\lambda)/\gamma} \quad (\gamma = p, q, n \in \mathbb{N} \setminus \{1\}). \quad (8)$$

For $0 \leq 2 - \gamma < \lambda \leq 2$ ($\gamma = p, q$), setting $u = \ln t / \ln n$ in (8), we have

$$\begin{aligned} \omega_\lambda(\gamma, n) &< \int_0^\infty \frac{1}{t(\ln nt)^\lambda} \left(\frac{\ln n}{\ln t} \right)^{(2-\lambda)/\gamma} dt \\ &= (\ln n)^{1-\lambda} \int_0^\infty \frac{1}{(1+u)^\lambda} u^{(\lambda-2)/\gamma} du. \end{aligned} \quad (9)$$

Since for the β function $B(p, q)$, we have (cf. [5])

$$B(p, q) = \int_0^\infty \frac{1}{(1+u)^{p+q}} u^{-1+p} du = B(q, p) \quad (p, q > 0), \quad (10)$$

then by (9) and (10), we obtain

$$\omega_\lambda(\gamma, n) < B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) (\ln n)^{1-\lambda} \quad (\gamma = p, q, n \in \mathbb{N} \setminus \{1\}). \quad (11)$$

In view of (7), (8), and (11), we have (3).

For $\varepsilon > 0$, such that $(2 - \lambda + \varepsilon)/p < 1$, setting

$$\tilde{a}_n = \frac{1}{n^{1-r}(\ln n)^{(2-\lambda+\varepsilon)/p}}, \quad \tilde{b}_n = \frac{1}{n^{1-s}(\ln n)^{(2-\lambda+\varepsilon)/q}} \quad (n \in \mathbb{N} \setminus \{1\}), \quad (12)$$

then we have

$$\begin{aligned} &\varepsilon \left\{ \sum_{n=2}^{\infty} n^{p(1-r)-1} (\ln n)^{1-\lambda} \tilde{a}_n^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} n^{q(1-s)-1} (\ln n)^{1-\lambda} \tilde{b}_n^q \right\}^{1/q} \\ &= \varepsilon \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{1+\varepsilon}} < \varepsilon \left[\frac{1}{2(\ln 2)^2} + \frac{1}{3\ln 3} + \sum_{n=4}^{\infty} \frac{1}{n(\ln n)^{(1+\varepsilon)}} \right] \\ &< \varepsilon \left[1 + \int_e^\infty \frac{1}{t(\ln t)^{1+\varepsilon}} dt \right] \\ &= \varepsilon + 1. \end{aligned} \quad (13)$$

Putting $u = \ln x / \ln y$ in the following, we find

$$\begin{aligned}
 & \varepsilon \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{\tilde{a}_m \tilde{b}_n}{m^r n^s (\ln mn)^\lambda} \\
 &= \varepsilon \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{1}{mn (\ln mn)^\lambda} \cdot \frac{1}{(\ln m)^{(2-\lambda+\varepsilon)/p}} \cdot \frac{1}{(\ln n)^{(2-\lambda+\varepsilon)/q}} \\
 &> \varepsilon \int_e^\infty \left[\int_e^\infty \frac{1}{xy (\ln xy)^\lambda} \cdot \frac{1}{(\ln x)^{(2-\lambda+\varepsilon)/p}} \cdot \frac{1}{(\ln y)^{(2-\lambda+\varepsilon)/q}} dx \right] dy \\
 &= \varepsilon \int_e^\infty \frac{1}{y (\ln y)^{1+\varepsilon}} \left[\int_{1/\ln y}^\infty \frac{1}{(1+u)^\lambda} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
 &= \varepsilon \int_e^\infty \frac{1}{y (\ln y)^{1+\varepsilon}} \left[\int_0^{1/\ln y} \frac{1}{(1+u)^\lambda} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
 &\quad - \varepsilon \int_e^\infty \frac{1}{y (\ln y)^{1+\varepsilon}} \left[\int_0^{1/\ln y} \frac{1}{(1+u)^\lambda} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
 &\geq B \left(\frac{p+\lambda-2}{p} - \frac{\varepsilon}{p}, \frac{q+\lambda-2}{q} + \frac{\varepsilon}{q} \right) \\
 &\quad - \varepsilon \int_e^\infty \frac{1}{y \ln y} \left[\int_0^{1/\ln y} \left(\frac{1}{u} \right)^{(2-\lambda+\varepsilon)/p} du \right] dy \\
 &= B \left(\frac{p+\lambda-2}{p} - \frac{\varepsilon}{p}, \frac{q+\lambda-2}{q} + \frac{\varepsilon}{q} \right) - \frac{\varepsilon}{[1 - (2-\lambda+\varepsilon)/p]^2}. \tag{14}
 \end{aligned}$$

