

COMPLEMENTED SUBSPACES OF p -ADIC SECOND DUAL BANACH SPACES

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ABSTRACT. Let K be a non-archimedean non-trivially valued complete field. In this paper we study Banach spaces over K . Some of main results are as follows:

- (1) The Banach space $BC((l^\infty)_1)$ has an orthocomplemented subspace linearly homeomorphic to c_0 .
- (2) The Banach space $BC((c_0)_1)$ has an orthocomplemented subspace linearly homeomorphic to l^∞ .

KEY WORDS AND PHRASES. non-archimedean valued fields, non-archimedean (p -adic) Banach spaces, polar spaces, spherically complete, complemented subspaces.

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1. INTRODUCTION.

Throughout this paper K is a non-archimedean non-trivially valued complete field with a valuation $|\cdot|$, and E, F are Banach spaces over K with a non-archimedean norm denoted by $\|\cdot\|$. Let $L(E, F)$ be the space consisting of all continuous linear maps of E to F . The dual space of E is $E' = L(E, K)$. The dual operator $T' \in L(F', E')$ of $T \in L(E, F)$ is defined as usual. If there exists a linear isometry from E onto F , then E and F are said to be isomorphic and we denote $E \sim F$. For a Banach space E , if there exists a (ortho)complemented subspace of F which is isomorphic to E , then E is said to be (ortho)complemented in F . Let S be a topological space and let $BC(S)$ be the Banach space consisting of all bounded continuous functions $S \rightarrow K$ with a norm

$$\|f\| = \sup\{|f(s)| : s \in S\} \quad (f \in BC(S)). \quad (1.1)$$

Let E'' be the second dual Banach space of E and let $J_E : E \rightarrow E''$ be the natural map.

DEFINITION. If J_E is linearly homeomorphic from E into E'' , then E is said to be polar (see [6]).

DEFINITION. A Banach space E is said to be strongly polar if every continuous seminorm p on E satisfies the following equality (see [7]).

$$p = \sup\{|f| : f \in E', |f| \leq p\} \quad (1.2)$$

These spaces were first introduced by Schikhof [5] for locally convex topological spaces over K and were studied by some authors (e.g. [1], [2]).

DEFINITION. Let D be a subspace of E . If every $x' \in D'$ has an extension $\bar{x}' \in E'$, then D has the weak extension property in E . In addition, if \bar{x}' can be chosen such that $\|\bar{x}'\| = \|x'\|$, then we say that D has the extension property in E .

For any $r > 0$ we put $E_r = \{x \in E : \|x\| \leq r\}$. Let π denote an arbitrary fixed element of K with $0 < |\pi| < 1$. Other terms will be used as in Rooij [4]. In this paper we deal with complemented subspaces of $BC((E')_1)$ and E'' . Throughout this paper, when we consider a subset $(E')_r$ ($r > 0$) of E' , $(E')_r$ is assumed to have the weak $*$ topology. In section 2 we show that there exists a Banach space E such that $BC((l^\infty)_1)$ is linearly homeomorphic to $c_0 \oplus E$. And in section 3, we show that there exists a Banach space F such that $BC((c_0)_1)$ is linearly homeomorphic to $l^\infty \oplus F$.

2. COMPLEMENTED SUBSPACES OF $BC(S)$.

For every $T \in L(E, BC(S))$, for every $s \in S$ and for every $x \in E$, let

$$(\psi_T(s))(x) = (T(x))(s). \quad (2.1)$$

Then the map $\psi_T(s)$ is a linear functional on E . Since $\|\psi_T(s)\| \leq \|T\|$, $\psi_T(s) \in (E')_{\|T\|}$.

Hence ψ_T is a weak * continuous map from S to $(E')_{\|T\|}$. Conversely, for every weak * continuous map $\psi : S \rightarrow (E')_r$ ($r > 0$), let

$$(T_\psi(x))(s) = (\psi(s))(x) \quad (x \in E, s \in S). \quad (2.2)$$

Then $T_\psi(x)$ is a map from S to K . Since for each $x \in E$

$$\sup\{ |(T_\psi(x))(s)| : s \in S \} \leq r \|x\|, \quad (2.3)$$

$T_\psi(x) \in BC(S)$. Hence T_ψ is a linear map from E to $BC(S)$. By (2.3), $\|T_\psi\| \leq r$. It follows that $T_\psi \in L(E, BC(S))$.

