

A GENERALIZATION OF AN INEQUALITY OF ZYGMUND

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ABSTRACT. The well known Bernstein inequality states that if D is a disk centered at the origin with radius R and if $p(z)$ is a polynomial of degree n , then $\max_{z \in D} |p'(z)| \leq \frac{n}{R} \max_{z \in D} |p(z)|$ with equality iff $p(z) = AZ^n$. However it is true that we have the following better inequality:

$$\max_{z \in D} |p'(z)| \leq \frac{n}{R} \max_{z \in D} |\operatorname{Re} p(z)|$$

with equality iff $p(z) = AZ^n$.

This is a consequence of a general equality that appears in Zygmund [7] (and which is due to Bernstein and Szegő): For any polynomial $p(z)$ of degree n and for any $1 \leq p < \infty$ we have

$$\left\{ \int_0^{2\pi} |p'(e^{ix})|^p dx \right\}^{1/p} \leq A_p n \left\{ \int_0^{2\pi} |\operatorname{Re} p(e^{ix})|^p dx \right\}^{1/p}$$

where $A_p = \pi^{1/2} \frac{\Gamma(\frac{1}{2}p + 1)}{\Gamma(\frac{1}{2}p + \frac{1}{2})}$ with equality iff $p(z) = AZ^n$.

In this note we generalize the last result to domains different from Euclidean disks by showing the following: If $g(e^{ix})$ is differentiable and if $p(z)$ is a polynomial of degree n then for any $1 \leq p < \infty$ we have

$$\left\{ \int_0^{2\pi} |g(e^{i\theta})p'(g(e^{i\theta}))|^p d\theta \right\}^{1/p} \leq A_p n \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re}(p(e^{i\beta}g(e^{i\theta})))|^p d\theta \right\}^{1/p}$$

with equality iff $p(z) = Az^n$.

We then obtain some conclusions for Schlicht Functions.

Key Words and Phrases: Bernstein inequality, Bernstein-Szegő inequality, Krzyz problem, Dirichlet kernel, trigonometric interpolation

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1. INTRODUCTION.

The classical result of Bernstein as it appears in [2] is Bernstein Inequality. If D is a Euclidean disk and P is a polynomial of degree n over \mathbb{C} , then

$$\|P'\|_D \leq \frac{n}{\text{tr}(D)} \|P\|_D \quad (1)$$

where $\|f\|_D = \sup_D |f(z)|$ and $\text{tr}(D)$ is the transfinite diameter of D (which is the disk's radius in this case).

This result was generalized to various directions. The following theorem appears in [1]. Let $0 \leq k \leq 1$ and let E be a closed k -quasidisk, then

THEOREM. For any polynomial P of degree n we have

$$\left| \frac{p(z_1) - p(z_2)}{z_1 - z_2} \right| \leq c_1 \frac{n^{1+k}}{\text{tr}(E)} \|p\|_E, \quad z_1, z_2 \in E \quad (2)$$

and

$$\|p'\|_E \leq c_2 \frac{n^{1+k}}{\text{tr}(E)} \|p\|_E \quad (3)$$

where $c_1 = 2^{-k} e^{\left(\frac{\pi}{4} + 1\right)}$ and $c_2 = 2^{-k} e$.

Another direction of generalization arises naturally in the following:

Let β be the class of all analytic functions $f(z) = \sum_{k=0}^{\infty} a_k z^k$ in $|z| < 1$ such that $0 < |f(z)| < 1$. A problem posed by Krzyz [4] is to determine $A_n = \max_{\beta} |a_n|$, $n \geq 1$ [3]. The conjecture (which is still unsolved) is that $A_n = \frac{2}{e}$ and that it is attained only by rotations of

$$g_n(z) = \exp \left(- \frac{z^{n+1} - 1}{z^{n+1}} \right).$$

Let $f(z)$ be an extremal function for A_n .

CONJECTURE. $|f(0)| \leq \frac{1}{e}$ and equality holds only for rotations of g_n .

A theorem which indicates that this conjecture may be true is:

THEOREM [5]. If $n = 2p + 1$ and if $a_1 = a_3 = \dots = a_{2p-1} = 0$, then $|a_0| \leq \frac{1}{e}$. Equality sign occurs iff $|a_n| = \frac{2}{e}$

The proof of this uses the following generalization of (1): Let $D(0,1) = \{z \in \mathbb{C} \mid |z| < 1\}$ and let p be any polynomial of degree n over \mathbb{C} , then

$$\|p'\|_{D(0,1)} \leq n \| \text{Re } p \|_{D(0,1)} \quad (4)$$

This follows from an inequality of Zygmund [7].

