

## A GENERALIZATION OF AN INEQUALITY OF ZYGMUND

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**ABSTRACT.** The well known Bernstein inequality states that if  $D$  is a disk centered at the origin with radius  $R$  and if  $p(z)$  is a polynomial of degree  $n$ , then  $\max_{z \in D} |p'(z)| \leq \frac{n}{R} \max_{z \in D} |p(z)|$  with equality iff  $p(z) = AZ^n$ .

However it is true that we have the following better inequality:

$$\max_{z \in D} |p'(z)| \leq \frac{n}{R} \max_{z \in D} |\operatorname{Re} p(z)|$$

with equality iff  $p(z) = AZ^n$ .

This is a consequence of a general equality that appears in Zygmund [7] (and which is due to Bernstein and Szegö): For any polynomial  $p(z)$  of degree  $n$  and for any  $1 \leq p < \infty$  we have

$$\left\{ \int_0^{2\pi} |p'(e^{ix})|^p dx \right\}^{1/p} \leq A_p^n \left\{ \int_0^{2\pi} |\operatorname{Re} p(e^{ix})|^p dx \right\}^{1/p}$$

where  $A_p^n = \pi^{1/2} \frac{\Gamma(\frac{1}{2}p + 1)}{\Gamma(\frac{1}{2}p + \frac{1}{2})}$  with equality iff  $p(z) = AZ^n$ .

In this note we generalize the last result to domains different from Euclidean disks by showing the following: If  $g(e^{ix})$  is differentiable and if  $p(z)$  is a polynomial of degree  $n$  then for any  $1 \leq p < \infty$  we have

$$\left\{ \int_0^{2\pi} |g(e^{i\theta})p'(g(e^{i\theta}))|^p d\theta \right\}^{1/p} \leq A_p^n \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re} \{p(e^{i\beta}g(e^{i\theta}))\}|^p d\theta \right\}^{1/p}$$

with equality iff  $p(z) = Az^n$ .

We then obtain some conclusions for Schlicht Functions.

**Key Words and Phrases:** Bernstein inequality, Bernstein-Szegö inequality, Krzyz problem, Dirichlet kernel, trigonometric interpolation

## 1. INTRODUCTION.

The classical result of Bernstein as it appears in [2] is Bernstein Inequality. If  $D$  is a Euclidean disk and  $P$  is a polynomial of degree  $n$  over  $\mathbb{C}$ , then

$$\|P'\|_D \leq \frac{n}{\text{tr}(D)} \|P\|_D \quad (1)$$

where  $\|f\|_D = \sup_D |f(z)|$  and  $\text{tr}(D)$  is the transfinite diameter of  $D$  (which is the disk's radius in this case).

This result was generalized to various directions. The following theorem appears in [1]. Let  $0 \leq k \leq 1$  and let  $E$  be a closed  $k$ -quasidisk, then

**THEOREM.** For any polynomial  $P$  of degree  $n$  we have

$$\left| \frac{P(z_1) - P(z_2)}{z_1 - z_2} \right| \leq c_1 \frac{n^{1+k}}{\text{tr}(E)} \|P\|_E, \quad z_1, z_2 \in E \quad (2)$$

and

$$\|P'\|_E \leq c_2 \frac{n^{1+k}}{\text{tr}(E)} \|P\|_E \quad (3)$$

where  $c_1 = 2^{-k} e(\frac{\pi}{4} + 1)$  and  $c_2 = 2^{-k} e$ .

Another direction of generalization arises naturally in the following:

Let  $\beta$  be the class of all analytic functions  $f(z) = \sum_{k=0}^{\infty} a_k z^k$  in  $|z| < 1$  such that  $0 < |f(z)| < 1$ . A problem posed by Krzyz [4] is to determine  $A_n = \max_{\beta} |a_n|$ ,  $n \geq 1$  [3]. The conjecture (which is still unsolved) is that  $A_n = \frac{2}{e}$  and that it is attained only by rotations of

$$g_n(z) = \exp \left( -\frac{z^{n-1}}{z^{n+1}} \right).$$

Let  $f(z)$  be an extremal function for  $A_n$ .

