ON ELASTIC WAVES IN A THINLY-LAYERED LAMINATED MEDIUM WITH STRESS COUPLES UNDER INITIAL STRESS

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ABSTRACT: The present work is concerned with a simple transformation rule in finding out the composite elastic coefficients of a thinly layered laminated medium whose bulk properties are strongly anisotropic with a microelastic bending rigidity. These elastic coefficients which were not known completely for a layered laminated structure, are obtained suitably in terms of initial stress components and Lame's constants λ_i, μ_i of initially isotropic solids. The explicit solutions of the dynamical equations for a prestressed thinly layered laminated medium under horizontal compression in a gravity field are derived. The results are discussed specifying the effects of hydrostatic, deviatoric and couple stresses upon the characteristic propagation velocities of shear and compression wave modes.

1. INTRODUCTION

Biot [1] has indicated that some of the basic properties of anisotropic elastic media are provided by analyzing a laminated medium superposed of thin adhering layers which are alternately hard and soft. He also suggested that an equivalent continuous anisotropic elastic medium can approximately be used to provide useful insight into some of the basic features of the statical or dynamical problem of elasticity. The validity of such an approximation is based upon the fact that rigidity contrast of the layers is not too large, and that the thickness of the layers remains sufficiently

small with respect to the wavelength of the displacement field. Such approximation depends also on the type of problem considered and in many cases it requires some additional refinements.

In a classic work on the theory of deformation of anisotropic elastic solids, Biot [2] suggested that a skin effect is associated with anisotropy. He showed that near a free surface or a surface of discontinuity certain components of the stress field vary rapidly from zero to a maximum value within a thin skin. The concentration of stress also occurs where certain stress components are amplified with the skin thickness. In order to make an additional refinement of his theory, Biot [3] has proposed the effect of couple stresses in elasticity and viscoelasticity of an initially stressed anisotropic solids. He showed that this theory is intended to provide an approximate continuous model valid over a wide range for the mechanics of layered laminated media. This theory when applied to a wide variety of geological structures provides remarkably simple and useful results, and hence can be used for a better understanding of the involved physical features.

When a laminated medium of compressible material is replaced by a continuous medium of anisotropic properties, Biot's [1] analysis gives the composite elastic coefficients. This study is found to be somewhat similar to that used by Postma [4], and also to that considered by Helbig [5] in the theory of acoustic propagation in a laminated medium composed of layers of isotropic materials and is initially stress free. The laminated medium is obtained by superposition of thin adherent layers which are assumed to be alternately hard and soft. In these studies, Biot [1] obtained elastic coefficients in terms of the unknown individual material coefficients after deformation. Recently, Tolstoy [6] has made some slight modifications of Biot's formulation and then obtained simple explicit solutions of the dynamical equations for a prestressed isotropic homogeneous solid under horizontal compression in a gravitational field.

In spite of the above progress, several problems concerning the propagation of elastic waves in a thinly-layered laminated medium with stress couples under initial stresses remain fully or partially understood. This paper is intended to address some of these problems.

The main purpose of this paper is to discuss how to obtain the composite elastic coefficients after deformation in terms of Lame's constants and prestress components of the individual layers before deformation. After determination of such coefficients, an attempt is made to find the explicit solutions of the dynamical equations

for a pre stressed thinly-layered laminated medium under horizontal compression in a gravitational field. The bending rigidity of the laminated medium is also taken into account by introducing stress couples.

2.   BASIC ASSUMPTIONS AND STRESS-STRAIN RELATIONS

We consider a thinly laminated medium composed of thin adhering layers which are alternately hard and soft. These hard and soft materials occupy, respectively, fractions α_1 and α_2 of the total thickness. If the rigidity contrast of the layers is not too large, and if the layer thickness remains sufficiently small with respect to the wavelength of the deformation field, then such a layered medium behaves approximately like an elastic continuum with anisotropic properties even though the individual layers may be isotropic.

