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ON ANTI-COMMUTATIVE SEMIRINGS

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ABSTRACT. An anticommutative semiring is completely characterized by the types of multiplications that are permitted. It is shown that a semiring is anticommutative if and only if it is a product of two semirings R_1 and R_2 such that R_1 is left multiplicative and R_2 is right multiplicative.

KEY WORDS AND PHRASES. Semiring, anticommutative, isomorphism.
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A semiring is a non-empty set R equipped with two binary operations, called addition + and multiplication (denoted by juxtaposition), such that R is multiplicatively a semigroup, additively a commutative semigroup and multiplication is distributive across the addition both from the left and the right.

A semiring R is called anti-commutative if and only if for arbitrary $x, y \in R$ the relation $x \neq y$ always implies $xy \neq yx$.

Let R_1 and R_2 be semirings, then $R_1 \times R_2$ is the semiring with the following operations:

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$$

$$(x_1, x_2) \cdot (y_1, y_2) = (x_1 y_1, x_2 y_2).$$

Suppose R is a commutative semigroup under +, and if we define multiplication in R of type

(T_1) $xy = x$ for all $x, y \in R$

or

(T_2) $xy = y$ for all $x, y \in R$,

then it is easily seen that R is an anti-commutative semiring.

A natural question that arises is the following: Suppose R is an anti-commutative semiring. Does the multiplication in R have to be of type (T_1) or (T_2)? to answer this question, we prove the following:

THEOREM 1. A semiring R is anti-commutative if and only if R is isomorphic to $R_1 \times R_2$, where R_1 is a semiring with multiplication of type (T_1) and R_2 is a seniring with multiplication of type (T_2).

We shall need the following lemma, whose proof is contained in [1,p.75], to prove Theorem 1.

LEMMA. Let R be an anti-commutative semiring, then for arbitrary x, y, z, $\in$ R we have

(i) $x^2 = x$

(ii) $xyz = xz$

PROOF OF THEOREM 1. Since R is non empty, let $a \in R$. Set $R_1 = Ra$ and $R_2 = aR$. By using the lemma, it is obvious that Ra and aR are semirings and multiplication in Ra is of type (T_1) and multiplication in aR is of type (T_2).

Let $f: R \to Ra \times aR$, such that for each $x \in R$,

$f(x) = (xa, ax)$.

then for $y \in R$, $f(y) = (ya, ay)$.

$$\begin{aligned} f(x+y) &= ((x+y)a, a(x+y)) = (xa + ya, ax + ay) \\ &= (xa, ax) + (ya, ay) \\ &= f(x) + f(y). \\ f(xy) &= (xya, axy) \\ &= (xaya, axay) \quad \text{[By part (ii) of the Lemma]} \\ &= f(x)f(y). \end{aligned}$$

Thus, f is a homomorphism.

To show f is an isomorphism, let us define $g: Ra \times aR \to R$, such that $g(xa,ay) = xy$.

Then

$$(gof)(x) = g(f(x)) = g(xa,ax) = xa^2x = x^2 = x,$$

and

$$(fog)(xa,ay) = f[g(xa,ay)] = f(xy) = (xya,axy) = (xa,ay).$$

This shows that f is an isomorphism.

The proof for the converse is left to the reader.

THEOREM 2. Let R be an anti-commutative semiring. Then for an arbitrary $x \in R$, $x + x = x$.

PROOF: As in the proof of Theorem 1, we have

$$x = g(xa,ax).$$

Thus,

$$\begin{aligned} x + x &= g(xa + xa,\ ax + ax) \\ &= g(x^2a + x^2a,\ ax^2 + ax^2) \\ &= g(x(x + x)a,\ a(x + x)x) \\ &= g(xa,\ ax) \\ &= x. \end{aligned}$$

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