**CONJECTURE.**  $|f(0)| \leq \frac{1}{e}$  and equality holds only for rotations of  $g_n$ .

A theorem which indicates that this conjecture may be true is:

**THEOREM [5].** If  $n = 2p + 1$  and if  $a_1 = a_3 = \dots = a_{2p-1} = 0$ , then  $|a_0| \leq \frac{1}{e}$ . Equality sign occurs iff  $|a_n| = \frac{2}{e}$

The proof of this uses the following generalization of (1): Let  $D(0,1) = \{z \in \mathbb{C} \mid |z| < 1\}$  and let  $p$  be any polynomial of degree  $n$  over  $\mathbb{C}$ , then

$$\|P'\|_{D(0,1)} \leq n \|\text{Re } p\|_{D(0,1)} \quad (4)$$

This follows from an inequality of Zygmund [7].

**THEOREM.** For any polynomial  $p$  of degree  $n$  and for any  $1 \leq p < \infty$  we have

$$\left\{ \int_0^{2\pi} |p'(e^{ix})|^p dx \right\}^{1/p} \leq A_p^n \left\{ \int_0^{2\pi} |\operatorname{Re} p(e^{ix})|^p dx \right\}^{1/p} \quad (5)$$

where

$$A_p^n = \pi^{1/2} \frac{\Gamma(\frac{1}{2}p + 1)}{\Gamma(\frac{1}{2}p + \frac{1}{2})} \quad (6)$$

and equality occurs in (5) iff  $p(z) = Az^n$ .

In this note we indicate a way to generalize (5) to domains  $E$  other than  $D(0,1)$  by using the same ideas as in Zygmund's proof applied to  $p \circ g$  where  $g$  is a quite general mapping  $D(0,1) \rightarrow E$ .

## 2. RESULTS.

**THEOREM 1.** Let  $g$  be a complex valued function of  $e^{ix}$ ,  $0 \leq x \leq 2\pi$ . Suppose that  $\{\arg g(e^{ix}) \mid 0 \leq x \leq 2\pi\} \supseteq [0, 2\pi/n]$  and that  $\frac{dg(e^{ix})}{dx}$  exists, then for any non-negative, non-decreasing convex function  $\chi$ , for any  $\alpha \in \mathbb{R}$  and for any polynomial  $P$  of degree  $n$  over  $\mathbb{C}$  we have

$$\int_0^{2\pi} \chi \left( n^{-1} \left| \operatorname{Im} \left\{ e^{i\alpha} g(e^{i\theta}) p'(g(e^{i\theta})) \right\} \right| \right) d\theta \leq \max_{\beta} \left\{ \int_0^{2\pi} \chi \left( \left| \operatorname{Re} \{ p(e^{i\beta} g(e^{i\theta})) \} \right| \right) d\theta \right\} \quad (7)$$

equality occurs in (7) iff  $p(z) = Az^n$ .

We remark that the consequences of Theorem 1 hold true even if the condition

$$\{\arg g(e^{ix}) \mid 0 \leq x \leq 2\pi\} \supseteq [0, 2\pi/n]$$

is dropped.

We will indicate at the end of Section 4 how to prove that.

With the notations of Theorem 1 we have

**THEOREM 2.** If  $1 \leq p < \infty$ , then

$$\left\{ \int_0^{2\pi} |g(e^{i\theta}) p'(g(e^{i\theta}))|^p d\theta \right\}^{1/p} \leq A_p^n \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re} \{ p(e^{i\beta} g(e^{i\theta})) \}|^p d\theta \right\}^{1/p} \quad (8)$$

with equality iff  $p(z) = Az^n$ .

As a consequence we derive an analogous theorem to (1),

**THEOREM 3.** If  $E$  is a simply connected domain such that  $0 \in E$ , and if  $G : D(0,1) \rightarrow E$  is a Riemann mapping normalized by  $G(0) = 0$ , then for every  $1 \leq p < \infty$  and every  $0 \leq r < 1$  we have

$$\left\{ \int_0^{2\pi} |P'(G(re^{i\theta}))|^p d\theta \right\}^{1/p} \leq \frac{4 A_p^n}{r |G'(0)|} \max_{\beta} \left\{ \int_0^{2\pi} |\operatorname{Re} \{ P(e^{i\beta} G(re^{i\theta})) \}|^p d\theta \right\}^{1/p} \quad (9)$$

This last inequality is not sharp.