We consider the case of a compressible material. The stress-strain relations for the composite medium are

$$t_{11} = C_{11}\, e_{xx} + C_{12}\, e_{yy} \tag{2.1}$$

$$t_{22} = C_{12}\, e_{xx} + C_{22}\, e_{yy} \tag{2.2}$$

$$t_{12} = 2L\, e_{ey} \tag{2.3}$$

where L is the composite slide modulus. Due to compressibility the strain component e_{yy} is not the same in each layer. However, e_{xx} and t_{22} remain the same in both materials. The normal stresses in the hard material and (Biot [1], page 190),

$$t_{11}^{(1)} = a_1\, e_{xx} + b_1\, e_{yy}^{(1)} \tag{2.4}$$

$$t_{22} = b_1\, e_{xx} + c_1\, e_{yy}^{(1)} \tag{2.5}$$

For the soft material, the normal stress components are

$$t_{11}^{(2)} = a_2\, e_{xx} + b_2\, e_{yy}^{(2)} \tag{2.6}$$

$$t_{22} = b_2\, e_{xx} + c_2\, e_{yy}^{(2)} \tag{2.7}$$

The average stress e_{yy} is

$$e_{yy} = \alpha_1 e_{yy}^{(1)} + \alpha_2 e_{yy}^{(2)} \tag{2.8}$$

The average strain t_{11} is

$$t_{11} = \alpha_1 t_{11}^{(1)} + \alpha_2 t_{11}^{(2)} \tag{2.9}$$

The composite elastic coefficients C_{ij} in equations (2.1)-(2.3) now obtained by eliminating the four variables $t_{11}^{(1)}$, $t_{11}^{(2)}$, $e_{yy}^{(1)}$, $e_{yy}^{(2)}$ from the six equations (2.4)-(2.9). They are

$$C_{11} = \alpha_1 a_1 + \alpha_2 a_2 - \frac{\alpha_1 \alpha_2 (b_1 - b_2)^2}{\alpha_1 c_2 + \alpha_2 c_1} \tag{2.10}$$

$$C_{12} = \frac{\alpha_1 b_1 c_2 + \alpha_2 b_2 c_1}{\alpha_1 c_2 + \alpha_2 c_1} , \quad C_{22} = \frac{c_1 c_2}{\alpha_1 c_2 + \alpha_2 c_1} \tag{2.11ab}$$

and

$$L = 1/(\frac{\alpha_1}{L_1} + \frac{\alpha_2}{L_2}) \tag{2.12}$$

For convenience we also define the alternative stress components t_{ij} (Biot [1], page 61)

$$t_{11} = s_{11} + S_{11} e_{yy} - S_{12} e_{xy} \tag{2.13}$$

$$t_{22} = s_{22} + S_{22} e_{xx} - S_{12} e_{xy} \tag{2.14}$$

$$t_{12} = s_{12} + \frac{1}{2} S_{12}(e_{xx} + e_{yy}) - \frac{1}{2} (S_{11} + S_{22}) e_{xy} \tag{2.15}$$

3.   THE HARD AND SOFT INITIALLY ISOTROPIC MATERIALS

We assume that the acceleration due to gravity points downward and the medium lies in the space $y < 0$. The external gravitational field has the components $(0, -g, 0)$.

The principal initial stress components are

$$s_{11}^1 = -P_1 + \rho g y$$
$$s_{22}^1 = \rho_1 g y \tag{3.1ab}$$

The relations between initial stress and initial strain give

$$\epsilon_{11}^1 = \frac{(\lambda_1 + 2\mu_1)S_{11}^1 - \lambda_1 S_{22}^1}{4\mu_1(\lambda_1 + \mu_1)}$$ (3.2ab)

$$\epsilon_{22}^1 = \frac{(\lambda_1 + 2\mu_1)S_{22}^1 - \lambda_1 S_{11}^1}{4\mu_1(\lambda_1 + \mu_1)}$$

The existence of initial stress introduces anisotropy defined by the elastic coefficients B_{ij}^1 (>2). These elastic coefficients will be functions of the Lame constants λ_1, μ_1 of the isotropic unstressed state and the prestrains ϵ_{ij}^1. In linear approximation following Biot (Ref. [1], page 111) we can write

$$B_{11}^1 = (2\mu_1 + \lambda_1)(1 + \epsilon_{11}^1 - \epsilon_{22}^1)$$ (3.3)

$$B_{22}^1 = (2\mu_1 + \lambda_1)(1 + \epsilon_{22}^1 - \epsilon_{11}^1)$$ (3.4)

$$B_{12}^1 = \lambda_1 - S_{11}^1$$ (3.5)

$$Q_1 = \mu_1 + \frac{1}{2}(\lambda_1 + \mu_1)\epsilon_{11}^1 + \frac{1}{2}(\lambda_1 + \mu_1)\epsilon_{22}^1$$ (3.6)

a. Thinly layered laminated medium composed of hard and soft materials.