For the natural map $J_E : E \rightarrow E''$ and for every $x \in E$, let $R_E(x)$ denote the restriction of $J_E(x)$ to $(E')_1$, that is,

$$R_E(x) = J_E(x)|_{(E')_1}. \quad (2.4)$$

Then R_E is a linear map from E into $BC((E')_1)$. Since for every $x \in E$

$$\begin{aligned} \|R_E(x)\| &= \sup\{ |(R_E(x))(x')| : x' \in (E')_1 \} \\ &\leq \sup\{ \|x'\| \|x\| : x' \in (E')_1 \} \\ &\leq \|x\|, \end{aligned} \quad (2.5)$$

we have $\|R_E\| \leq 1$ and $R_E \in L(E, BC((E')_1))$.

The next theorem follows from Schikhof [7].

THEOREM 1. Let E be a strongly polar Banach space and let D be a closed subspace of E . Then for each $\epsilon > 0$, each $f \in D'$ can be extended to an $\bar{f} \in E'$ with $|\bar{f}(x)| \leq (1+\epsilon)\|f\|\|x\|$ ($x \in E$).

A norm $\|\cdot\|_p$ on E is said to be polar if

$$\|\cdot\|_p = \sup\{ |f| : f \in E', |f| \leq \|\cdot\|_p \}. \quad (2.6)$$

We recall that if E is polar, then there exists a polar norm $\|\cdot\|_p$ on E such that it is equivalent to the original norm $\|\cdot\|$ (see [1, p.75]), and so there exists a real number d ($d \geq 1$) such that for every $x \in E$ $\|x\| \leq \|x\|_p \leq d\|x\|$.

THEOREM 2. Let E be a polar Banach space. Then there exists a real number c ($c > 1$) satisfying the following (1) and (2).

- (1) For each finite-dimensional subspace D of E and for each $f \in D'$ there exists an extension $\bar{f} \in E'$ such that $\|\bar{f}\| \leq c\|f\|$.
- (2) For each finite-dimensional subspace D of E there exists a projection $P : E \rightarrow D$ with $\|P\| \leq c$.

PROOF. (1) Since $f \in D'$, it is trivial that $f \in (D, \|\cdot\|_p)'$. Let $\epsilon > 0$ be an arbitrarily given real number and put $c = (1+\epsilon)d$. By Theorem 2.1 in Garcia [1], there exists an extension $\bar{f} \in (E, \|\cdot\|_p)'$ such that $\|\bar{f}\|_p \leq (1+\epsilon)\|f\|_p$. Then we have that $\|\bar{f}\|/d \leq (1+\epsilon)\|f\|$.

(2) Using again Theorem 2.1 in [1], there exists a projection $P : E \rightarrow D$ such that $\|P\|_p \leq 1+\epsilon$. It follows that $\|P\| \leq d\|P\|_p \leq c$.

THEOREM 3. If E is a polar space, then R_E is a linear homeomorphism. And if the norm on E is polar, then R_E is a linear isometry.

PROOF. In section 1, it is proved that for all $x \in E$

$$\|R_E(x)\| \leq \|x\|. \quad (2.7)$$

Note that for every $x' \in E'$, $x' \neq 0$, there exists an integer m with $|\pi|^{m+1} \leq \|x'\| \leq |\pi|^m$, then

$$\begin{aligned} |\pi| \frac{|x'(x)|}{\|x'\|} &\leq |\pi|^{-m} |x'(x)| = |(\pi^{-m} x')(x)| \\ &= |(R_E(x))(\pi^{-m} x')| \leq \|R_E(x)\|. \end{aligned} \quad (2.8)$$

From (2.7) and (2.8) it follows that

$$|\pi| \|J_E(x)\| \leq \|R_E(x)\| \leq \|x\|. \quad (2.9)$$

Since E is polar, J_E is a homeomorphism, so is R_E . Next, if the norm $\|\cdot\|$ of E is polar, then for all $x \in E$ we have

$$\begin{aligned} \|x\| &= \sup\{|x'(x)| : x' \in E', \|x'\| \leq 1\} \\ &= \sup\{|x'(x)| : x' \in (E')_1\} = \|R_E(x)\|. \end{aligned} \quad (2.10)$$

Therefore R_E is an isometry.

COROLLARY 4. (1) For any strongly polar space E , R_E is a linear isometry.

(2) For any topological space S , $R_{BC(S)}$ is a linear isometry.

THEOREM 5. For every $T \in L(E, BC(S))$, there exists a $\bar{T} \in L(BC((E')_1), BC(S))$ such that $\bar{T} \circ R_E = T$. In particular, if $\|T\| = 1$, then \bar{T} satisfies $\|\bar{T}\| = 1$.