If the constant factor $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ in (3) is not the best possible, then there exists a positive number K (where $K < B((p+\lambda-2)/p, (q+\lambda-2)/q)$), such that (3) is valid if we change $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ to K . In particular, we have

$$\begin{aligned}
 & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{\tilde{a}_m \tilde{b}_n}{m^r n^s (\ln mn)^\lambda} \\
 &< K \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} \tilde{a}_n)^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-s} \tilde{b}_n)^q \right\}^{1/q}. \tag{15}
 \end{aligned}$$

In virtue of (13), (14), and (15), we have

$$B \left(\frac{p+\lambda-2}{p} - \frac{\varepsilon}{p}, \frac{q+\lambda-2}{q} + \frac{\varepsilon}{q} \right) - \frac{\varepsilon}{[1 - (2-\lambda+\varepsilon)/p]^2} < K(\varepsilon+1). \tag{16}$$

Setting $\varepsilon \rightarrow 0^+$, it follows that $B((p+\lambda-2)/p, (q+\lambda-2)/q) \leq K$, which contradicts the fact that $K < B((p+\lambda-2)/p, (q+\lambda-2)/q)$. Hence the constant factor $B((p+\lambda-2)/p, (q+\lambda-2)/q)$ in (3) is the best possible. This proves the theorem. \square

REMARK 2. (i) For $\lambda = r = s = 1$, (3) changes to (1), it follows that (3) is a generalization of (1) with three parameters and (4), (5), and (6) are its some particular results with the best constant factors.

(ii) For $\lambda = 2$, (6) changes to

$$\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{(\ln mn)^2} < \left\{ \sum_{n=2}^{\infty} \frac{1}{n \ln n} (n a_n)^p \right\}^{1/p} \left\{ \sum_{n=2}^{\infty} \frac{1}{n \ln n} (n b_n)^q \right\}^{1/q}, \quad (17)$$

which is a new inequality with a best constant factor 1.

(iii) For $p = q = 2$, (5) and (6) change, respectively, to

$$\begin{aligned} \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s \ln mn} &< \pi \left\{ \sum_{n=2}^{\infty} n^{1-2r} a_n^2 \sum_{n=2}^{\infty} n^{1-2s} b_n^2 \right\}^{1/2}; \\ \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{(\ln mn)^\lambda} &< B\left(\frac{\lambda}{2}, \frac{\lambda}{2}\right) \left\{ \sum_{n=2}^{\infty} n (\ln n)^{1-\lambda} a_n^2 \sum_{n=2}^{\infty} n (\ln n)^{1-\lambda} b_n^2 \right\}^{1/2}, \end{aligned} \quad (18)$$

which are new inequalities similar to (cf. [1, 2])

$$\begin{aligned} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{m^r n^s \ln e^{3/4} mn} &< \pi \left\{ \sum_{n=1}^{\infty} n^{1-2r} a_n^2 \sum_{n=1}^{\infty} n^{1-2s} b_n^2 \right\}^{1/2}; \\ \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{a_m b_n}{(m+n)^\lambda} &< B\left(\frac{\lambda}{2}, \frac{\lambda}{2}\right) \left\{ \sum_{n=1}^{\infty} n^{1-\lambda} a_n^2 \sum_{n=1}^{\infty} n^{1-\lambda} b_n^2 \right\}^{1/2} \quad (0 < \lambda \leq 2). \end{aligned} \quad (19)$$