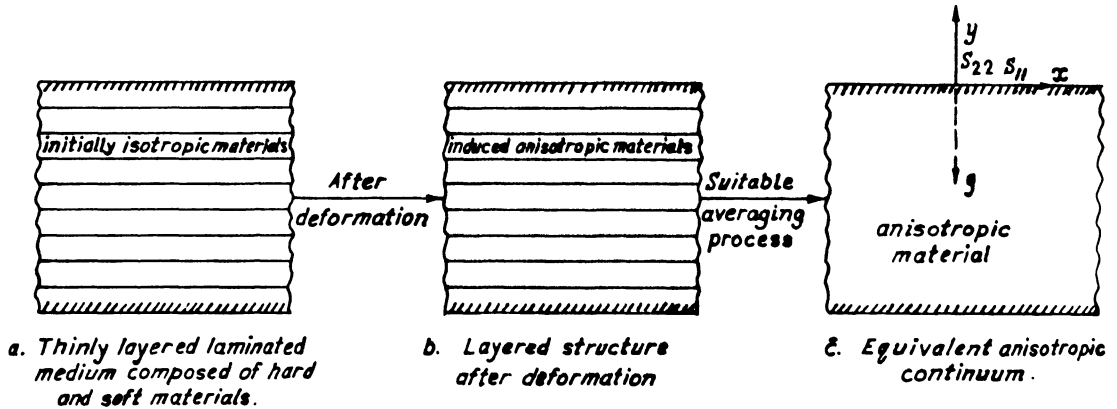b. Layered structure after deformation

c. Equivalent anisotropic continuum.

Fig. 1 : Geometry of the problem.

Using (3.2ab), equations (3.3)-(3.6) become

$$B_{11}^1 = (2\mu_1 + \lambda_1)(1 + \frac{S_{11}^1 - S_{22}^1}{2\mu_1})$$ (3.7)

$$B_{22}^1 = (2\mu_1 + \lambda_1)(1 + \frac{S_{22}^1 - S_{11}^1}{2\mu_1})$$ (3.8)

$$B_{12}^1 = \lambda_1 - S_{11}^1 \tag{3.9}$$

$$Q = \mu_1 + \frac{1}{4}(S_{11}^1 + S_{22}^1) \tag{3.10}$$

Also $B_{11}^1 = C_{11}^1, \quad B_{12}^1 = C_{12}^1 + P_1 - \rho_1 gy;$

$$B_{21}^1 = C_{12}^1 - \rho_1 gy = B_{12}^1 - P_1 \tag{3.11abc}$$

$$C_{22}^1 = B_{22}^1, \quad Q^1 = L_1 - \frac{1}{4}P_1 + \frac{1}{2}\rho_1 gy$$

or $B_{11}^1 = a_1, \quad B_{12}^1 = b_1 + P_1 - \rho_1 gy$

$$B_{21}^1 = b_1 - \rho_1 gy = B_{12}^1 - P_1 \tag{3.12abc}$$

$$B_{22}^1 = c_1, \quad A^1 = L_1 - \frac{1}{4}P_1 + \frac{1}{2}\rho_1 gy$$

Hence, from (3.1ab), (3.7)-(3.10), and (3.12abc), it follows that

$$a_1 = (\lambda_1 + 2\mu_1)(1 - \frac{P_1}{2\mu_1}),$$

$$b_1 = \lambda_1 \tag{3.13abcd}$$

$$c_1 = (2\mu_1 + \lambda_1)(1 + \frac{P_1}{2\mu_1})$$

$$L_1 = \mu_1$$

Following exactly the above derivation the elastic coefficients for the soft material are obtained as

$$a_2 = (\lambda_2 + 2\mu_2)(1 - \frac{P_2}{2\mu_2})$$

$$b_2 = \lambda_2 \tag{3.14abcd}$$

$$c_2 = (\lambda_2 + 2\mu_2)(1 + \frac{P_2}{2\mu_2})$$

$$L_2 = \mu_2$$

4.   THE COMPOSITE ANISOTROPIC MATERIAL

Using relations (2.10)-(2.11ab), (3.13abcd) and (3.14abcd) we obtain the composite elastic coefficients

$$C_{11} = \alpha_1(\lambda_1+2\mu_1)(1 - \frac{P_1}{2\mu_1}) + \alpha_2(\lambda_2+2\mu_2)(1 - \frac{P_2}{2\mu_2})$$

