Internat. J. Math. & Math. Sci.
VOL. 15 NO. 4 (1992) 789-794

DARCY-BRINKMAN FREE CONVECTION ABOUT A WEDGE AND A CONE SUBJECTED TO A MIXED THERMAL BOUNDARY CONDITION

G. RAMANAIAH

and

V. KUMARAN

Department of Mathematics
Anna University, Madras 600 025, INDIA.

(Received January 29, 1991 and in revised form July 26, 1991)

ABSTRACT. The Darcy-Brinkman free convection near a wedge and a cone in a porous medium with high porosity has been considered. The surfaces are subjected to a mixed thermal boundary condition characterized by a parameter m; m=0,1,∞ correspond to the cases of prescribed temperature, prescribed heat flux and prescribed heat transfer coefficient respectively. It is shown that the solutions for different m are dependent and a transformation group has been found, through which one can get solution for any m provided solution for a particular value of m is known. The effects of Darcy number on skin friction and rate of heat transfer are analyzed.

KEYWORDS AND PHRASES. Free convection, Boundary layer, Porous media.
1980 AMS SUBJECT CLASSIFICATION CODES. 76R10, 76D10, 76S05.

1. INTRODUCTION.

The problem of free convection adjacent to a heated vertical surface has received a great deal of attention. These studies assume that the surface is subjected to a prescribed temperature or a prescribed heat flux. In the existing literature these two cases have been studied independently. The present paper aims to present a unified treatment of these cases. It also includes the case of prescribed heat transfer coefficient hitherto not considered by earlier researchers.

Further the free convection on heated surfaces subjected to mixed thermal boundary condition has not received sufficient attention. In this paper we shall consider Darcy-Brinkman free convection [1,2] on a wedge and a cone in a porous medium with high porosity. The free convection on a vertical plate subjected to a prescribed temperature and prescribed heat flux are obtained as special cases.

2. ANALYSIS.

The configuration of free convection adjacent to a wedge and a cone is shown in Fig. 1. The surfaces are subjected to a mixed thermal boundary conditions. The boundary layer equations governing the Darcy-Brinkman free convection are

$$(r^n u)_x + (r^n v)_y = 0, \quad n = 0 \text{ for wedge} \quad = 1 \text{ for cone} \tag{2.1}$$

$$u\,u_x + v\,u_y = \sigma\,u_{yy} - (\sigma/K)\,u + g\beta(T-T_\infty)\cos\alpha \tag{2.2}$$

$$u\,T_x + v\,T_y = (\sigma/P_r)\,T_{yy} \tag{2.3}$$

with boundary conditions,

$$u = 0,\ v = 0,\ a_0\,(T-T_\infty) - a_1\,T_y = a_2 x \quad \text{at } y = 0 \tag{2.4}$$

$$u \longrightarrow 0,\ T \longrightarrow T_\infty \quad \text{as } y \longrightarrow \infty \tag{2.5}$$

where u,v are the velocity components along x and y directions respectively. T is the temperature and T_∞ is the ambient temperature. The symbols g, β, σ and P_r denote gravitational acceleration, coefficient of thermal expansion, kinematic viscosity of the ambient fluid and Prandtl number respectively. a_0, $a_1 \geq 0$, $a_2 \geq 0$ are prescribed constants.

Introducing the following nondimensional quantities,

$$y = \phi L, \quad u = 4\,\sigma x\, f'(\phi)/L^2, \quad v = -\,4\sigma\,(n+1)\, f(\phi)/L$$

$$T = T_\infty + \frac{4\,\sigma^2 x}{L^4\, g\beta \cos\alpha}\,\theta(\phi), \quad Da = K/L^2, \text{ the Darcy number} \tag{2.6}$$

where L is to be determined from the thermal boundary condition (2.4) in a manner to be explained [3]. Equations (2.2) - (2.5) become

$$f''' + 4\,((n+1)\,ff'' - f'^2) - Da^{-1}f' + \theta = 0 \tag{2.7}$$

$$\theta'' + 4\,P_r\,((n+1)\,f\theta' - f'\theta) = 0 \tag{2.8}$$

$$f(0) = f'(0) = f'(\infty) = \theta(\infty) = 0 \tag{2.9}$$

$$(1-m)\,\theta(0) - m\,\theta'(0) = 1 \tag{2.10}$$

where primes denote differentiation with respect to ϕ,

$m = a_1/(a_1 + La_0)$ and L is the positive root of the equation,

$$a_2 g\beta \cos\alpha\; L^5/(4\sigma^2) - a_0 L - a_1 = 0 \tag{2.11}$$

This equation has a unique positive root by Descartes rule of signs for $a_1 \geq 0$, $a_2 \geq 0$. The local Nusselt number defined by