Returning to the function  $g$  of Theorem 1 we add

**COROLLARY.**

$$\max_{\alpha} \left\{ \int_0^{2\pi} \chi \left( | \operatorname{Im} \{ e^{i\alpha} g(e^{i\theta}) \} | \right) d\theta \right\} = \max_{\beta} \left\{ \int_0^{2\pi} \chi \left( | \operatorname{Re} \{ e^{i\beta} g(e^{i\theta}) \} | \right) d\theta \right\} \quad (10)$$

$$\left\{ \int_0^{2\pi} |g(e^{i\theta})|^p d\theta \right\}^{1/p} \leq A_p \max_{\beta} \left\{ \int_0^{2\pi} | \operatorname{Re} \{ e^{i\beta} g(e^{i\theta}) \} |^p d\theta \right\}^{1/p} \quad (11)$$

The last corollary can be seen directly, but, it shows that we cannot drop "max" on the right hand of the above inequalities since it is easy to find a  $g$  such that  $\| \operatorname{Re} g \|_p \leq 1$  while  $\lim_{p \rightarrow \infty} \| g \|_p = \infty$ .

### 3. PREPARATIONS.

Let  $p(z) = c_0 + c_1 z + \dots + c_n z^n$  be a polynomial of degree  $n$ , where  $c_0 \in \mathbb{R}$ . We denote

$$S(z) = \frac{1}{2}(p(z) + \overline{p(z)}) , \quad \tilde{S}(z) = \frac{1}{2i}(p(z) - \overline{p(z)}) \quad (12)$$

Let  $g$  be a complex valued function of  $e^{ix}$ ,  $x \in \mathbb{R}$  such that

$\{\arg g(e^{ix}) | 0 \leq x \leq 2\pi\} \supseteq [0, \frac{2\pi}{n}]$  and such that  $\frac{dg}{dx}(e^{ix})$  exists. We denote

$$g(e^{ix}) = R(x)e^{i\phi(x)}, \quad R(x) = |g(e^{ix})|, \quad \phi(x) = \arg g(e^{ix}) \quad (13)$$

$$S(x, t) = c_0 + \sum_{\nu=1}^n R^\nu(x)(a_\nu \cos \nu t + b_\nu \sin \nu t) \quad (14)$$

$$\tilde{S}(x, t) = \sum_{\nu=1}^n R^\nu(x)(a_\nu \sin \nu t - b_\nu \cos \nu t)$$

where  $c_0, a_1, \dots, a_n, b_1, \dots, b_n \in \mathbb{R}$

where the coefficients  $a, b$  are such that

$$S(x, \phi(x)) = S(g(e^{ix})), \quad \tilde{S}(x, \phi(x)) = \tilde{S}(g(e^{ix})). \quad (15)$$

As in Zygmund we denote the modified Dirichlet kernel and it's conjugate kernel by  $D_n^*(u)$ ,  $\tilde{D}_n^*(u)$  respectively. Thus

$$D_n^*(u) = \frac{1}{2} \sum_{\nu=1}^{n-1} \cos \nu u + \frac{1}{2} \cos nu = \frac{\sin nu}{2 \tan \frac{1}{2} u} \quad (16)$$

$$\tilde{D}_n^*(u) = \sum_{\nu=1}^{n-1} \sin \nu u + \frac{1}{2} \sin nu = (1 - \cos nu) \frac{1}{2} \cot \frac{1}{2} u.$$

We will also need the zeros of  $\cos nt$

$$u_\nu = (2\nu-1)\pi/2n, \nu = 1, 2, \dots, 2n \quad (17)$$

$\phi_{2n}(t)$  will be a step function which has jumps  $\frac{\pi}{n}$  at the points  $u_\nu$ . By (3.6), (3.21) on pages 10, 11 [7] we have

**THEOREM (Zygmund)**

$$S(x, u) = a_n R^n(x) \cos nu + \frac{1}{\pi} \int_0^{2\pi} S(x, t) D_n^*(t-u) d\phi_{2n}(t) \quad (18)$$

$$\tilde{S}(x, u) = a_n R^n(x) \sin nu + \frac{1}{\pi} \int_0^{2\pi} S(x, t) \tilde{D}_n^*(t-u) d\phi_{2n}(t)$$

Thus for any real number  $\alpha$  we have

$$\begin{aligned} S(g(e^{ix})) \cos \alpha - \tilde{S}(g(e^{ix})) \sin \alpha &= a_n R^n(x) \cos[n\phi(x)+\alpha] + \\ &+ \frac{1}{\pi} \int_0^{2\pi} S(x, t) \left\{ \frac{\sin[n(\phi(x)-t)+\alpha]-\sin \alpha}{2 \tan \frac{1}{2}(\phi(x)-t)} \right\} d\phi_{2n}(t) \end{aligned} \quad (19)$$