PROOF. At first, we notice that $(E')_1$ is supposed to carry the weak $*$ topology. To show theorem, we may assume that $\|T\| \leq 1$. Then ψ_T is a weak $*$ continuous map from S into $(E')_1$. Define

$$\bar{T} : BC((E')_1) \rightarrow BC(S), \quad (2.11)$$

by

$$\bar{T}(f) = f \circ \psi_T \quad (f \in BC((E')_1)). \quad (2.12)$$

For every $x \in E$ and for every $s \in S$, we have

$$(T(R_E(x)))(s) = (R_E(x))(\psi_T(s)) = (\psi_T(s))(x) = (T(x))(s). \quad (2.13)$$

Then $\bar{T} \circ R_E = T$. Further,

$$\begin{aligned} \|\bar{T}\| &= \sup\left\{\frac{\sup\{|f(\psi_T(s))| : s \in S\}}{\|f\|} : f \in BC((E')_1)\right\} \\ &\leq 1. \end{aligned} \quad (2.14)$$

Hence if $\|T\| = 1$, then

$$1 = \|T\| \leq \|\bar{T} \circ R_E\| \leq \|\bar{T}\| \|R_E\| \leq \|\bar{T}\| \leq 1. \quad (2.15)$$

The proof is complete.

LEMMA 6. Let E , F and X be Banach spaces. Let $A : E \rightarrow X$ be a linear homeomorphism onto X and $H : E \rightarrow F$ be a linear homeomorphism into F . If there exists an $\bar{A} \in L(F, X)$ such that $\bar{A} \circ H = A$, then the closed subspace $H(E)$ of F is complemented. In particular, if A and H are linear isometries and $\|\bar{A}\| = 1$, then E is orthocomplemented in F .

PROOF. Put $P = H \circ A^{-1} \circ \bar{A} : F \rightarrow H(E) \subset F$. Then P is a projection onto $H(E)$. If A and H are linear isometries and $\|\bar{A}\| = 1$, then $\|P\| \leq 1$. Hence P is an orthoprojection.

THEOREM 7. Let E be of countable type. Then $R_E(E)$ is complemented in $BC((E')_1)$. Especially, c_0 is orthocomplemented in $BC((l^\infty)_1)$.

PROOF. If E is finite-dimensional, then the assertion of this theorem is clear. Hence we may assume E is infinite-dimensional. Since E is of countable type, E is a polar space. Then by Theorem 3 the map $R_E : E \rightarrow BC((E')_1)$ is a linear homeomorphism into $BC((E')_1)$. Further, since E is infinite-dimensional, for an infinite compact ultrametrizable space S , E is linearly homeomorphic to $BC(S)$ (see [4, p.190]). Let $H_0 : E \rightarrow BC(S)$ be a linear homeomorphism onto $BC(S)$. By Theorem 5, there exists an $\bar{H}_0 \in L(BC((E')_1), BC(S))$ such that $\bar{H}_0 \circ R_E = H_0$. Hence by Lemma 6, $R_E(E)$ is complemented in $BC((E')_1)$. If $E = c_0$, then the above H_0 can be taken as a linear isometric from c_0 onto $BC(S)$. Since c_0 is strongly polar, by Corollary 4, the map R_{c_0} is linearly isometric. Hence by Theorem 5, there exists an $\bar{H}_0 \in L(BC(((c_0)')_1), BC(S))$ with $\|\bar{H}_0\| = 1$. Thus, by Lemma 6, $R_{c_0}(c_0)$ is orthocomplemented in $BC(((c_0)')_1)$. Since $(c_0)' \sim 1^\infty$, $BC(((c_0)')_1) \sim BC((1^\infty)_1)$. Hence c_0 is orthocomplemented in $BC((1^\infty)_1)$.

The following corollary follows immediately from Theorem 7.

COROLLARY 8. Let E be of countable type. Then there exists a Banach space X such that $BC((1^\infty)_1)$ and $E \oplus X$ are linearly homeomorphic.

Since c_0 is linearly isometric to some $BC(S)$, the second part of Theorem 7 is a special case of the following corollary.

COROLLARY 9. For any topological space S , let $E = BC(S)$. Then E is orthocomplemented in $BC((E')_1)$.