$$- \frac{\alpha_1\alpha_2}{K}(\lambda_1-\lambda_2)^2 \tag{4.1a}$$

$$C_{12} = \frac{1}{K} [(\lambda_2+2\mu_2)(1 + \frac{P_2}{2\mu_2})\alpha_1\lambda_1 + (2\mu_1+\lambda_1)(1 + \frac{P_1}{2\mu_1})\alpha_2\lambda_2] \tag{4.1b}$$

$$C_{22} = \frac{1}{K} [(\lambda_1+2\mu_1)(\lambda_2+2\mu_2)(1 + \frac{P_1}{2\mu_1})(1 + \frac{P_2}{2\mu_2})] \tag{4.1c}$$

$$L = 1/(\frac{\alpha_1}{\mu_1} + \frac{\alpha_2}{\mu_2}) \tag{4.1d}$$

and

$$K = \alpha_1(\lambda_2+2\mu_2)(1 + \frac{P_2}{2\mu_2}) + \alpha_2(\lambda_1+2\mu_1)(1 + \frac{P_1}{2\mu_1}) \tag{4.1e}$$

5.   FORMULATION OF THE PROBLEM FOR THE COMPOSITE MATERIAL

The average initial stresses in the x and y directions are

$$S_{11} = \alpha_1 S_{11}^1 + \alpha_2 S_{11}^2 = -P + \rho gy$$

$$\tag{5.1ab}$$

$$S_{22} = \alpha_1 S_{22}^1 + \alpha_2 S_{22}^2 = \rho gy$$

where $P = \alpha_1 P_1 + \alpha_2 P_2$ and $\rho = \alpha_1\rho_1 + \alpha_2\rho_2$ \hfill (5.2ab)

It follows from (2.13)-(2.15) that

$$t_{11} = s_{11} + (-P+\rho gy)e_{yy}$$

$$t_{22} = s_{22} + \rho gy\, e_{xx} \tag{5.3abc}$$

$$t_{12} = s_{12} - (-\frac{1}{2} P+\rho gy)\, e_{xy}$$

From (2.1)-(2.3) and 5.3abc) equating t's we get after simplification,

$$s_{11} = B_{11} e_{xx} + B_{12} e_{yy}$$

$$S_{22} = B_{21} e_{xx} + B_{22} e_{yy} \qquad\qquad (5.4abc)$$

$$S_{12} = 2Q e_{xy}$$

where $B_{11} = C_{11}, \quad B_{12} = C_{12} + P - \rho gy$

$$B_{21} = C_{12} - \rho gy = B_{12} - P \qquad\qquad (5.5abc)$$

$$B_{22} = C_{22}, \quad Q = L - \frac{1}{4} P + \frac{1}{2} \rho gy$$

where C's are given by (4.1abcde).

6.   COUPLE STRESS ANALOGY

     If the hard layer is sufficiently stiff a couple stress of moment M per unit area is produced in the plane normal to the x axis. In this case $t_{12} \neq t_{21}$. Equilibrium of moments for an element of material requires the condition

$$T_{12} - t_{21} = \frac{\partial M}{\partial x} \qquad\qquad (6.1)$$

If we now introduce the effect of couple stress, the equilibrium equations are obtained as (Biot [3])

$$\frac{\partial t_{11}}{\partial x} + \frac{\partial t_{12}}{\partial y} = \rho \frac{\partial^2 u}{\partial t^2} \qquad\qquad (6.2)$$

$$\frac{\partial t_{12}}{\partial x} + \frac{\partial t_{22}}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2} + P \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 M}{\partial x^2} \qquad\qquad (6.3)$$

The value of the moment M in terms of the deformation is obtained as (Biot [3])

$$M = b \frac{\partial^2 v}{\partial x^2} \qquad\qquad (6.4)$$

where the couple stress coefficient is

$$b = \frac{1}{3} h^2 \alpha_1\alpha_2 \frac{(L_1-L_2)}{(\alpha_1 L_1 + \alpha_2 L_2)} (M_1\alpha_1 - M_2\alpha_2) \qquad\qquad (6.5)$$

with $h = h_1 + h_2$.