**4. A PROOF OF THEOREM 1.**

As in Zygmund, let  $x_0$  be a root of  $\sin[n\phi(x) + \alpha]$  such that  $\cos[n\phi(x_0) + \alpha] = 1$ . We differentiate (19) with respect to  $x$  and substitute  $x = x_0$ . By (12) we have

$$\frac{dS}{dx}(g(e^{ix})) = -\text{Im} \left\{ e^{ix} g'(e^{ix}) p' \left( g(e^{ix}) \right) \right\} \quad (20)$$

$$\frac{d\tilde{S}}{dx}(g(e^{ix})) = \text{Re} \left\{ e^{ix} g'(e^{ix}) p' \left( g(e^{ix}) \right) \right\}$$

This takes care of the left hand side of (19). On the right hand side we first differentiate  $R(x)$  and use:

$$\frac{R'(x)}{R(x)} = -\text{Im} \left\{ \frac{e^{ix} g'(e^{ix})}{g(e^{ix})} \right\},$$

$$\frac{\partial}{\partial t} \{ \tilde{S}(x, t) \} = \sum_{\nu=1}^n \nu R^\nu(x) (a_\nu \cos \nu t + b_\nu \sin \nu t),$$

$$\left. \frac{\partial \tilde{S}}{\partial t} \right|_{t=\phi(x)} = \text{Re} \left\{ g(e^{ix}) p' \left( g(e^{ix}) \right) \right\},$$

$$\left. \frac{\partial \tilde{S}}{\partial t} \right|_{t=\phi(x)} = \text{Im} \left\{ g(e^{ix}) p' \left( g(e^{ix}) \right) \right\},$$

$$-\text{Im} \left\{ \frac{e^{ix} g'(e^{ix})}{g(e^{ix})} \right\} \left\{ \text{Re} \left\{ g(e^{ix}) p' \left( g(e^{ix}) \right) \right\} \cos \alpha - \text{Im} \left\{ g(e^{ix}) p' \left( g(e^{ix}) \right) \right\} \sin \alpha \right\} \quad (21)$$

We now differentiate  $\phi(x)$  on the right hand side of (19). Using (3.22) on page 12 [7] we get

$$\operatorname{Re}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\} \frac{1}{n} \sum_{v=1}^{2n} \frac{(-1)^{v+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_v)} S(x_0, u_v) \quad (22)$$

where we have used  $\phi'(x_0) = \operatorname{Re}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\}$ .

Combining (20), (21), (22) with (19) gives

$$\begin{aligned} & -\operatorname{Im}\left\{e^{i(x_0+\alpha)} g'(e^{ix_0}) p'\left(g(e^{ix_0})\right)\right\} = \\ & -\operatorname{Im}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\} \operatorname{Re}\left\{e^{i\alpha} g(e^{ix_0}) p'\left(g(e^{ix_0})\right)\right\} + \\ & + \operatorname{Re}\left\{\frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}\right\} \frac{1}{n} \sum_{v=1}^{2n} \frac{(-1)^{v+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_v)} S(x_0, u_v) \end{aligned}$$

We now use the identity  $\operatorname{Im}(A \cdot B) = \operatorname{Re}(A) \operatorname{Im}(B) + \operatorname{Im}(A) \operatorname{Re}(B)$  with

$$A = \frac{e^{ix_0} g'(e^{ix_0})}{g(e^{ix_0})}, \quad B = e^{i\alpha} g(e^{ix_0}) p'\left(g(e^{ix_0})\right) \quad \text{and get finally}$$

$$\operatorname{Im}\left\{e^{i\alpha} g(e^{ix_0}) p'\left(g(e^{ix_0})\right)\right\} = -\frac{1}{n} \sum_{v=1}^{2n} \frac{(-1)^{v+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_v)} S(x_0, u_v) \quad (23)$$

This is a generalization of (3.22) on page 12 of [7]. Let

$$\beta_v = \left| \frac{(-1)^{v+1} + \sin \alpha}{4 \sin^2 \frac{1}{2} (\phi(x_0) - u_v)} \right|, \quad v = 1, 2, \dots, 2n \quad (24)$$

then

$$\beta_1 + \beta_2 + \dots + \beta_{2n} = n^2 \quad (25)$$

We use (23) with  $R(\theta + x - x_0) e^{i(\phi(\theta) + \phi(x) - \phi(x_0))}$  in place of  $g(e^{ix})$  (see (13)) and get

$$\left| \operatorname{Im}\left\{e^{i\alpha} g(e^{i\theta}) p'\left(g(e^{i\theta})\right)\right\} \right| \leq \frac{1}{n} \sum_{v=1}^{2n} \beta_v \left| \operatorname{Re}\left\{P\left(e^{i(u_v - \phi(x_0))} g(e^{i\theta})\right)\right\} \right|$$