$$Nu = -\,x\,T_y/(T-T_\infty)\,\big|_{y=0} \text{ becomes,}$$

$$Nu = -\,(x/L)\,\theta'(0)/\theta(0) \tag{2.12}$$

If μ is viscosity, the stress at the surface is given by

$$\Gamma = \mu\,u_y\,\big|_{y=0} = 4\mu\sigma x\, f''(0)/L^3 \tag{2.13}$$

3. DISCUSSION AND CONCLUSIONS.

The solution of the boundary value problem (2.7) - (2.10) has been obtained by shooting method for different values of $m \geq 0$. The values of $\theta(0)$, $-\theta'(0)$, $-\theta'(0)/\theta(0)$ and $f''(0)$ are given in the tables 1 & 2 for n = 0, and 1 respectively with $P_r = 0.733$. It is seen that for n = 1 (cone) the surface temperature $\theta(0)$, the surface heat flux $-\theta'(0)$, the heat transfer coefficient $-\theta'(0)/\theta(0)$ and the surface stress $f''(0)$ increase with m. $\theta(0)$ decreases with increase in Darcy number, whereas the dimensionless Nusselt number $-\theta'(0)/\theta(0)$ increases with increasing Da. All these effects are more pronounced for n = 0 (wedge) than for n = 1 (cone). It is observed that the porous medium transports larger amount of energy compared to the corresponding fluid medium (Da^{-1} = 0).

Da^{-1}	m	θ(0)	-θ'(0)	-θ'(0)/θ(0)	f''(0)
	0	1.00000	0.75859	0.75859	0.54935
	0.5	1.12302	0.87698	0.78092	0.59930
0	1	1.24736	1.00000	0.80169	0.64841
	10	2.37825	2.24043	0.94205	1.05207
	∞	3.01972	3.01972	1.00000	1.25843
	0	1.00000	0.75786	0.75786	0.54854
	0.5	1.12342	0.87658	0.78027	0.59862
0.01	1	1.24823	1.00000	0.80113	0.64788
	10	2.38334	2.24500	0.94196	1.05274
	∞	3.02647	3.02647	1.00000	1.25945
	0	1.00000	0.75132	0.75132	0.54133
	0.5	1.12949	0.87051	0.77072	0.59291
0.1	1	1.25597	1.00000	0.79619	0.64324
	10	2.42888	2.28599	0.94117	1.05873
	∞	3.08689	3.08689	1.00000	1.26863

Table 1. Values of θ(0), -θ'(0), -θ'(0)/θ(0) and f''(0) for n = 0 (Wedge)

Da^{-1}	m	θ(0)	-θ'(0)	-θ'(0)/θ(0)	f''(0)
	0	1.00000	0.81449	0.81449	0.50853
	0.5	1.09141	0.90859	0.83250	0.54301
0	1	1.17840	1.00000	0.84861	0.57515
	10	1.87653	1.78887	0.95329	0.81533
	∞	2.27224	2.27224	1.00000	0.94115
	0	1.00000	0.81385	0.81385	0.50789
	0.5	1.09174	0.90826	0.83193	0.54248
0.01	1	1.17908	1.00000	0.84812	0.57474
	10	1.88006	1.79205	0.95319	0.81573
	∞	2.27700	2.27700	1.00000	0.94184
	0	1.00000	0.80193	0.80193	0.50079
	0.5	1.09790	0.90210	0.82166	0.53800
0.1	1	1.19229	1.00000	0.83872	0.57206
	10	1.91178	1.82060	0.95231	0.81938
	∞	2.38060	2.38060	1.00000	0.96445

Table 2. Values of θ(0), -θ'(0), -θ'(0)/θ(0) and f''(0) for n = 1 (Cone)

It is interesting to note that the solutions corresponding to different values of m are dependent as stated in the following properties:

Property 1 : The equations (2.7) - (2.9) are invariant under the transformation,

$$\phi^* = A\,\phi,\ Da^* = A^2\,Da,\ f^*(\phi^*,Da^*) = f(\phi,Da)/A,\ \theta^*(\phi^*,Da^*) = \theta(\phi,Da)/A^4 \qquad (3.1)$$

where A is any positive real number.