$$M_i = \frac{1}{4C_i} (a_i c_i - b_i^2)$$

$$= \frac{1}{4(\lambda_i + 2\mu_i)(1 + \frac{1}{2\mu_i})} [(\lambda_i + 2\mu_i)^2 (1 - \frac{P_i^2}{4\mu_i^2}) - \lambda_i^2], \quad i = 1,2. \tag{6.6}$$

When one layer is much more rigid than the other the couple stress coefficient has the simple value

$$b = \frac{1}{3} h^2 M \quad \text{with} \quad M = M_1 \alpha_1 \tag{6.7}$$

7. GOVERNING DYNAMICAL EQUATIONS AND THEIR SOLUTIONS

In view of (5.3abc)-(5.4abc), equations (6.2)-(6.3) become

$$C_{11} \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 v}{\partial x \partial y} + L \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{7.1}$$

$$C_{22} \frac{\partial^2 v}{\partial y^2} + B \frac{\partial^2 v}{\partial x \partial y} + (L-P) \frac{\partial^2 v}{\partial x^2} = \rho \frac{\partial^2 v}{\partial t^2} + b \frac{\partial^4 v}{\partial x^4} . \tag{7.2}$$

where $\quad B = C_{12} + L$ and $L = Q + \frac{P}{4} - \frac{1}{2} g\rho y \tag{7.3ab}$

It is noted that the effect of the buoyancy terms in equations (7.1)-(7.2) is appreciably smaller than that of pre-stressed values for a period of half a minute or less. In dealing with seismic wave problems, we can neglect the buoyancy terms in (7.1)-(7.2) compared with the pre-stressed terms. Using now (5.5abc) and neglecting the buoyancy terms in the above equations, it turns out that

$$B_{11} \frac{\partial^2 u}{\partial x^2} + (B_{12} - \frac{3}{4} P + Q) \frac{\partial^2 v}{\partial x \partial y} + (Q + \frac{1}{4} P) \frac{\partial^2 u}{\partial y^2} = \rho \frac{\partial^2 u}{\partial t^2} \tag{7.4}$$

$$B_{22} \frac{\partial^2 v}{\partial y^2} + (Q - \frac{3}{4} P) \frac{\partial^2 v}{\partial x^2} + (B_{12} + Q - \frac{3}{4} P) \frac{\partial^2 u}{\partial x \partial y} = \rho \frac{\partial^2 v}{\partial t^2} + b \frac{\partial^4 v}{\partial x^4} \tag{7.5}$$

we next introduce the classic potentials ϕ, ψ defined by

$$u = \phi_x - \psi_y, \quad \text{and} \quad v = \phi_y + \psi_x \tag{7.6ab}$$

In view of (7.6ab), equations (7.4)-(7.5) assume the form

$$B_{11}(\phi_{xxx}-\psi_{yxx}) + (B_{12} - \frac{3}{4} P+Q)(\phi_{yyx}+\psi_{xxy})$$

$$+ (Q + \frac{1}{4} P)(\phi_{xyy}-\psi_{yyy}) = \rho(\phi_{xtt}-\psi_{ytt}) \tag{7.7}$$

$$B_{12}(\phi_{yyy}+\psi_{xyy}) + (Q - \frac{3}{4} P)(\phi_{yxx}+\psi_{xxx}) + (B_{12}+Q - \frac{3}{4} P)(\phi_{xxy}-\psi_{yyx})$$

$$= \rho(\phi_{ytt}+\psi_{xtt}) + b(\phi_{yxxxx}+\psi_{xxxxx}) \tag{7.8}$$

For some interesting solutions we take the trial functions as

$$\phi = \phi(a_1x+B_1y\pm c_1t) \tag{7.9}$$

$$\psi = \psi(a_2x+b_2y\pm c_2t) \tag{7.10}$$

Assuming now a pure shear wave travelling in the x-direction, we set

$$\phi = 0, \quad a_2 = 1, \quad b_2 = 0, \quad \psi = \psi(x\pm c_2t) \tag{7.11}$$

Then $$u = 0, \quad v = \psi_x \tag{7.12}$$

and

$$\frac{\psi''''}{\psi''''} = (Q - \frac{3}{4} P - \rho c_2^2)/b \tag{7.13}$$

Thus when the functional form of ψ is known then (7.13) corresponds to a shear mode of the SV type ($u = 0$) propagating in the S_{11} i.e. x-direction, with velocity c_2.

