Internat. J. Math. & Math. Sci.
VOL. 15 NO. 1 (1992) 205-206

REMARKS ON DERIVATIONS ON SEMIPRIME RINGS

MOHAMAD NAGY DAIF

Department of Mathematics
Faculty of Education
Umm Al-Qura University
Taif, Saudi Arabia

and

HOWARD E. BELL

Department of Mathematics
Brock University
St. Catharines, Ontario
Canada L2S 3A1

(Received December 31, 1990 and in revised form May 10, 1991)

ABSTRACT. We prove that a semiprime ring R must be commutative if it admits a derivation d such that (i) $xy + d(xy) = yx + d(yx)$ for all x,y in R, or (ii) $xy - d(xy) = yx - d(yx)$ for all x,y in R. In the event that R is prime, (i) or (ii) need only be assumed for all x,y in some nonzero ideal of R.

KEY WORDS AND PHRASES. Derivation, semiprime ring, prime ring, commutative, central ideal, integral domain, direct sum.
1980 AMS SUBJECT CLASSIFICATION CODES. 16A15, 16A70.

1. INTRODUCTION.

In the past fifteen years, there has been an ongoing interest in derivations on prime or semiprime rings; and many of the results have involved commutativity. (See [1] for a partial bibliography.) In this brief note, we explore the commutativity implications of the existence on R of a derivation d satisfying the following:

(*) there exists a nonzero ideal K of R such that either $xy + d(xy) = yx + d(yx)$ for all x,y in K, or $xy - d(xy) = yx - d(yx)$ for all x,y in K.

2. THE PRINCIPAL RESULTS.

Our principal results in this note are

THEOREM 1. If R is any prime ring admitting a derivation d satisfying (*), then R is commutative.

THEOREM 2. Let R be a semiprime ring admitting a derivation d for which either $xy + d(xy) = yx + d(yx)$ for all x,y in R or $xy - d(xy) = yx - d(yx)$ for all x,y in R. Then R is commutative.

In fact, both of these theorems are consequences of a third theorem, which is reminiscent of the results in [1].

THEOREM 3. If R is a semiprime ring admitting a derivation d satisfying (*), the K is a central ideal.

3. PROOFS.

The proof of Theorem 3 hinges on the following lemma.

LEMMA 1. Let R be a semiprime ring and I a nonzero ideal of R. If z in R centralizes the set [I,I], then z centralizes I.

PROOF. Let z centralizes [I,I]. Then for all x,y in I, we have z[x,xy]=[x,xy]z, which can be rewritten as zx[x,y] = x[x,y]z; hence [z,x][x,y] = 0 for all x,y in I. Replacing y by yz, we get [z,x]I[z,x] = {0}. Since I is an ideal, it follows that, [z,x]IR[z,x]I = {0} = I[z,x]RI[z,x], so that [z,x]I = I[z,x] = {0}. Thus, [[z,x],x]=0 for all x in I; and by Theorem 3 of [2], z centralizes I.

For ease of reference, we include a second lemma, which is well-known.

LEMMA 2. (a) If R is a prime ring with a nonzero central ideal, then R is commutative.

(b) If R is a semiprime ring, the center of a nonzero ideal is contained in the center of R.

PROOF OF THEOREM 3. We suppose first that

$$xy + d(xy) = yx + d(yx) \text{ for all } x,y \text{ in } K, \tag{1}$$

which can be rewritten as

$$[x,y] = -d([x,y]) \text{ for all } x,y \text{ in } K. \tag{2}$$

Now for all x,y,z in K, we have [x,y]z + d([x,y]z) = z[x,y] + d(z[x,y]), which yields

$$[x,y]z + d([x,y])z + [x,y]d(z) = z[x,y] + d(z)[x,y] + zd([x,y]);$$

and applying (2) we conclude that

$$[x,y]d(z) = d(z)[x,y] \text{ for all } x,y,z \text{ in } K. \tag{3}$$

By Lemma 1, we see that d(K) centralizes K; and it follows from (1) that [x,y] is in the center of K for all x,y in K. Another application of Lemma 1 shows that the ideal K is commutative; hence by Lemma 2(b), K is in the center of R. In the event that xy-d(xy) = yx - d(yx) for all x,y in K, it is equally easy to establish (3), therefore our proof is complete.

Theorem 2 is immediate from Theorem 3, and Theorem 1 follows from Theorem 3 and Lemma 2(a).

We remark, in conclusion, that under the hypotheses of Theorem 3 we cannot hope to prove commutativity of R. Consider $R = R_1 \oplus R_2$, where R_1 is an integral domain, R_2 is a prime ring which is not commutative, and d is the "direct sum" of derivations on the summands R_1 and R_2.

ACKNOWLEDGEMENT. H. H. Bell was supported by the Natural Sciences and Engineering Research Council of Canada, Grant No. A3961.

REFERENCES

1. BELL, H.E. and MARTINDALE III, W.S. Centralizing Mappings of Semiprime Rings, Canad. Math. Bull. 30 (1987), 92-101.

2. BELL, H.E. and NADA, I. On Some Center-like Subsets of Rings, Arch. Math. 48 (1987), 381-387.

Mathematical Problems in Engineering

Special Issue on Modeling Experimental Nonlinear Dynamics and Chaotic Scenarios

Call for Papers

Thinking about nonlinearity in engineering areas, up to the 70s, was focused on intentionally built nonlinear parts in order to improve the operational characteristics of a device or system. Keying, saturation, hysteretic phenomena, and dead zones were added to existing devices increasing their behavior diversity and precision. In this context, an intrinsic nonlinearity was treated just as a linear approximation, around equilibrium points.

Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from "Qualitative Theory of Differential Equations," allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the *Mathematical Problems in Engineering* aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	December 1, 2008
First Round of Reviews	March 1, 2009
Publication Date	June 1, 2009

Guest Editors

José Roberto Castilho Piqueira, Telecommunication and Control Engineering Department, Polytechnic School, The University of São Paulo, 05508-970 São Paulo, Brazil; piqueira@lac.usp.br

Elbert E. Neher Macau, Laboratório Associado de Matemática Aplicada e Computação (LAC), Instituto Nacional de Pesquisas Espaciais (INPE), São Josè dos Campos, 12227-010 São Paulo, Brazil ; elbert@lac.inpe.br

Celso Grebogi, Center for Applied Dynamics Research, King's College, University of Aberdeen, Aberdeen AB24 3UE, UK; grebogi@abdn.ac.uk

Hindawi Publishing Corporation
http://www.hindawi.com