PROOF. Let $I : E \rightarrow BC(S)$ be the identity. Then there exists an $\bar{I} \in L(BC((E')_1), BC(S))$ such that $\bar{I} \circ R_E = I$ and $\|\bar{I}\| = \|I\| = 1$. By Corollary 4, $R_E : E \rightarrow BC((E')_1)$ is linearly isometric. Put $P = R_E \circ I^{-1} \circ \bar{I}$. Then P is an orthoprojection of $BC((E')_1)$ onto $R_E(E)$. Hence E is orthocomplemented in $BC((E')_1)$.

COROLLARY 10. The Banach space $BC((c_0)_1)$ contains an orthocomplemented subspace linearly homeomorphic to 1^∞ . In particular if K is spherically complete, then the Banach space $BC((c_0)_1)$ contains an orthocomplemented subspace linearly isometric to 1^∞ .

PROOF. Suppose that K is not spherically complete. Applying the extended version of Corollary 9 to $S = \mathbb{N}$ (\mathbb{N} denotes the set of all natural numbers) and observing that $E = 1^\infty$ and $E' = c_0$, we can obtain this corollary. Furthermore, if K is spherically complete, then so is 1^∞ ; it follows easily that the second part holds.

3. COMPLEMENTED SUBSPACES IN SECOND DUAL SPACES.

Let $T \in L(E, F')$. Then T determines a map

$$\phi_T : F \rightarrow E' \quad (3.1)$$

defined by $(\phi_T(y))(x) = (T(x))(y)$ ($x \in E, y \in F$). Clearly, ϕ_T is linear and $\|\phi_T\| \leq \|T\|$.

Hence $\phi_T \in L(F, E')$. Let D be a closed subspace and let D^\perp be the annihilator of D in F' , i.e. $D^\perp = \{x' \in F' : x'(d) = 0, d \in D\}$. A subset A of E is said to be compactoid if for every $\epsilon > 0$, there exists a finite subset X of E such that $A \subset B_\epsilon + \text{Co}(X)$, where $B_\epsilon = \{x \in E : \|x\| \leq \epsilon\}$ and $\text{Co}(X)$ is the absolutely convex hull of X . Let $T \in L(E, F)$. If $T(E_1)$ is compactoid in F , then T is said to be compact. A Banach space E is said to be (0) -space if every $T \in L(E, c_0)$ is compact.

PROPOSITION 11. Let E, F be Banach spaces and let D be a closed subspace of F . Then for every $T \in L(E, D^\perp)$, there exists a $\bar{T} \in L(E'', D^\perp)$ such that $\bar{T} \circ J_E = T$ and $\|\bar{T}\| = \|T\|$.

PROOF. Let $J_{E'} : E' \rightarrow E''$ be the canonical map. Define an operator

$$\bar{T} : E'' \rightarrow D^\perp \quad (3.2)$$

by $(\bar{T}(x''))(y) = (J_E, (\phi_T(y))(x''))$ ($y \in F$, $x'' \in E''$). For every $x'' \in E''$, $\bar{T}(x'')$ is a linear functional on F and $\|\bar{T}(x'')\| \leq \|T\| \|x''\|$, so $\bar{T}(x'') \in F'$. For every $y \in D$ and for every $x \in E$,

$$(\phi_T(y))(x) = (T(x))(y) = 0. \quad (3.3)$$

Hence $(\bar{T}(x''))(y) = 0$. This means that $\bar{T}(x'') \in D^\perp$. It follows that $\bar{T} \in L(E'', D^\perp)$ and $\|\bar{T}\| \leq \|T\|$. Further, for every $x \in E$ and for every $y \in F$,

$$\begin{aligned} ((\bar{T} \circ J_E)(x))(y) &= (J_E, (\phi_T(y))(J_E(x))) \\ &= (J_E(x))(\phi_T(y)) \\ &= (\phi_T(y))(x) \\ &= (T(x))(y). \end{aligned} \quad (3.4)$$

Hence $\bar{T} \circ J_E = T$. Therefore we have

$$\|T\| \leq \|\bar{T}\| \|J_E\| \leq \|\bar{T}\|. \quad (3.5)$$

Thus we complete the proof.

The following corollary is immediate from Proposition 11.

COROLLARY 12. Let E and F be Banach spaces. For every $T \in L(E, F')$, there exists a $\bar{T} \in L(E'', F')$ such that $\bar{T} \circ J_E = T$ and $\|\bar{T}\| = \|T\|$.

PROOF. In Proposition 11, put $D = \{0\}$. Then $D^\perp = F'$.