THEOREM 3. If $p > 1$, $1/p + 1/q = 1$, $\{a_n\}$ is a nonnegative sequence of real numbers, $2 - \min\{p, q\} < \lambda \leq 2$ and $r \in \mathbb{R}$, such that

$$0 < \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p < \infty, \quad (20)$$

then

$$\begin{aligned} \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^\lambda} \right]^p \\ < \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p, \end{aligned} \quad (21)$$

where the constant factor $[B((p+\lambda-2)/p, (q+\lambda-2)/q)]^p$ is the best possible. Equation (21) is equivalent to (3). In particular,

(i) for $r = 1/q$,

$$\begin{aligned} \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^{1/q} (\ln mn)^\lambda} \right]^p \\ < \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} a_n^p; \end{aligned} \quad (22)$$

(ii) for $\lambda = 1$,

$$\sum_{n=2}^{\infty} \frac{1}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r \ln mn} \right]^p < \left[\frac{\pi}{\sin(\pi/p)} \right]^p \sum_{n=2}^{\infty} \frac{1}{n} (n^{1-r} a_n)^p; \quad (23)$$

(iii) for $r = 0$,

$$\begin{aligned} & \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{(\ln mn)^{\lambda}} \right]^p \\ & < \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (na_n)^p. \end{aligned} \quad (24)$$

PROOF. Setting

$$b_n = (\ln n)^{(p-1)(\lambda-1)} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r n^{1/p} (\ln mn)^{\lambda}} \right]^{p-1} \quad (n = 2, 3, \dots), \quad (25)$$

then by (3), for $s = 1/p$, we have

$$\begin{aligned} 0 & < \left[\sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q \right]^p = \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^{\lambda}} \right]^p \right\}^p \\ & = \left[\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^{1/s} (\ln mn)^{\lambda}} \right]^p \\ & \leq \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p \left\{ \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q \right\}^{p-1}. \end{aligned} \quad (26)$$

Hence, we have

$$\begin{aligned} 0 & < \sum_{n=2}^{\infty} (\ln n)^{1-\lambda} b_n^q = \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^{\lambda}} \right]^p \\ & \leq \left[B\left(\frac{p+\lambda-2}{p}, \frac{q+\lambda-2}{q}\right) \right]^p \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-r} a_n)^p < \infty. \end{aligned} \quad (27)$$

In view of (3), neither (26) nor (27) keeps the form of equality. Hence, (21) is valid.

On the other hand, if (21) is valid, by Hölder's inequality, we have

$$\begin{aligned}
 & \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{a_m b_n}{m^r n^s (\ln mn)^\lambda} \\
 &= \sum_{n=2}^{\infty} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r n^{1/p} (\ln mn)^\lambda (\ln n)^{(1-\lambda)/q}} \right] [(\ln n)^{(1-\lambda)/q} n^{1/p-s} b_n] \\
 &\leq \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{(p-1)(\lambda-1)}}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m^r (\ln mn)^\lambda} \right]^p \right\}^{1/p} \\
 &\quad \times \left\{ \sum_{n=2}^{\infty} \frac{(\ln n)^{1-\lambda}}{n} (n^{1-s} b_n)^q \right\}^{1/q}. \tag{28}
 \end{aligned}$$

In view of (21), we have (3). Hence (21) is equivalent to (3). If the constant factor in (21) is not the best possible, then by (28), we can get a contradiction that the constant factor in (3) is not the best possible. This proves the theorem. \square

REMARK 4. For $r = 1$, (23) changes to

$$\sum_{n=2}^{\infty} \frac{1}{n} \left[\sum_{m=2}^{\infty} \frac{a_m}{m \ln mn} \right]^p < \left[\frac{\pi}{\sin(\pi/p)} \right]^p \sum_{n=2}^{\infty} \frac{1}{n} a_n^p, \tag{29}$$

which is equivalent to (1). Both constant factors in (1) and (29) are the best possible.

ACKNOWLEDGMENT. This research was supported by the Faculty Research Council of the University of Texas-Pan American.

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