THEOREM. For any polynomial p of degree n and for any $1 \leq p < \infty$ we have

$$\left\{ \int_0^{2\pi} |p'(e^{ix})|^p dx \right\}^{1/p} \leq A_p n \left\{ \int_0^{2\pi} |\operatorname{Re} p(e^{ix})|^p dx \right\}^{1/p} \quad (5)$$

where

$$A_p = \pi^{1/2} \frac{\Gamma(\frac{1}{2}p + 1)}{\Gamma(\frac{1}{2}p + \frac{1}{2})} \quad (6)$$

and equality occurs in (5) iff $p(z) = Az^n$.

In this note we indicate a way to generalize (5) to domains E other than $D(0,1)$ by using the same ideas as in Zygmund's proof applied to $p \circ g$ where g is a quite general mapping $D(0,1) \rightarrow E$.

2. RESULTS.

THEOREM 1. Let g be a complex valued function of e^{ix} , $0 \leq x \leq 2\pi$. Suppose that $\{\arg g(e^{ix}) | 0 \leq x \leq 2\pi\} \supseteq [0, 2\pi/n]$ and that $\frac{dg(e^{ix})}{dx}$ exists, then for any non-negative, non-decreasing convex function χ , for any $\alpha \in \mathbb{R}$ and for any polynomial P of degree n over \mathbb{C} we have

$$\int_0^{2\pi} \chi \left(n^{-1} |\operatorname{Im} \{ e^{i\alpha} g(e^{i\theta}) P'(g(e^{i\theta})) \}| \right) d\theta \leq \max_{\beta} \left\{ \int_0^{2\pi} \chi \left(|\operatorname{Re} \{ P(e^{i\beta} g(e^{i\theta})) \}| \right) d\theta \right\} \quad (7)$$

equality occurs in (7) iff $p(z) = Az$.

We remark that the consequences of Theorem 1 hold true even if the condition

$$\{\arg g(e^{ix}) | 0 \leq x \leq 2\pi\} \supseteq [0, 2\pi/n]$$

is dropped.

We will indicate at the end of Section 4 how to prove that.

With the notations of Theorem 1 we have

THEOREM 2. If $1 \leq p < \infty$, then

$$\left\{ \int_0^{2\pi} |g(e^{i\theta}) P'(g(e^{i\theta}))|^p d\theta \right\}^{1/p} \leq A_p n \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re} \{ P(e^{i\beta} g(e^{i\theta})) \}|^p d\theta \right\}^{1/p} \quad (8)$$

with equality iff $p(z) = Az^n$.

As a consequence we derive an analogous theorem to (1),

THEOREM 3. If E is a simply connected domain such that $0 \in E$, and if $G : D(0,1) \rightarrow E$ is a Riemann mapping normalized by $G(0) = 0$, then for every $1 \leq p < \infty$ and every $0 \leq r < 1$ we have

$$\left\{ \int_0^{2\pi} |P'(G(re^{i\theta}))|^p d\theta \right\}^{1/p} \leq \frac{4 A_p n}{r |G'(0)|} \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re} \{ P(e^{i\beta} G(re^{i\theta})) \}|^p d\theta \right\}^{1/p} \quad (9)$$

This last inequality is not sharp.

Returning to the function g of Theorem 1 we add

COROLLARY.

$$\max_{\alpha} \left\{ \int_0^{2\pi} \chi \left(\left| \operatorname{Im} \{ e^{i\alpha} g(e^{i\theta}) \} \right| \right) d\theta \right\} = \max_{\beta} \left\{ \int_0^{2\pi} \chi \left(\left| \operatorname{Re} \{ e^{i\beta} g(e^{i\theta}) \} \right| \right) d\theta \right\} \quad (10)$$

$$\left\{ \int_0^{2\pi} |g(e^{i\theta})|^p d\theta \right\}^{1/p} \leq A_p \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re} \{ e^{i\beta} g(e^{i\theta}) \}|^p d\theta \right\}^{1/p} \quad (11)$$

The last corollary can be seen directly, but, it shows that we cannot drop "max" on the right hand of the above inequalities since it is easy to find a g such that $\| \operatorname{Re} g \|_p \leq 1$ while $\lim_{p \rightarrow \infty} \|g\|_p = \infty$.