In general, for plane harmonic waves along the x-direction, we write

$$\psi = \exp[i(x\pm c_2t)] \tag{7.14}$$

Then, from equation (7.13), we obtain

$$c_{sx}^2 = (Q+b - \frac{3}{4} P)/\rho \tag{7.15}$$

Evidently, the shear wave velocity, c_{sx} depends on the initial stress, couple stress and hydrostatic stress.

Likewise a shear wave travelling in the vertical y-direction corresponds to

$$\phi = 0, \quad a_2 = 0, \quad b_2 = 1, \quad \psi(y \pm c_2 t) \tag{7.16}$$

Then it follows from (7.7)-(7.8) that the shear wave velocity is

$$c_{sy}^2 = (Q + P/4)/\rho \tag{7.17}$$

In this case of shear waves travelling in the y-direction, there is no effect of couple stress.

We also note

$$c_{sy}^2 - c_{sx}^2 = c_s^2 (P-b)/Q \tag{7.18}$$

For pure compressional modes, we take

$$\psi = 0, \quad a_1 = 1, \quad b_1 = 0, \quad \phi = \phi(x \pm c_1 t) \tag{7.19}$$

so that the wave velocity is

$$c_{px}^2 = B_{11}/\rho \tag{7.20}$$

In the case of harmonic waves the velocity also remains the same.

Likewise there exists a pure P-mode travelling in the vertical direction,

$$\psi = 0, \quad a_1 = 0, \quad b_1 = 1, \quad \phi = \phi(y \pm c_1 t)$$

In view of equation (7.8), we have

$$c_{py}^2 = B_{22}/\rho \tag{7.21}$$

and hence

$$c_{py}^2 - c_{px}^2 = \frac{B_{22} - B_{11}}{\rho} \tag{7.22}$$

As in the case of a single isotropic layer (Tolstoy [6]) there are however, apart from the principal axes of pre-stress, two further conjugate directions for which uncoupled i.e. pure modes of shear of the pre-stress field occur.

We assume

$$\psi = 0, \quad \phi = \phi(a_1 x + b_1 y \pm c_1 t),$$

where

$$a_1 = \sin \theta, \quad b_1 = \cos \theta$$

It turns out that

$$c_1^2 = \frac{B_{11} a_1^2 + (B_{12} - P/2 + 2Q) b_1^2}{\rho} \qquad (7.23)$$

and

$$\tan^2\theta = \frac{-(m+n) + [(m-n)^2 + 4mr)]^{1/2}}{2} \qquad (7.24)$$

where

$$m = (B_{22} - B_{12} - 2Q + P/2), \quad n = (B_{12} + 2Q - \frac{3}{2} P - B_{11}), \quad r = \frac{b}{\phi} \qquad (7.25ab)$$

A similar uncoupled solution exists for shear modes. We assume the solutions as

$$\phi = 0, \quad \psi = \psi(a_2 x + b_2 \pm c_2 t) \qquad (7.26ab)$$

where

$$a_2 = \sin \theta. \quad b_2 = \cos \theta \qquad (7.27ab)$$

Setting

$$\psi = e^{i(a_2 x + b_2 y \pm c_2 t)}, \quad a_2 = \sin \theta, \quad b_2 = \cos \theta$$

We obtain

$$c_2^2 = \frac{a_2^2 (B_{11} - B_{12} + \frac{3}{4} P - Q) + (Q + \frac{P}{4}) b_2^2}{\rho} \qquad (7.28)$$

and

$$\tan^2 \theta = \frac{-B_1 + (B_1^2 - 4A_1A_2)^{1/2}}{2A_2} \qquad (7.29)$$

where

$$A_1 = B_{22} - B_{12} - 2Q + \frac{1}{2}P, \ B_1 = B_{22} - B_{11} - P, \ A_2 = B_{12} - B_{11} + b + 2Q - \frac{3}{2}P \quad (7.30abc)$$