Using the assumptions on  $\chi$ , (25) and applying Jensen's inequality we get

$$\chi \left( n^{-1} \left| \operatorname{Im} \left\{ e^{i\alpha} g(e^{i\theta}) p' \left( g(e^{i\theta}) \right) \right\} \right| \right) \leq \frac{1}{n^2} \sum_{\nu=1}^{2n} \beta_\nu \chi \left( \left| \operatorname{Re} \left\{ P \left( e^{i(u_\nu - \phi(x_0))} g(e^{i\theta}) \right) \right\} \right| \right)$$

Integration with respect to  $\theta$  gives (7). The equality assertion follows from Zygmund. This completes the proof of Theorem 1.  $\square$

To prove that the consequence of Theorem 1 hold true even if we drop the condition

$$\{\arg g(e^{ix}) | 0 \leq x \leq 2\pi\} \supset [0, 2\pi/n]$$

we can use (3,23) in [7] with the following

$$S(\theta) = c_0 + \sum_1^n (a_\nu \cos \nu\theta + b_\nu \sin \nu\theta) R^\nu \quad \text{where } x_0 = -\frac{\alpha}{n}.$$

Then for  $R \geq 0$ ,  $0 \leq \theta$ ,  $\alpha \leq 2\pi$  we get

$$\left| \operatorname{Im} \left( e^{i\alpha} \operatorname{Re} e^{i\theta} p' \left( \operatorname{Re} e^{i\theta} \right) \right) \right| \leq \frac{1}{n} \sum_1^{2n} \beta_\nu \left| \operatorname{Re} p \left( \operatorname{Re} e^{i(\theta+u_\nu + \frac{\alpha}{n})} \right) \right|,$$

where the  $\beta_\nu$  are independent of  $R, \theta$ . From that we proceed as in the proof of Theorem 1.

## 5. A PROOF OF THEOREM 2.

Let  $\chi(t) = t^p$  in (7). We get

$$\int_0^{2\pi} \left| \operatorname{Im} \left\{ e^{i\alpha} g(e^{i\theta}) p' \left( g(e^{i\theta}) \right) \right\} \right|^p d\theta \leq n^p \max_\beta \left\{ \int_0^{2\pi} \left| \operatorname{Re} \left\{ P \left( e^{i\beta} g(e^{i\theta}) \right) \right\} \right|^p d\theta \right\}$$

Let  $g(e^{i\theta}) p' \left( g(e^{i\theta}) \right) = A(\theta) + iB(\theta)$  then we have

$$\int_0^{2\pi} |B(\theta) \cos \alpha + A(\theta) \sin \alpha|^p d\theta \leq n^p \max_\beta \left\{ \int_0^{2\pi} \left| \operatorname{Re} \left\{ P \left( e^{i\beta} g(e^{i\theta}) \right) \right\} \right|^p d\theta \right\}$$

As in Zygmund we integrate this with respect to  $\alpha$  over  $0 \leq \alpha \leq 2\pi$ , change the order of integration on the left hand side and use

$$\int_0^{2\pi} |a \cos \alpha + b \sin \alpha|^p d\alpha = (a^2 + b^2)^{p/2} \int_0^{2\pi} |\sin \alpha|^p d\alpha$$

to get

$$\begin{aligned} & \left\{ \int_0^{2\pi} \left| g(e^{i\theta}) p' \left( g(e^{i\theta}) \right) \right|^p d\theta \right\}^{1/p} \\ & \leq \left\{ \frac{2\pi}{\int_0^{2\pi} |\sin \alpha|^p d\alpha} \right\}^{1/p} n \max_\beta \left\{ \int_0^{2\pi} \left| \operatorname{Re} \left\{ P \left( e^{i\beta} g(e^{i\theta}) \right) \right\} \right|^p d\theta \right\}^{1/p} \end{aligned}$$

this proves (8) and completes the proof of Theorem 2.  $\square$

## 6. PROOFS OF THEOREM 3 AND THE COROLLARY.

By the normalization  $G(0) = 0$  we can use Theorem 2 with

$g(e^{ix}) = G(re^{ix})$ . We apply Koebe's  $\frac{1}{4}$ -theorem [6] to get

$\frac{r|G'(0)|}{4} \leq |G(re^{i\theta})|$ . This bounds the left hand side of (8) from below

and proves (9).  $\square$

(10) follows from (7) with  $p(z) = z$  applied to  $g$  and to  $ig$ .

(11) follows from (8) with  $p(z) = z$ .  $\square$

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