3. PREPARATIONS.

Let $p(z) = c_0 + c_1 z + \dots + c_n z^n$ be a polynomial of degree n , where $c_0 \in \mathbb{R}$. We denote

$$S(z) = \frac{1}{2}(p(z) + \overline{p(\overline{z})}), \quad \tilde{S}(z) = \frac{1}{2i}(p(z) - \overline{p(\overline{z})}) \quad (12)$$

Let g be a complex valued function of e^{ix} , $x \in \mathbb{R}$ such that

$\{\arg g(e^{ix}) | 0 \leq x \leq 2\pi\} \supseteq [0, \frac{2\pi}{n}]$ and such that $\frac{dg}{dx}(e^{ix})$ exists. We denote

$$g(e^{ix}) = R(x)e^{i\phi(x)}, \quad R(x) = |g(e^{ix})|, \quad \phi(x) = \arg g(e^{ix}) \quad (13)$$

$$S(x, t) = c_0 + \sum_{\nu=1}^n R^{\nu}(x)(a_{\nu} \cos \nu t + b_{\nu} \sin \nu t) \quad (14)$$

$$\tilde{S}(x, t) = \sum_{\nu=1}^n R^{\nu}(x)(a_{\nu} \sin \nu t - b_{\nu} \cos \nu t)$$

where $c_0, a_1, \dots, a_n, b_1, \dots, b_n \in \mathbb{R}$,

where the coefficients a, b are such that

$$S(x, \phi(x)) = S(g(e^{ix})), \quad \tilde{S}(x, \phi(x)) = \tilde{S}(g(e^{ix})). \quad (15)$$

As in Zygmund we denote the modified Dirichlet kernel and its conjugate kernel by $D_n^*(u)$, $\tilde{D}_n^*(u)$ respectively. Thus

$$D_n^*(u) = \frac{1}{2} \sum_{\nu=1}^{n-1} \cos \nu u + \frac{1}{2} \cos nu = \frac{\sin nu}{2 \tan \frac{1}{2} u} \quad (16)$$

$$\tilde{D}_n^*(u) = \sum_{\nu=1}^{n-1} \sin \nu u + \frac{1}{2} \sin nu = (1 - \cos nu) \frac{1}{2} \cot \frac{1}{2} u.$$

We will also need the zeros of $\cos nt$

$$u_\nu = (2\nu-1)\pi/2n, \quad \nu = 1, 2, \dots, 2n \quad (17)$$

$\phi_{2n}(t)$ will be a step function which has jumps $\frac{\pi}{n}$ at the points u_ν . By (3.6), (3.21) on pages 10, 11 [7] we have

THEOREM (Zygmund)

$$S(x, u) = a_n R^n(x) \cos nu + \frac{1}{\pi} \int_0^{2\pi} S(x, t) D_n^*(t-u) d\phi_{2n}(t) \quad (18)$$

$$\tilde{S}(x, u) = a_n R^n(x) \sin nu + \frac{1}{\pi} \int_0^{2\pi} S(x, t) \tilde{D}_n^*(t-u) d\phi_{2n}(t)$$

Thus for any real number α we have

$$\begin{aligned} S(g(e^{ix})) \cos \alpha - \tilde{S}(g(e^{ix})) \sin \alpha &= a_n R^n(x) \cos[n\phi(x) + \alpha] + \\ &+ \frac{1}{\pi} \int_0^{2\pi} S(x, t) \left\{ \frac{\sin[n(\phi(x)-t) + \alpha] - \sin \alpha}{2 \tan \frac{1}{2}(\phi(x) - t)} \right\} d\phi_{2n}(t) \end{aligned} \quad (19)$$

4. A PROOF OF THEOREM 1.

As in Zygmund, let x_0 be a root of $\sin[n\phi(x) + \alpha]$ such that $\cos[n\phi(x_0) + \alpha] = 1$. We differentiate (19) with respect to x and substitute $x = x_0$. By (12) we have

$$\frac{dS}{dx}(g(e^{ix})) = -\operatorname{Im} \left\{ e^{ix} g'(e^{ix}) p' \left(g(e^{ix}) \right) \right\} \quad (20)$$

$$\frac{d\tilde{S}}{dx}(g(e^{ix})) = \operatorname{Re} \left\{ e^{ix} g'(e^{ix}) p' \left(g(e^{ix}) \right) \right\}$$