PROPOSITION 13. Let E be a Banach space and let D be a closed subspace of E . If D is linearly homeomorphic (resp. isometric) to some dual space and is complemented (resp. orthocomplemented) in E , then $J_E(D)$ is complemented (resp. orthocomplemented) in E'' . In particular, if K is not spherically complete and D is of countable type and complemented in E , then $J_E(D)$ is complemented in E'' .

PROOF. Let D be a complemented closed subspace of E , linearly homeomorphic to a dual Banach space F' . By Lemma 4.23, (ii) and (iii), in Rooij [4], J_D is a homeomorphism and there exists a projection of D'' onto $J_D(D)$, so there is a $Q \in L(D'', D)$ with $Q \circ J_D = I_D$ (= the identity map of D). As D is complemented in E , there is a projection $P : E \rightarrow D$. Then $J_E \circ Q \circ P'' \in L(E'', J_E(D))$. As

$$\begin{aligned} (Q \circ P'') \circ J_E &= Q \circ (P'' \circ J_E) = Q \circ (J_D \circ P) \\ &= (Q \circ J_D) \circ P = I_D \circ P = P, \end{aligned} \quad (3.6)$$

for $x \in D$ we have

$$(J_E \circ Q \circ P'')(J_E(x)) = J_E(P(x)) = J_E(x), \quad (3.7)$$

so $J_E \circ Q \circ P''$ is the identity on $J_E(D)$. Thus $J_E \circ Q \circ P''$ is a projection of E'' onto $J_E(D)$. If D is orthocomplemented in E'' and linearly isometric to F' , we obtain $\|Q\| \leq 1$ and $\|P\| \leq 1$, whence $\|J_E \circ Q \circ P''\| \leq 1$. In particular, if K is not spherically complete and D is of countable type, then D is linearly homeomorphic to $(l^\infty)'$ or K^n , where n is some positive integer. Hence by the first assertion of this proposition, we can complete the proof.

COROLLARY 14. Suppose K is not spherically complete. Let E be an infinite-dimensional polar space which is not a (0)-space and let F be an infinite-dimensional Banach space of countable type. Then there exists a Banach space X such that E'' is linearly homeomorphic to $F \oplus X$.

PROOF. By hypothesis, there exists an infinite-dimensional complemented subspace D of E which is of countable type (see [6, p.23]). It follows from Proposition 13 that there exists a subspace X of E'' such that $E'' = J_E(D) \oplus X$. Since E is a polar space, J_E is a linear homeomorphism. Therefore, $J_E(D)$ is of countable type. Hence $J_E(D)$ and F are linearly homeomorphic, so E'' is linearly homeomorphic to $F \oplus X$.

COROLLARY 15. The subspace $J_E(E)$ of E'' has the extension property in E'' .

PROOF. For every continuous linear $x' : J_E(E) \rightarrow K$ the function $\bar{x}' = J_E(x' \circ J_E)$ is a continuous linear function $E'' \rightarrow K$ extending x' and with $\|\bar{x}'\| \leq \|x'\|$, hence $\|\bar{x}'\| = \|x'\|$.

The following comment was given by the referee: From the proof of Corollary 15 we obtain a sort of "simultaneous extension", a linear isometry $x' \mapsto \bar{x}'$ of $(J_E(E))'$ onto E'' that assigns to every continuous linear function $J_E(E) \rightarrow K$ an extension $E'' \rightarrow K$. Further, the following question was asked by him: Under what circumstances is there an orthoprojection of E'' onto (the closure of) $J_E(E)$?

COROLLARY 16. Let D be a closed subspace of E . If J_D has an extension T from E into D'' . Then D has the weak extension property in E . In particular, if $\|T\| = \|J_D\|$, then D has the extension property in E .

PROOF. By Corollary 12, for every $f \in D'$, there exists an $\bar{f} \in D''$ such that $\bar{f} \circ J_D = f$ and $\|\bar{f}\| = \|f\|$. Put $g = \bar{f} \circ T$. Then $g \in E'$ and $g|_D = f$. Hence D has the weak extension property in E . If $\|T\| = \|J_D\|$, then by Corollary 12, for every $x \in E$

$$\begin{aligned} |g(x)| &= |(\bar{f} \circ T)(x)| \leq \|\bar{f}\| \|T\| \|x\| \\ &= \|\bar{f}\| \|J_D\| \|x\| \leq \|\bar{f}\| \|x\|. \end{aligned} \quad (3.8)$$

Hence it holds that $\|g\| \leq \|\bar{f}\| = \|f\| \leq \|g\|$.

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