

COINCIDENCE POINTS IN UNIFORM SPACES

T. KUBIAK

Institute of Mathematics
A. Mickiewicz University
Matejki 48/49
60-769 Poznan, POLAND

and

Y.J. CHO

Department of Mathematics
Gyeongsang National University
Jinju 660-701, KOREA

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ABSTRACT. In this note, we give a coincidence point theorem in sequentially complete Hausdorff uniform spaces. Our result reduces to a result of Acharya [1].

KEY WORDS AND PHRASES. Uniformity, sequentially Hausdorff uniform spaces, fixed points, and coincidence points.

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1. INTRODUCTION.

Let (X, \mathfrak{U}) be a uniform space and let \mathbb{N} denote the set of all natural numbers. If p is a pseudo-metric on X , we let

$$V_{(p,r)} = \{(x,y) \in X \times X : p(x,y) < r\} (r > 0).$$

For a family P of pseudo-metrics on X generating the uniformity \mathfrak{U} ,

$$\mathfrak{G} = \{ \bigcap_{i=1}^n V_{(p_i, r_i)} : p_i \in P, r_i > 0, n \in \mathbb{N} \}$$

is a base of \mathfrak{U} .

If $V = \bigcap_{i=1}^n V_{(p_i, r_i)}$ and $k > 0$, define $kV = \bigcap_{i=1}^n V_{(p_i, kr_i)}$. If $Y \subset X$, then $\mathfrak{U}|Y = \{U^Y : U \in \mathfrak{U}\}$ is a uniformity on Y , where $U^Y = U \cap (Y \times Y)$. Moreover, $\mathfrak{G}|Y = \{V^Y : V \in \mathfrak{G}\}$ is a base for $\mathfrak{U}|Y$. For more details, see [1] and [2]. We need the following lemma and a theorem of Acharya ([1]) for our main theorem:

LEMMA 1.1. Let $Y \subset X, V \in \mathfrak{G}$ and $\alpha, \beta > 0$. Then we have the following :

- (1) $\alpha(\beta V) = (\alpha\beta)V$.
- (2) $\alpha V \circ \beta V \subset (\alpha + \beta)V$.
- (3) $\alpha V \subset \beta V$ if $\alpha < \beta$.
- (4) $(x,y) \in \alpha V_{(p,r_1)} \circ \beta V_{(p,r_2)}$ implies $p(x,y) < \alpha r_1 + \beta r_2$.
- (5) $\alpha V^Y = (\alpha V)^Y$.

Recall also that for each $V \in \mathfrak{G}$ there exists a pseudo-metric p (called the Minkowski pseudo-metric of V such that $V = V_{(p,1)}$.

THEOREM 1.1. Let (X, \mathfrak{U}) be a sequentially complete Hausdorff uniform space and let $f: X \rightarrow X$ be a mapping. Let $0 < \alpha < 1$. If for every $V \in \mathfrak{G}$ and $x, y \in X$,

$$(fx, fy) \in \alpha V \text{ if } (x, y) \in V, \quad (1.1)$$

then f has a unique fixed point in X .

2. MAIN RESULTS.

Now, by using Lemma 1.1 and Theorem 1.1 we give our main theorem:

THEOREM 2.1. Let (X, \mathfrak{U}) be a uniform space. Let A be a set in $X, S, T: A \rightarrow X$ be mappings and $S(A) \subset T(A) = Y$. If $(Y, \mathfrak{U}|_Y)$ is a sequentially complete Hausdorff uniform space, $0 < \alpha < 1$, and

$$(Sx, Sy) \in \alpha V \quad \text{if } (Tx, Ty) \in V \quad (2.1)$$

for every $V \in \mathfrak{G}$ and $x, y \in A$, then there exists a point $u \in A$ such that $Tu = Su$.

PROOF. As in [3], define $fa = S(T^{-1}\{a\})$ for $a \in Y$. Then f is a well-defined self-mapping of Y . For if $b_1, b_2 \in fa$, there exist $x_1, x_2 \in T^{-1}\{a\}$ such that $b_1 = Sx_1$ and $b_2 = Sx_2$. By (2.1), for any $V \in \mathfrak{G}$ we have $(b_1, b_2) = (Sx_1, Sx_2) \in \alpha V$ if $(Tx_1, Tx_2) \in V$. and since $(Tx_1, Tx_2) = (a, a) \in V$, it follows that $(b_1, b_2) \in \alpha V \subset V$. Since $S(A) = Y$, we have $(b_1, b_2) \in V^Y$ for each $V \in \mathfrak{G}$. Thus, since $(Y, \mathfrak{U}|_Y)$ is Hausdorff, it follows that $b_1 = b_2$.

Now, let $a, b \in Y$, $x \in T^{-1}\{a\}$ and $y \in T^{-1}\{b\}$. By (2.1), $(a, b) = (Tx, Ty) \in V$ implies $(fa, fb) = (Sx, Sy) \in \alpha V$ for every $V \in \mathfrak{G}$. Hence, by Lemma 1.1(5), for each $a, b \in Y$ and $V \in \mathfrak{G}$, $(a, b) \in V^Y$ implies $(fa, fb) \in (\alpha V)^Y = \alpha V^Y$. Thus the mapping $f: Y \rightarrow Y$ satisfies all the conditions of Theorem 2.1. Consequently, there exists a unique point $a \in Y$ with $fa = a$. Therefore, for each $u \in T^{-1}\{a\}$, we have $Su = S(T^{-1}\{a\}) = fa = a = Tu$, that is, u is a coincidence point of S and T . This completes the proof.

REMARK. If we put $A = X$ and $T = id_X$ (: the identity mapping on X), then Theorem 2.1 reduces to Theorem 1.1.

REFERENCES

1. ACHARYA, S.P. Some results on fixed points in uniform spaces, Yokohama Math. J. **22** (1974), 105-116.
2. CSASZAR, A. General Topology, Akademiai Kiado, Budapest, 1978.
3. GOEBEL, K. A coincidence theorem, Bull. Acad. Pol. Math. **16** (1968), 733-735.

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K. K. Lai, Department of Management Sciences, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong; mskkklai@cityu.edu.hk