This takes care of the left hand side of (19). On the right hand side we first differentiate $R(x)$ and use:

$$\frac{R'(x)}{R(x)} = -\operatorname{Im} \left\{ \frac{e^{ix} g'(e^{ix})}{g(e^{ix})} \right\},$$

$$\frac{\partial}{\partial t} \{ \tilde{S}(x, t) \} = \sum_{\nu=1}^n \nu R^\nu(x) (a_\nu \cos \nu t + b_\nu \sin \nu t),$$

$$\left. \frac{\partial \tilde{S}}{\partial t} \right|_{t=\phi(x)} = \operatorname{Re} \left\{ g(e^{ix}) p' \left(g(e^{ix}) \right) \right\},$$

$$\left. \frac{\partial \tilde{S}}{\partial t} \right|_{t=\phi(x)} = \operatorname{Im} \left\{ g(e^{ix}) p' \left(g(e^{ix}) \right) \right\},$$

$$-\operatorname{Im} \left\{ \frac{e^{ix} g'(e^{ix})}{g(e^{ix})} \right\} \left\{ \operatorname{Re} \left\{ g(e^{ix}) p' \left(g(e^{ix}) \right) \right\} \cos \alpha - \operatorname{Im} \left\{ g(e^{ix}) p' \left(g(e^{ix}) \right) \right\} \sin \alpha \right\} \quad (21)$$

We now differentiate $\phi(x)$ on the right hand side of (19). Using (3.22) on page 12 [7] we get

$$\operatorname{Re}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\} \frac{1}{n} \sum_{\nu=1}^{2n} \frac{(-1)^{\nu+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_\nu)} S(x_0, u_\nu) \quad (22)$$

where we have used $\phi'(x_0) = \operatorname{Re}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\}$.

Combining (20), (21), (22) with (19) gives

$$\begin{aligned} & - \operatorname{Im}\left\{e^{i(x_0+\alpha)} g'(e^{ix_0})_{p'}\left(g(e^{ix_0})\right)\right\} = \\ & - \operatorname{Im}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\} \operatorname{Re}\left\{e^{i\alpha} g(e^{ix_0})_{p'}\left(g(e^{ix_0})\right)\right\} + \\ & + \operatorname{Re}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\} \frac{1}{n} \sum_{\nu=1}^{2n} \frac{(-1)^{\nu+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_\nu)} S(x_0, u_\nu) \end{aligned}$$

We now use the identity $\operatorname{Im}(A \cdot B) = \operatorname{Re}(A)\operatorname{Im}(B) + \operatorname{Im}(A)\operatorname{Re}(B)$ with

$$A = \frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}, \quad B = e^{i\alpha} g(e^{ix_0})_{p'}\left(g(e^{ix_0})\right) \quad \text{and get finally}$$

$$\operatorname{Im}\left\{e^{i\alpha} g(e^{ix_0})_{p'}\left(g(e^{ix_0})\right)\right\} = -\frac{1}{n} \sum_{\nu=1}^{2n} \frac{(-1)^{\nu+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_\nu)} S(x_0, u_\nu) \quad (23)$$

This is a generalization of (3.22) on page 12 of [7]. Let

$$\beta_\nu = \left| \frac{(-1)^{\nu+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_\nu)} \right|, \quad \nu = 1, 2, \dots, 2n \quad (24)$$

then

$$\beta_1 + \beta_2 + \dots + \beta_{2n} = n^2 \quad (25)$$

We use (23) with $R(\theta + x - x_0)e^{i(\phi(\theta)+\phi(x)-\phi(x_0))}$ in place of $g(e^{ix})$ (see (13)) and get

$$\left| \operatorname{Im}\left\{e^{i\alpha} g(e^{i\theta})_{p'}\left(g(e^{i\theta})\right)\right\} \right| \leq \frac{1}{n} \sum_{\nu=1}^{2n} \beta_\nu \left| \operatorname{Re}\left\{P\left(e^{i(u_\nu - \phi(x_0))} g(e^{i\theta})\right)\right\} \right|$$