Property 2 : If f(ø,Da), θ(ø,Da) is the solution of the boundary value problem (2.7) - (2.10) for any particular value of m, say m_0, then the solution for any m is given by the equations (3.1) provided A is the positive root of the equation,

$$A^5 - (1-m)\,A\,\theta(0,Da) + m\,\theta'(0,Da) = 0 \qquad (3.2)$$

Property 3 : If the solution of the boundary value problem (2.7) - (2.10) is same for any two distinct values of m, then the solution is same for all values of m.

The mixed boundary conditions (2.10) includes the following as special cases :

1. Prescribed Temperature (PT) : $a_0 > 0$, $a_1 = 0$, $a_2 > 0$.
Hence $L = ((4\sigma^2 a_0)/(a_2 g\beta \cos\alpha))^{1/4}$, $m = 0$ and equation (2.10) becomes

$$\theta(0) = 1 \qquad (3.3)$$

2. Prescribed Heat Flux (PHF) : $a_0 = 0$, $a_1 > 0$, $a_2 > 0$.
Hence $L = ((4\sigma^2 a_1)/(a_2 g\beta \cos\alpha))^{1/5}$, $m = 1$ and equation (2.10) becomes

$$\theta'(0) = -1 \qquad (3.4)$$

3. Prescribed Heat Transfer Coefficient (PHTC) : $a_0 < 0$, $a_1 > 0$, $a_2 = 0$.
Hence $L = -a_1/a_0$, $m = \infty$ and equation (2.10) becomes

$$\theta(0) + \theta'(0) = 0 \qquad (3.5)$$

——>	PT	PHF	PHTC
PT	1	$[-\theta'(0,Da)]^{1/5}$	$-\theta'(0,Da)$
PHF	$[\theta(0,Da)]^{1/4}$	1	$1/\theta(0,Da)$
PHTC	$[\theta(0,Da)]^{1/4}$	$[\theta(0,Da)]^{1/5}$	1

Table 3. Values of A for transition

Table 3 gives the values of the parameter A required for transition from one case to the other. The transition is illustrated by the following example for cone case (n = 1) with $Da^{-1} = 0.1$.

1. For PT we have, $\theta'(0) = -0.80193$, $f''(0) = 0.50079$ which gives
$A = 0.95681$, $Da^{-1} = 0.10923$, $\theta(0) = 1.19314$, $f''(0) = 0.57171$ for PHF
$A = 0.80193$, $Da^{-1} = 0.15550$, $\theta(0) = 2.41799$, $f''(0) = 0.97106$ for PHTC
2. For PHF we have, $\theta(0) = 1.19229$, $f''(0) = 0.57206$ which gives
$A = 1.04495$, $Da^{-1} = 0.09158$, $\theta'(0) = -0.80264$, $f''(0) = 0.50137$ for PT
$A = 0.83872$, $Da^{-1} = 0.14216$, $\theta(0) = 2.40940$, $f''(0) = 0.96959$ for PHTC
3. For PHTC we have, $\theta'(0) = -2.38060$, $f''(0) = 0.96445$ which gives
$A = 1.24214$, $Da^{-1} = 0.06481$, $\theta'(0) = -0.80506$, $f''(0) = 0.50323$ for PT
$A = 1.18943$, $Da^{-1} = 0.07068$, $\theta(0) = 1.18943$, $f''(0) = 0.57315$ for PHF

In Table 4, critical values of P_r for different values of Darcy number, for which the solution is independent of m (property 3) are given. An interesting aspect of this value of P_r, say P_r^c is that it bifurcates the class of solutions for different P_r as follows:

For $P_r > P_r^c$, the values of $\theta(0)$, $-\theta'(0)$ and $-\theta'(0)/\theta(0)$ decrease with m whereas they increase with m for $P_r < P_r^c$.

n	Da^{-1}	0	0.001	0.01	0.1
0	P_r	1.70954	1.71004	1.71450	1.75903
	$f''(0)$	0.47737	0.47728	0.47648	0.46863
1	P_r	1.36790	1.36823	1.37116	1.40052
	$f''(0)$	0.45549	0.45542	0.45476	0.44836

Table 4. Critical values of P_r for different Darcy numbers

The results of free convection on a vertical plate subjected to prescribed temperature or prescribed heat flux can be obtained from the present study as special cases of $m = 0$ or 1 respectively when $n = 0$, $\alpha = 0$ and $Da^{-1} = 0$.