Thus we obtain explicit expressions for the effects of hydrostatic stress, deviatoric stress and couple stress upon the characteristic propagation velocities of shear and compression modes by using equations (7.15), (7.17), (7.20) and (7.21) with relations (4.1) and (5.5abc);

$$c_{sx}^2 = \frac{1}{\rho}(L+b-P + \frac{1}{2}\rho gy) \qquad (7.31)$$

$$c_{sy}^2 = \frac{1}{\rho}(L + \frac{1}{2}y\rho g) \qquad (7.32)$$

$$c_{px}^2 = \frac{1}{\rho}[\alpha_1(\lambda_1 + 2\mu_1)(1 - \frac{P_1}{2\mu_1}) + \alpha_2(\lambda_2 + 2\mu_2)(1 - \frac{P_2}{2\mu_2}) - \frac{\alpha_1\alpha_2}{K}(\lambda_1 - \lambda_2)^2] \qquad (7.33)$$

$$c_{py}^2 = \frac{1}{K\rho}[(\lambda_1 + 2\mu_1)(\lambda_2 + 2\mu_2)(1 + \frac{P_1}{2\mu_1})(1 + \frac{P_2}{2\mu_2})] \qquad (7.34)$$

where $K = \alpha_1(\lambda_2 + 2\mu_2)(1 + \frac{P_2}{2\mu_2}) + \alpha_2(\lambda_1 + 2\mu_2)(1 + \frac{P_1}{2\mu_1}) \qquad (7.35)$

8.   SUMMARY AND CONCLUSION

The theory presented in this paper enables us to determine the composite elastic coefficients C_{ij}. These coefficients were not known earlier for a laminated structure. After deformation, the state of initial stresses (different for hard and soft materials) introduces anisotropy. The corresponding elastic coefficients B_{ij} are obtained in section 3. When B_{ij} are known for two different materials, it is also possible to find out the corresponding C_{ij}'s through equations (3.11abc). On the other hand, when C_{ij}'s are known for two different materials, then Biot's (1, p. 190] analysis can be used to obtain the composite elastic coefficients.

In some cases, the use of the equivalent anisotropic continuum with elastic coefficients in equations (4.1a)-(4.1e) are not sufficient. So the bending rigidity of the laminations are taken into account by introducing stress couples through

equations (4.11)-(4.12) as stated by Biot [3]. The corresponding equilibrium equations are obtained in section 6.

Based upon the equilibrium equations for the composite elastic material and using an analysis similar to Tolstoy [6], the velocities of the P and SV wave modes are obtained explicitly.

The effects of couple stresses and initial stresses are found to be large for values approaching the instability value. The effect on the propagation velocity can then be quite large. As for the effects of gravity, these seem to be sensible for a very weak (low rigidity) sediments and for a very low frequency at which buoyancy effects are significant. This point deserves further attention and will be discussed in a subsequent paper.

9. CONCLUDING REMARKS

Although the present theory is developed in the context of plane strain deformation, it can be extended to three-dimensional problems of transverse isotropic materials with rectangular, triangular and circular plane form. Also, further extension of results is possible for a thermoclastic laminated medium. Such problems will be treated in a subsequent paper.

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As a multidisciplinary field, financial engineering is becoming increasingly important in today's economic and financial world, especially in areas such as portfolio management, asset valuation and prediction, fraud detection, and credit risk management. For example, in a credit risk context, the recently approved Basel II guidelines advise financial institutions to build comprehensible credit risk models in order to optimize their capital allocation policy. Computational methods are being intensively studied and applied to improve the quality of the financial decisions that need to be made. Until now, computational methods and models are central to the analysis of economic and financial decisions.

However, more and more researchers have found that the financial environment is not ruled by mathematical distributions or statistical models. In such situations, some attempts have also been made to develop financial engineering models using intelligent computing approaches. For example, an artificial neural network (ANN) is a nonparametric estimation technique which does not make any distributional assumptions regarding the underlying asset. Instead, ANN approach develops a model using sets of unknown parameters and lets the optimization routine seek the best fitting parameters to obtain the desired results. The main aim of this special issue is not to merely illustrate the superior performance of a new intelligent computational method, but also to demonstrate how it can be used effectively in a financial engineering environment to improve and facilitate financial decision making. In this sense, the submissions should especially address how the results of estimated computational models (e.g., ANN, support vector machines, evolutionary algorithm, and fuzzy models) can be used to develop intelligent, easy-to-use, and/or comprehensible computational systems (e.g., decision support systems, agent-based system, and web-based systems)

This special issue will include (but not be limited to) the following topics:

- **Computational methods**: artificial intelligence, neural networks, evolutionary algorithms, fuzzy inference, hybrid learning, ensemble learning, cooperative learning, multiagent learning

- **Application fields**: asset valuation and prediction, asset allocation and portfolio selection, bankruptcy prediction, fraud detection, credit risk management

- **Implementation aspects**: decision support systems, expert systems, information systems, intelligent agents, web service, monitoring, deployment, implementation

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