Using the assumptions on χ , (25) and applying Jensen's inequality we get

$$\chi \left(n^{-1} \left| \operatorname{Im} \left\{ e^{i\alpha} g(e^{i\theta})_{p'} \left(g(e^{i\theta}) \right) \right\} \right| \right) \leq \frac{1}{n^2} \sum_{\nu=1}^{2n} \beta_{\nu} \chi \left(\left| \operatorname{Re} \left\{ P \left(e^{i(u_{\nu} - \phi(x_0))} g(e^{i\theta}) \right) \right\} \right| \right)$$

Integration with respect to θ gives (7). The equality assertion follows from Zygmund. This completes the proof of Theorem 1. \square

To prove that the consequence of Theorem 1 hold true even if we drop the condition

$$\{\arg g(e^{ix}) | 0 \leq x \leq 2\pi\} \supset [0, 2\pi/n]$$

we can use (3.23) in [7] with the following

$$S(\theta) = c_0 + \sum_1^n (a_{\nu} \cos \nu\theta + b_{\nu} \sin \nu\theta) R^{\nu} \quad \text{where} \quad x_0 = -\frac{\alpha}{n}.$$

Then for $R \geq 0$, $0 \leq \theta$, $\alpha \leq 2\pi$ we get

$$\left| \operatorname{Im} \left(e^{i\alpha} \operatorname{Re}^{-i\theta} p'(\operatorname{Re}^{i\theta}) \right) \right| \leq \frac{1}{n} \sum_1^{2n} \beta_{\nu} \left| \operatorname{Re} p \left(\operatorname{Re}^{i(\theta + u_k + \frac{\alpha}{n})} \right) \right|,$$

where the β_{ν} are independent of R, θ . From that we proceed as in the proof of Theorem 1.

5. A PROOF OF THEOREM 2.

Let $\chi(t) = t^p$ in (7). We get

$$\int_0^{2\pi} \left| \operatorname{Im} \left(e^{i\alpha} g(e^{i\theta})_{p'} \left(g(e^{i\theta}) \right) \right) \right|^p d\theta \leq n^p \max_{\beta} \left\{ \int_0^{2\pi} \left| \operatorname{Re} \left\{ P \left(e^{i\beta} g(e^{i\theta}) \right) \right\} \right|^p d\theta \right\}$$

Let $g(e^{i\theta})_{p'} \left(g(e^{i\theta}) \right) = A(\theta) + iB(\theta)$ then we have

$$\int_0^{2\pi} |B(\theta) \cos \alpha + A(\theta) \sin \alpha|^p d\theta \leq n^p \max_{\beta} \left\{ \int_0^{2\pi} \left| \operatorname{Re} \left\{ P \left(e^{i\beta} g(e^{i\theta}) \right) \right\} \right|^p d\theta \right\}$$

As in Zygmund we integrate this with respect to α over $0 \leq \alpha \leq 2\pi$, change the order of integration on the left hand side and use

$$\int_0^{2\pi} |a \cos \alpha + b \sin \alpha|^p d\alpha = (a^2 + b^2)^{p/2} \int_0^{2\pi} |\sin \alpha|^p d\alpha$$

to get

$$\begin{aligned} & \left\{ \int_0^{2\pi} \left| g(e^{i\theta})_{p'} \left(g(e^{i\theta}) \right) \right|^p d\theta \right\}^{1/p} \\ & \leq \left\{ \frac{2\pi}{\int_0^{2\pi} |\sin \alpha|^p d\alpha} \right\}^{1/p} n \max_{\beta} \left\{ \int_0^{2\pi} \left| \operatorname{Re} \left\{ P \left(e^{i\beta} g(e^{i\theta}) \right) \right\} \right|^p d\theta \right\}^{1/p} \end{aligned}$$

this proves (8) and completes the proof of Theorem 2. \square

6. PROOFS OF THEOREM 3 AND THE COROLLARY.

By the normalization $G(0) = 0$ we can use Theorem 2 with

$g(e^{ix}) = G(re^{ix})$. We apply Koebe's $\frac{1}{4}$ -theorem [6] to get

$$\frac{r|G'(0)|}{4} \leq |G(re^{i\theta})|. \quad \text{This bounds the left hand side of (8) from below}$$

and proves (9). \square

(10) follows from (7) with $p(z) = z$ applied to g and to ig .

(11) follows from (8) with $p(z) = z$. \square

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