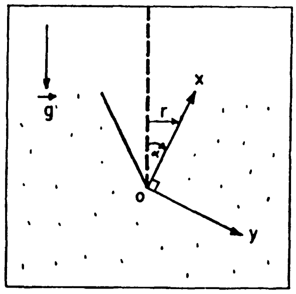

Fig.1. Configuration of the Physical System.

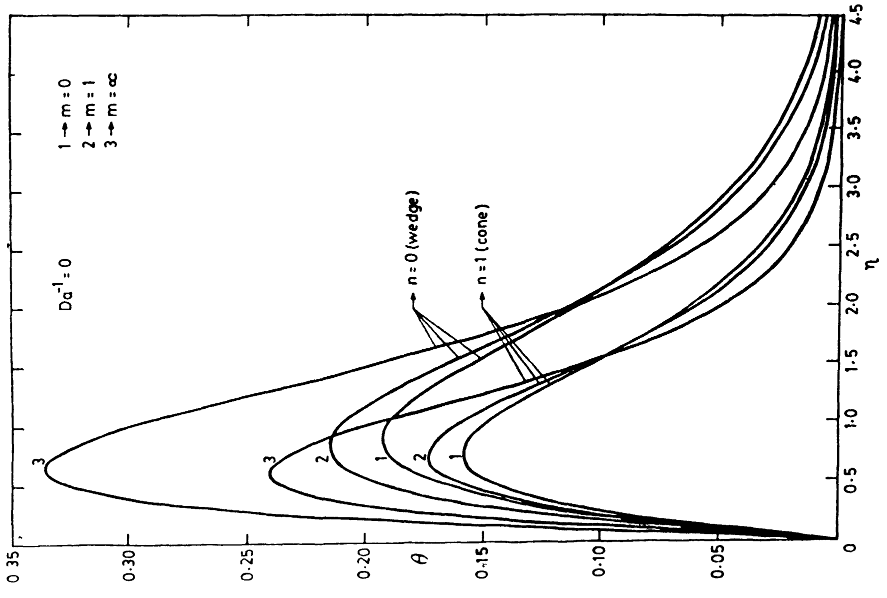

Fig. 2 Temperature profiles for $Da^{-1} = 0$

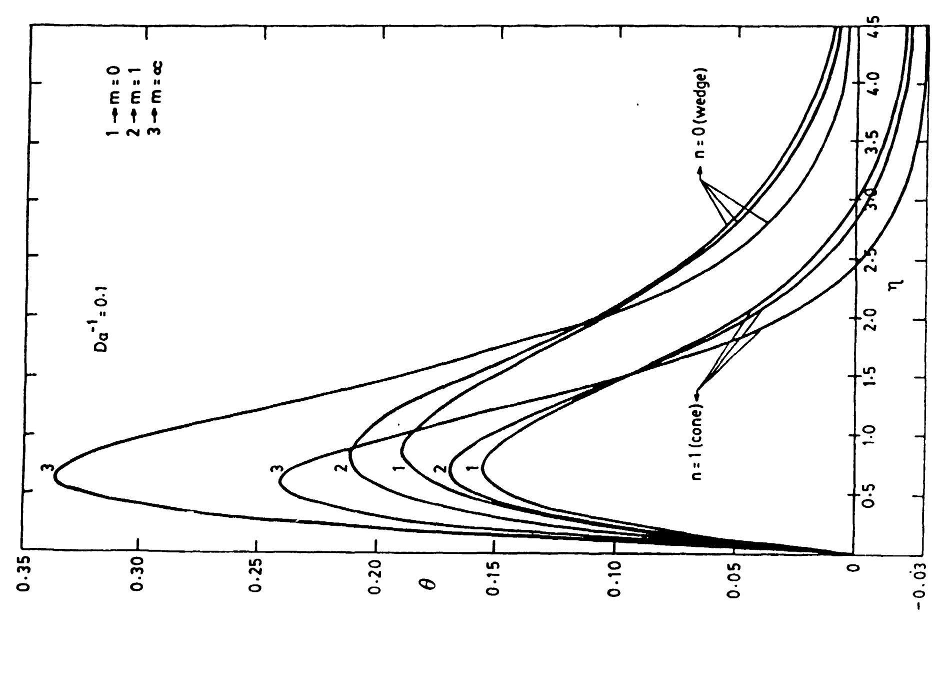

Fig. 3 Temperature profiles for $Da^{-1} = 0.1$

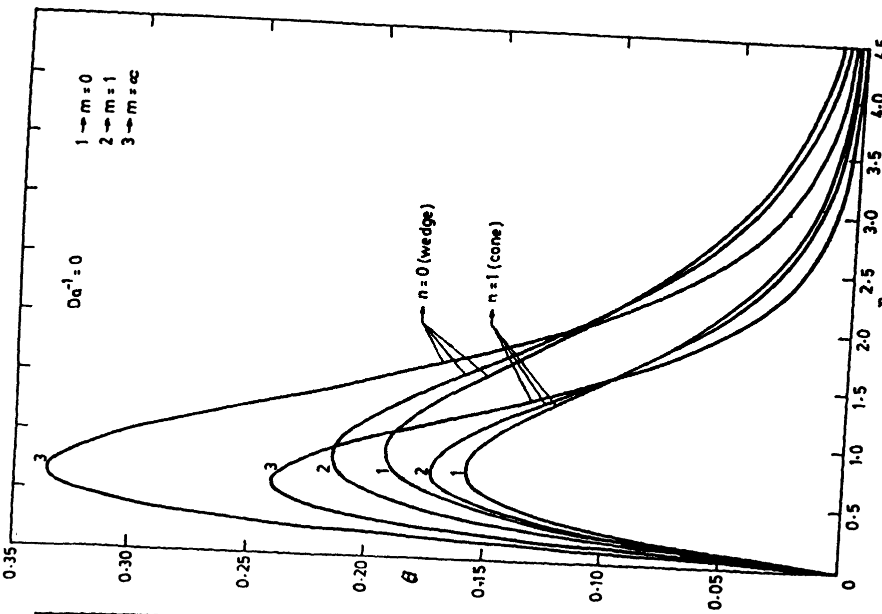

Fig. 4 Velocity profiles for $Da^{-1} = 0$

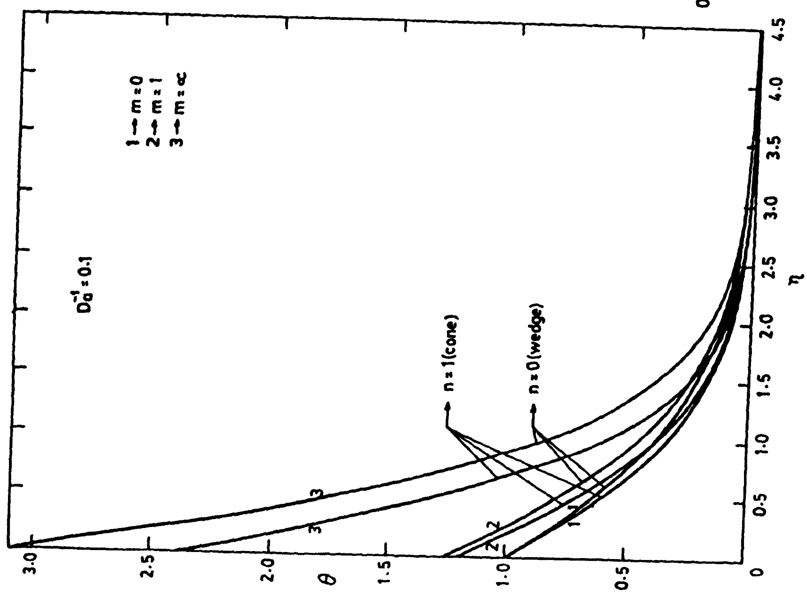

Fig. 5 Velocity profiles for $Da^{-1} = 0.1$

ACKNOWLEDGEMENT. One of the authors (V.K) is grateful to the Council of Scientific and Industrial Research, New Delhi, India for the award of Senior Research Fellowship to him.

REFERENCES

1. ASHOK K.SEN. Natural Convection in a Shallow Porous Cavity-the Brinkman Model, Int.J.Heat Mass Transfer, 30 (1987), 855-868.

2. NIRMAL C.SACHETI. Application of Brinkman Model in Viscous Incompressible Flow Through a Porous Channel, J.Math.Phy.Sci. 17 (1983), 567-578.

3. RAMANAIAH, G. and MALARVIZHI, G. Free Convection on a Horizontal Plate Subjected to a Mixed Thermal Boundary Condition, Vol.2, Proceedings of the First International Conference on Advanced Computational Methods in Heat Transfer, Computational Mechanics Publications, Southampton, Boston (1990), 77-86.

Mathematical Problems in Engineering

Special Issue on Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

Edson Denis Leonel, Departamento de Estatística, Matemática Aplicada e Computação, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil ; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru